Experiments
Hyrogen, Dirac theory

Lecture 2

23 October 2013

Our “road map”

Lectures

- 1 16.10.2012 Preliminary Discussion / Introduction

Basics concepts, Dirac sea, Creation of Particles

- 2 23.10.2013 Dirac Theory
- 3 30.10.2013 Atomic Decay Modes and Radiation Properties
- 4 06.11.2013 Interaction of Photons with Matter
- *5* 13.11.2013 Interaction of Charged Particles with Matter
- 6 20.11.2013 Key Experiments

Sources of High Energetic Radiation

- 7 27.11.2013 Nuclei and their Decay Modes
- 8 04.12.2013 Cosmic Radiation

Detectors

- 9 11.12.2013 Photon-, x-ray-, gamma-detectors
- 10 18.12.2013 Particle Detectors

Applications

- 11 08.01.2014 Radiation and their Biological Effectiveness
- 12 15.01.2014 Application of Charged Particle to Cancer Therapy

Novel Accelerators

- 13 22.01.2014 Novel Photon Sources
- 14 29.01.2014 Modern Accelerators for Ions and Exotic Nuclei

Summary

- 15 05.02.2014 Excursion to GSI

Exercises

Basics concepts, Dirac sea, Creation of Particles

- 1 31.10.2013
- 2 14.11.2013
- 3 28.11.2013

Sources of High Energetic Radiation

- 4 12.12.2013

Detectors

- 5 09.01.2014

Applications

- 6 23.01.2014

Novel Accelerators

- 7 06.02.2014

Our “road map”

Lectures

Presentation topics:

- Discovery of antimatter(positron, antiproton, anti-hydrogen)
- PET
- Monte-Carlo-Simulation (high-energy electrons interacting with matter)

Exercises

Basics concepts, Dirac sea, Creation of Particles

- 1 31.10.2013
- 2 14.11.2013
- 3 28.11.2013

Sources of High Energetic Radiation

- 4 12.12.2013

Detectors

- 5 09.01.2014

Applications

- 6 23.01.2014

Novel Accelerators

- 7 06.02.2014

Our "road map"

Presentation topics:

- Discovery of antimatter(positron, antiproton, anti-hydrogen)
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- Monte-Carlo-Simulation (high-energy electrons interacting with matter)

Exercises

http://web-docs.gsi.de/~stoe_exp/lectures/lectures.php

oe_exp/lectures/lectures.php

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Lectures & Seminars

Interaction of high-energy radiation with matter

Prof. Dr. Thomas Stöhlker

Wednesday, 10.15 - 12.00 (Lecture)
Thursday, 14.15 - 16.00 (Exercises, biweekly)
Room: Seminar room 205, Helmholtz Institute Jena, Fröbelstieg 3

Transperencies presented during the lecture:

17/10/2012: Lecture Introduction Part 1 Lecture Introduction Part 2

PASSWORD: dirac2012

Friedrich-Schiller-Universität

HELMHOLTZ | GEMEINSCHAFT

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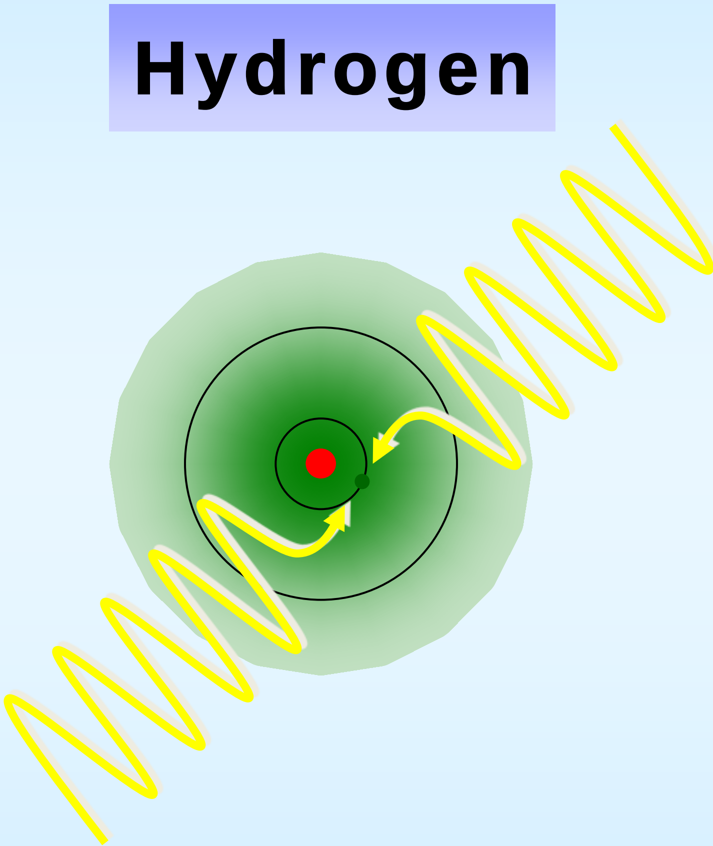
GSI

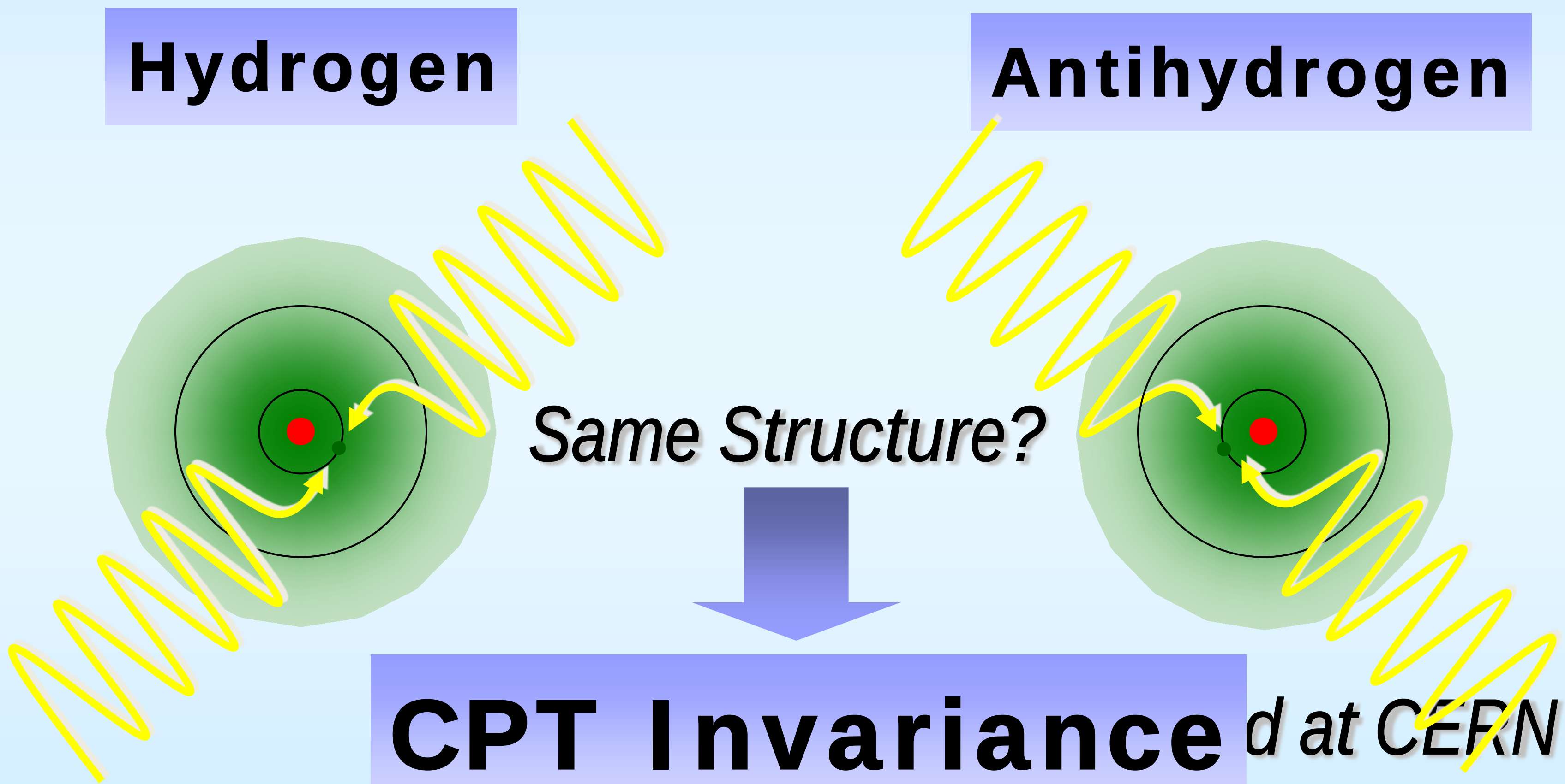
Contents

Hydrogen atom



$$\Delta E / E \approx 10^{-14}$$

Ultracold & Trapped $\bar{p}$

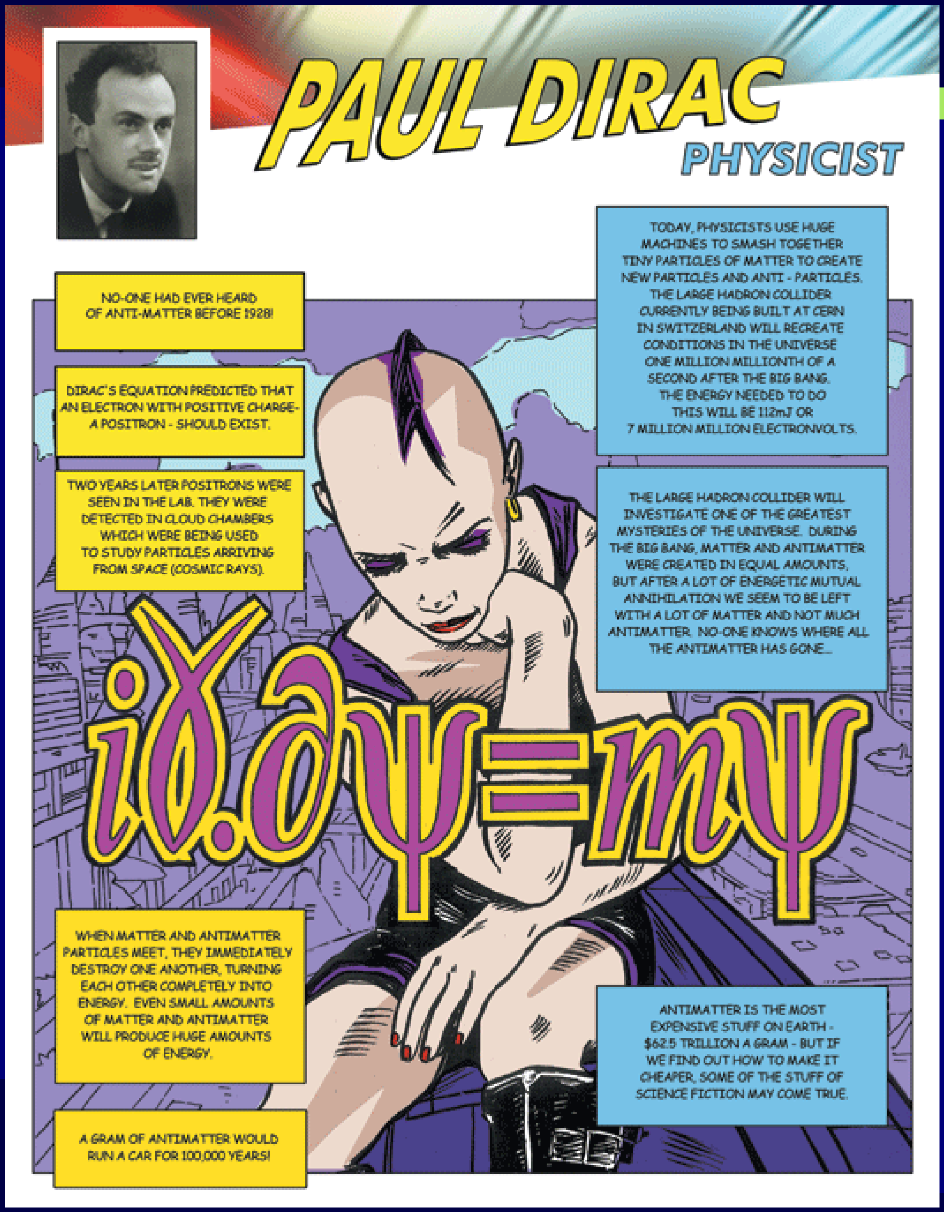


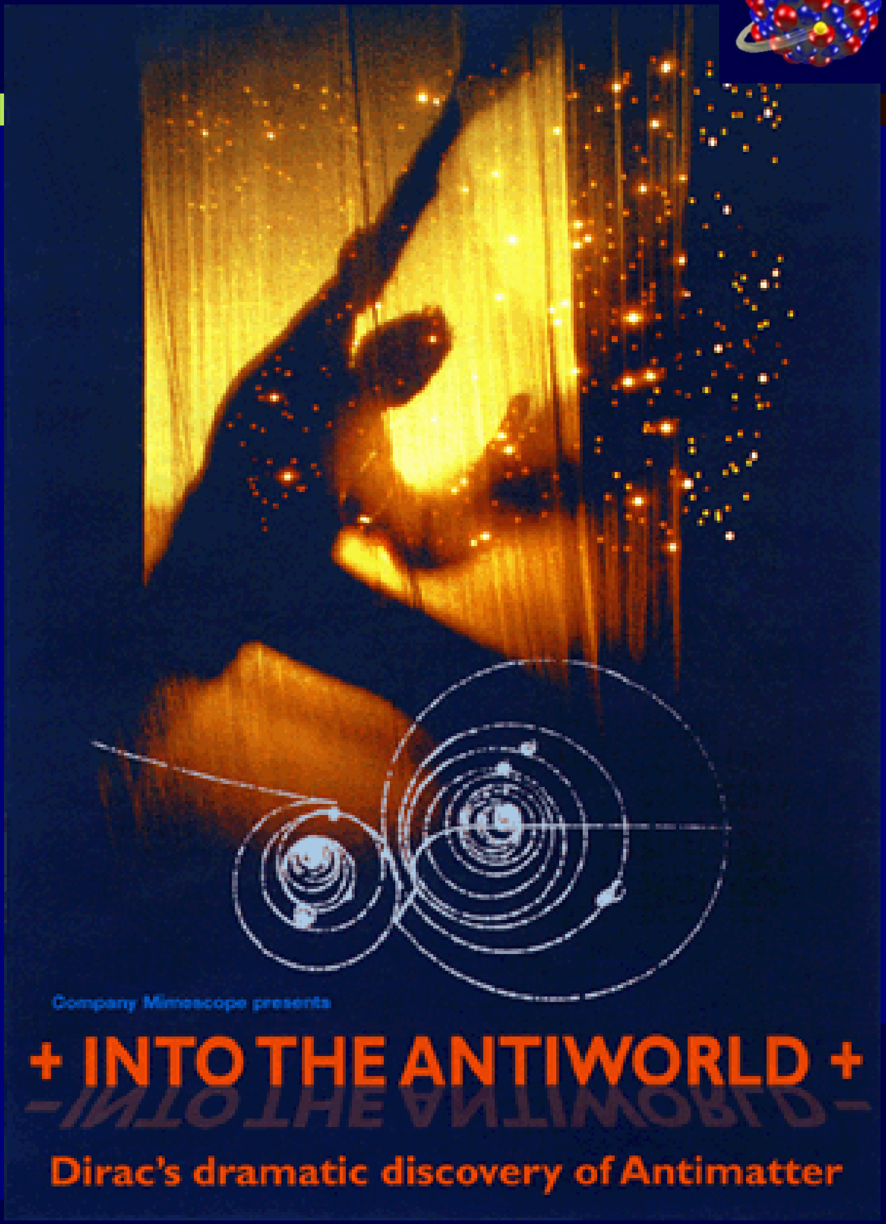
CPT Invariance

$$\Delta E / E \approx 10^{-14}$$

d at CERN
t: 1996
s: 2002


DER SPIEGEL
DAS TOR ZU EINER
ANDEREN WELT
Physiker
entschlüsseln
das Geheimnis der
Anti-Materie


PAUL DIRAC
PHYSICIST
NO-ONE HAD EVER HEARD OF ANTI-MATTER BEFORE 1928!
DIRAC'S EQUATION PREDICTED THAT AN ELECTRON WITH POSITIVE CHARGE - A POSITRON - SHOULD EXIST.
TWO YEARS LATER POSITRONS WERE SEEN IN THE LAB. THEY WERE DETECTED IN CLOUD CHAMBERS WHICH WERE BEING USED TO STUDY PARTICLES ARRIVING FROM SPACE (COSMIC RAYS).
TODAY, PHYSICISTS USE HUGE MACHINES TO SMASH TOGETHER TINY PARTICLES OF MATTER TO CREATE NEW PARTICLES AND ANTI - PARTICLES. THE LARGE HADRON COLLIDER CURRENTLY BEING BUILT AT CERN IN SWITZERLAND WILL RECREATE CONDITIONS IN THE UNIVERSE ONE MILLION MILLIONTH OF A SECOND AFTER THE BIG BANG. THE ENERGY NEEDED TO DO THIS WILL BE 112mJ OR 7 MILLION MILLION ELECTRONVOLTS.
THE LARGE HADRON COLLIDER WILL INVESTIGATE ONE OF THE GREATEST MYSTERIES OF THE UNIVERSE. DURING THE BIG BANG, MATTER AND ANTIMATTER WERE CREATED IN EQUAL AMOUNTS, BUT AFTER A LOT OF ENERGETIC MUTUAL ANNIHILATION WE SEEM TO BE LEFT WITH A LOT OF MATTER AND NOT MUCH ANTIMATTER. NO-ONE KNOWS WHERE ALL THE ANTIMATTER HAS GONE...
iγ.∂ψ=mψ
WHEN MATTER AND ANTIMATTER PARTICLES MEET, THEY IMMEDIATELY DESTROY ONE ANOTHER, TURNING EACH OTHER COMPLETELY INTO ENERGY. EVEN SMALL AMOUNTS OF MATTER AND ANTIMATTER WILL PRODUCE HUGE AMOUNTS OF ENERGY.
ANTIMATTER IS THE MOST EXPENSIVE STUFF ON EARTH - $62.5 TRILLION A GRAM - BUT IF WE FIND OUT HOW TO MAKE IT CHEAPER, SOME OF THE STUFF OF SCIENCE FICTION MAY COME TRUE.
A GRAM OF ANTIMATTER WOULD RUN A CAR FOR 100,000 YEARS!


+ INTO THE ANTIWORLD +
Dirac's dramatic discovery of Antimatter

The Nobel Prize in Physics 1933

"for the discovery of new productive forms of atomic theory"

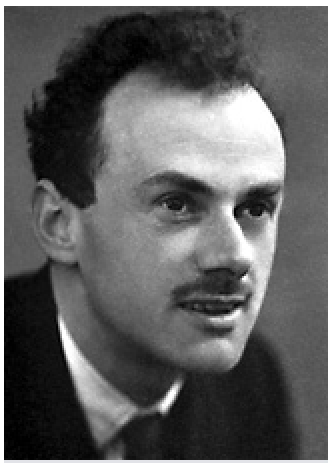
Erwin Schrödinger

1/2 of the prize

Austria

Berlin University
Berlin, Germany

b. 1887
d. 1961



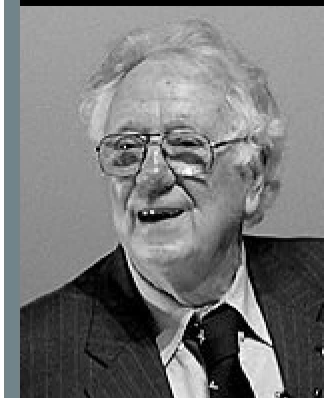
Paul Adrien Maurice Dirac

1/2 of the prize

United Kingdom

University of Cambridge
Cambridge, United Kingdom

b. 1902
d. 1984



The Nobel Prize in Physics 1936

"for his discovery of cosmic radiation"

"for his discovery of the positron"

Victor Franz Hess

Carl David Anderson

Printer Friendly
Comments & Questions
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The 1936 Prize in:
Physics
Prev. year Next year

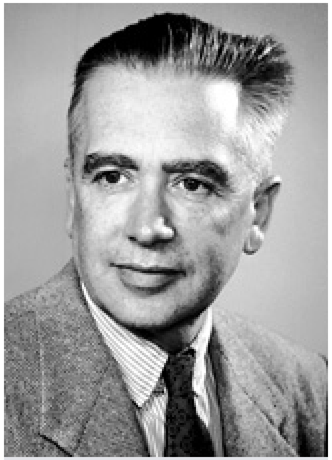
The Nobel Prize in Physics 1936
Presentation Speech

Victor F. Hess
Biography
Nobel Lecture
Documentary

Carl D. Anderson
Biography
Nobel Lecture
Documentary
Banquet Speech

The Nobel Prize in Physics 1959

"for their discovery of the antiproton"



Emilio Gino Segrè

1/2 of the prize

USA

University of California
Berkeley, CA, USA

b. 1905
(in Tivoli, Italy)
d. 1989

Owen Chamberlain

1/2 of the prize

USA

University of California
Berkeley, CA, USA

b. 1920
d. 2006

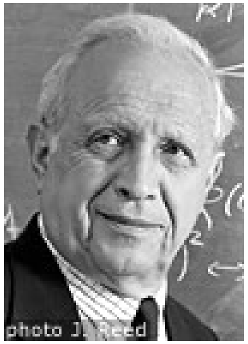
Media Player

Documentary about Doris Lessing

The Nobel Prize in Physics 2005

"for his contribution to the quantum theory of optical coherence"

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"



Roy J. Glauber

1/2 of the prize

USA

Harvard University
Cambridge, MA, USA

b. 1925

John L. Hall

1/4 of the prize

USA

University of Colorado, JILA;
National Institute of Standards and Technology
Boulder, CO, USA

b. 1934

Theodor W. Hänsch

1/4 of the prize

Germany

Max-Planck-Institut für Quantenoptik
Garching, Germany;
Ludwig-Maximilians-Universität
Munich, Germany

b. 1941

http://nobelprize.org/physics/laureates/2005/index.html

Hydrogenic spectrum

- One needs three quantum numbers to define the state of hydrogen (hudrogen-like) atom:

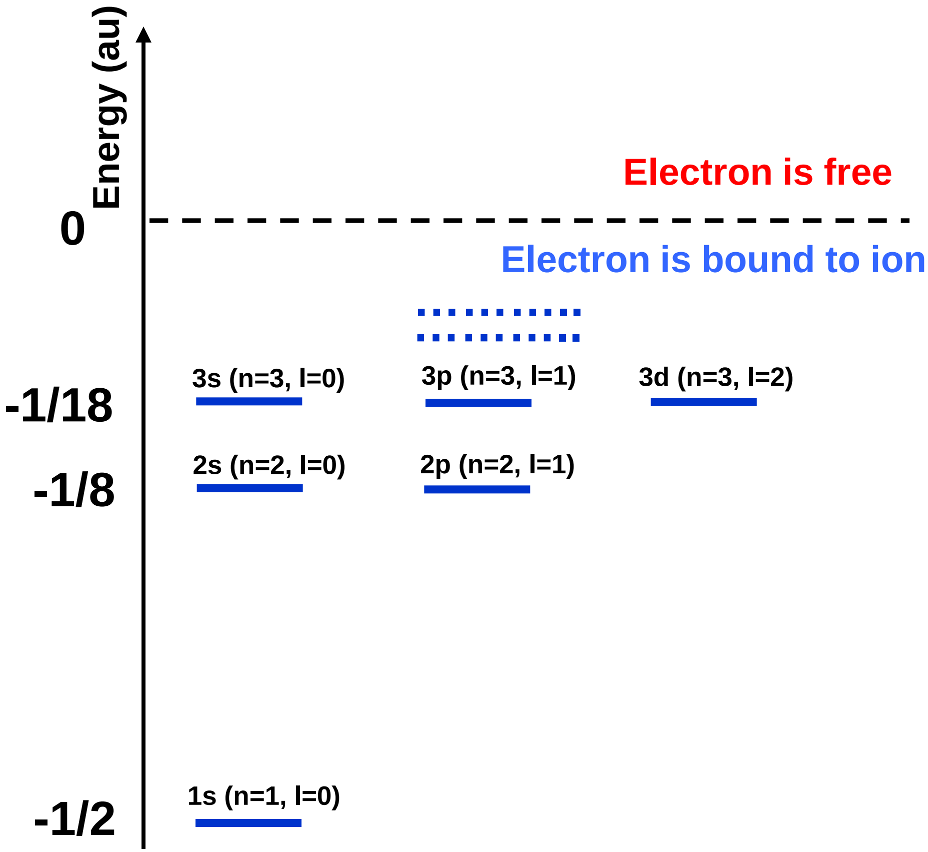
  - n = 1, 2, 3… (principal)
  - l = 0, … n-1 (orbital)
  - m = -l, …. +l (magnetic)

- The energy depends only on the principal quantum number:

$$E_n = -\frac{Z^2}{2n^2}$$

- i.e. in nonrelativistic theory the states are degenerate (l, m)!

$$\psi(\mathbf{r}) = \psi(r,\theta,\varphi) = R_{nl}(r)Y_{lm}(\theta,\varphi)$$



How to remove degeneracy?
We have to break the symmetry of the system!

Hydrogenic spectrum

Energy eigenvalue of the hydrogen atom

$$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2 n^2} = \frac{-13.6}{n^2}\,\mathrm{eV}$$

Schrödinger equation $\quad U \propto -\frac{1}{r}$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi_{nlm} + U\psi_{nlm} = E_n\psi_{nlm}$$

The solution (energy) for a central Coulomb-potential only depends on the quantum number n, but not on l or m. States with the same n are degenerated, what means they have the same energy.

(In many-electron atoms the degeneracy disappears because of a non-central Coulomb-potential.)

Atomic Units

for electrons $m_e = 1 \quad \Rightarrow \quad v = p$

scaling properties $r = \frac{n^2}{Z} \qquad v = \frac{Z}{n} \qquad E = \frac{1}{2} Z^2$

fine-structure constant $c = \frac{1}{\alpha} = 137.036$

$$\alpha = \left(\frac{e^2}{\hbar \cdot c} \right)_{gauss} ; \alpha = \left(\frac{e^2}{4\pi \cdot \varepsilon_o \cdot \hbar \cdot c} \right)_{SI}$$

$$\alpha = 1/137.03599911(46)$$

Atomic Units

Atomic Units		SI-Units
$\hbar = 1$	atomic Planck constant	$1.05 * 10^{-34}$ Js
$m_e = 1$	atomic mass unit	$9.1 * 10^{-31}$ kg
$e = 1$	atomic charge unit	$1.6 * 10^{-19}$ C
$4\pi\varepsilon_0 = 1$	dielectric constant	

The Bohr-radius defines the atomic length unit

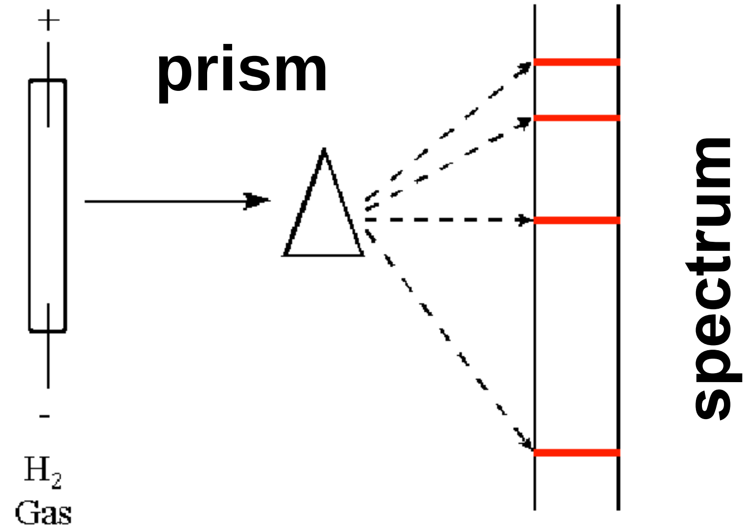
$a_0 = 0{,}53 \cdot 10^{-8}$ cm : 1 a.u.

The atomic energy unit is 27.21 eV and is called Hartree

For the ionization-energy of the hydrogen atom follows

1/2 Hartree = 13.6 eV = 1 Rydberg

The hydrogen spectrum



Balmer-spectrum

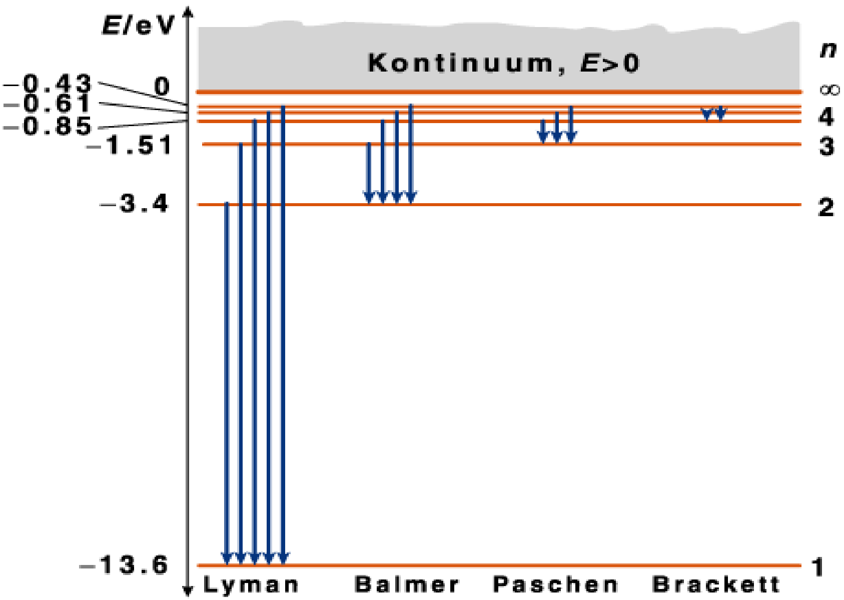
A lot of stars have spectra which are identical to the absorption spectrum of hydrogen. In 1885 Balmer developed an empirical formula to calculate the frequency of these lines

$$\nu_m = R\left(\frac{1}{4} - \frac{1}{m^2}\right) \qquad \lambda_m = \frac{c}{\nu_m} \tag{1}$$

where m $\geq$ 3 and R are constants (Rydberg-frequency). This formula describes for m=3, 4, ... a continuous serial of lines of the frequencies ν_m (resp. the wavelengths λ_m) known as Balmer-series. In general these lines are described in the following way:

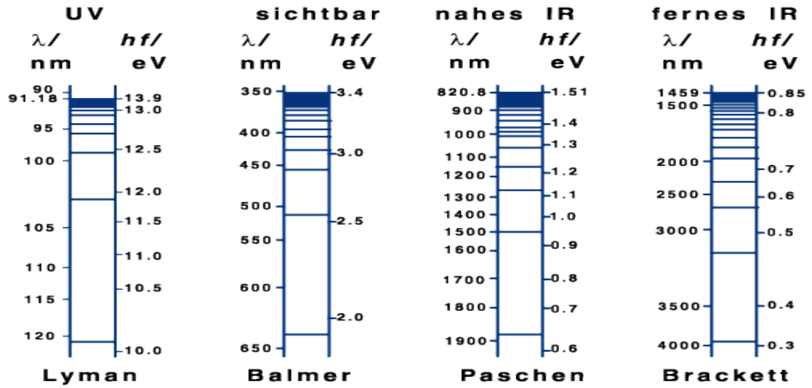
H_α (m=3), H_β (m=4), ...

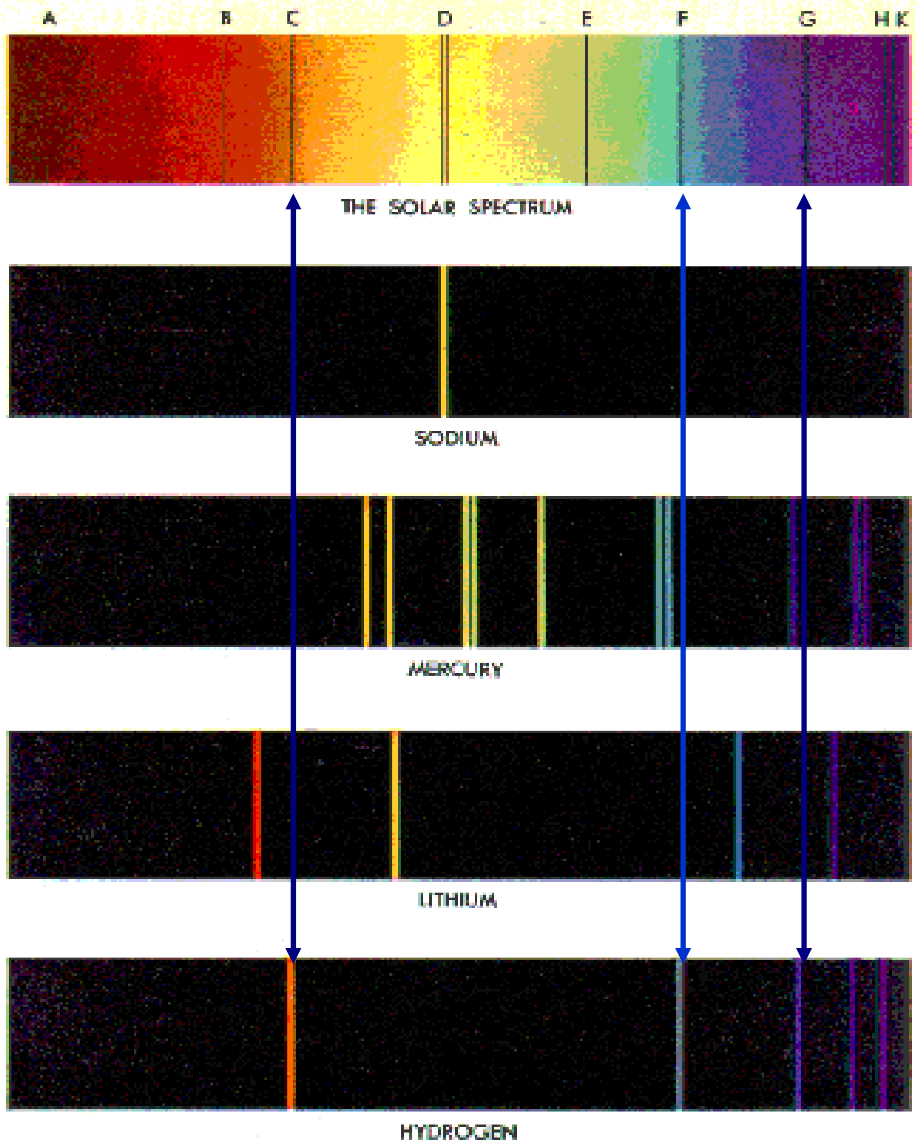
The spectrum of atomic hydrogen



$$\Delta E = E_i - E_j = \frac{-e^4 m_e}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{i^2} - \frac{1}{j^2}\right).$$

For the ground state of hydrogen (n=1) the eigenvalue is ***-13.6 eV***. The excitation-energies can be calculated as differences between the energy levels by using the **Rydberg-formula.**





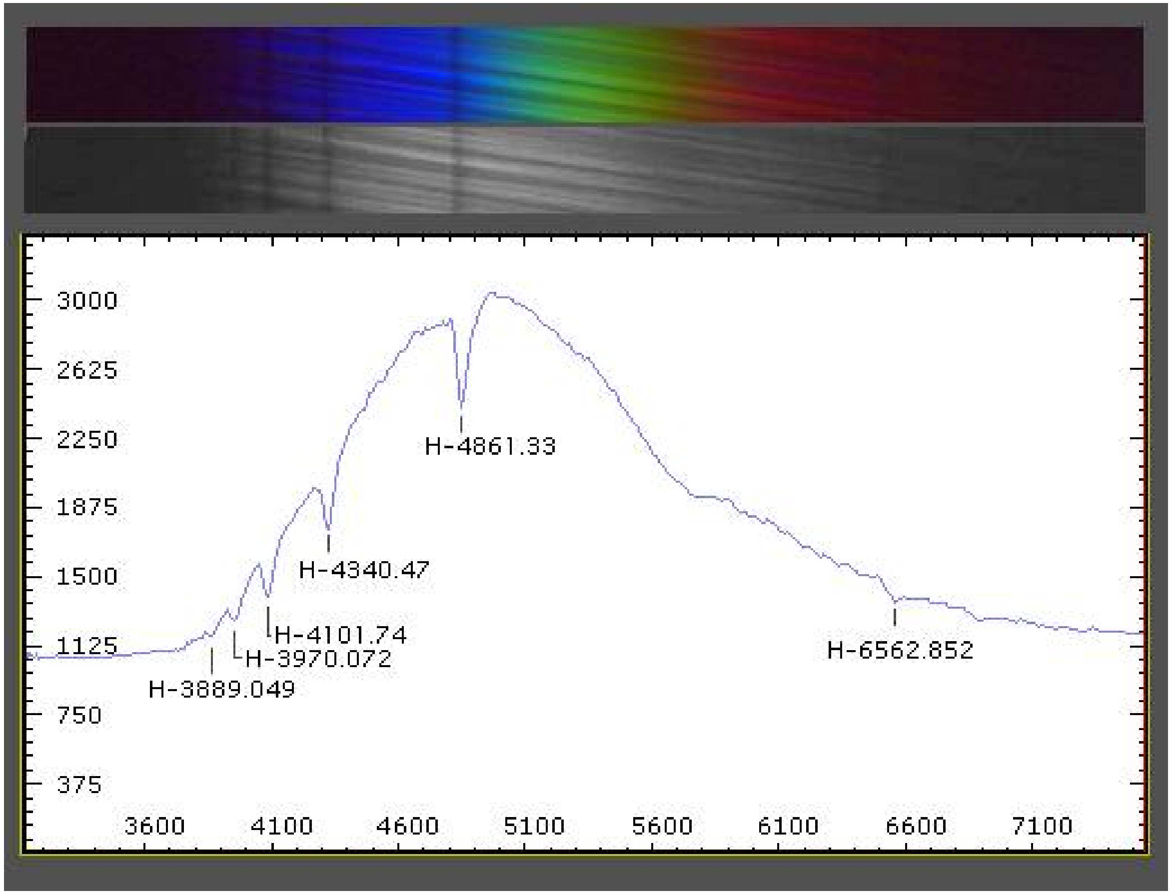
sodium

mercury

lithium

hydrogen

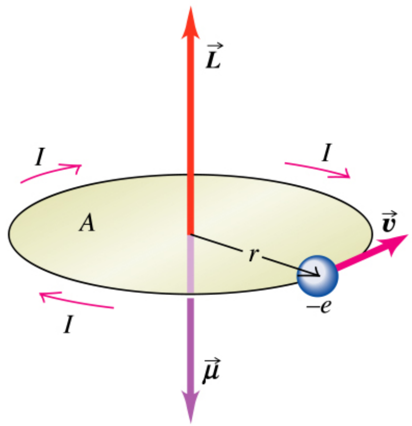
solar spectrum **(top) with absorption-lines of sodium (D) und hydrogen, in comparison to calibration lines of some elements**



Spectrum of Sirius depending on the wavelenght [in A = 10^{-8} cm]
with a multitude of hydrogen (H) –absorption lines from the Balmer-series.

Magnetic moments

Orbital magnetic dipole moment



In classical electrodynamics:

$$|\mu| = \boldsymbol{I} \cdot \boldsymbol{A}$$

current — vector area of the current loop

$$|\mu| = \boldsymbol{I} \cdot \boldsymbol{A} = \frac{q}{T}\pi r^2 = \frac{qv}{2\pi r}\pi r^2 = \frac{q}{2m} mvr = \frac{q}{2m} L$$

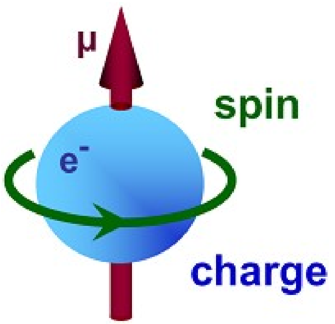
In quantum mechanics, for electron: q=-e

$$\hat{\boldsymbol{\mu}}_l = -\mu_0 \hat{\mathbf{L}} / \hbar, \qquad \mu_0 = \frac{e\hbar}{2m_e}$$

Bohr magneton

Spin magnetic moment

classical picture



$$\hat{\boldsymbol{\mu}}_s = -g_s \mu_0 \hat{\mathbf{S}} / \hbar$$

Gyromagnetic rato

Stern-Gerlach-experiment

The z-component of the angular momentum

(Stern-Gerlach experiment 1922)

example: d-state with n=3

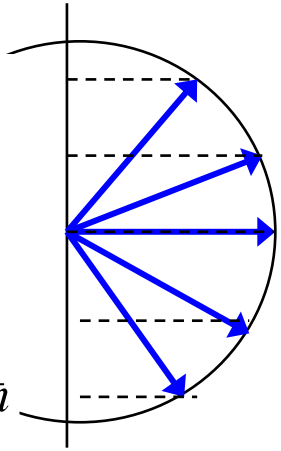
$L_z = 2\hbar$

$L_z = \hbar$

$L_z = 0$

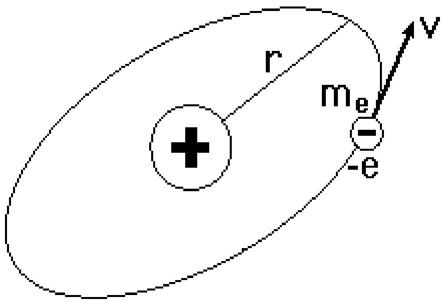
$L_z = -\hbar$

$L_z = -2\hbar$



$$\vec{\mu} = \frac{\mathrm{e}}{2\mathrm{m}}\left(\mathrm{m}\vec{\mathrm{v}} \times \vec{\mathrm{r}}\right) = \mathrm{e}/2\mathrm{m} \cdot \vec{\mathrm{L}}$$

Magnetic moment = current x area



In a magnetic field $\vec{B} = B_z \cdot \vec{e}$ is the ***magnetic* energy** of an electron

$$E = \vec{\mu} \cdot \vec{B} = -\mu_z B_z = -\frac{e}{2m} L_z B_z$$

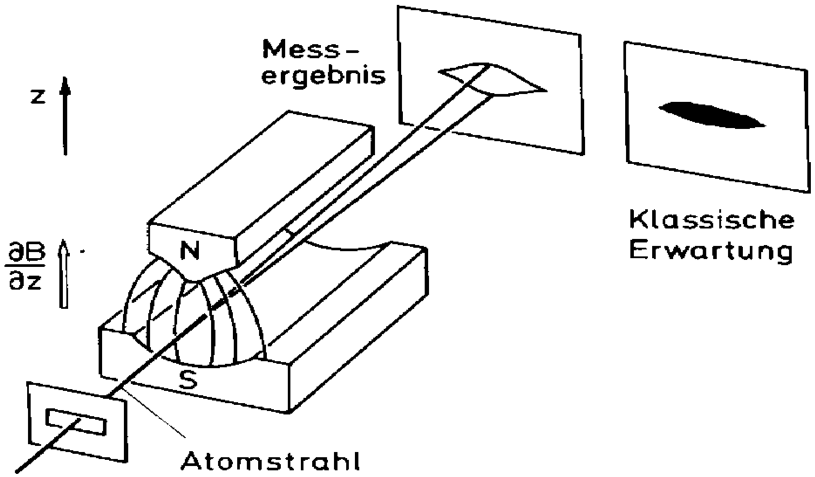
Is $\mathbf{B_z}$ **inhomogeneous** ($\partial/\partial z\, B_z \neq 0$), the electron feels a force proportional to L_z

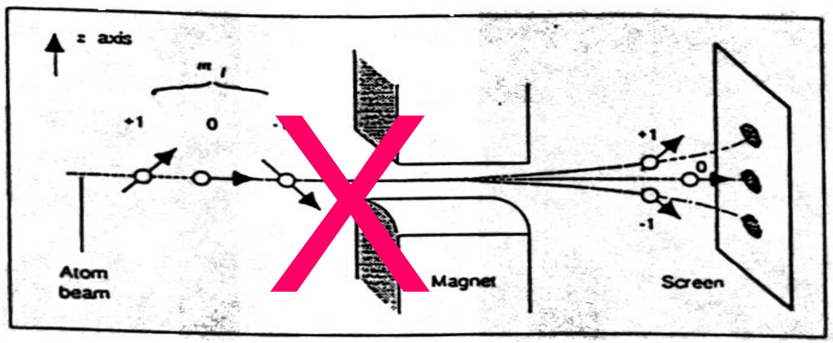
$$F_z \propto -L_z \frac{\partial}{\partial z} B_z$$

Stern-Gerlach Experiment

Stern and Gerlach used silver atoms (Ag, Z=47)
electron configuration: $_{36}$Kr + 4 d^{10} + $5s^1$; accordingly one valence electron in the 5s-shell

$$F_z \propto -L_z \partial / \partial z B_z$$

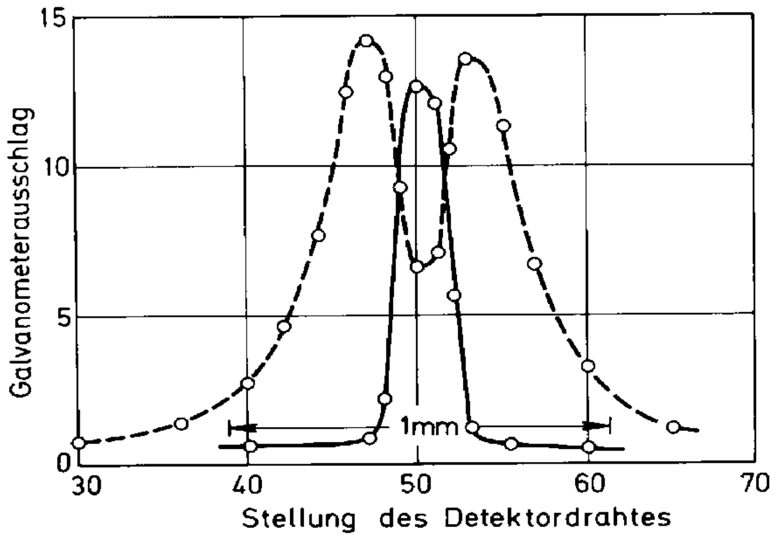




Stern-Gerlach-experiment: In an inhomogeneous magnetic field a beam of silver atoms is diverted and splitted into two beam parts. The magnetic field possesses a gradient of 10 T/cm and a length of 10 cm.

Stern and Gerlach assumed L=1 for the electron and therefore expected a splitting into three parts with

$$m_z = -1, 0, 1$$



Observed intensity of the silver atom beam as a function of the distance to the beam axis: with (dashed line) and without (solid line) magnetic field

Only two lines were observed !!!

existance of a quantization direction

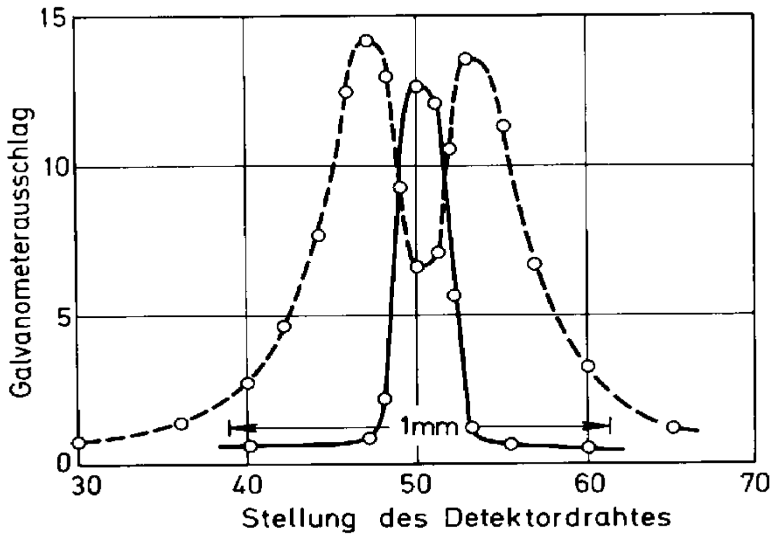
Contrary to the expectation, an even splitting was observed

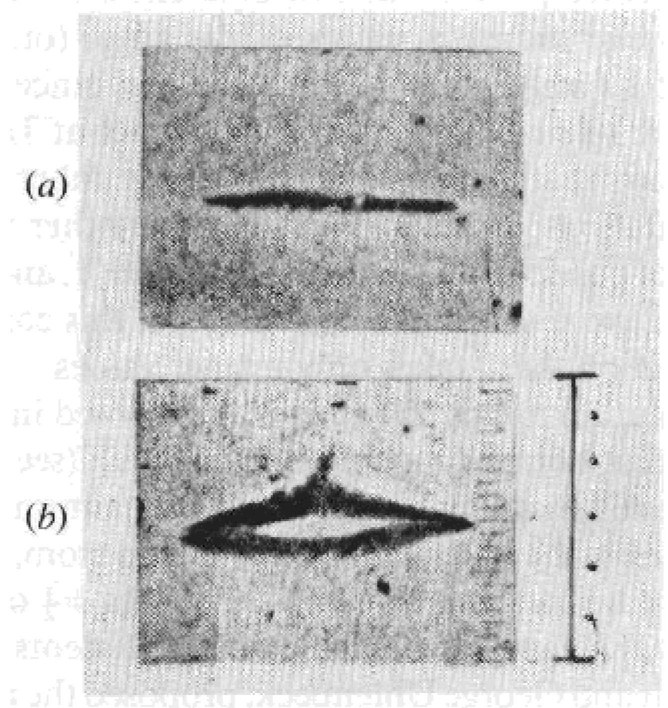
From today's point of view it is known that the assumption **L = 1** for the valence electron in the silver atom was wrong. The 47th electron occupies the 5s -shell and therefore L = 0.
Assuming this, a **single spot** would have been expected instead of **two**!
In 1925 Goudsmit, Uhlenbeck and **Pauli** found the solution to this problem by postulating the '***exclusion principle'***

Besides the known quantum numbers n, l, m there must be a fourth quantum number

no two electrons of one atom are equal in all four quantum numbers

Stern-Gerlach Experiment: The experimental result





DIRAC theory
(relativistic formulation of quantum mechanics)

Schrödinger's wave function (1926) was the first 'highlight" of the new quantum mechanics. But there was still a problem: the **theory of special relativity** was **not** included.

Hamilton-operator of a free electron according to Dirac

$$H = \alpha \cdot p + \beta m_e$$

with the operators α and β (4 x 4 matrix). The corresponding eigenvalue-equation is:

$$H|\Psi> = E|\Psi>$$

with the two solutions

$$E = +c\sqrt{(p^2 + m^2c^2)}$$

$$E = -c\sqrt{(p^2 + m^2c^2)}$$

Unexpected Antiparticles (Dirac)

1928

Since half the solutions must be rejected as referring to the charge $+e$ on the electron, the correct number will be left to account for duplexity phenomena.

1930

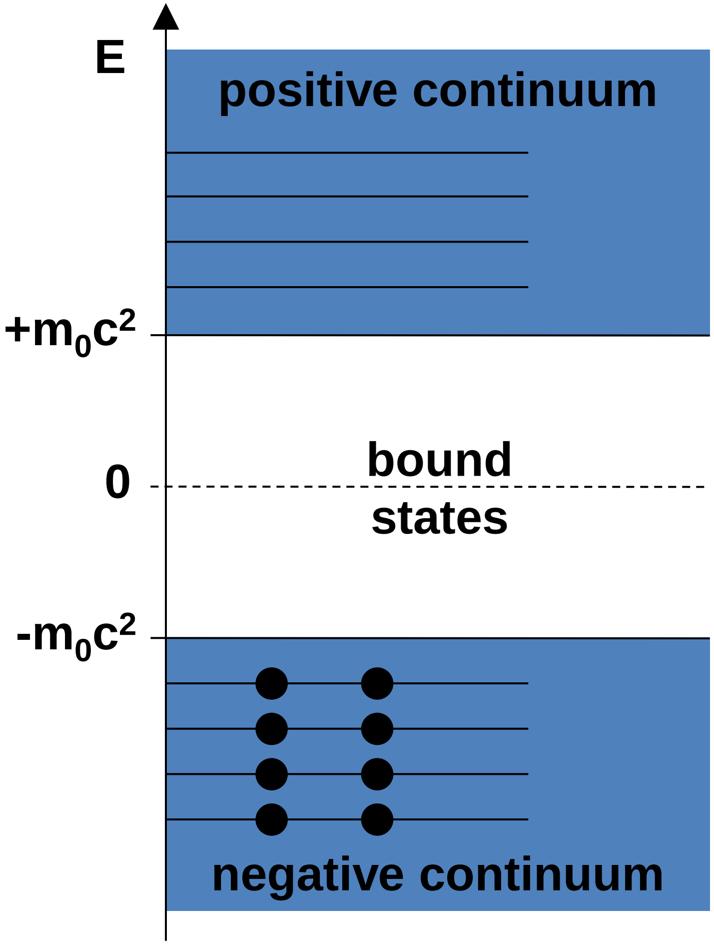
would fill it, and will thus correspond to its possessing a charge $+e$. We are therefore led to the assumption that *the holes in the distribution of negative-energy electrons are the protons*. When an electron of positive energy drops into

1931

nearly all, of the negative-energy states for electrons are occupied. A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We should not expect to find any of

Presumably the protons will have their own negative-energy states, all of which normally are occupied, an unoccupied one appearing as an anti-proton.

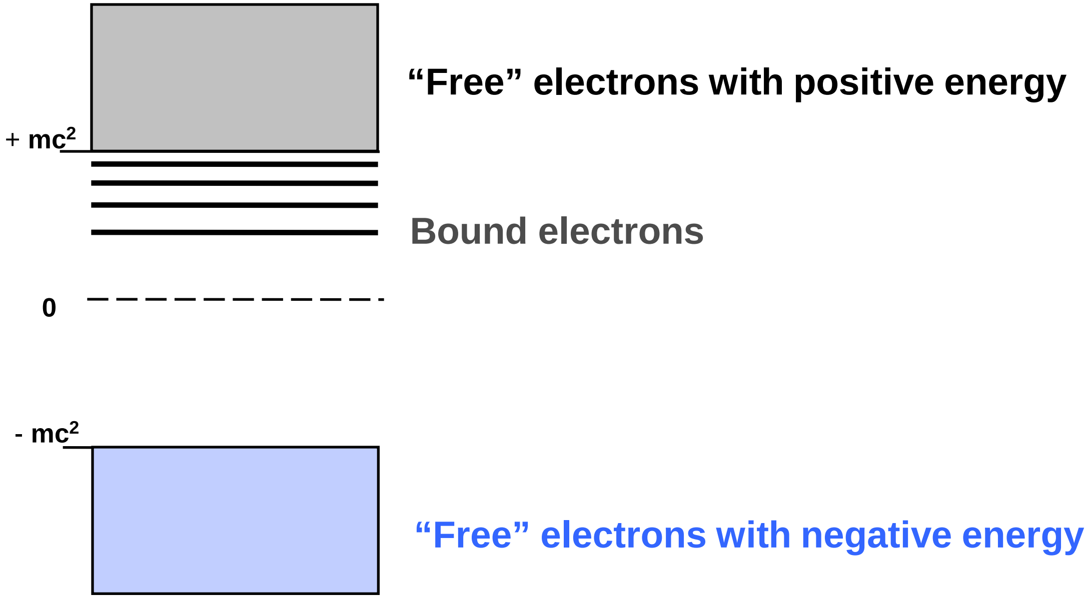
Prediction of anti-matter



Dirac, Anderson, the Positron and the anti-matter. In his famous equation Paul Dirac combined (**1929**) the fundamental equation of quantum mechanics, the Schrödinger-equation with the theory of special relativity. He did not discard the negative energy –solutions of his equation as unphysical but interpreted them as states of the anti-particle of the electron (positron, having the same mass but opposite charge). In **1932** Carl Anderson discovered the positron the first time in the cosmic radiation. This was the proof of the existence of 'anti-matter', with incalculable consequences for the future of physics.

Energy spectrum of the Dirac particle

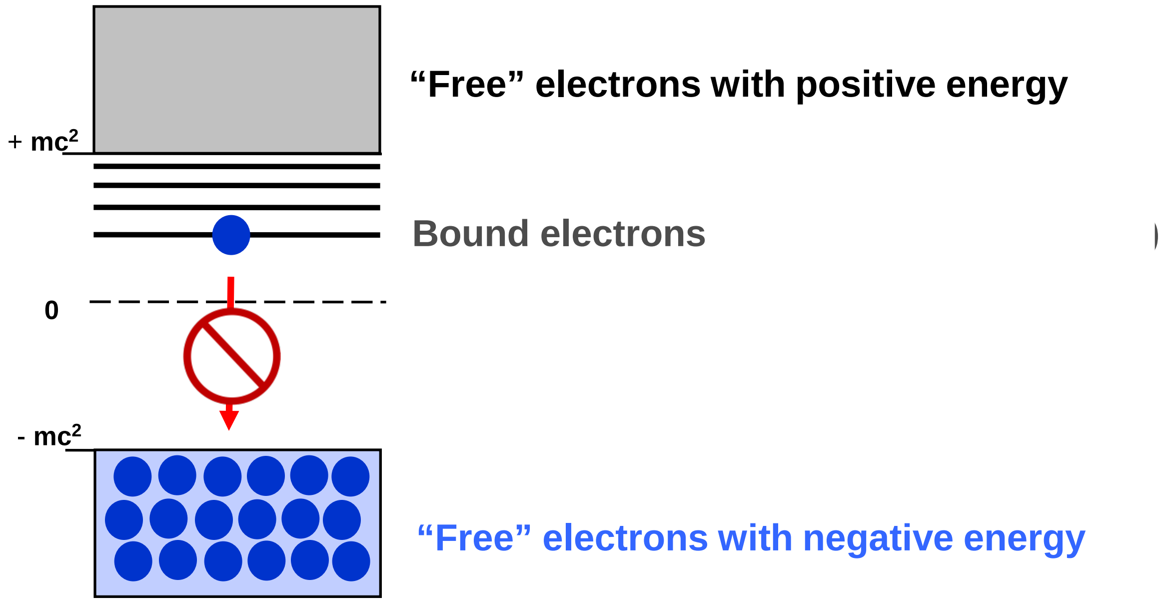
- **For the free particles we found:** $E_{\pm}(p) = \pm\sqrt{(m_e c^2)^2 + (pc)^2}$
  - **Energy of positive energy particles:** $E_{+}(p) > m_e c^2$
  - **Energy of negative energy particles:** $E_{-}(p) < -m_e c^2$



Where is the problem here?

Dirac sea

- In 1930 Paul Dirac have proposed a theoretical model of the vacuum as an infinite sea of particles possessing negative energy.

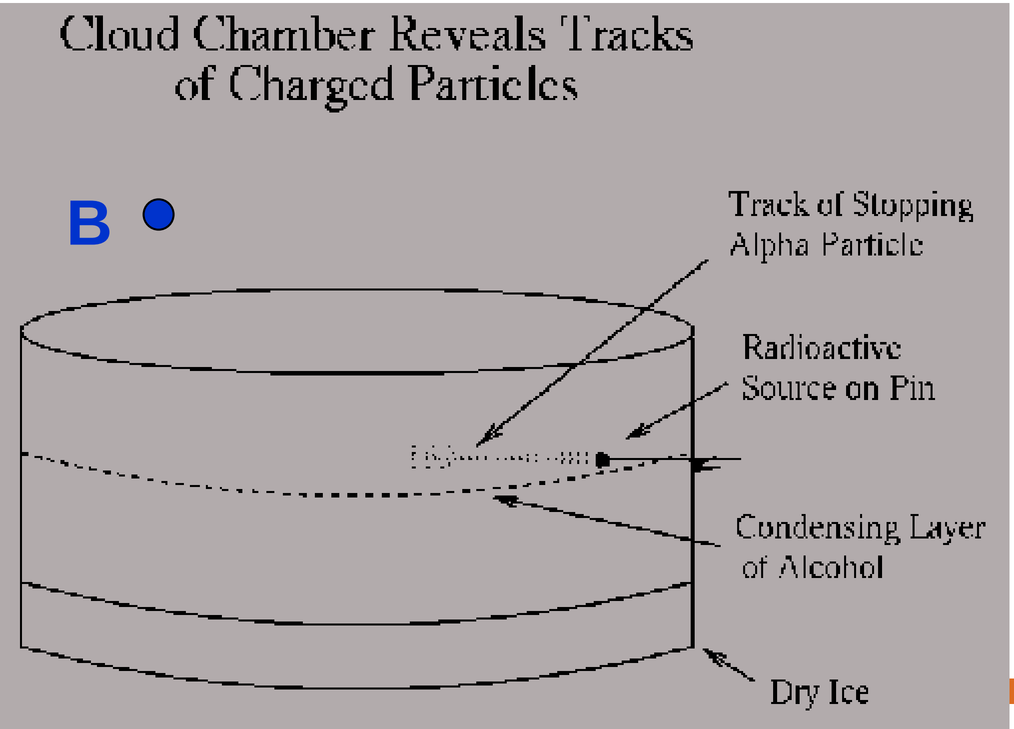


- Since all the states in Dirac sea are occupied "our" electron can not go down from the domain of positive energies. (Pauli principle.)

Discovery of the positron (Carl David Anderson 1905 – 1991)

Detector (cloud chamber: Wilson 1910)

Cloud chamber is filled with over-saturated watersteam, which condensates along the track of an energetic, ionizing particle. In addition a strong **magnetic field B** is applied to the cloud chamber. A **charged particle** will be forced on a **circular,** in general case an **ellipsoidal track** by the B field which crosses the interaction plane perpendicularly.



Charged particle in magnetic field:
(momentum **mv** and **B** are perpendicular)

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow rB = B\rho = \frac{mv}{q}$$

$B\rho$ magnetic ridigity

detector in magnetic field: particles are moving on a circular track

Issues to be considered by the experiment

1. What can be used as a **source for positrons**?

2. How is it possible to determine the **signature of the charge**?
   (Did the particle come from the 'top' or from 'bottom'?)

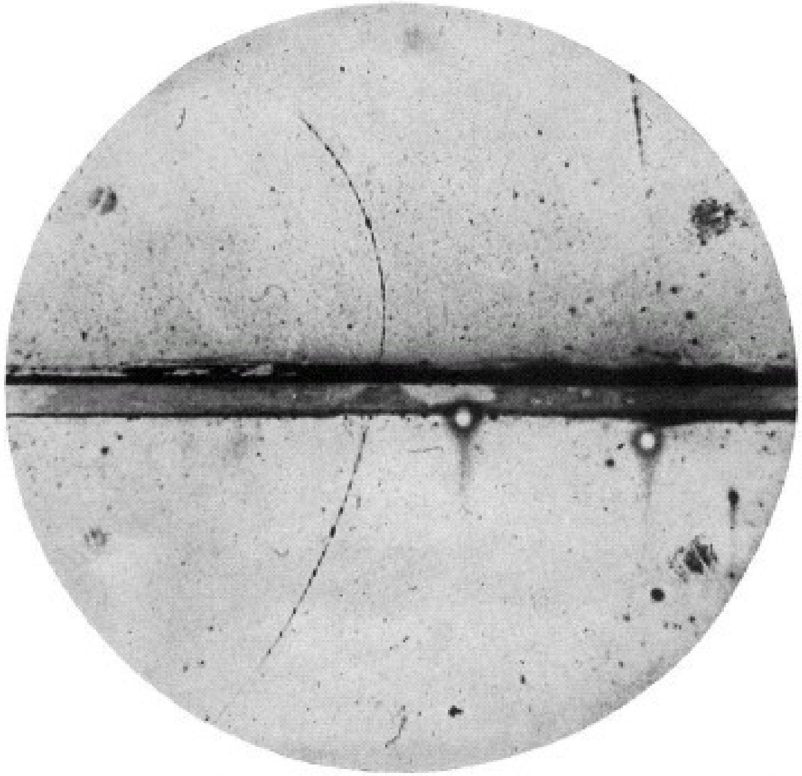
3. How can the 'new' **positron** with electron mass $\mathbf{m_e}$ and positive charge $\mathbf{q = +e}$ be distinguished from a **proton** in case, only momentum-measurement is possible?

Solutions:

1. Cosmic radiation

2. Cloud chamber: separated by a lead-plate of 6mm thickness, which extenuates the energy of the cosmic particle. As a result, the radius of curvature has to be smaller before passing the plate then afterwards. This gives the incoming-direction.

3. for protons and positrons with given momentum **mv** the range of coverage in the cloud chamber differs a lot!



The first confirmation of a positron

Cloud chamber photograph by Andersen

- Phys. Rev. 43, 491 (1933)
- Nobel prize 1936 together with Victor Hess !

The first **'fingerprint' of anti-matter**. Anderson discovers the trace of a positron in his cloud chamber (in the middle one can see a lead-plate of 6mm thickness).

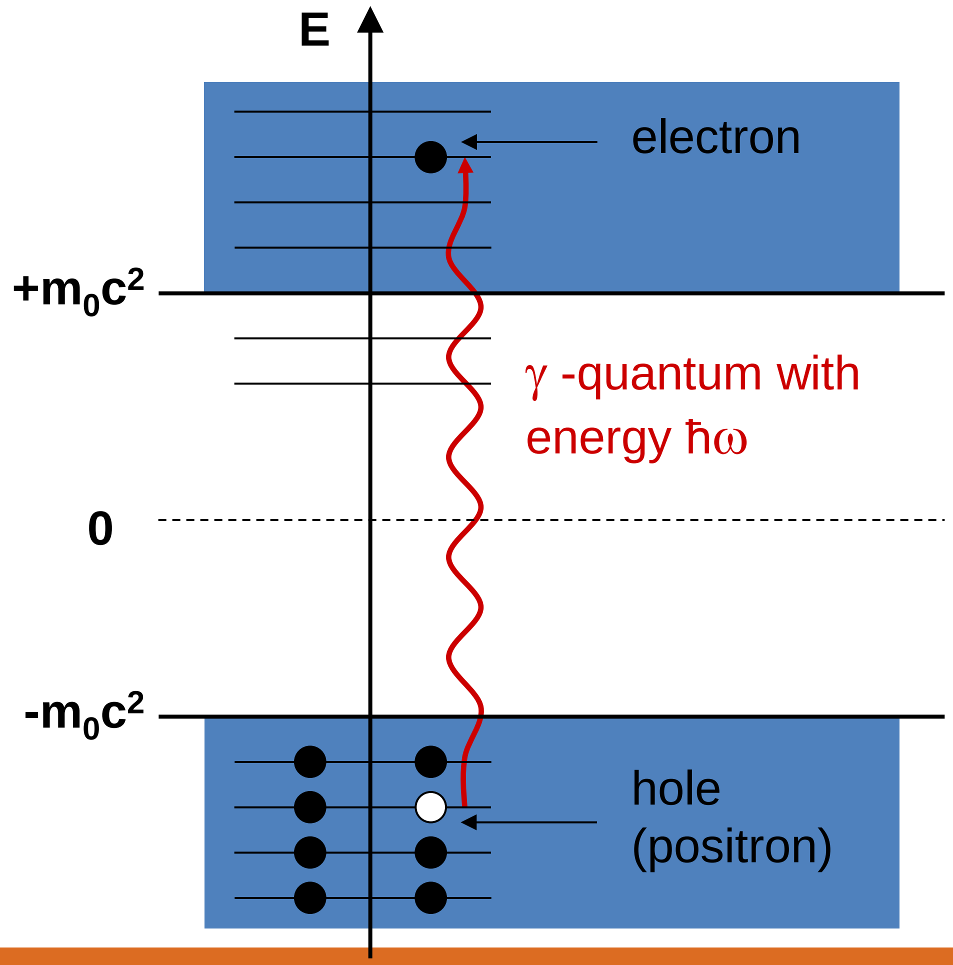
1. The **upper** part of the bending gives information about the **incoming-direction.**

2. The **lower** part gives the **positive charge** of the particle by its bending-direction.

3. By analyzing the **radius of curvature** before and after the transition the momentum can be estimated

Electron-positron pair production

In order to produce electron-positron pairs we would need:
at least two times the rest mass of the elecreon !!!



- energy of about

$$\hbar\omega \approx mc^2 \approx 1MeV$$

to induce pair production

In case a whole in the negative continuum excits, an electron will immediately fill the vacancy and two 511 keV quanta are emitted.

Application in tomography

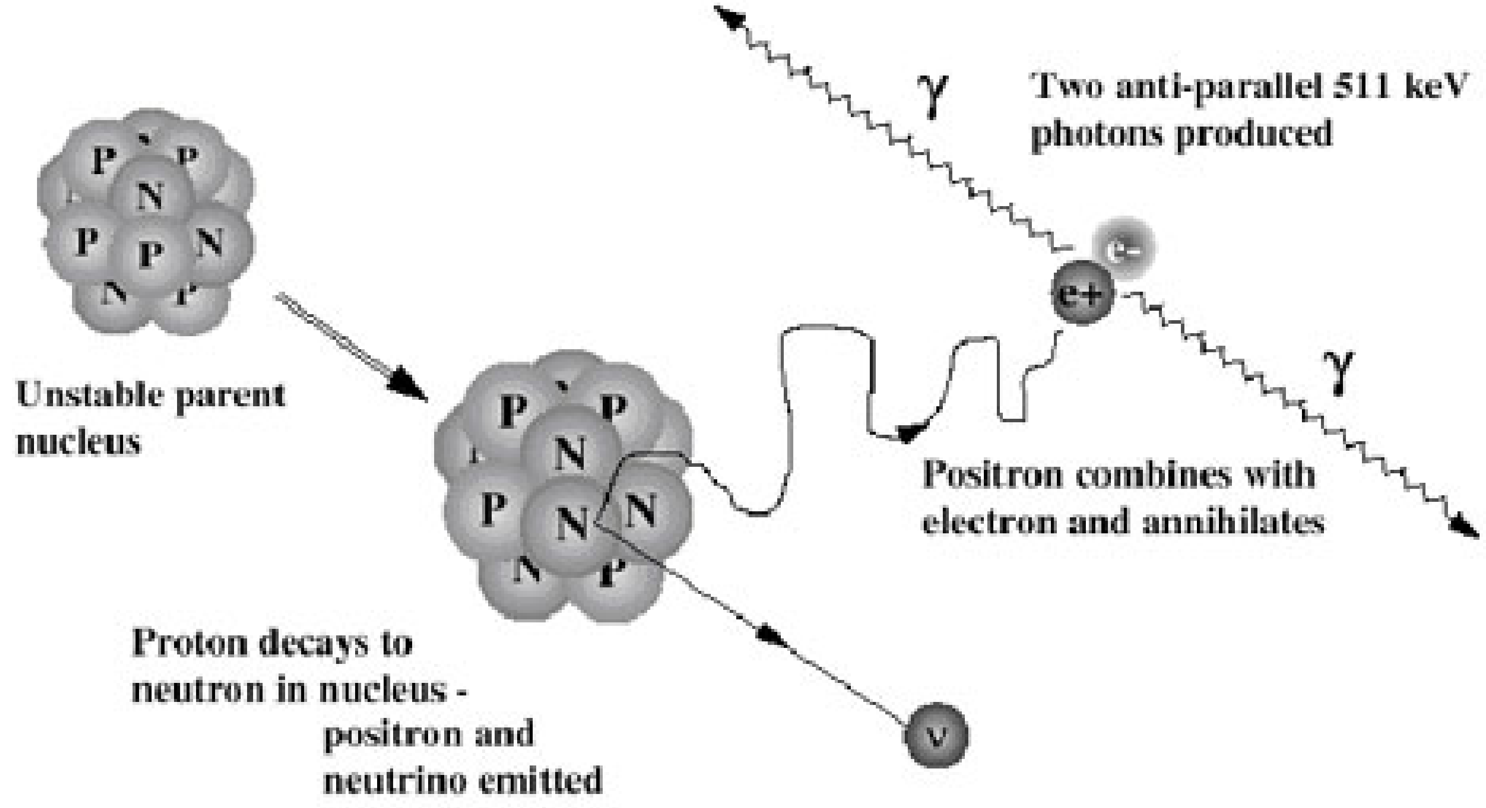
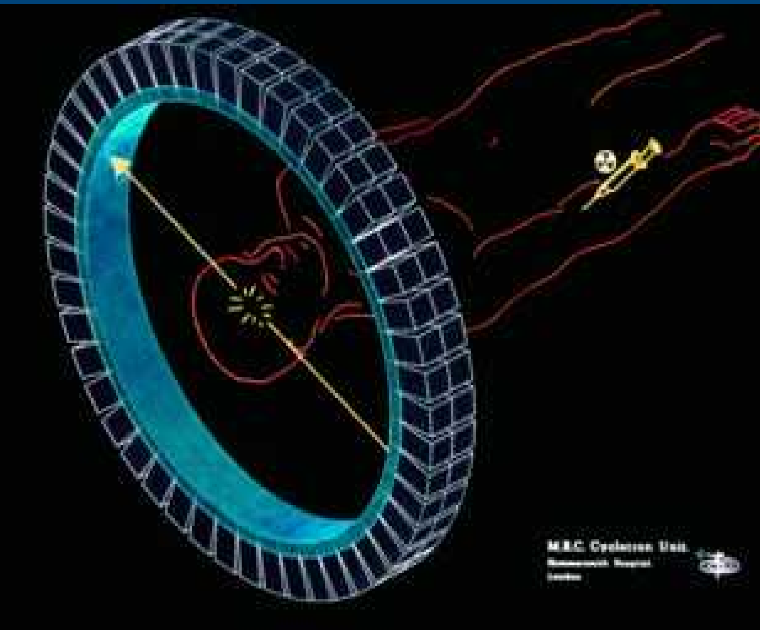


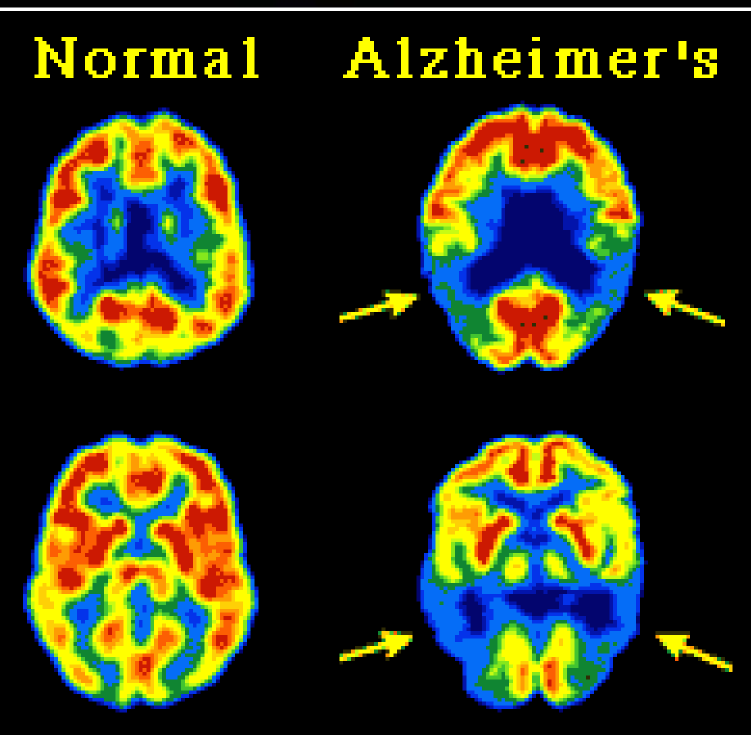
Figure 1.1. Positron emission and annihilation.

Production of a positron during β+ decay and annihilation afterwards



Positron emission tomography (**PET**):
β+ -active C-, O-, or Fluor-nuclei are injected into the brain. There the local brain activity can be measured by detecting the collinear 511 keV photons of the **electron-positron elimination-radiation**

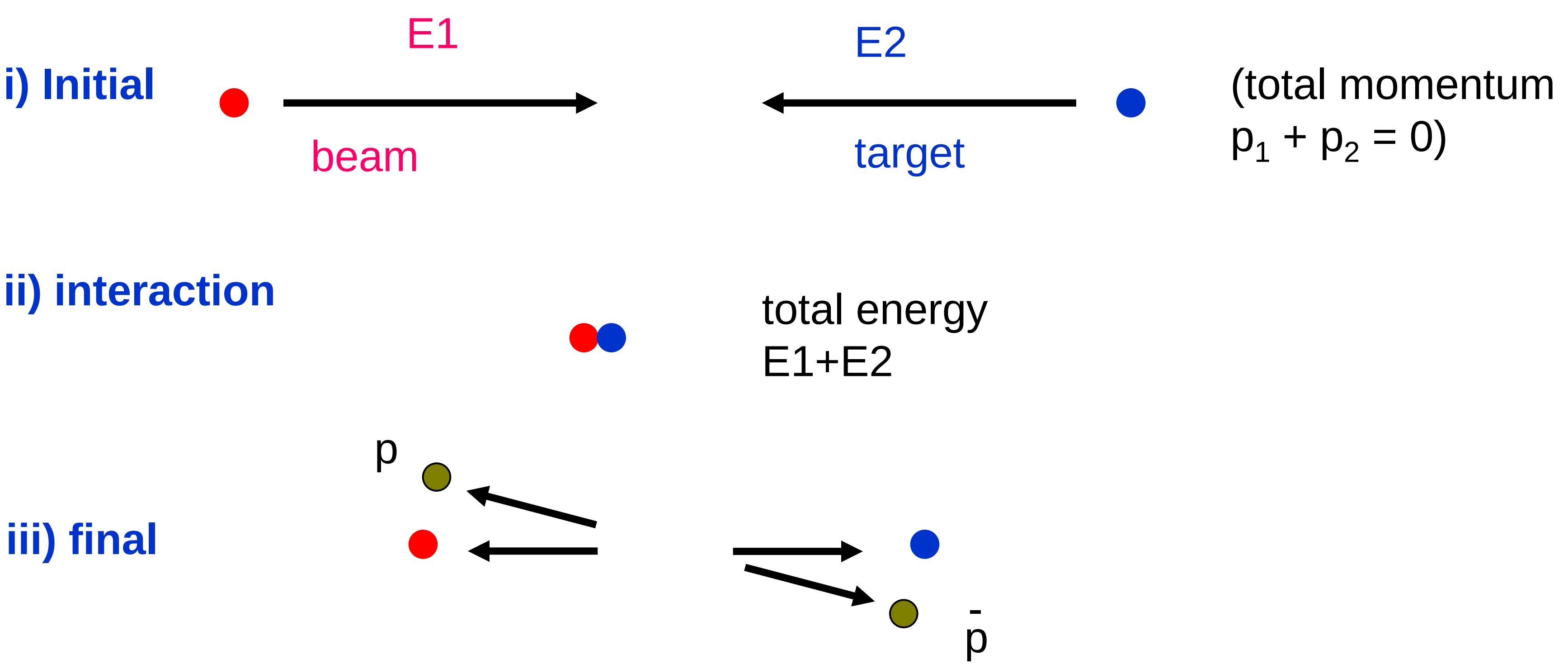
PET - camera made of segmented (position sensitive) γ-detectors



Matter ⇔ Energy $E=mc^2$

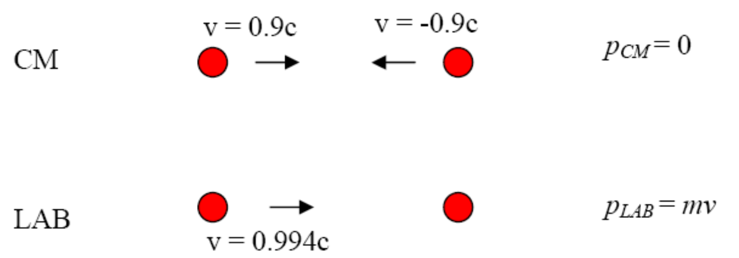
Collision processes of high-energetic particles (cm system), particle production (antiproton production)

$$p + p \Rightarrow p + p + (p + \bar{p})$$



Total energy: $E1 + E2 = mc^2 > 4\, m_0\, c^2$ ***Threshold !!!***

$$E^2 = \left(m_0 c^2\right)^2 + c^2 p^2$$



The same collision as viewed in the CM and LAB frames.

Comparing the center of mass energy with the lab energy at these high energies,

$$E_{lab} = (m + m_0)c^2$$

$$E_{cm}{}^2 - c^2 p_{cm}{}^2 = E_{lab}{}^2 - c^2 p_{lab}{}^2 = m_0^2\, c^4 = \text{invariant}$$

$$E_{cm}{}^2 = E_{lab}{}^2 - c^2 p_{lab}{}^2; \quad \text{but } p_{cm} = 0$$

$$E_{cm}{}^2 = m^2c^4 + 2mc^2 m_0 c^2 + m_0^2 c^4 - p_{lab}^2 c^2; \qquad p_{lab} \approx mc$$

$$E_{cm}{}^2 = m_0 c^2 \left(2mc^2 + m_0 c^2\right)$$

$$E_{cm}{}^2 = 2m_0 c^2 mc^2\,;\, m >> m_0$$

$$E_{cm} \approx \sqrt{2m_0 c^2 E_{lab}}$$

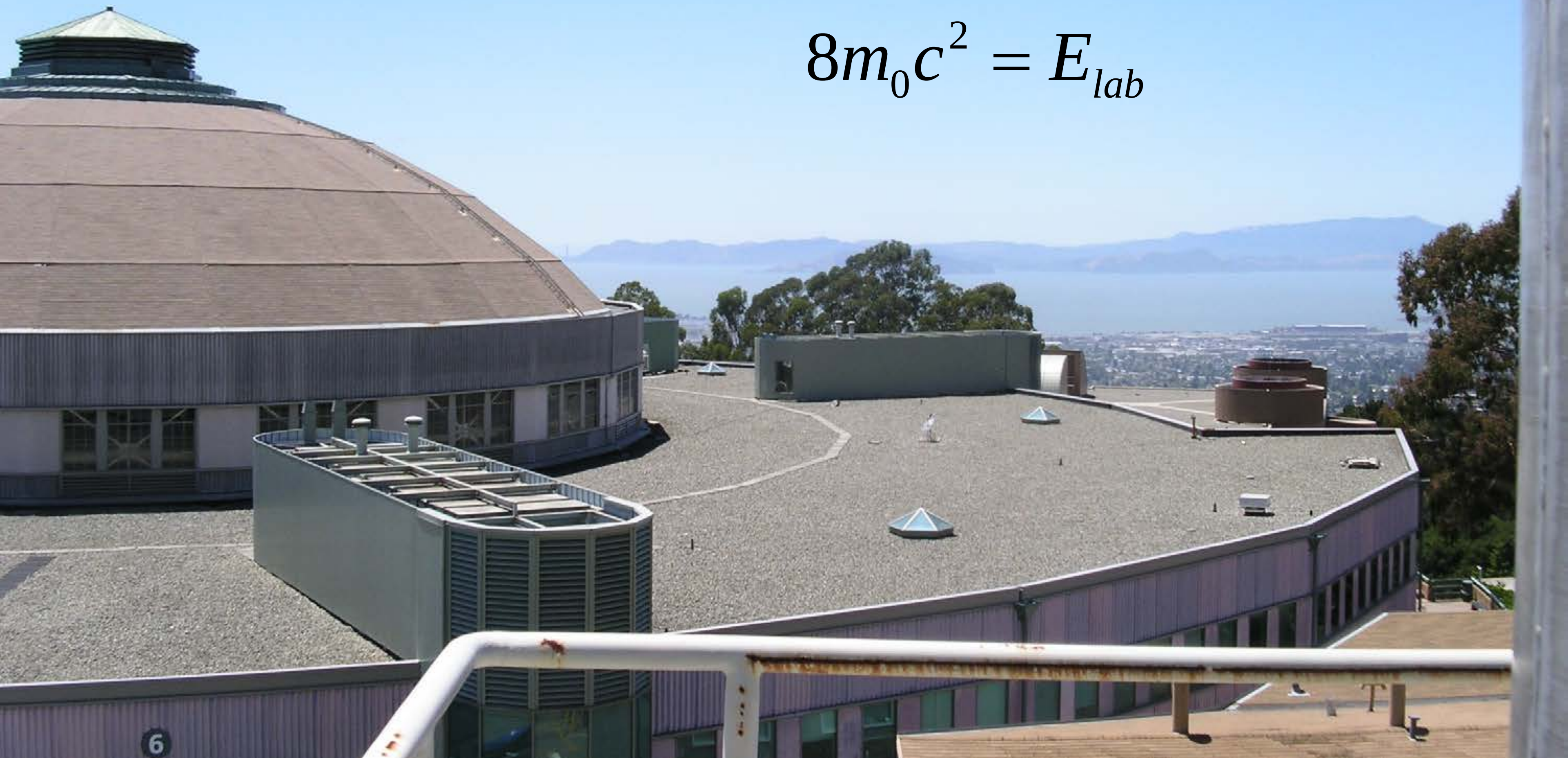
BEVALAC / Berkeley

$$E_{cm} \approx \sqrt{2m_0c^2E_{lab}}$$

particle production

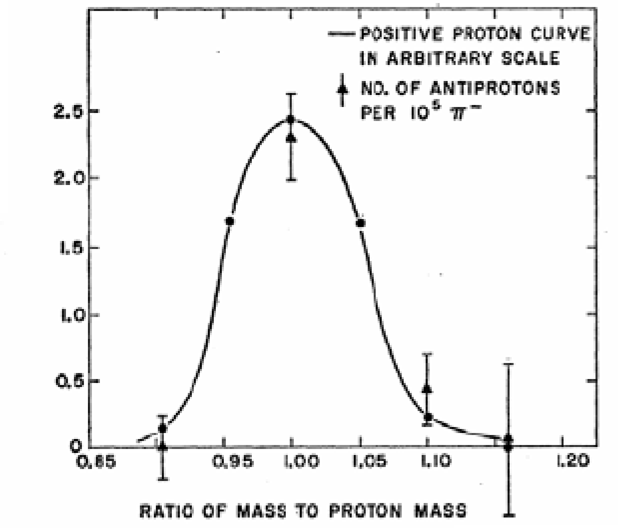
$$E_{cm} \geq 4m_0c^2 \Rightarrow E_{cm}^2 \approx 16m_0^2c^4 = 2m_0c^2E_{lab}$$

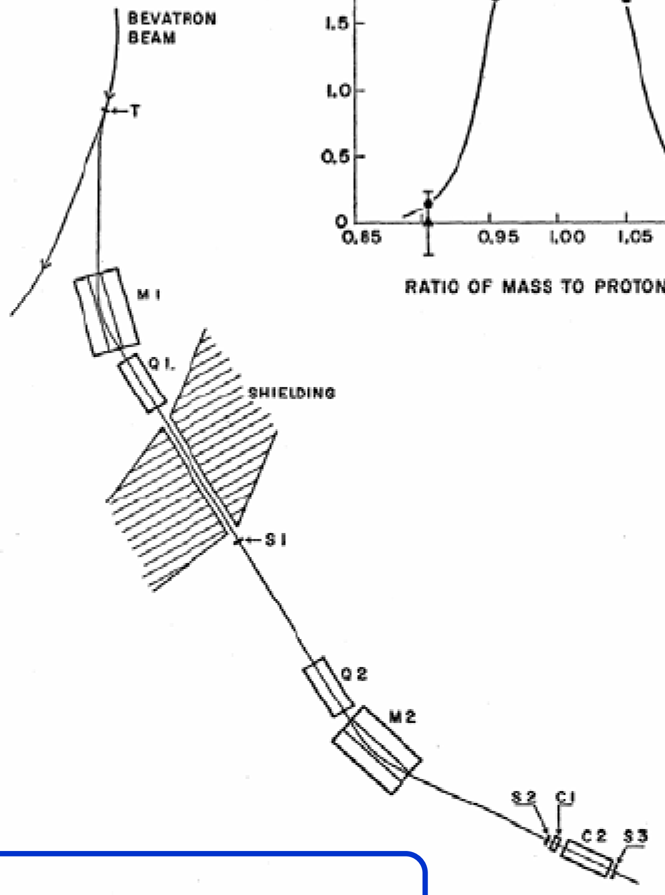
$$8m_0c^2 = E_{lab}$$



Discovery of the Antiproton

- *Bevatron 5.6 GeV*
  - Just at threshold!
- *Discrimination against π^-: measure*
  - Momentum
    - Magnets: 1.19 GeV
  - Velocity
    - TOF 51 vs. 40 ns
  - Cerenkov counter veto
- *60 events in 1955*
- *$\Delta m/m_p \sim 5\%$*

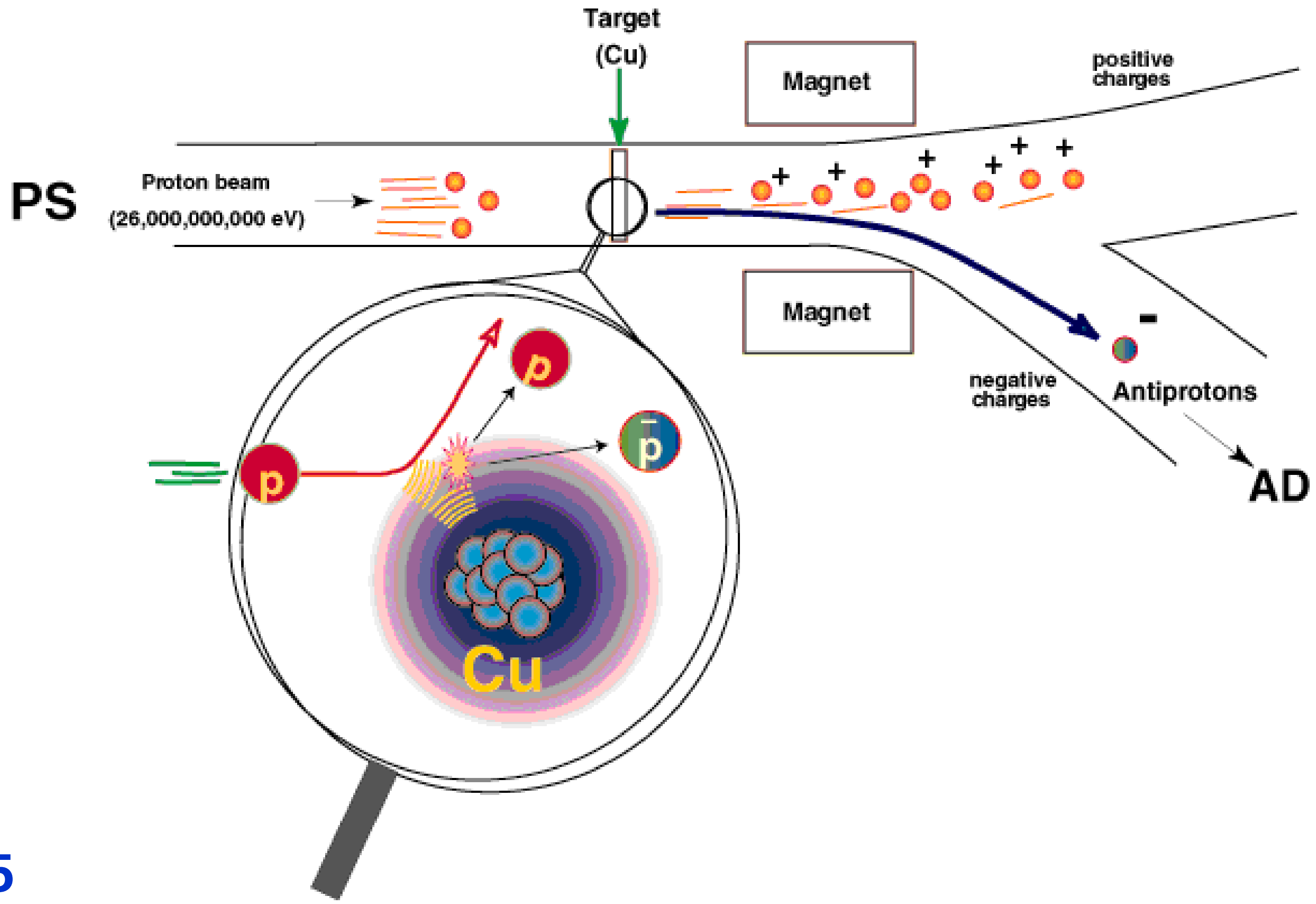




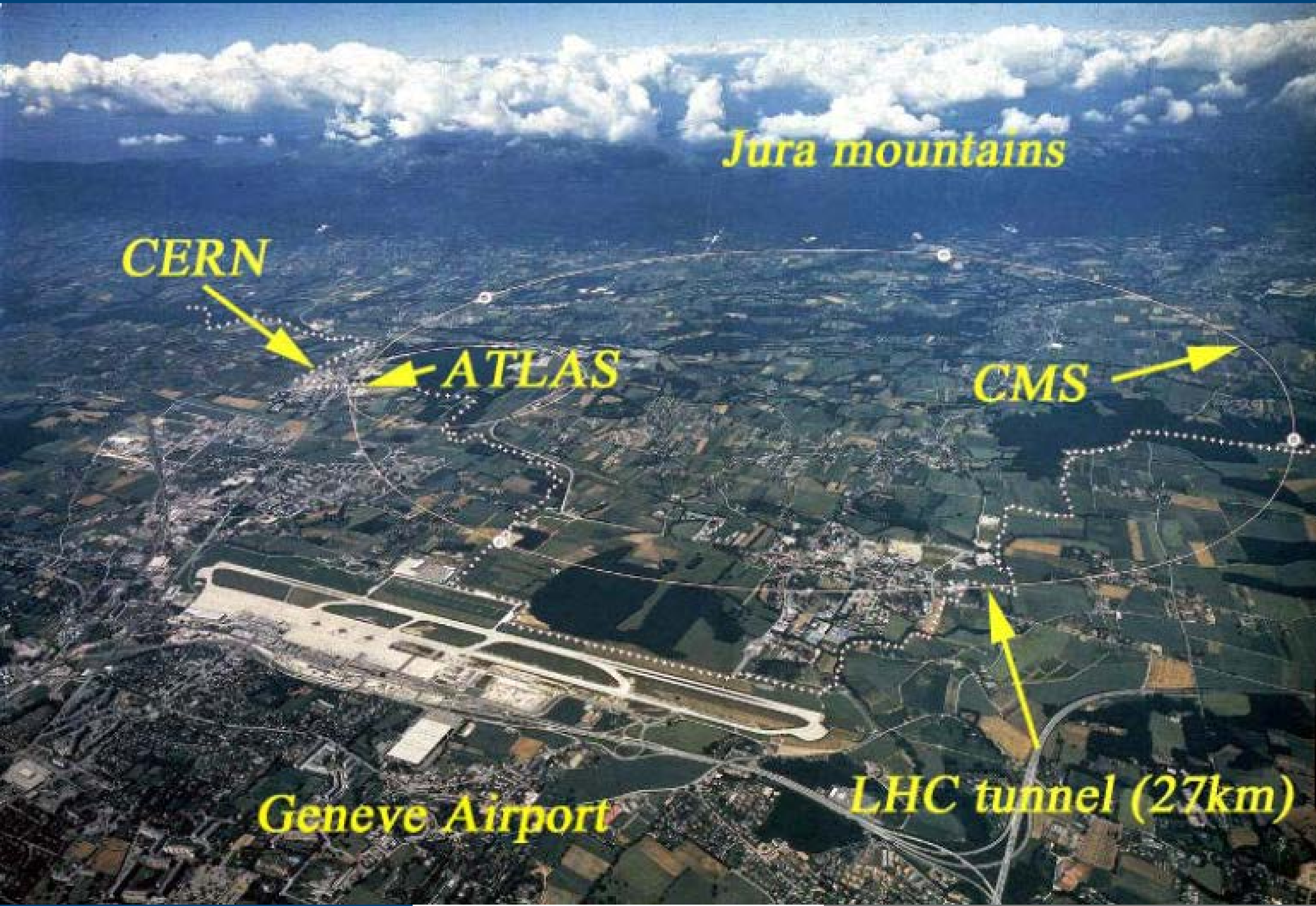
- O. Chamberlain, E. Segre, C. Wiegand, T. Ypsilantis, Phys. Rev. 100, 947 (1955)
- *Nobelprize Chamberlain & Segre 1959*

p + p => p + p + (p + p)

Principle of Antiproton Production

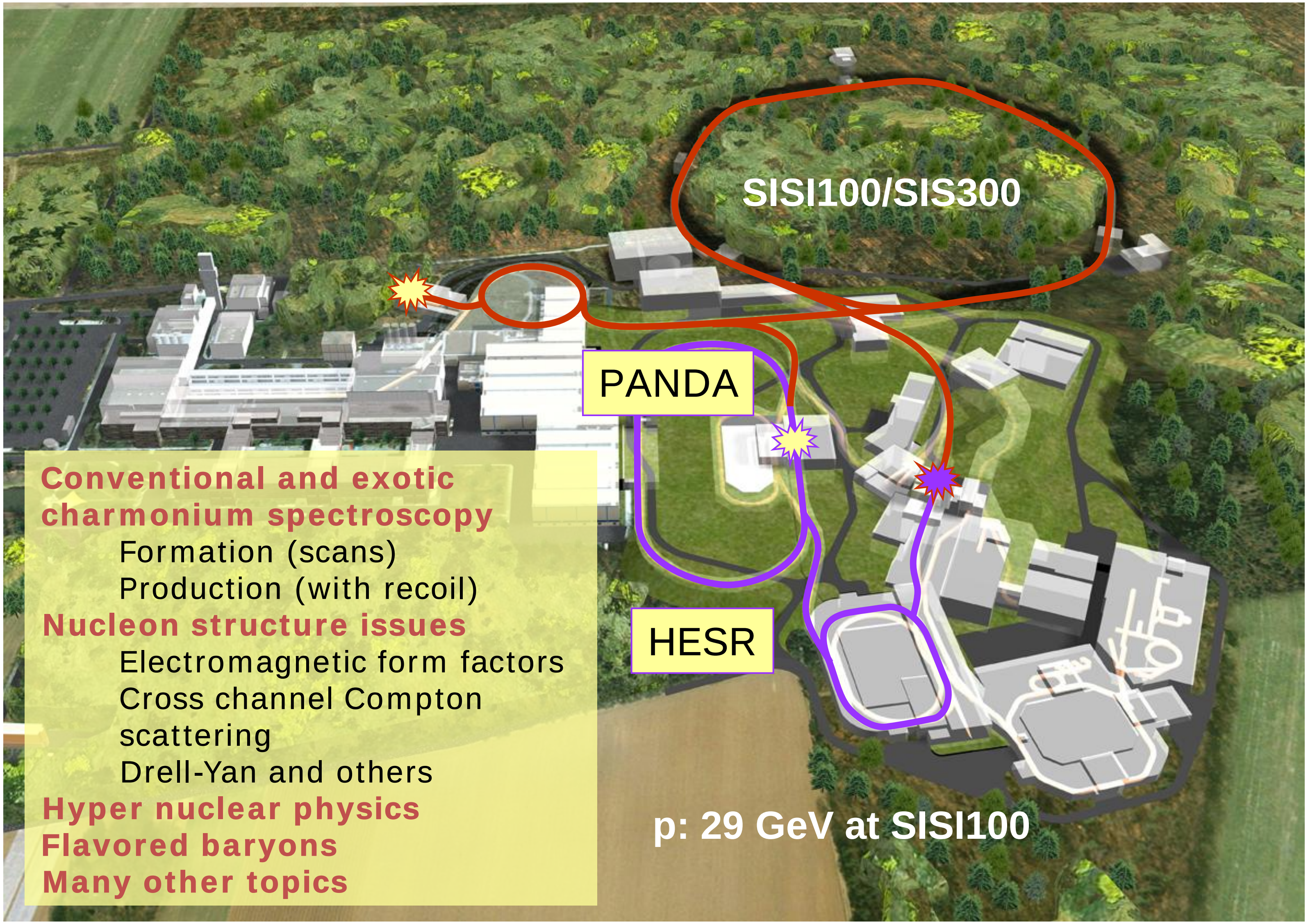


1995


Jura mountains
CERN
ATLAS
CMS
Geneve Airport
LHC tunnel (27km)

CN DE ES FI FR GB GR IN IT PL RO RU SE

Panda and the p-Facility @ FAIR



Conventional and exotic charmonium spectroscopy
- Formation (scans)
- Production (with recoil)

Nucleon structure issues
- Electromagnetic form factors
- Cross channel Compton scattering
- Drell-Yan and others

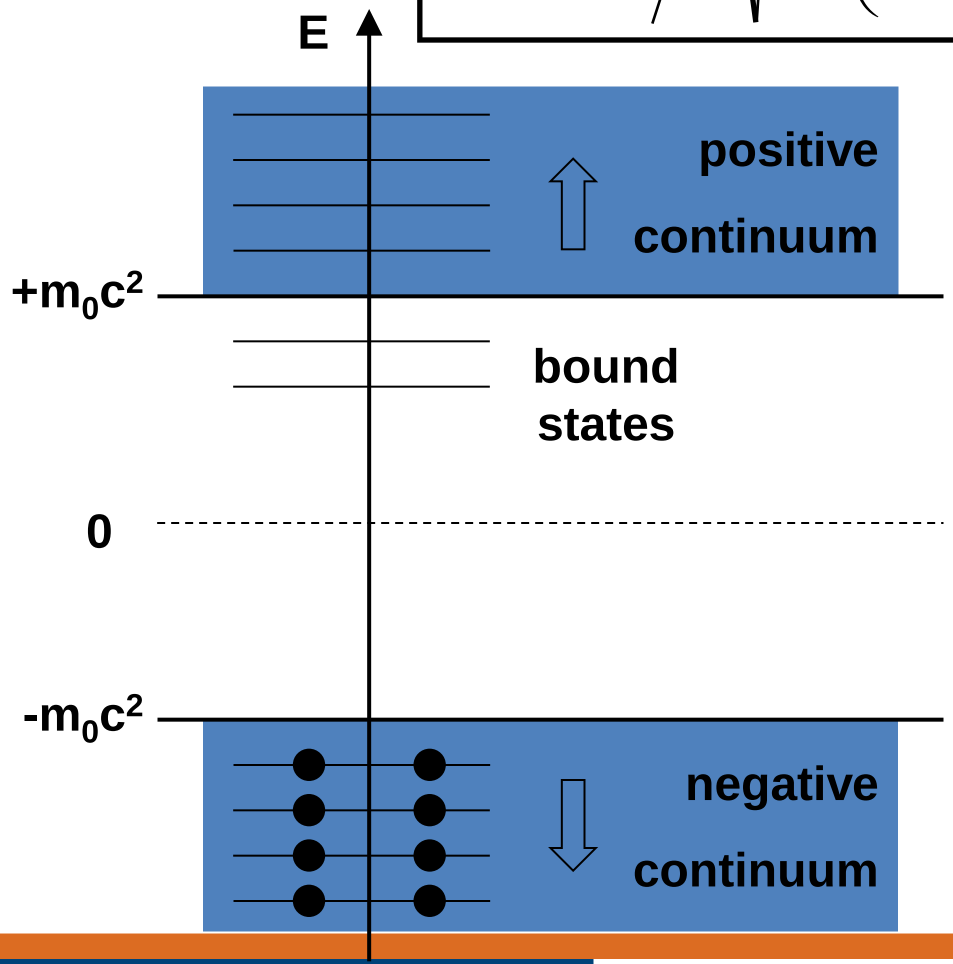
Hyper nuclear physics

Flavored baryons

Many other topics

DIRAC Theory
(relativistic formulation of quantum mechanics)

$$E_{nj} = mc^2 \Bigg/ \sqrt{1 + \left(\frac{Z\alpha}{n - |j + 1/2| + \sqrt{(j+1/2)^2 - (Z\alpha)^2}} \right)^2}$$

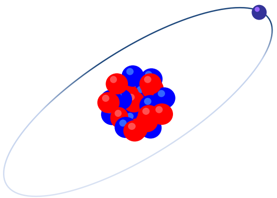


n:	principal quantum number
j:	total angular momentum
α:	Finestructure- or Sommerfeld-constant α = 1/137.036)
mc^2:	electron mass at rest (511 keV)

Dirac-Theory: *Relativistic complete description of quantum mechanics All states with same **n** and **j** are degenerated.*

Critical electromagnetic fields

- Let us come back to Dirac energy of a *single* hydrogen-like ion:

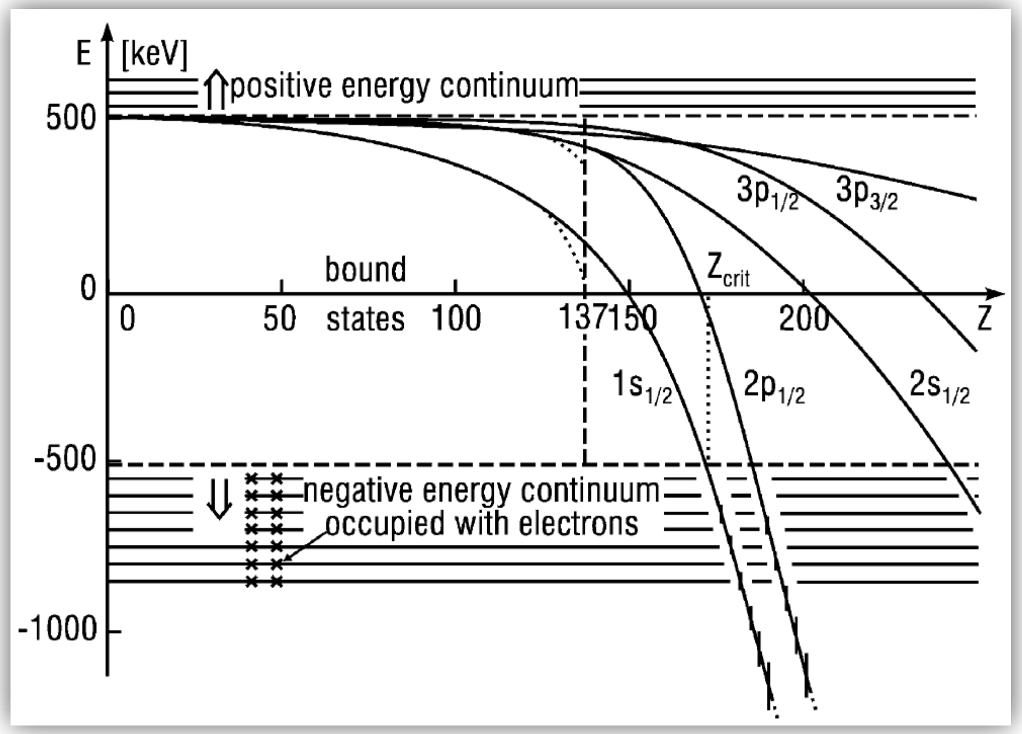


$$E_{1s} = mc^2\sqrt{1-(Z\alpha)^2}$$

- What happens if we increase the nuclear charge Z?

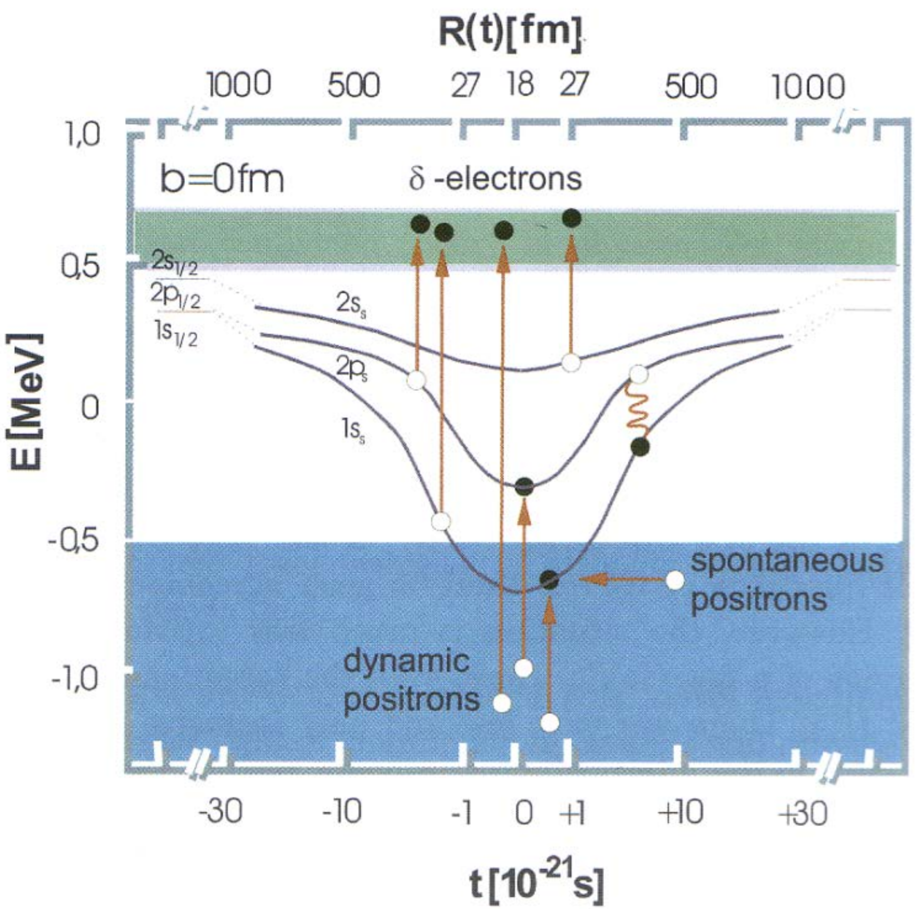
- If nuclear charge of the ion is greater than Z_{crit} the ionic levels can “dive” into Dirac’s negative continuum.

- Physical vacuum becomes unstable: creation of pairs may take place!



Supercritical fields: Formation of Quasi-Molecules

Merged Beams



A. Artemyev, et al. J. Phys. B **43** (2010) 235207