Fluctuations on the phase boundary

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in collaboration with

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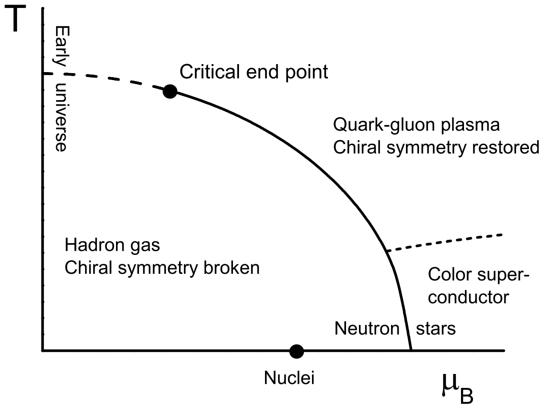
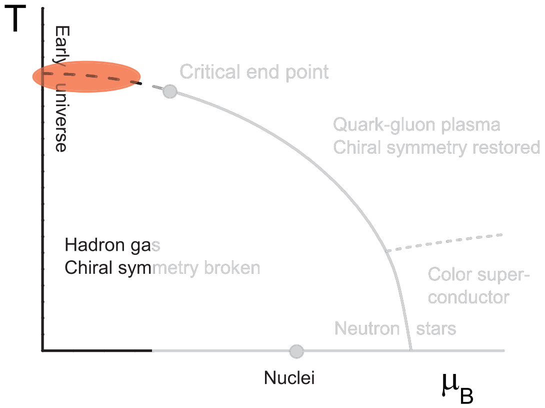
Exploring the QCD phase diagram



- Experiments
- Lattice QCD
- Model calculations
- Functional methods

- ~~Fluctuations at CEP~~
  Fluctuations at $\mu_B \simeq 0$

Focus on $\mu_B \simeq 0$



Use results from Lattice QCD and model calculations (FRG) to explore the critical dynamics of conserved charges at the QCD transition (so far on a qualitative level).

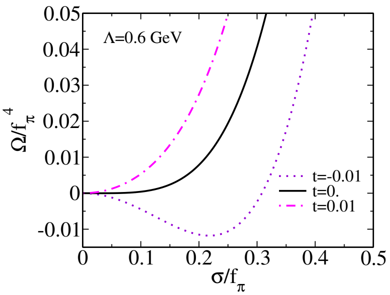
Critical fluctuations and critical slowing down

Critical fluctuations at 2^{nd} order phase transition

Landau theory:

$$\Omega(\sigma) - \Omega_{bg} \sim t\,\sigma^2 + \lambda\,\sigma^4$$

$$t = (T - T_c)/T_c \quad (\lambda > 0)$$



$\chi_\sigma = \langle \sigma^2 \rangle$ large at t = 0

Critical fluctuations:

- Low energy: small fraction of all modes
  - $\rightarrow$ EOS, expansion dynamics $\simeq$ unaffected
  - Example: $\Omega_{sing} \sim t^{2-\alpha}, \quad \alpha \simeq -.2$
  - $\rightarrow$ Need derivatives to "see" singularity!

$$\chi_n = \partial^n \Omega / \partial t^n \sim t^{2-\alpha-n}$$

(susceptibilites $\leftrightarrow$ fluctuations)

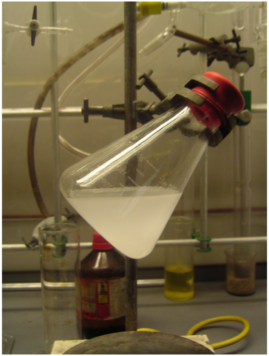
- Long wave length: critical slowing down
  - $\leftrightarrow$ Slow equilibration of soft modes
  - $\rightarrow$ Need time to see fully develop criticality!

Example: critical opalescence

Mixture of methanol and cyclohexane:

One phase ($T > T_c$):

Critical point ($T = T_c$):



Light scattered on critical fluctuations

separated fluids ↔ uniform mixture

index of refraction, $n_1 \neq n_2$

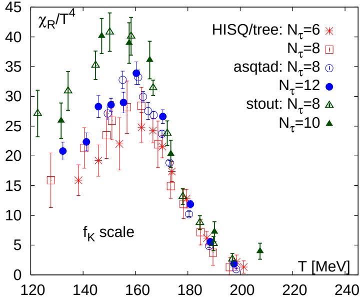
Critical fluctuations → phase boundary?

Fluctuations of order parameter → ∞ at 2^{nd} order transition
< ∞ at cross over transition

Fluctuations of order parameter (Chiral susceptibility)

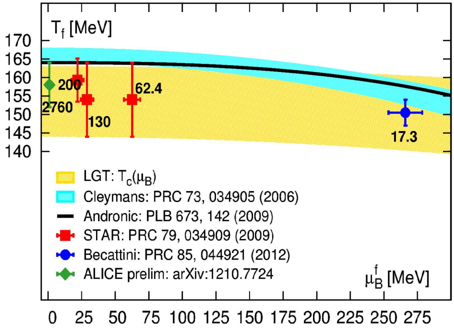
$$\chi_m = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}_{GC}(T,\mu,v)}{\partial m_q^2}$$

Freeze-out close to chiral cross over



(BNL-Bielefeld)

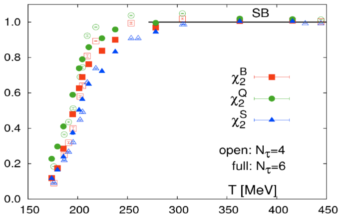
cross over, finite volume → no divergence

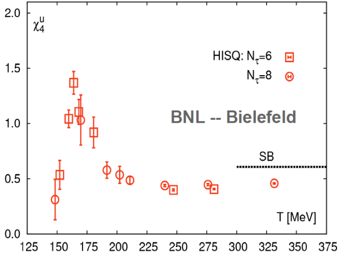


$$T_c(\mu) \simeq T_c(0)(1 - 0.06(\mu_q/T_c(0))^2)$$

Why conserved charges?

- Problems with fluctuations of order parameter
  - No direct measurement
  - Destroyed by subsequent interactions
- Fluctuations of conserved quantities
  - \+ Survive if
    - → Total system large
    - → Surface effects small
  - \+ Measurable event-by-event (difficult?)
  - \- Critical fluctuations suppressed → higher cumulants!





Fluctuations of conserved quantities

- Consider small subsystem v $\ll V$
- Grand canonical fluctuations

$$P(N, \mathrm{v}) = e^{\mu N} \frac{Z_C(T, N, \mathrm{v})}{\mathcal{Z}_{GC}(T, \mu, \mathrm{v})}$$

$$\langle N^n \rangle = \sum_N N^n P(N, \mathrm{v})$$

- Cumulants (gen. susceptibilities)

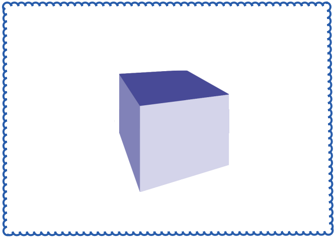
$$c_n = \frac{\partial^n \log \mathcal{Z}_{GC}(T, \mu, \mathrm{v})}{\partial(\mu/T)^n} \sim \chi_n^B \equiv \chi_B^{(n)}$$

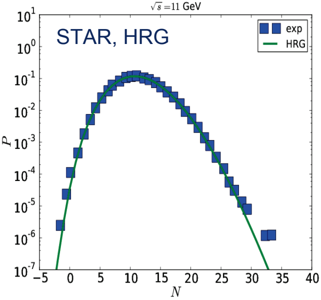
$$c_2 = \langle N^2 \rangle - \langle N \rangle^2 \equiv \langle (\delta N^2) \rangle \sim \mathrm{v}$$

$$\delta N = N - \langle N \rangle$$

$$c_4 = \langle \delta N^4 \rangle - 3 \langle (\delta N)^2 \rangle^2 \sim \mathrm{v}$$

"v" sub volume in phase space





Critical scaling

Chiral transition of 2-flavor QCD expected in universality class of 3d $O(4)$ spin model
(Pisarski & Wilczek 1984)

Singular part of free energy density:
$f = f_r + f_s$ where $f_s(t,h) = h^{1+1/\delta} f_f(z)$

Scaling variable (no other scale)
$z = t/h^{1/\beta\delta}$
$t = (T - T_c)/T_c$
$h = m_q/T_c$

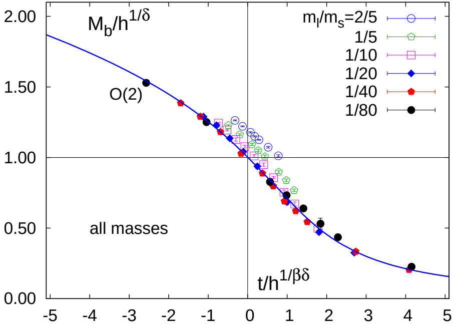
Order parameter
$M = -\partial f/\partial h \simeq h^{1/\delta} f_G(z)$
$f_G(z) = -(1 + 1/\delta) f(z) + (z/\beta\delta) f'(z)$
$M/h^{1/\delta} = f_G(z)$

Chiral susceptibility

$$\chi_m = \frac{\partial M}{\partial h} \sim h^{1/\delta - 1} \sim \xi^2 \to \infty \, (h \to 0)$$

$\beta \simeq .38, \delta \simeq 4.8$

(Ejiri *et al.*, HotQCD, 2009)



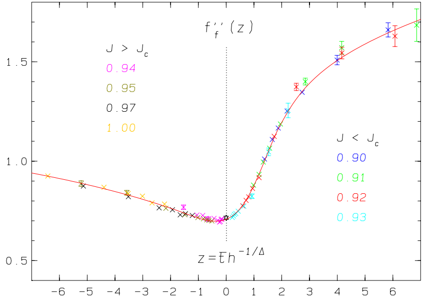
Physical m_π in $O(N)$ scaling regime!

$$G_\sigma(r) \sim \frac{1}{r} e^{-r/\xi}$$

Baryon-number susceptibilities ($\mu_q = 0$)

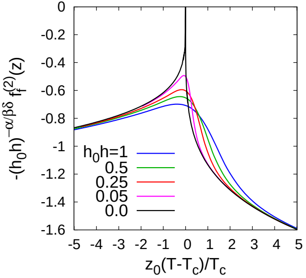
- generalize scaling parameter $t = (T - T_c)/T_c + \kappa(\mu_q/T_c)^2$ $z = t/h^{1/\beta\delta}$
- $\chi_{2n}^B = -\partial^{2n}(f/T^4)/\partial(\mu/T)^{2n} \sim -h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z) + \ldots$
- $\chi_4^B \sim -h^{-\alpha/\beta\delta} f_f^{(2)}(z)$ finite in chiral limit ($\alpha \simeq -0.2$)
- $\chi_6^B \sim -h^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z)$ $\sim \xi^{1.1}$
- $\chi_8^B \sim -h^{-(2+\alpha)/\beta\delta} f_f^{(4)}(z)$ $\sim \xi^{2.4}$

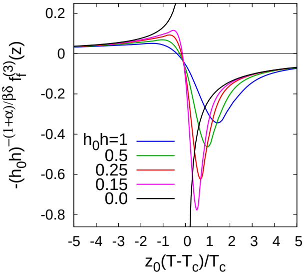
- O(4) order parameter scaling function (Karsch & Engels)

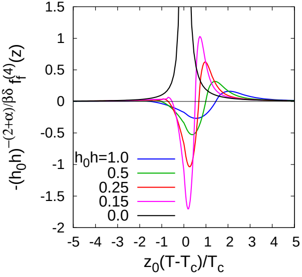


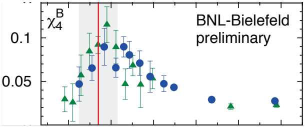
Baryon-number susceptibilities ($\mu_q = 0$)

- $\chi^B_{2n} \sim -h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z)$









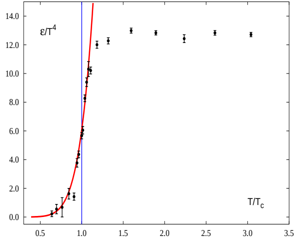
C. Schmidt, QM 2012

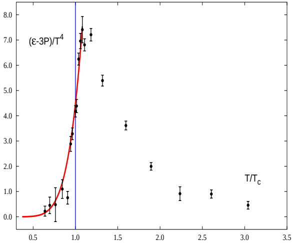
negative for $T \simeq T_c$

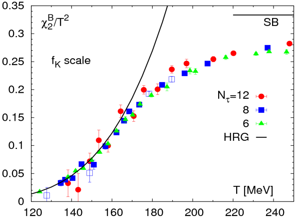
negative for $T \simeq T_c$

Baseline: HRG (no critical fluctuations)

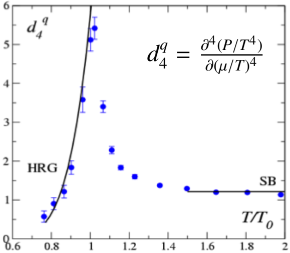
- The Hadron Resonance Gas yields good description of Lattice results below T_c.







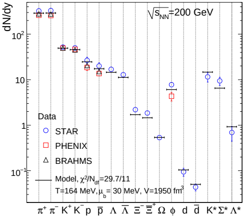
Bielefeld-Brookhaven



HRG vs. experiment

- HRG: good description of multiplicities…

Andronic *et al.*



- …and of fluctuations (event-by-event $p - \bar{p}$, STAR)

$$p_{HRG} = f(T)\cosh(\mu_B/T) + \ldots$$

$$\chi_B^{(n)} = \partial^n (p/T^4)/\partial(\mu_B/T)^n$$

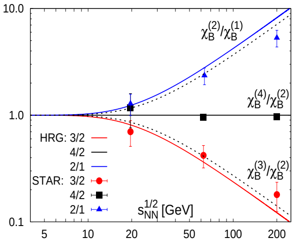
$$\chi_B^{(4)}/\chi_B^{(2)} \simeq 1$$

$$\chi_B^{(3)}/\chi_B^{(2)} \simeq \tanh(\mu_B/T)$$

$$\chi_B^{(2)}/\chi_B^{(1)} \simeq \coth(\mu_B/T)$$

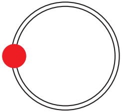
Critical fluctuations expected in higher susceptibilites!

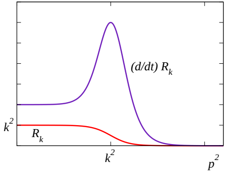
Karsch & Redlich



Effective models, FRG

- Effective models
  - Models respect the (global) symmetries of QCD
    chiral symmetry $SU(2)_L \otimes SU(2)_R$ ($\sim O(4)$), order parameter $\langle \bar{q}q \rangle$ (NJL, QM)
    + center symmetry $Z(3)$, order parameter Polyakov Loop $\langle L \rangle$ (PNJL, PQM)
  - Tune effective model to Lattice results,
    explore higher cumulants and $\mu_q \neq 0$.
- Functional Renormalization Group (FRG)
  - Include critical fluctuations:
    need non-perturbative approach, which respects symmetries → FRG
  - FRG yields scaling properties and proper critical exponents
    (universal quantities)

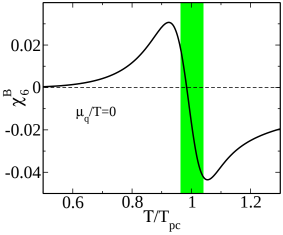
$\partial_k \Gamma_k[\phi] = \frac{1}{2}$ 

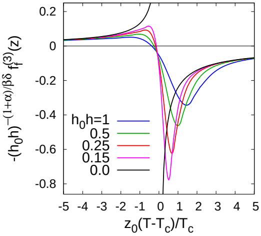


Use local potential approximation.

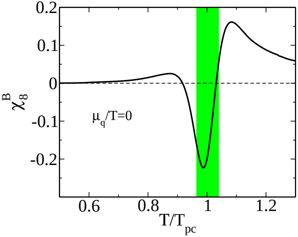
Net quark number density fluctuations $\delta N_q = N_q - \langle N_q \rangle$

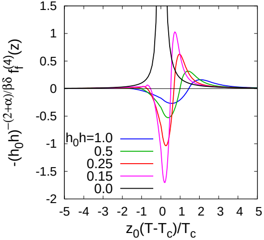
$$\chi_6^q = \frac{1}{vT^3}\left(\langle(\delta N_q)^6\rangle \cdots\right)$$





$$\chi_8^q = \frac{1}{vT^3}\left(\langle(\delta N_q)^8 \cdots\right)$$

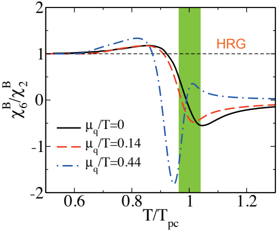


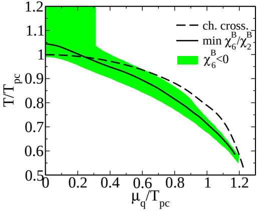


- Zeroes of χ_n not universal, but shape $\sim O(4)$ scaling functions.

Net quark number density fluctuations $\mu_q \neq 0$

$$\chi_6^q = \frac{1}{\mathrm{v}T^3} \left(\langle (\delta N_q)^6 \rangle \cdots \right)$$





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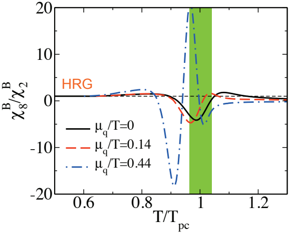
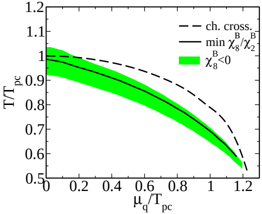
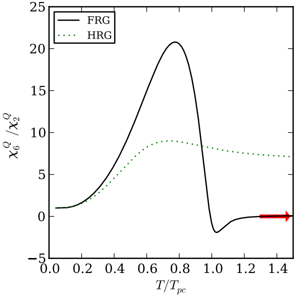


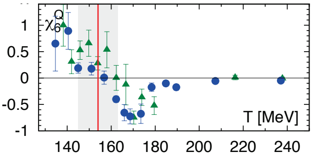


- Negative regions of χ_6 and χ_8 follow phase boundary.

Electric charge fluctuations

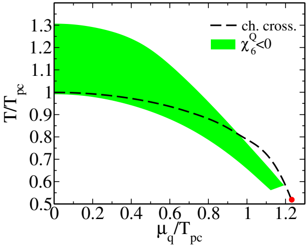
V.Skokov, B.F., K. Redlich





C. Schmidt, QM 2012

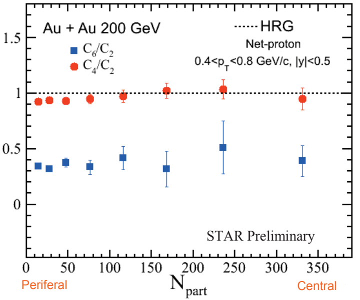
Quantitative understanding still needed.

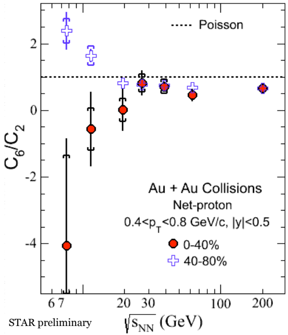


Electric charge fluctuations follow similar pattern as baryon fluctuations.

Freeze-out and the QCD transition

- If $T_{fo} \simeq T_{pc}$, expect measured χ_6/χ_2 and χ_8/χ_2 to deviate from HRG expectation, (< 0)
- Signature of cross-over transition (STAR@RHIC)?





- Clear deviation from HRG expectation.
- Need to understand dependence on centrality and on energy!

Cumulants and net-baryon probability distribution

- Canonical partition function

$$Z(N,T,\mathrm{v}) = \mathrm{Tr}_N\, e^{-\beta H}$$

- Grand-canonical partition function

$$\begin{aligned}\mathcal{Z}(\mu_q,T,\mathrm{v}) &= \mathrm{Tr}\, e^{-\beta(H-\mu_q N)} \\ &= \sum_N Z(N,T,\mathrm{v}) e^{\beta\mu N}\end{aligned}$$

- Probability distribution for finding net charge N in sub-volume

$$P(N) = \frac{1}{\mathcal{Z}(\mu_q,T,\mathrm{v})} Z(N,T,\mathrm{v})\, e^{\beta\mu N}$$

- Moments

$$\langle N^n \rangle = \sum_N N^n P(N) \sim \mathrm{v}^n$$

- Cumulants

$$\begin{aligned}\chi_2 &= \frac{1}{\mathrm{v}T^3}\left(\langle N^2\rangle - \langle N\rangle^2\right) \\ \chi_4 &= \frac{1}{\mathrm{v}T^3}\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2\end{aligned}$$

$$\chi_n \sim (\mathrm{v})^0 \qquad \text{(cancellations!)}$$

- Critical behavior in tail of distributions

Skellam distribution

- Boltzmann distribution for baryons and antibaryons (Poisson)

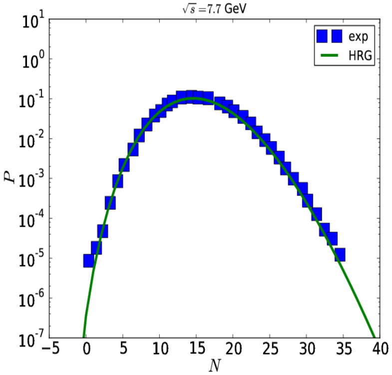
$$P(N_i) = \frac{1}{N_i!} \frac{\bar{N}_i^{N_i}}{e^{\bar{N}_i}}$$

$$\bar{N}_i = \langle N_i \rangle \quad (i = b, \bar{b})$$

- Distribution of net baryon number $(N = N_b - N_{\bar{b}})$

$$P(N) = \left(\frac{\bar{N}_b}{\bar{N}_{\bar{b}}}\right)^{N/2} I_N(2\sqrt{\bar{N}_b \bar{N}_{\bar{b}}})\, e^{-(\bar{N}_b + \bar{N}_{\bar{b}})}$$

- Skellam distribution describes fluctuations of net baryon number in HRG (no criticality)



- Central collisions, $\sqrt{s} = 7.7$ A GeV

Skellam distribution

- Boltzmann distribution for baryons and antibaryons (Poisson)

$$P(N_i) = \frac{1}{N_i!}\frac{\bar{N}_i^{N_i}}{e^{\bar{N}_i}}$$
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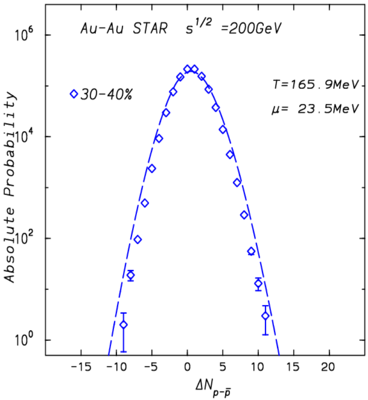
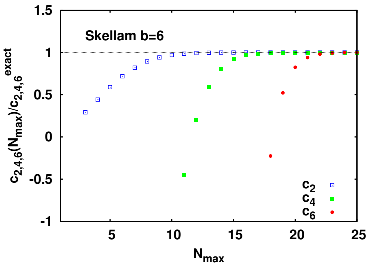


- Semi-central collisions, $\sqrt{s} = 200$ A GeV

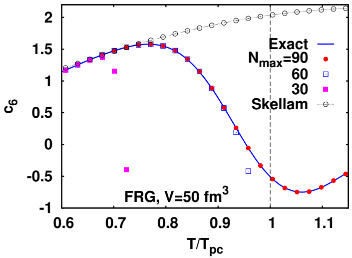
Sufficient reach in N?

In experiment: $\langle N^n \rangle = \sum_{N=-N_{max}}^{N_{max}} N^n P(N)$

- N_{max} large enough to reproduce cumulants?
- Skellam: convergence for $N_{max} \gtrsim a_n \sqrt{\bar{N}_b}$



- N_{max} large enough to see criticality?
- c_6 in PQM model (FRG)
- $N_{max} \sim N_{max}^{Skellam}$



Can be checked by varying N_{max} in data

Other things to worry about

- Conservation of total charge ($V < \infty$, $R \searrow$) (Bzdak, Koch, Skokov)
- Acceptance corrections (dilutes signal, p not B measured) (Bzdak & Koch)

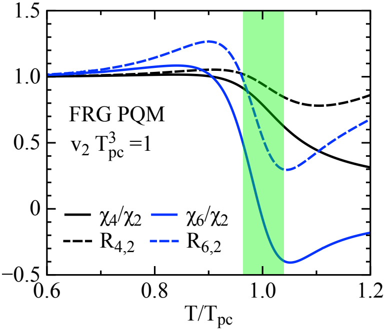
  Net electric charge easier!
- Non-critical fluctuations, e.g. of volume ($R \nearrow$)

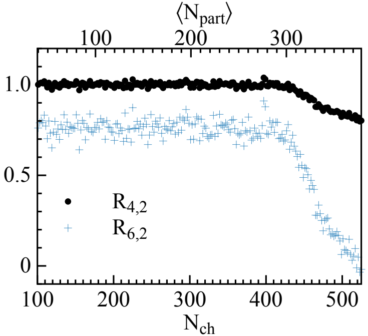
$$R_{6,2} = \frac{\chi_6}{\chi_2} + 15\chi_4 \cdot T^3 \mathrm{v}_2$$

$$\mathrm{v}_2 = (\langle \mathrm{v}^2 \rangle - \langle \mathrm{v} \rangle^2)/\langle \mathrm{v} \rangle$$

Glauber: fix N_{ch}

$\langle \mathrm{v} \rangle \sim \langle N_{part} \rangle$, $\langle \mathrm{v}^2 \rangle \sim \langle N_{part}^2 \rangle$



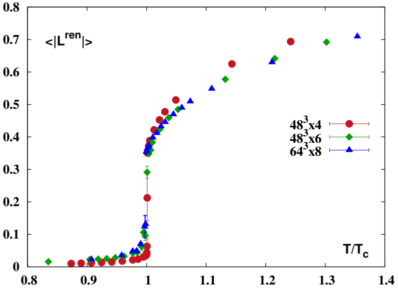


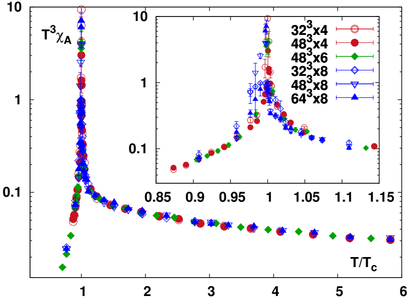
Tuning the models: gluon dynamics

Fluctuations associated with deconfinement transition?

- Order parameter of deconfinement ($M_{quark} \to \infty$)
  - Polyakov loop $L = \frac{1}{3}\mathrm{tr}\, e^{i\int A_4\, d\tau} = e^{-F_{quark}/T}$
  - In color SU(3): $L = L_R + iL_I$

- Fluctuations of order parameter:
  - $\chi_R = \langle L_R L_R\rangle - \langle L_R\rangle^2$
  - $\chi_I = \langle L_I L_I\rangle - \langle L_I\rangle^2$
  - $\chi_A = \langle | L \| L |\rangle - \langle | L |\rangle^2$



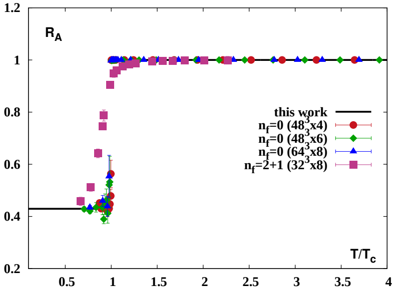


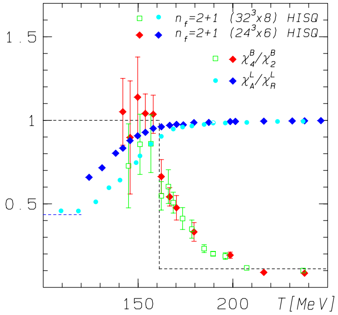
→ improve description of gluon dynamics in effective models

Signatures of deconfinement

Ratios of susceptibilities less dependent on lattice artefacts

- $R_A = \chi_A/\chi_R$
- $\chi_4^B/\chi_2^B = (B)^2$





χ_4^B/χ_2^B: deconf. of quarks, R_A: deconf. of gluons

Conclusions and outlook

- QCD in scaling regime → critical fluctuations potentially detectable
  (finite size and finite time effects!)
- The 6th- and 8th-order cumulants potential probes of chiral transition.
  Basically robust effect, but many issues still unclear
  Large cancellations in high cumulants, depend on the tails of the distribution
- Additional constraints from electric charge $P(Q)$ and strangeness $P(S)$ distributions.
- Experimental ratios show interesting qualitative effect,
  - but dependence on N_{part} and $\sqrt{s}$ not understood
  - worry about removing idealizations
  - tune effective models to reproduce also non-universal properties of QCD (gluon dynamics)
- Looking for CEP at finite μ:
  - Different universality class ($Z(2)$):
    $\rightarrow \chi_2 \rightarrow \infty$
    → characteristic difference in χ_4
    → χ_6 similar to $O(4)$
  - Lower beam energy: corrections due to $V < \infty$ more important!