Comparison Between the Nonrelativistic Dipole Approximation and the Exact Relativistic Theory for Radiative Recombination

O. Brînzanescu[1,2], J. Eichler[3], A. Ichihara[4], and Th. Stöhlker[1,5]
[1]Gesellschaft für Schwerionenforschung mbH Darmstadt
[2]National Institute for Laser, Plasma and Radiation Physics, 76900 Bucharest Magurele, Romania
[3]Bereich Theoretische Physik, Hahn-Meitner-Institut Berkin, 141109 Berlin, Germany
[4]Japan Atomic Energy Research Institute, Tokai-mura, Ibaraki 319-11, Japan
[5]Institut für Kernphysik, University of Frankfurt, D-60486 Frankfurt, Germany

As early as 1930, M. Stobbe [1] developed a theory for the photoelectric effect of a one-electron atom within a nonrelativistic dipole approximation. The cross section for K-shell ionization is given by an analytical formula which, owing to its simplicity, is still widely used today, both for photoionization and for its inverse reaction, radiative recombination (RR). The matrix element within this approach is given by:

$$M_{fi}^{Stobbe} = \left\langle \mathbf{p} m_s \left| \mathbf{r} \cdot \hat{\mathbf{u}}_\lambda \right| j_i \mu_i \right\rangle \qquad (1)$$

where $\mathbf{p}$ and m_s describe the asymptotic momentum and spin projection of the emitted electron, $\hat{\mathbf{u}}$ is the unit polarization vector of the photon, and j_i, μ_i define the initial bound state. On the other hand, in recent years, an exact relativistic formulation [3] has been applied to a large amount of experimental data e.g., [4]. The corresponding matrix element is given by:

$$M_{fi}^{exact} = \left\langle \mathbf{p} m_s \left| \boldsymbol{\alpha} \cdot \hat{\mathbf{u}}_\lambda e^{i\mathbf{k}\cdot\mathbf{r}} \right| k_i \mu_i \right\rangle \qquad (2)$$

with the initial and final states now defined as Coulomb-Dirac eigenstates. The quantity $\boldsymbol{\alpha}$ denotes the Dirac matrix and the exponential describes the photon wave function (retardation effect) with wave vector $\mathbf{k}$. For a low nuclear charge Z=10, Z=54, for a principal quantum number n = 5 and for a low projectile energy of 0.1 MeV/u (corresponding to a relative kinetic electron energy of 54.86 eV), it turns out that the nonrelativistic treatment and the relativistic formulation yield the same result within very close limits, as to be expected, see Fig.1. However, retaining low relative velocities but choosing a high nuclear charge Z=92, we obtain sizeable differences in the cross sections. This means that even for n=5, relativistic effects in U^{92+} survive or, possibly, retardation effects (higher powers in kr) play a role, owing to tighter binding. At a considerably higher energy of 68 MeV/u (corresponding to a relative kinetic electron energy of 37.3 keV), the differences between the approaches (1) and (2) become still larger, see Fig.2. At these larger relative velocities, the interaction of the current produced by the projectile motion and the electron spin can certainly not be neglected.

We also compare the differential cross sections for radiative recombination in the K, L, O-shells of U^{92+}. At a low relative velocity for which a nonrelativistic description is considered to be applicable, the angular distributions differ markedly. In the K-shell case, within a complete description, there is a finite cross section at forward and backward angles owing to spin-flip effects, and on the other hand the cross section is shifted to forward angles compared to a nonrelativistic $\sin^2\theta$ distribution. In summary, we find that even for nonrelativistic relative velocities but high nuclear charges there are remaining differences between a nonrelativistic and a relativistic description.

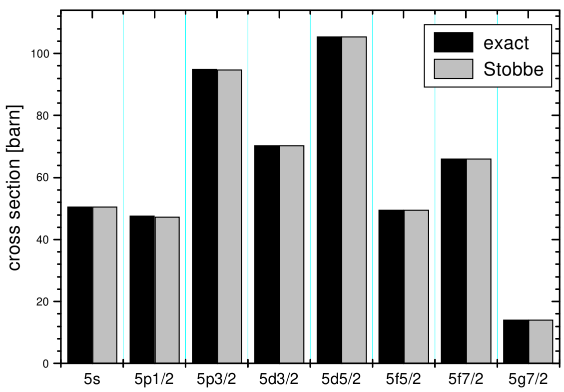

Fig.1. Comparison of total cross sections calculated in the Stobbe (nonrelativistic dipole) approximation [1] and in the exact relativistic formulation [2] for radiative recombination into the n = 5 subshells of Ne^{10+} ions moving with 0.1 MeV/u towards electrons at rest (corresponding to a relative kinetic electron energy of 54.86 eV).

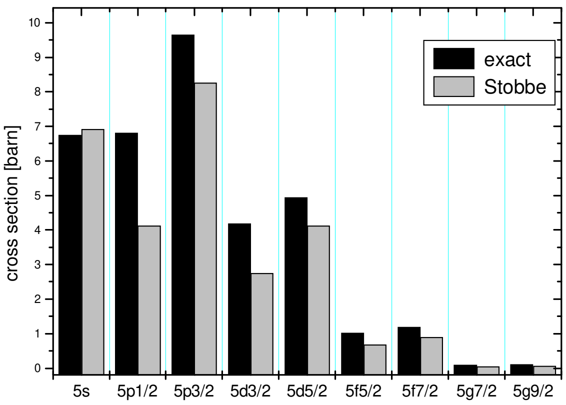

Fig.2. Same as Fig.1., but for U^{92+} ions moving with 68 MeV/u (coresponding to a relative kinetic electron enery of 37.3 keV).

References

[1] Stobbe, M., Ann. Phys. 7 , 661 (1930).
[2] Ichihara, A., Shirai, T., Eichler, J., Phys. Rev. A **49**, 1875 (1994).
[3] Eichler, J., Ichihara, A., Shirai, T., Phys. Rev. A **51**, 3027 (1995).
[4] Stöhlker, Th. et al., Phys. Rev. A. **51**, 2098 (1995); Stöhlker, Th., Kozhuharov, C., Mokler, P.H., Warczak, A., Comments At. Mol. Phys. **33**, 271 (1997)