# Hydrodynamical models at various collision energies


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**FIAS — Frankfurt Institute for Advanced Studies**


**EMMI Nuclear and Quark Matter seminar**

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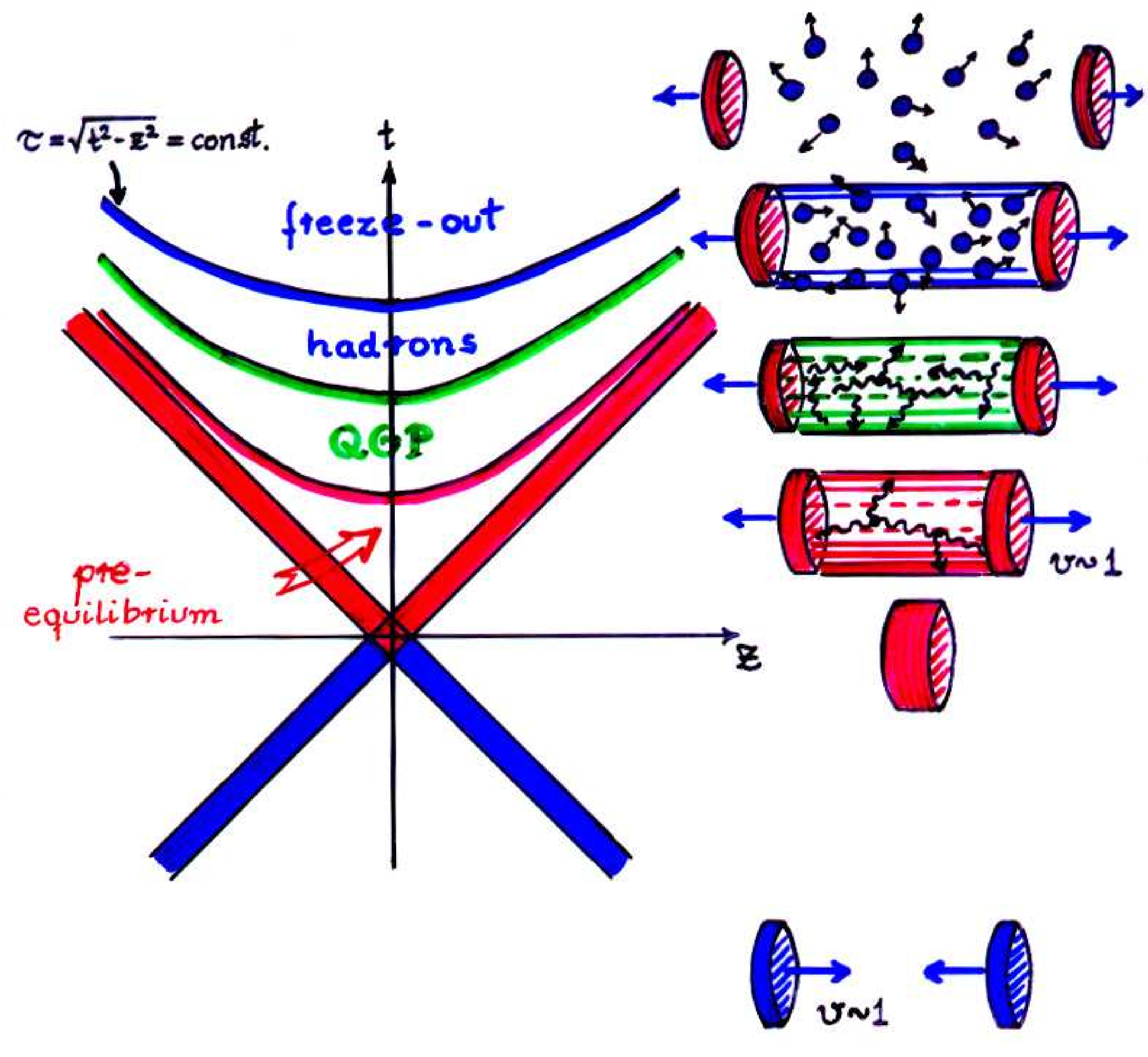
# The space-time picture:

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# Conservation laws

**Conservation of energy and momentum:**

$$\partial_\mu T^{\mu\nu}(x) = 0$$

**Conservation of charge(s):**

$$\partial_\mu N_i^\mu(x) = 0$$

**Local conservation of particle number and energy-momentum.**

$$\Longleftrightarrow$$ **Hydrodynamics!**

# Ideal hydrodynamics

**perfect local equilibrium:**

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - Pg^{\mu\nu}$$
$$N^\mu = nu^\mu$$

- **no dissipation**

- **entropy is conserved**

**matter characterized by:** **equation of state** $P = P(e,n)$

- **EoS by lattice QCD**

# Dissipative hydrodynamics

$$T^{\mu\nu} = eu^\mu u^\nu - (P+\Pi)\Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}$$

**relativistic Navier-Stokes:** $\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$

- **unstable and acausal**

**Israel-Stewart:** $\pi^{\mu\nu}$ **independent dynamical variable**

$$\left\langle u^\lambda \partial_\lambda \pi^{\mu\nu} \right\rangle = \frac{\pi^{\mu\nu}_{\mathrm{NS}} - \pi^{\mu\nu}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\lambda u^\lambda$$

- $\eta$: **shear viscosity coefficient**
- $\tau_\pi$: **shear relaxation time**
- $\pi^{\mu\nu}(t=0)$**?**

# $\eta/s$

**Ideal:**

$$(\epsilon + P)Du^\mu = \nabla^\mu P$$

**where** $D = u^\mu \partial_\mu$**,** $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$**,** $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

**Viscous:**

$$\begin{aligned}(\epsilon + P)Du^\mu &= \nabla^\mu P - \Delta^\mu{}_\alpha \partial_\beta \pi^{\alpha\beta} \\ Du^\mu &= \frac{1}{\epsilon + P}\nabla^\mu P - \frac{2\eta}{\epsilon + P}\Delta^\mu{}_\alpha \partial_\beta \left[\nabla^{\langle\alpha} u^{\beta\rangle} + \cdots\right] + \cdots\end{aligned}$$

$\mu = 0 \Longrightarrow Ts = \epsilon + P :$

$$Du^\mu = \frac{1}{\epsilon + P}\nabla^\mu P - \frac{2}{T}\frac{\eta}{s}\Delta^\mu{}_\alpha \partial_\beta \left[\nabla^{\langle\alpha} u^{\beta\rangle} + \cdots\right] + \cdots$$

# Problems

Differential equations $\Rightarrow$ what are the boundary conditions?

hydro does not know where to start nor where to stop. . .

• primary collision far from equilibrium
$\Rightarrow$ initial state "soon" after primary collision
• assume thermalization

# modeling the initial state of fluid

**MC-Glauber**: pure phenomenology
density $\propto$ # participants or # binary collisions

**MC-KLN**: Color Glass Condensate using $k_T$-factorization

**IP-Glasma**: Color Glass based, classical Yang-Mills evolution

- all initialize nucleon positions by sampling Woods-Saxon distribution

- all assume thermalization

# Problems

Differential equations $\Rightarrow$ what are the boundary conditions?

hydro does not know where to start nor where to stop. . .

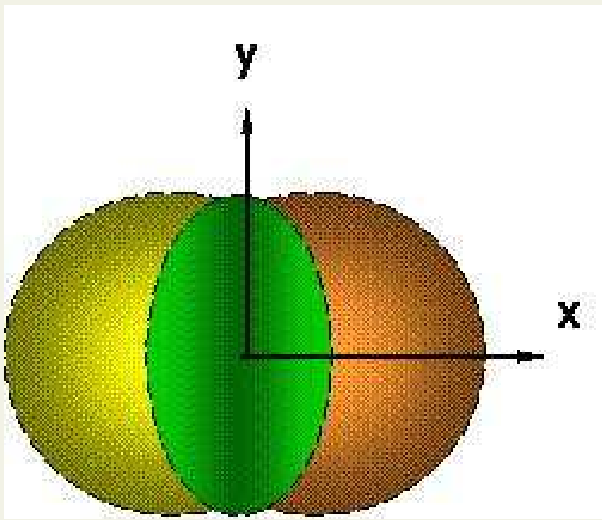
• primary collision far from equilibrium
$\Rightarrow$ initial state "soon" after primary collision
• assume thermalization

• no fluid observed in detectors. . .
• freeze-out: system begins to behave as free particles
$\Rightarrow$ $T_{\mathrm{fo}}$ a free parameter, use $p_T$ distributions to fix

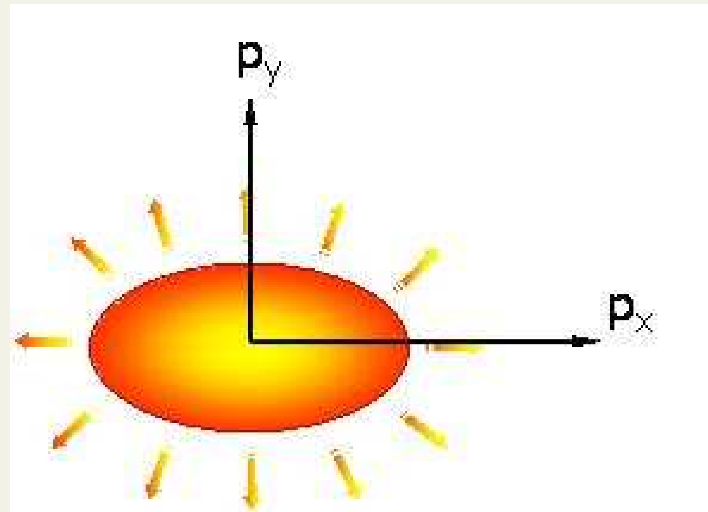
OR: hybrid models
• hadron cascade to describe late dilute stage
• switch at $T_{\mathrm{sw}}$ - again a free parameter. . .

# Elliptic flow

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy



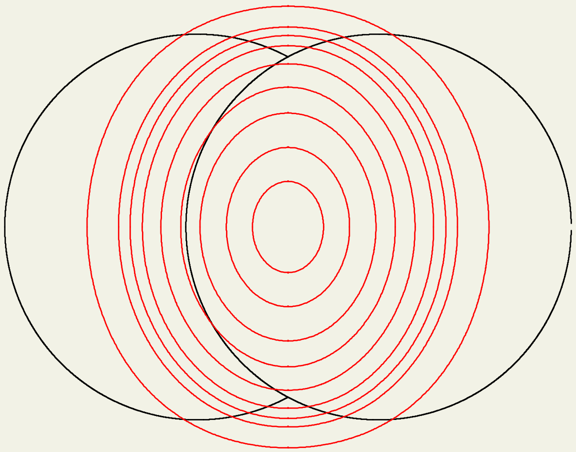

$\rightarrow$



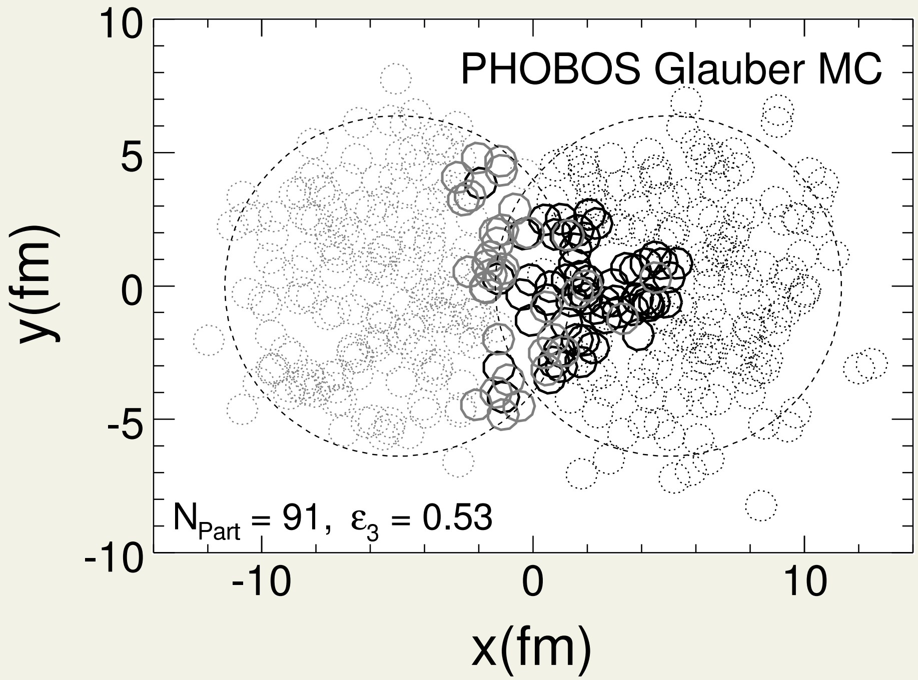

$$\epsilon_2 \equiv \frac{\int r\mathrm{d}r\mathrm{d}\phi\, r^2 \cos(2\phi) e(r,\phi)}{\int r\mathrm{d}r\mathrm{d}\phi\, r^2 e(r,\phi)} \rightarrow v_2 \equiv \frac{\int \mathrm{d}\phi \cos(2\phi) \frac{\mathrm{d}N}{\mathrm{d}\phi}}{\int \mathrm{d}\phi \frac{\mathrm{d}n}{\mathrm{d}\phi}}$$

**sensitive to speed of sound** $c_s^2 = \partial p/\partial e$ **and shear viscosity** $\eta$

# event-by-event



$\Longrightarrow$



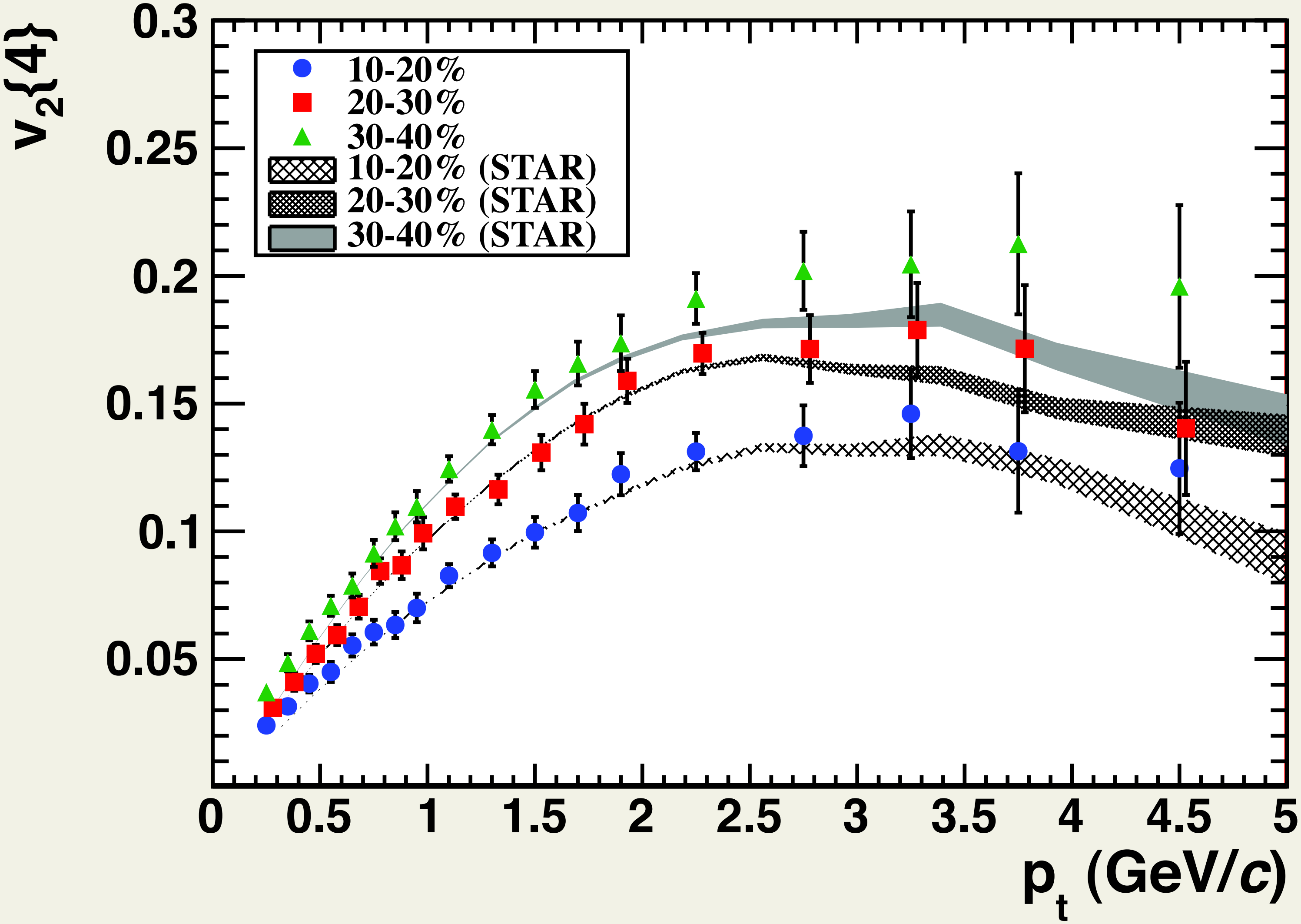

**Alver and Roland, Phys.Rev.C81:054905,2010**

- **shape fluctuates event-by-event**
- **all coefficients $v_n$ finite**

$$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}\phi} = \frac{\mathrm{d}N}{\mathrm{d}y}\left[1 + \sum_n 2v_n \cos(2(\phi - \Psi_n))\right]$$

# $v_2$ at LHC

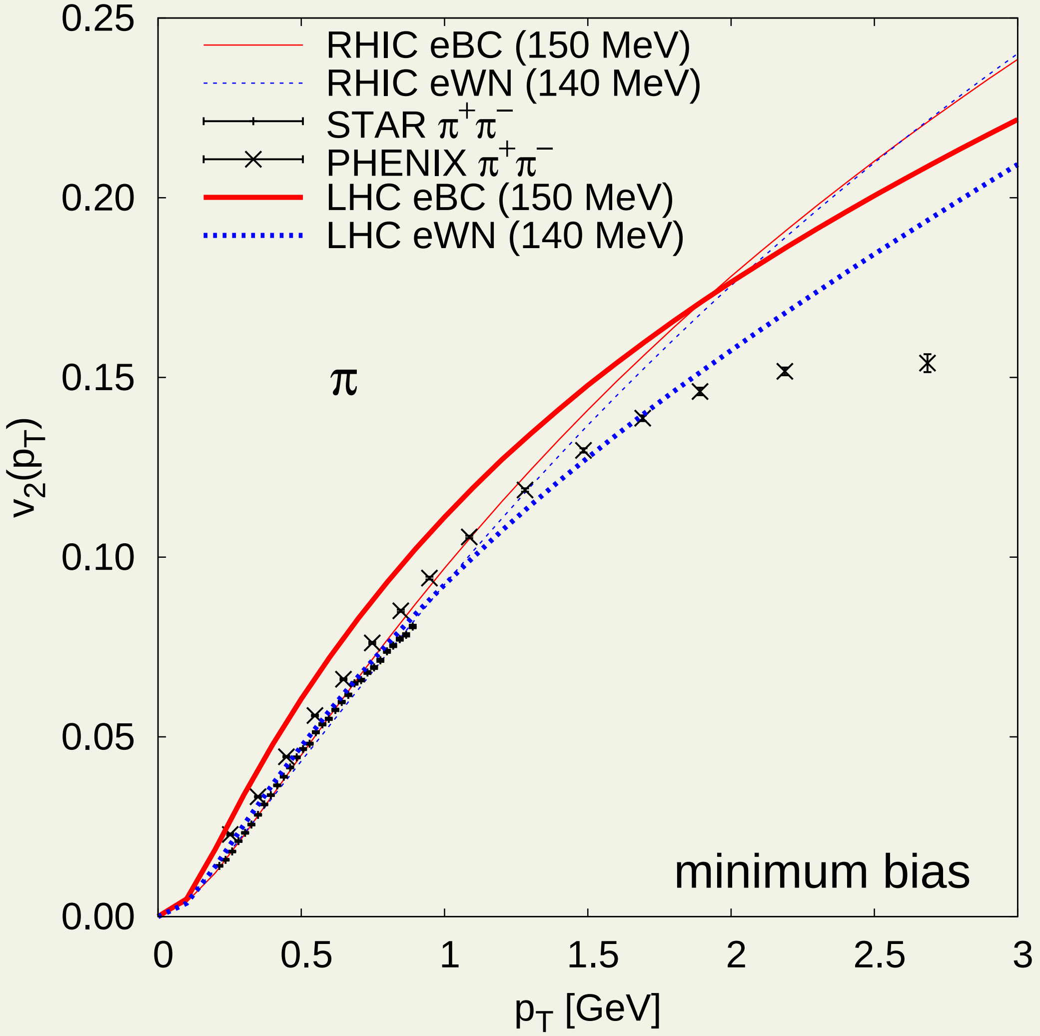
Aamodt *et al.* [ALICE collaboration], PRL 105, 252302




- surprisingly similar $v_2(p_T)$ than at RHIC

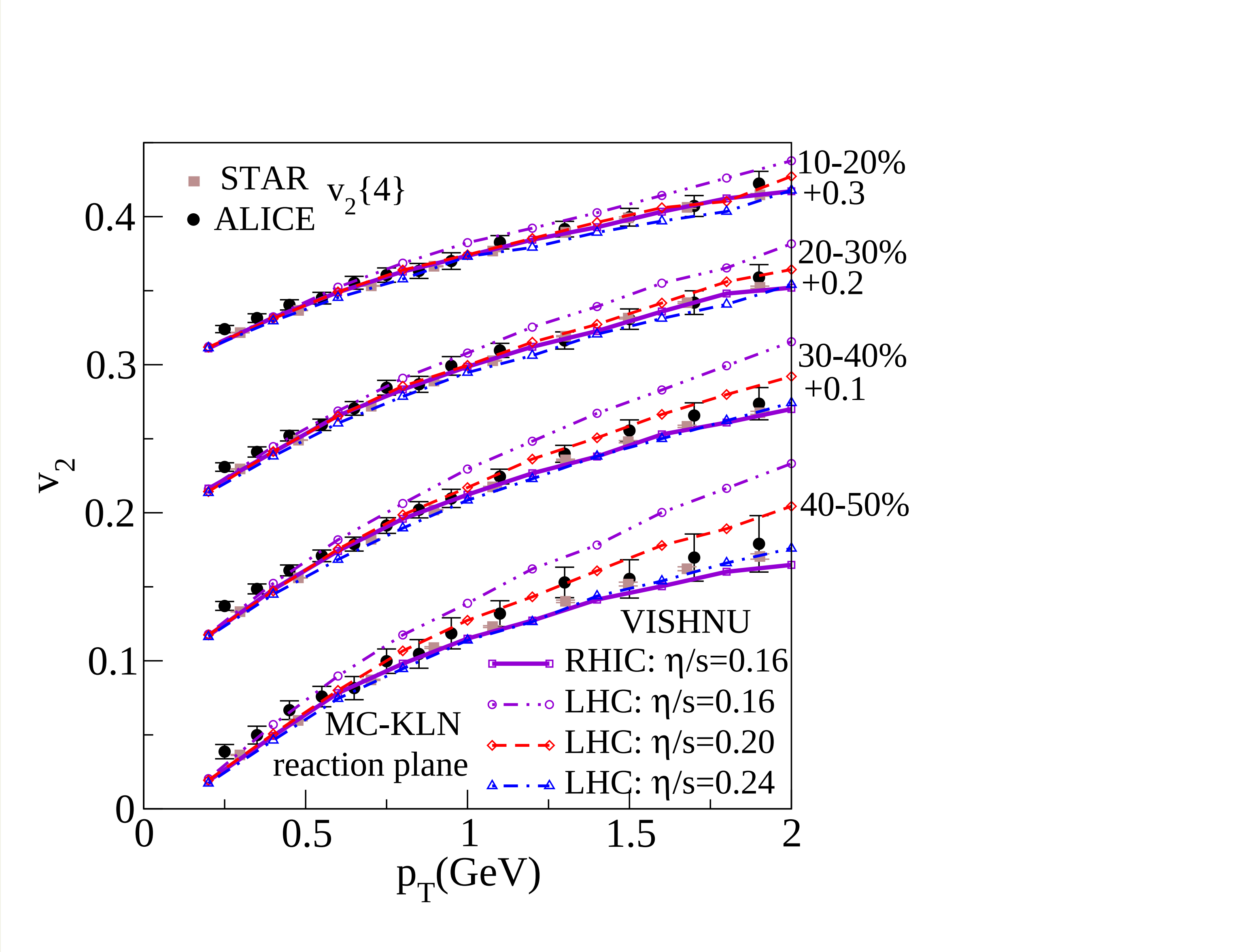
# ideal fluid prediction

**Niemi *et al*, PRC79, 024903 (2009)**



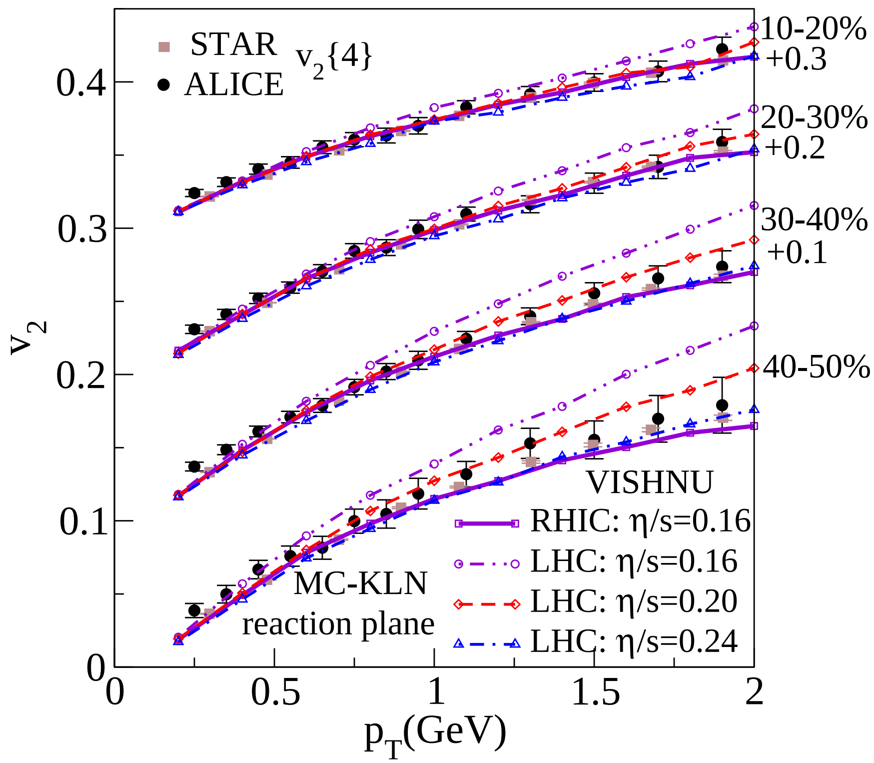

- $v_2(p_T)$ **may or may not change**

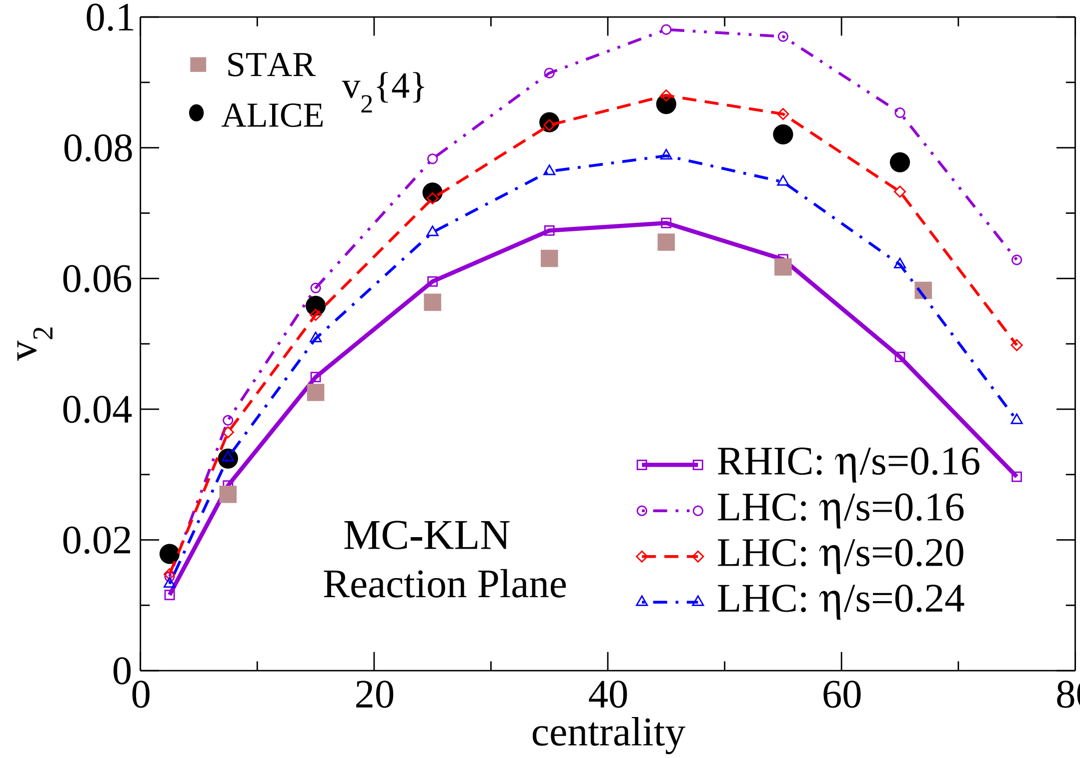
# hybrid model fit




- **both RHIC and LHC data reproduced**

# $\eta/s$ at RHIC and LHC

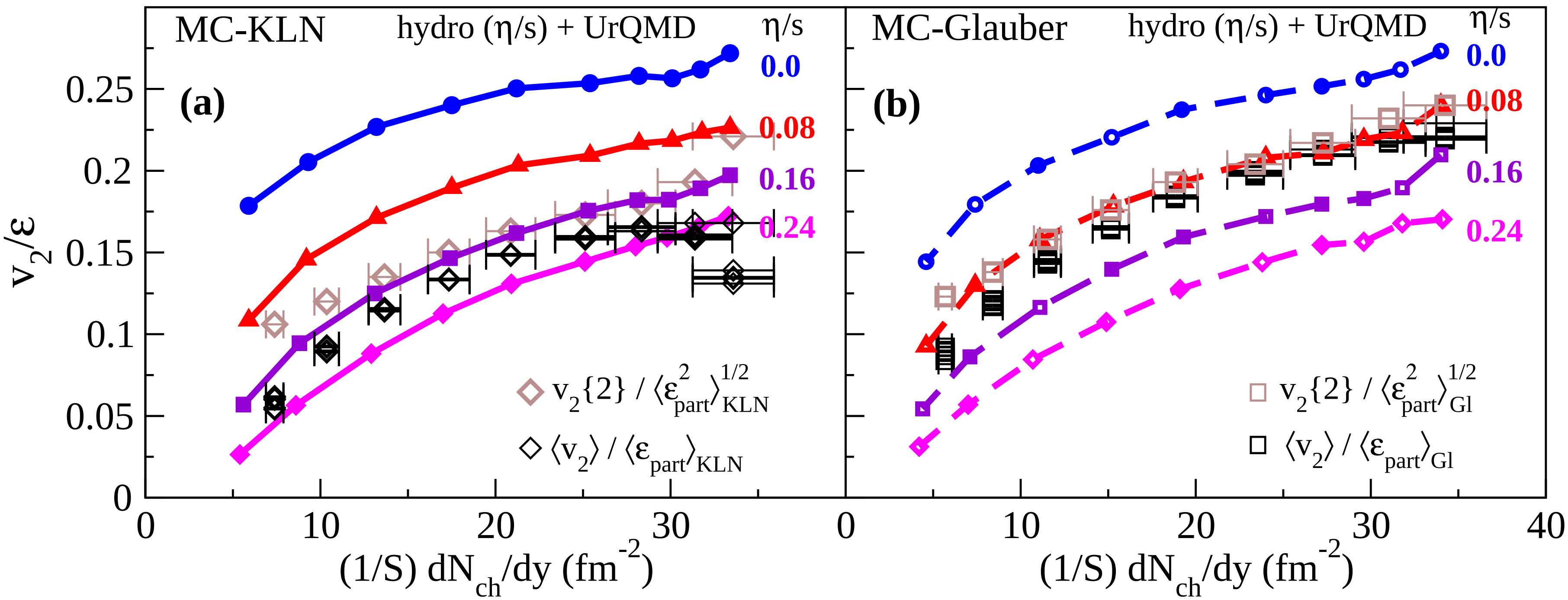



- **RHIC:** $\eta/s \lesssim 0.16$
- **LHC:** $\eta/s \approx 0.20$–$0.24$

- **sign of temperature dependence of $\eta/s$?**

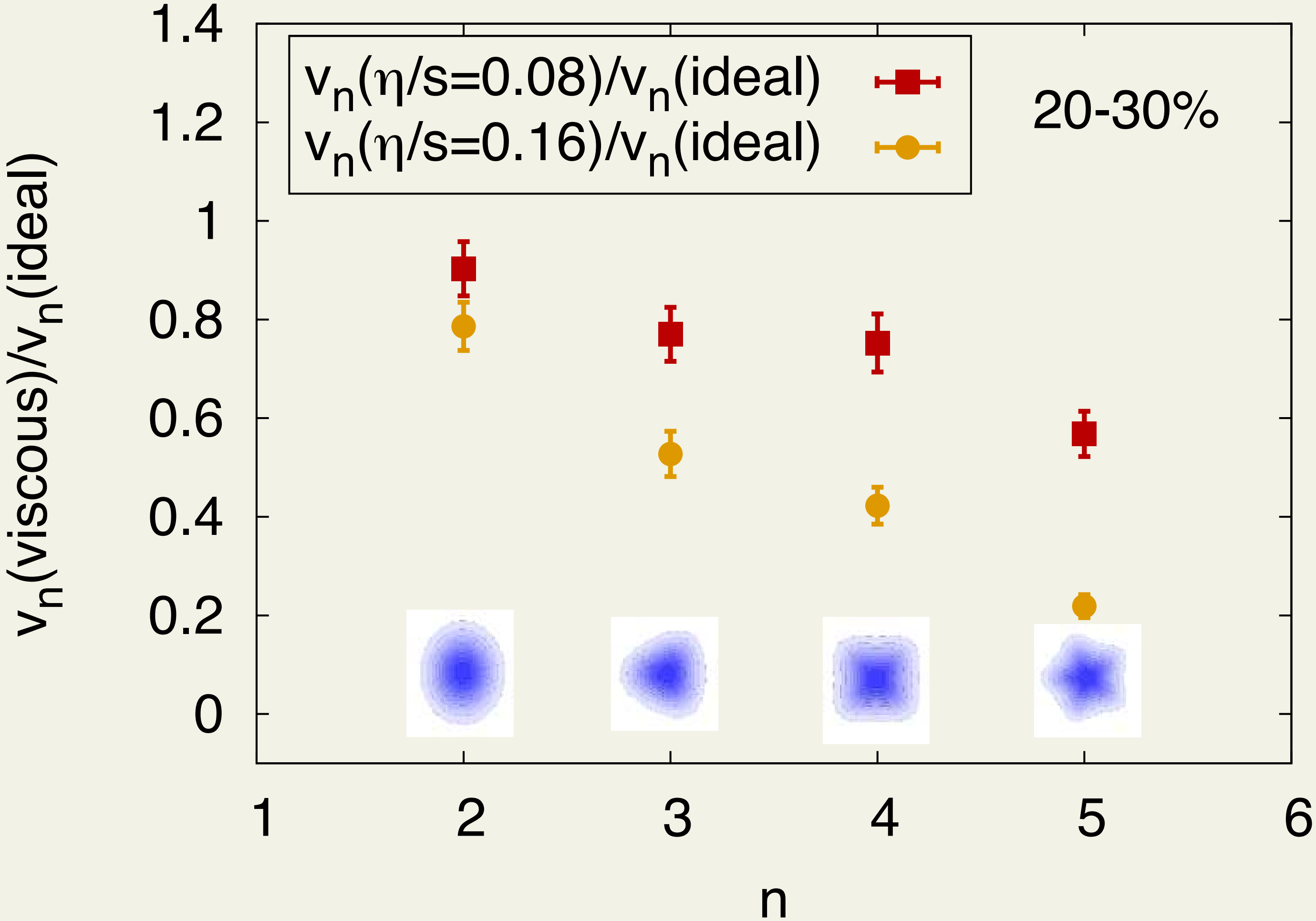
# $\eta/s$ depends on initialization




- MC-KLN favours larger $\eta/s$ than MC-Glauber
- factor 2 uncertainty

# Sensitivity to $\eta/s$

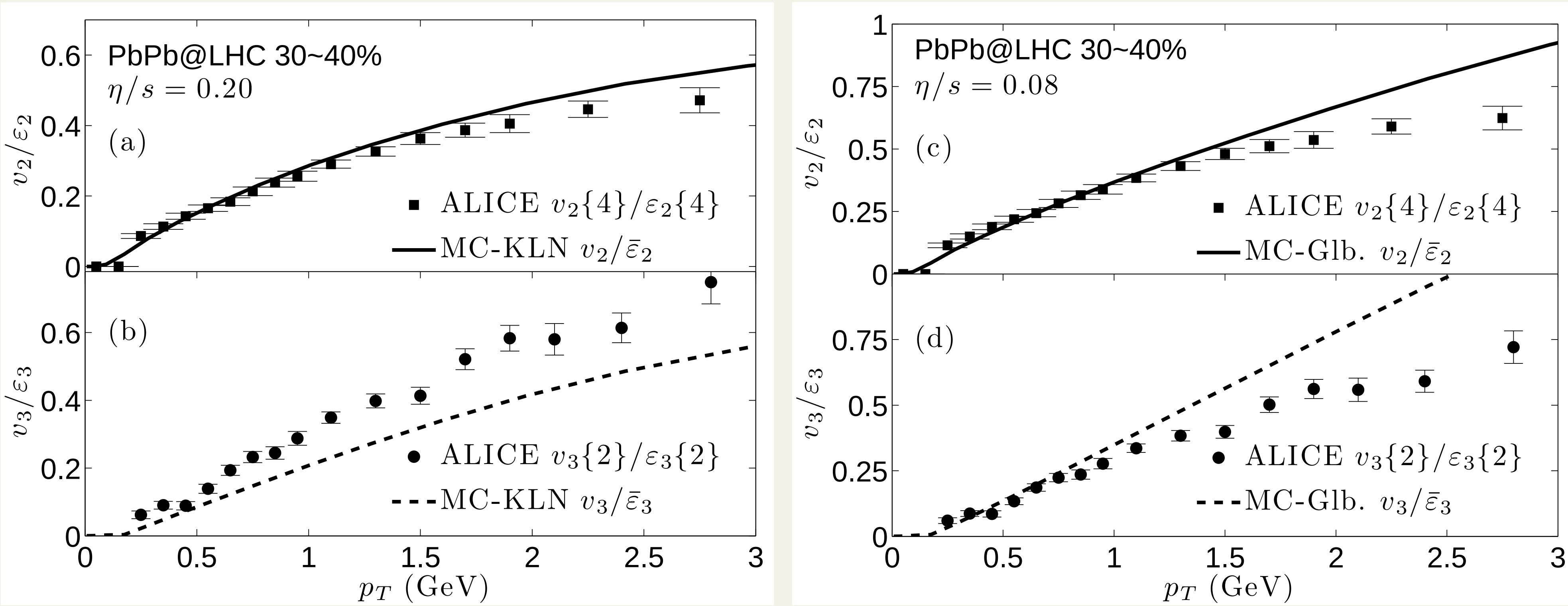
Schenke *et al.* PRC 85, 024901




- **higher coefficients are suppressed more by dissipation**

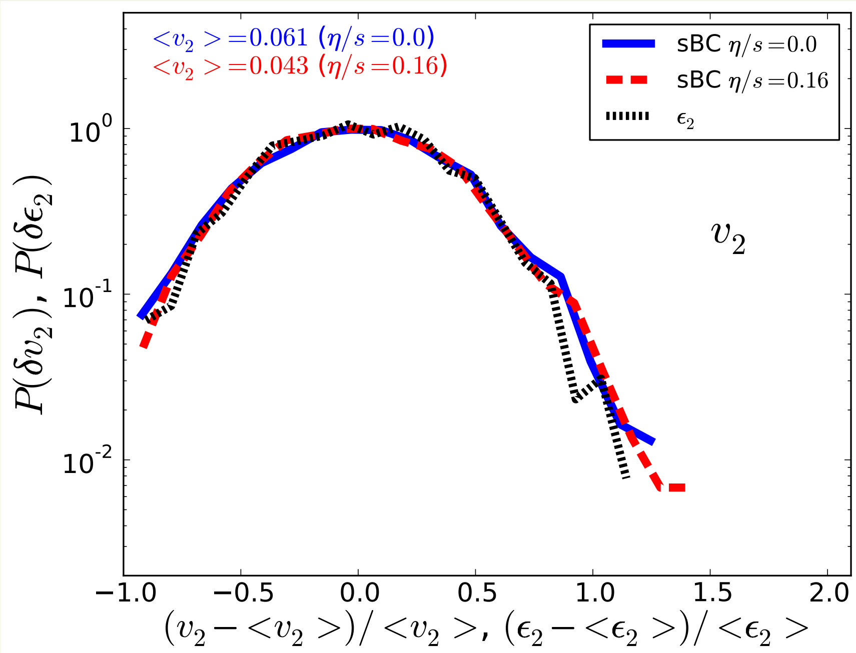
# MC-KLN vs. MC-Glauber

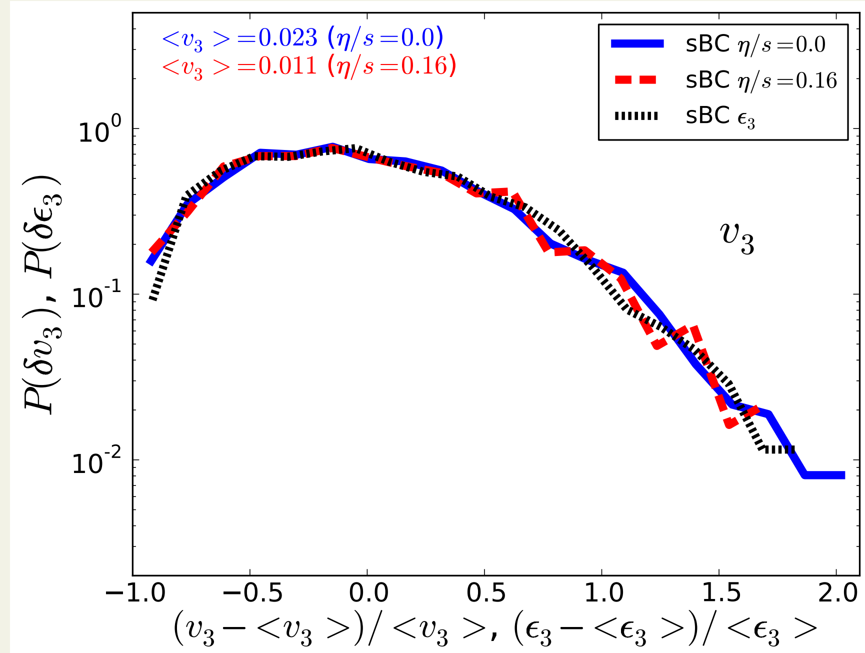
**Qiu *et al.* PLB 707, 151**

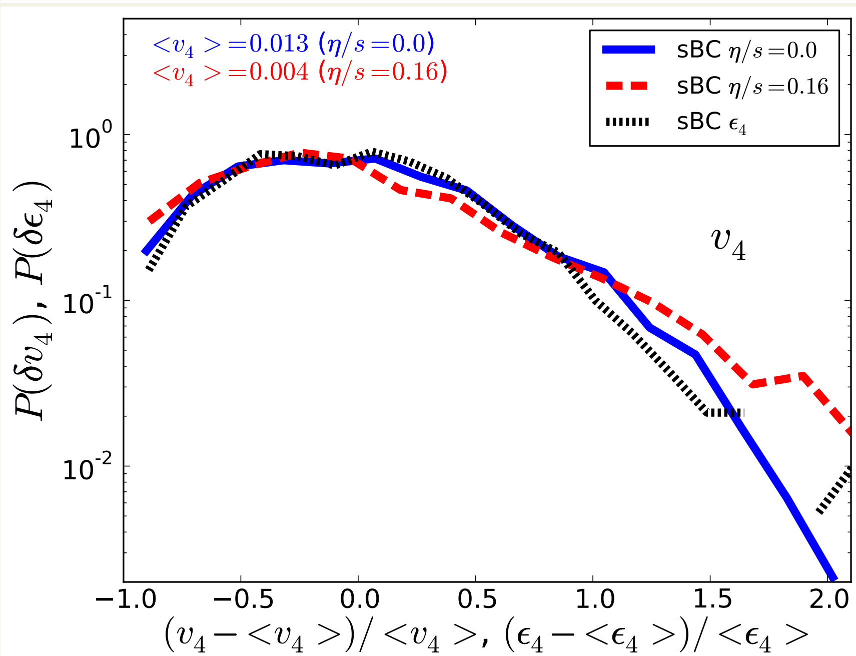



- **models can be distinguished**
- **MC-Glauber slightly favoured**

# Relative fluctuations. . .



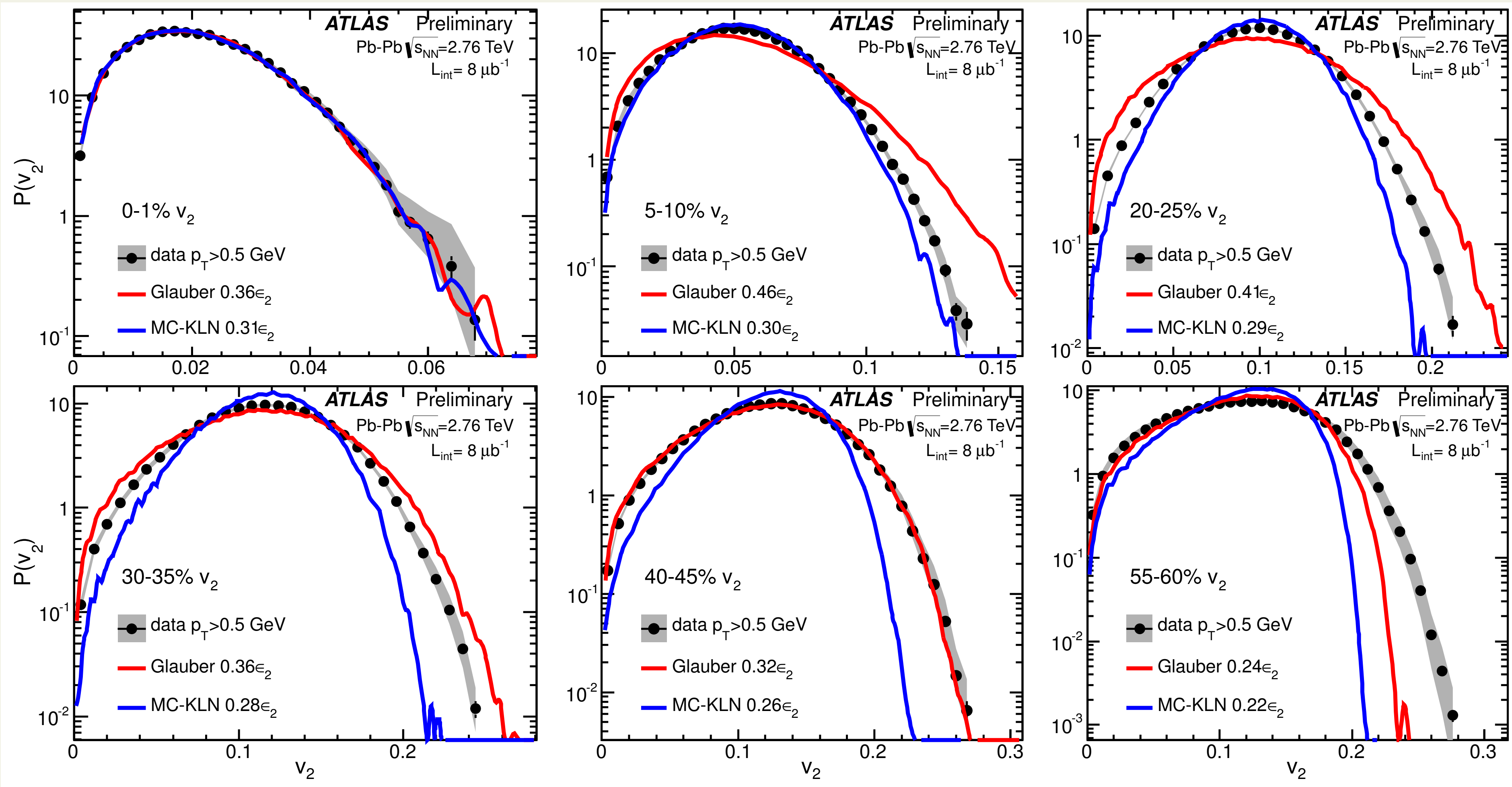




**Niemi *et al.* arXiv:1212.1008**

- $\delta v_n \approx \delta\epsilon_n$ **independent of $\eta/s$**
- **measurement of initial state**

# . . . have been measured

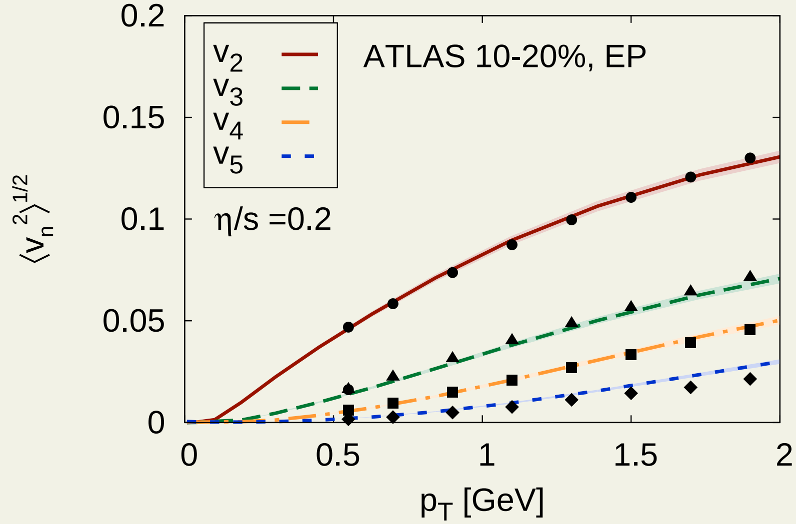
J. Jia [ATLAS Collaboration], arXiv:1209.4232

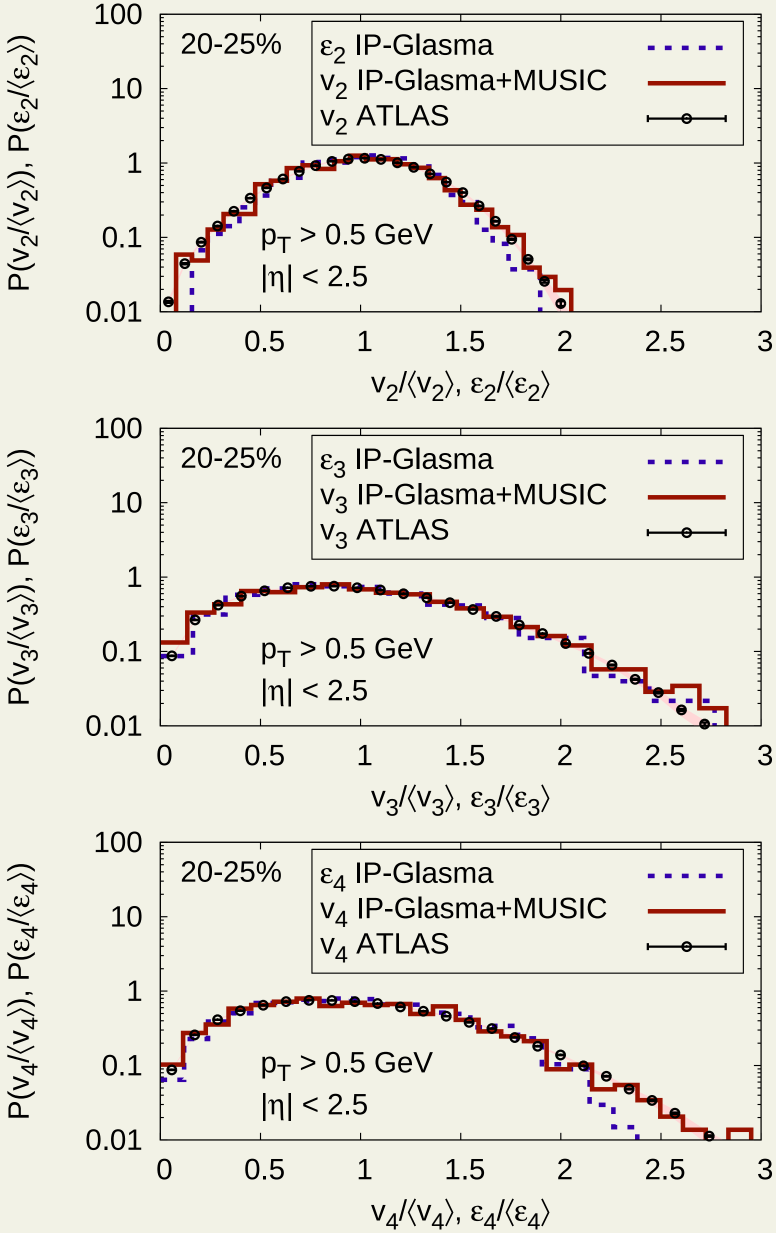



- **Neither MC-Glauber nor MC-KLN fits the data!**

# but IP-Glasma does

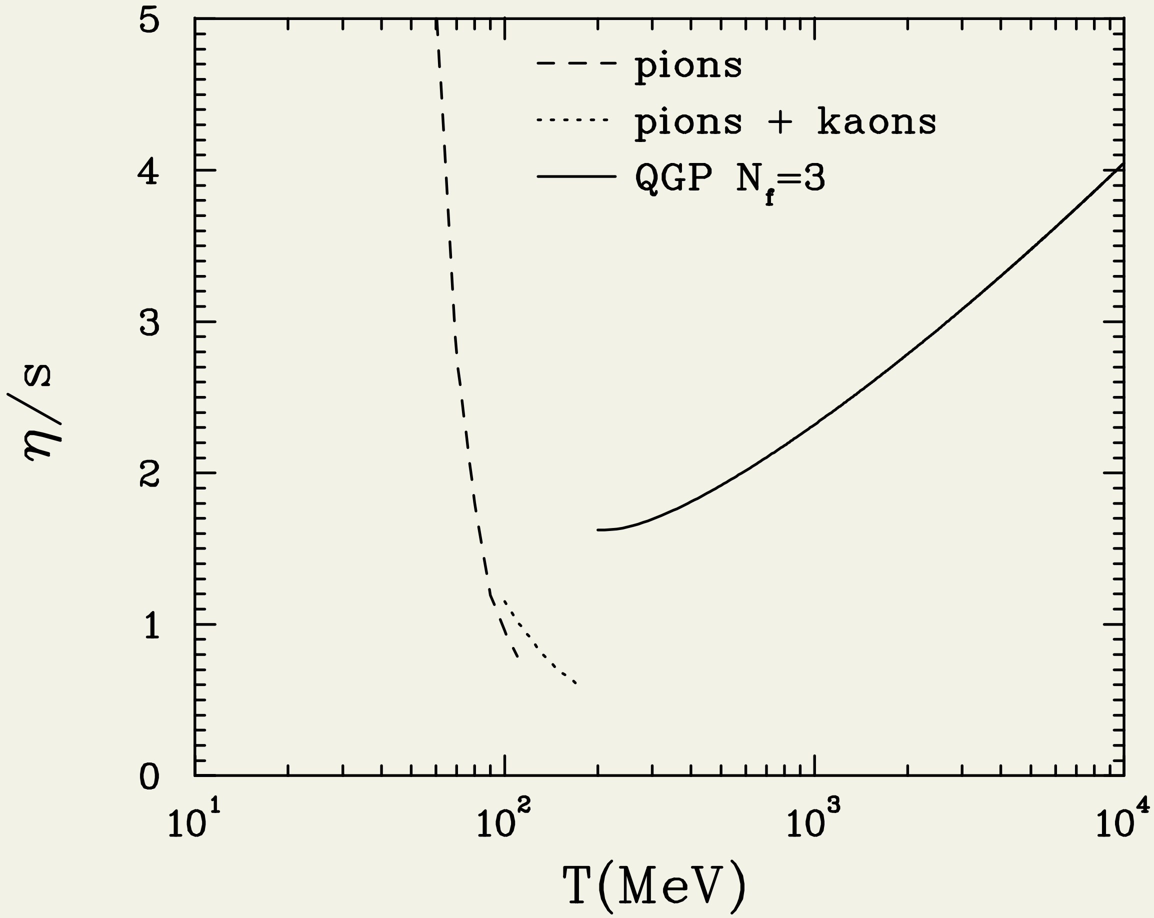
**Schenke *et al.* Phys.Rev.Lett.110:012302,2013**



# $\eta/s(T)$

**Kapusta, McLerran and Csernai, Phys.Rev.Lett.97:152303,2006:**

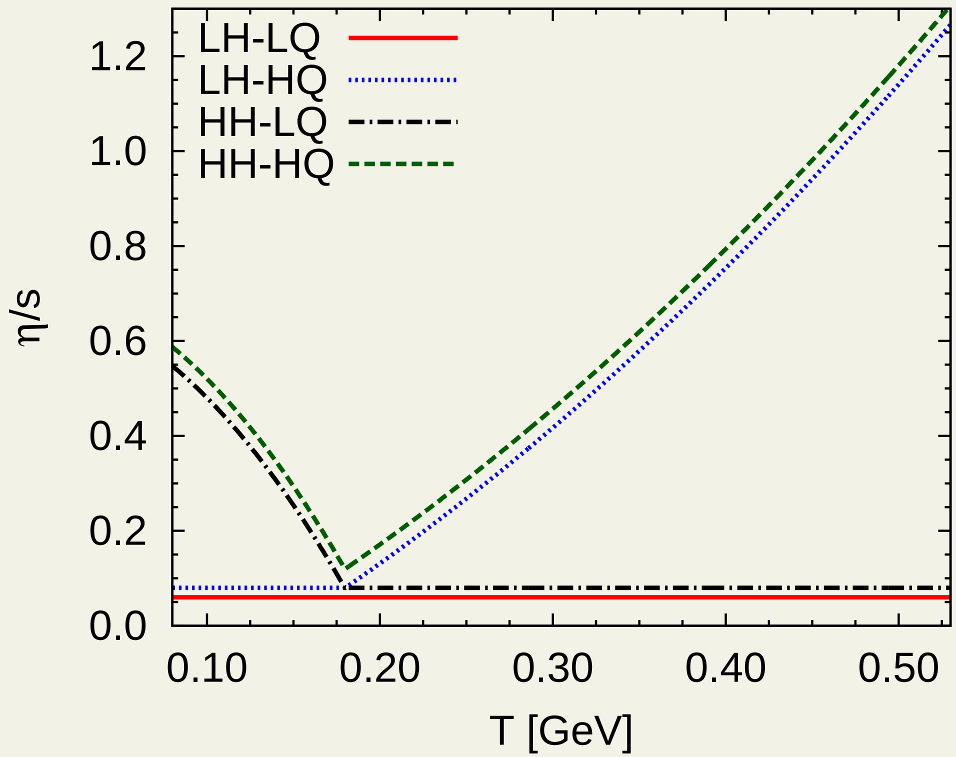


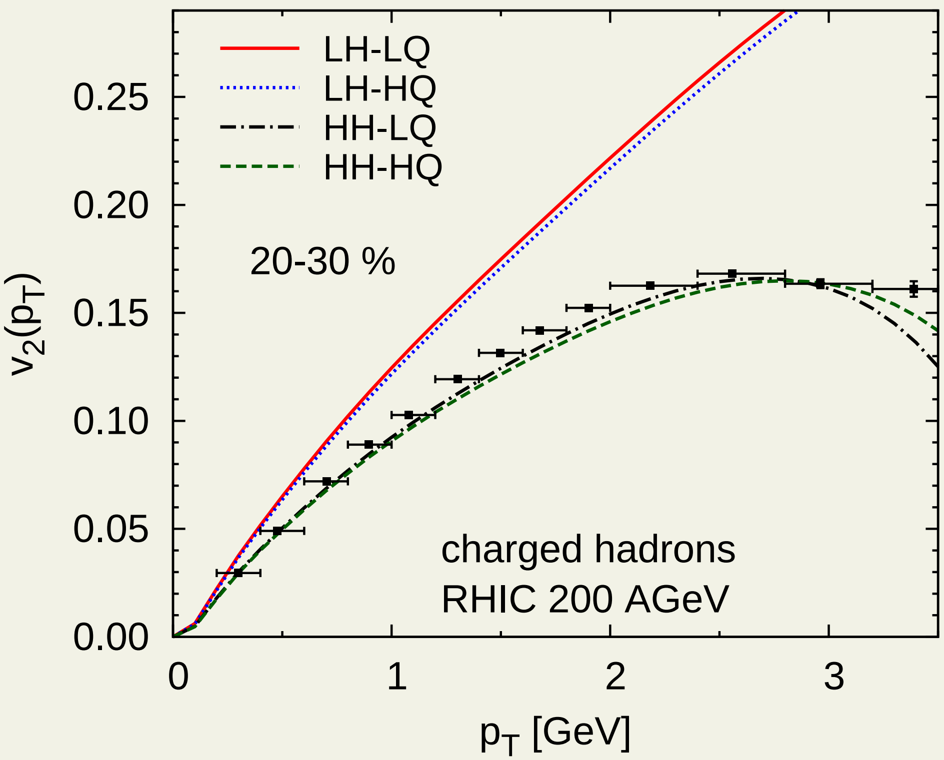

**Low T** (*Prakash et al.*) **using experimental data for 2-body interactions**

**High T** (*Yaffe et al.*) **using perturbative QCD**

# $\eta/s(T)$ at RHIC

**Niemi *et al.* Phys.Rev.C86:014909,2012**

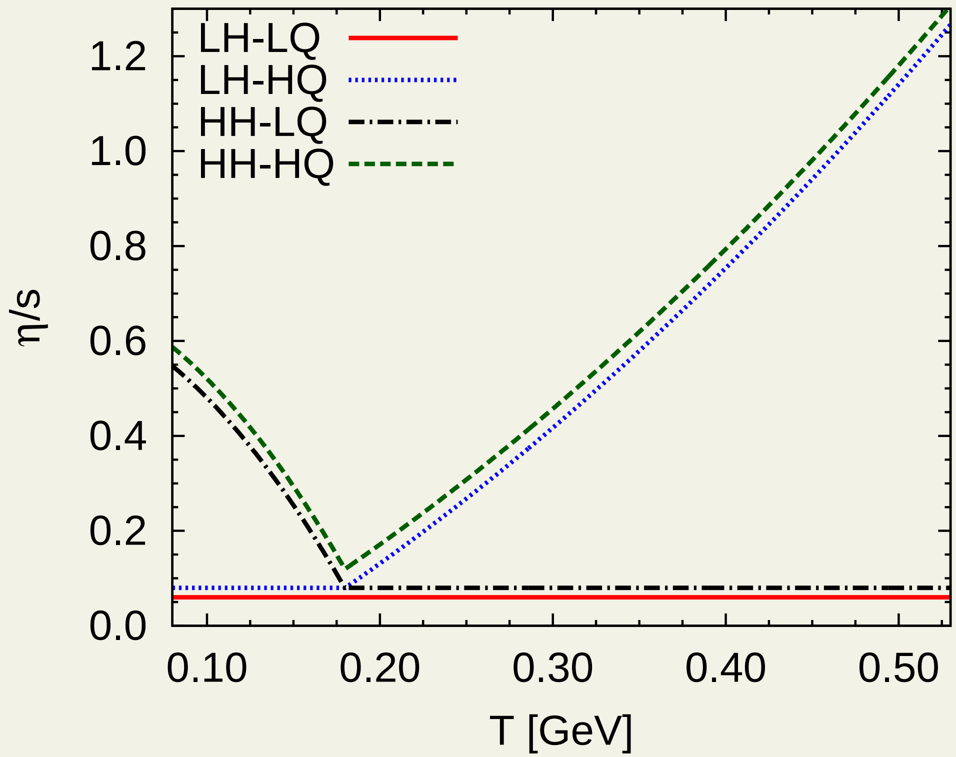


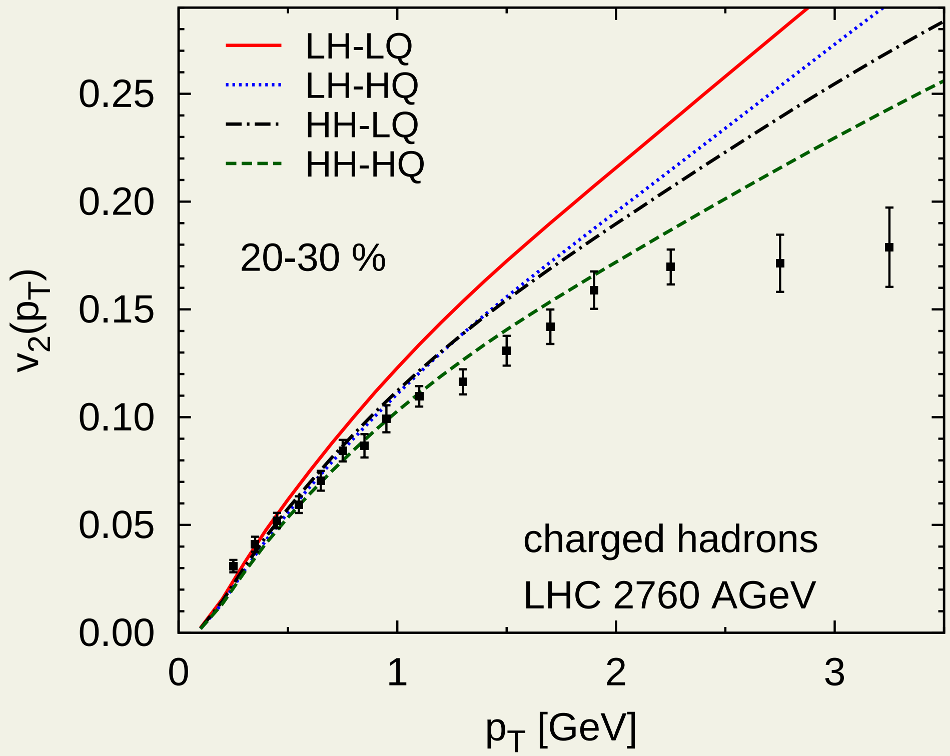




- $v_2(p_T)$ **(almost) insensitive to plasma viscosity!**
- $v_2(p_T)$ **dominated by hadronic viscosity**

# $\eta/s(T)$ at LHC ($\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV)

**Niemi** *et al.* **Phys.Rev.C86:014909,2012**

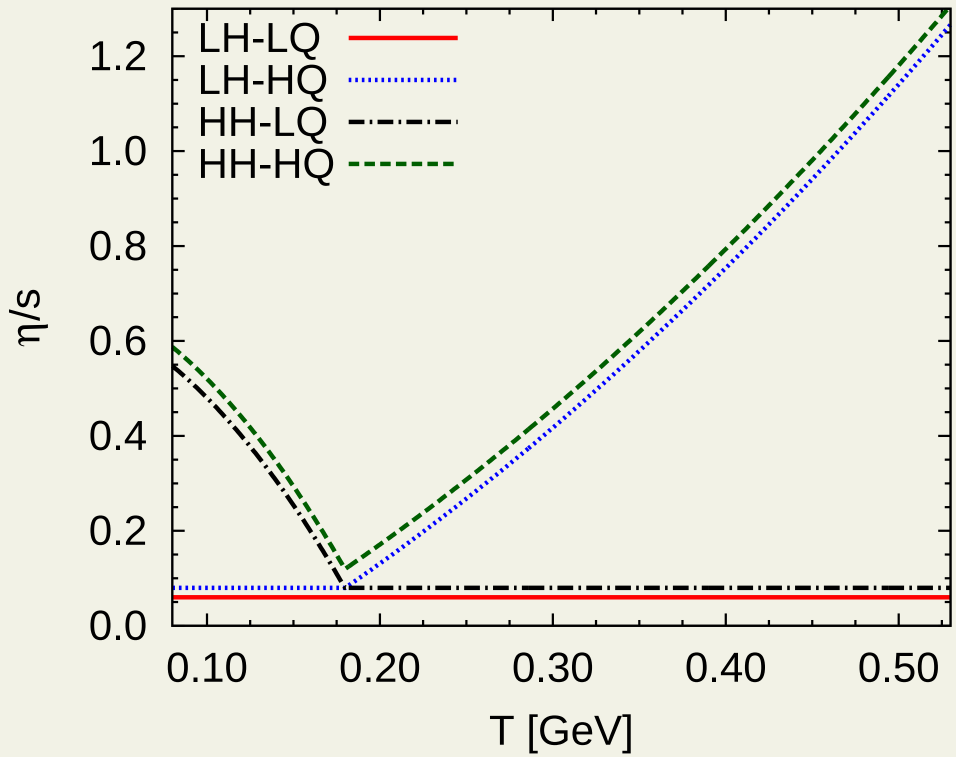


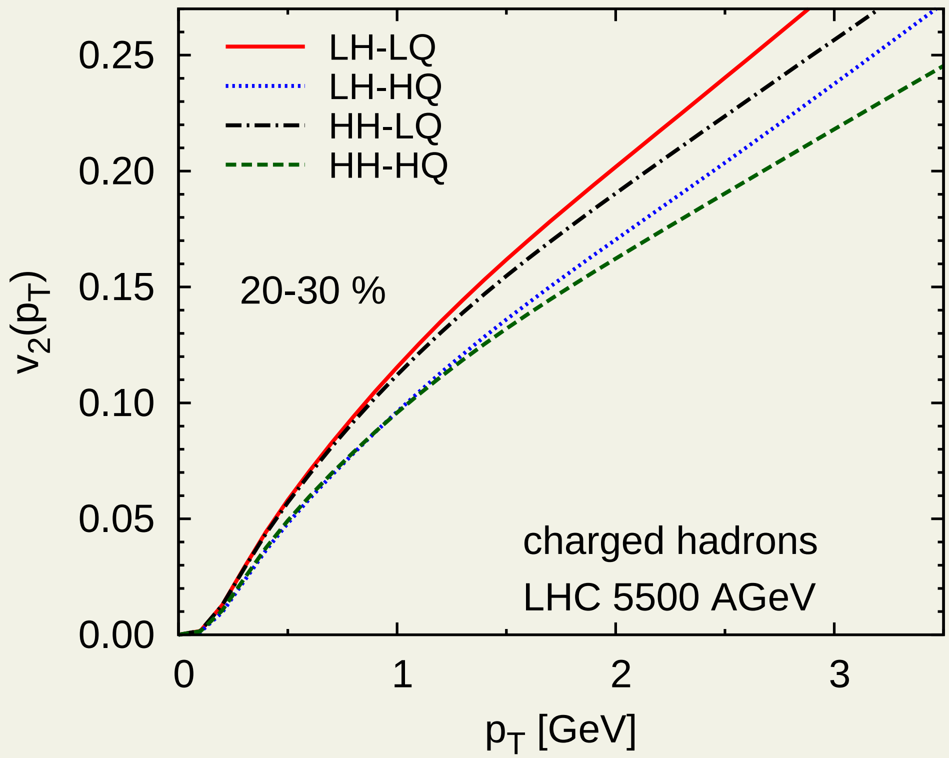




- **both plasma and hadronic** $\eta/s$ **affect** $v_2(p_T)$

# $\eta/s(T)$ at LHC ($\sqrt{s_{\mathrm{NN}}} = 5.5$ TeV)

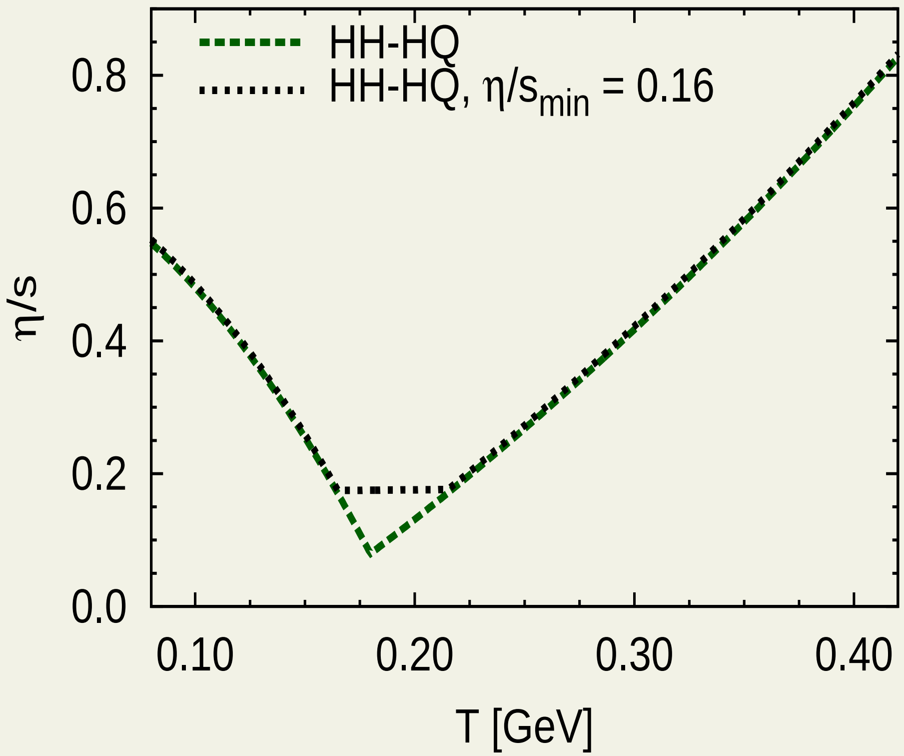
**Niemi *et al.* Phys.Rev.C86:014909,2012**

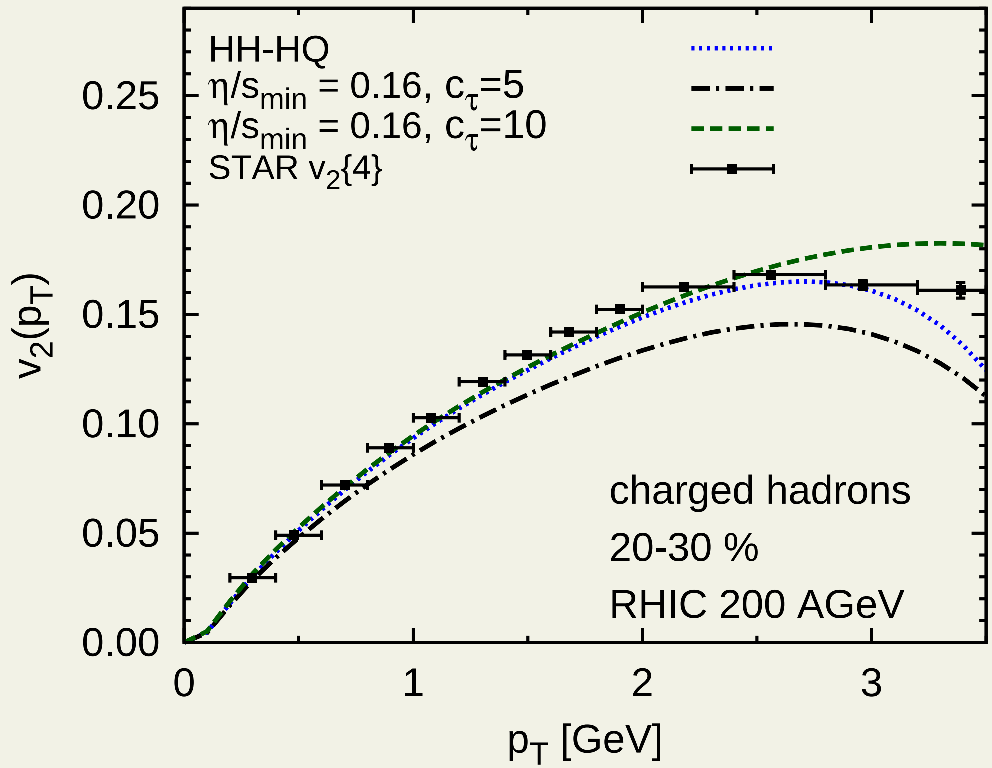



- $v_2(p_T)$ **dominated by plasma viscosity**

Niemi *et al.* Phys.Rev.C86:014909,2012




- $\tau_\pi = c_\tau \eta/(e+P)$
- $v_2$ **sensitive to both** $\eta$ **and** $\tau_\pi$
- **how to disentangle?**

# Lower energies — SPS, FAIR, NICA. . .

# Is hydro applicable?

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$$\partial_\mu T^{\mu\nu}(x) = 0$$

- **conservation laws are always applicable!**

# Is hydro applicable?

$$\partial_\mu T^{\mu\nu}(x) = 0$$

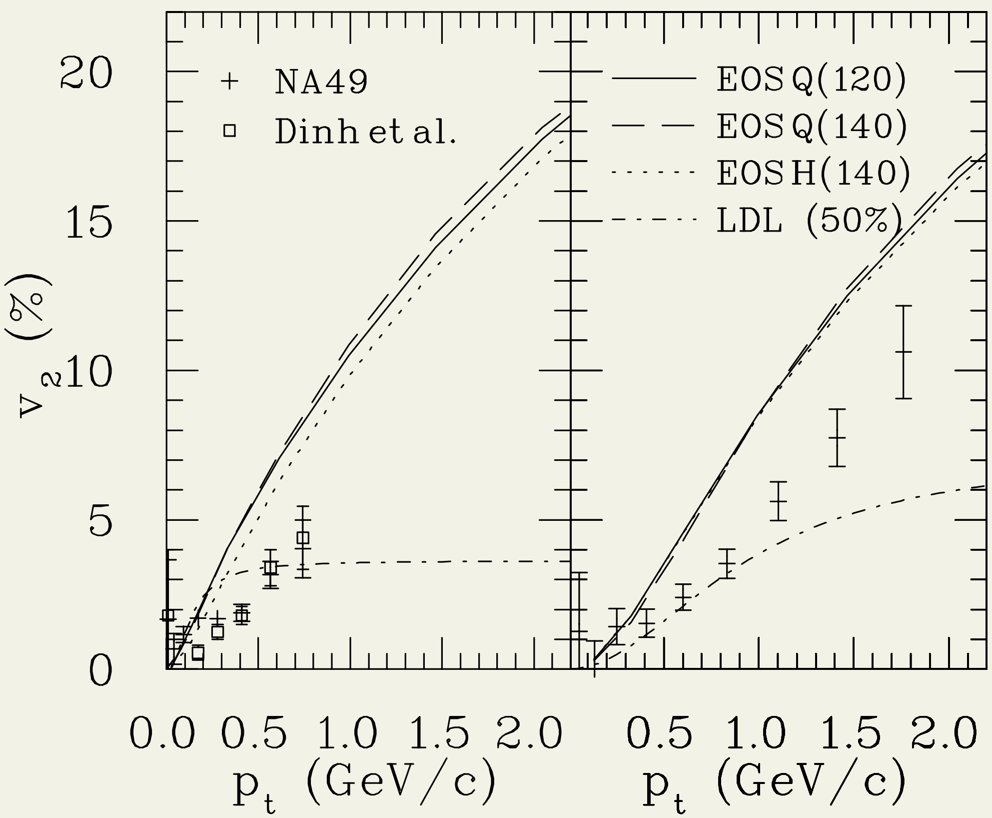
- **conservation laws are always applicable!**

- **but what are**

$$T^{\mu\nu}?$$

$$T^{\mu\nu} \Longleftrightarrow f(p,x)?$$

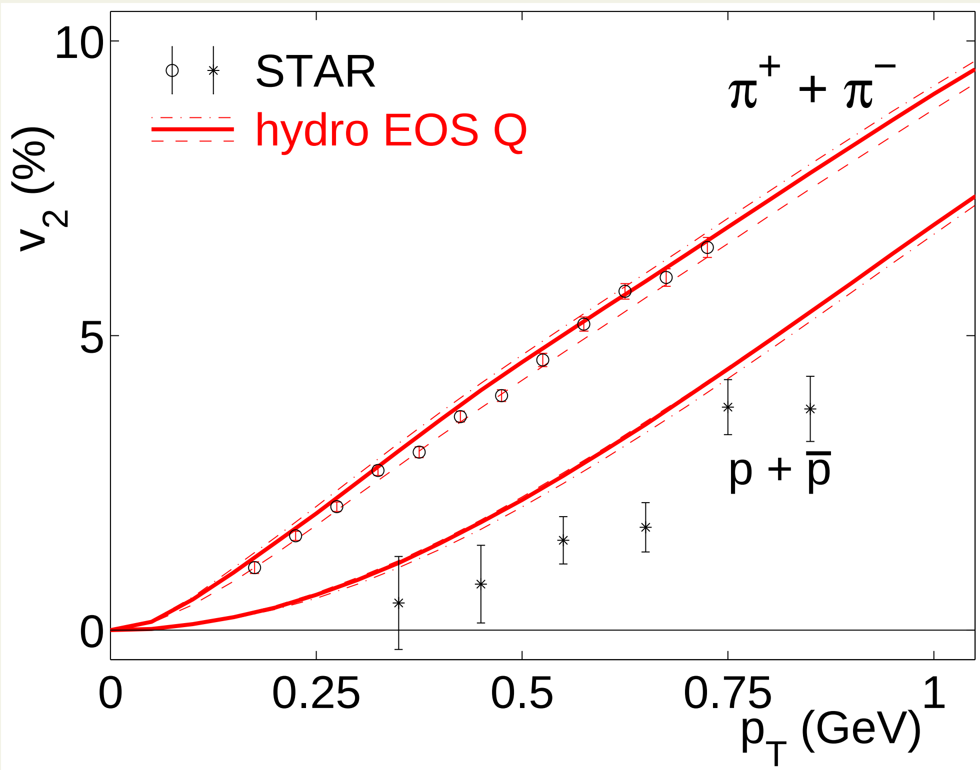
# Is hydro applicable?




**Kolb** *et al.*, **Phys.Lett.B500:232,2001**

- **at SPS hydro not so good. . .**
- **but it was ideal hydro. . .**

# Success at RHIC?




**Kolb *et al.***

**at RHIC**

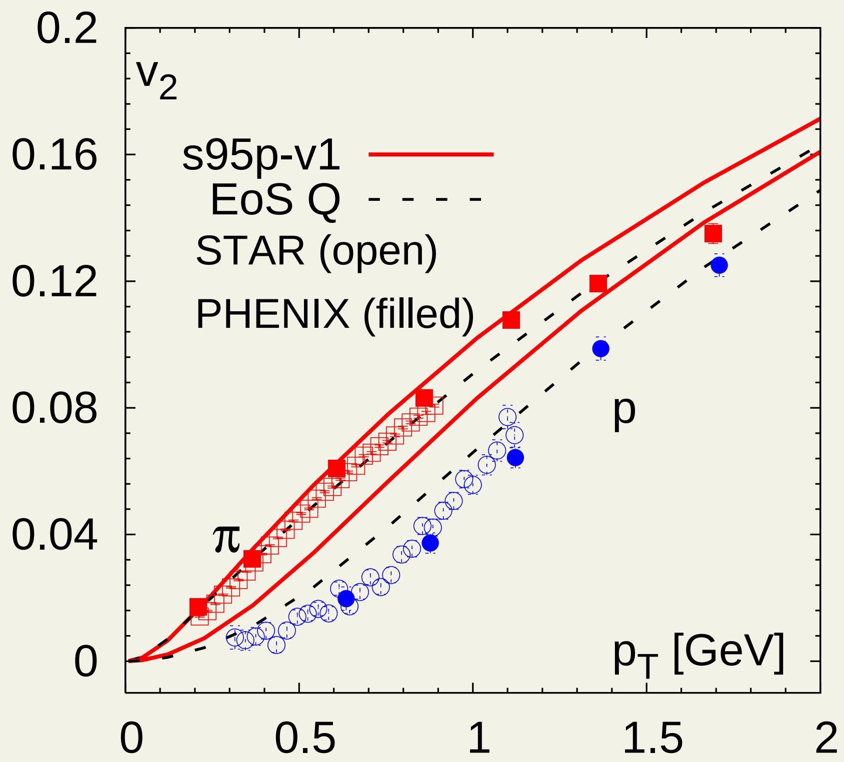
- **hydro seemed to work**

**BUT**

- **1st order phase transition**
- **yields not reproduced**

$\Longrightarrow$ **wrong EoS**

# Success at RHIC?

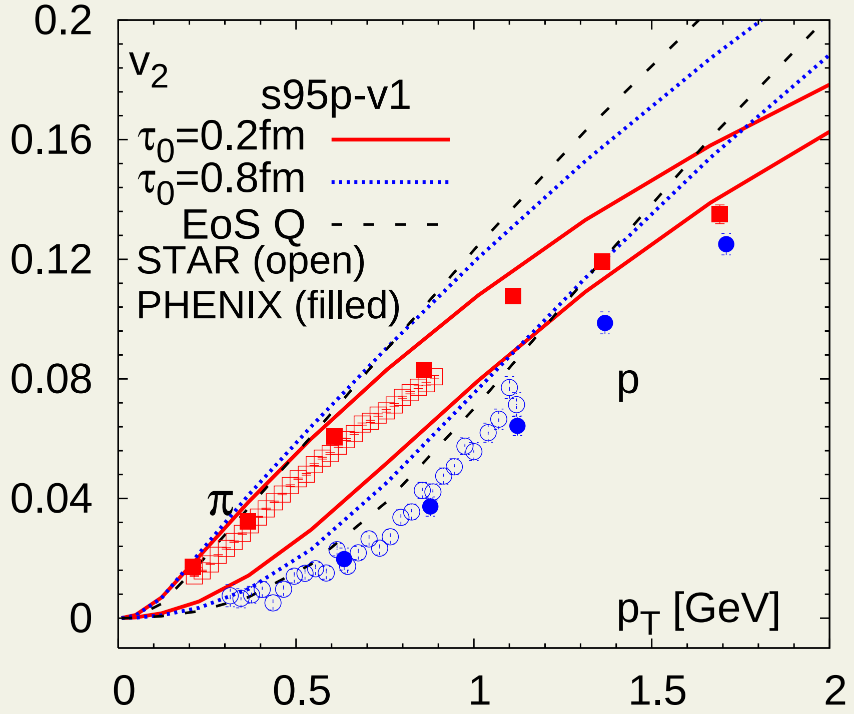



**Huovinen & Petreczky, NPA837:26,2010**

**lattice based EoS**

**proton $v_2(p_T)$ not reproduced**

# Success at RHIC?



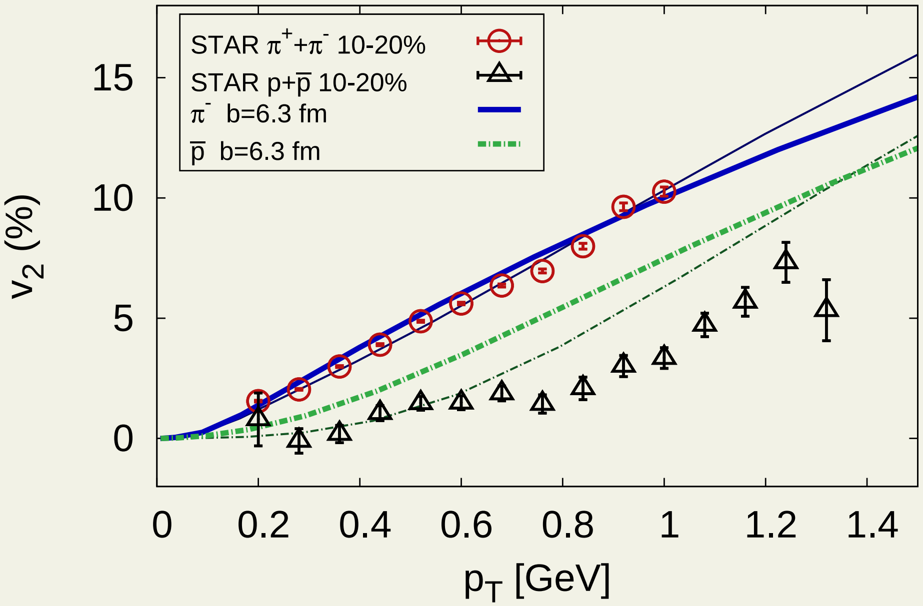

**Huovinen & Petreczky, NPA837:26,2010**

**lattice based EoS**

$T_{\mathrm{chem}} \neq T_{\mathrm{kin}}$

**proton $v_2(p_T)$ not reproduced**
**pion $v_2(p_T)$ not reproduced**

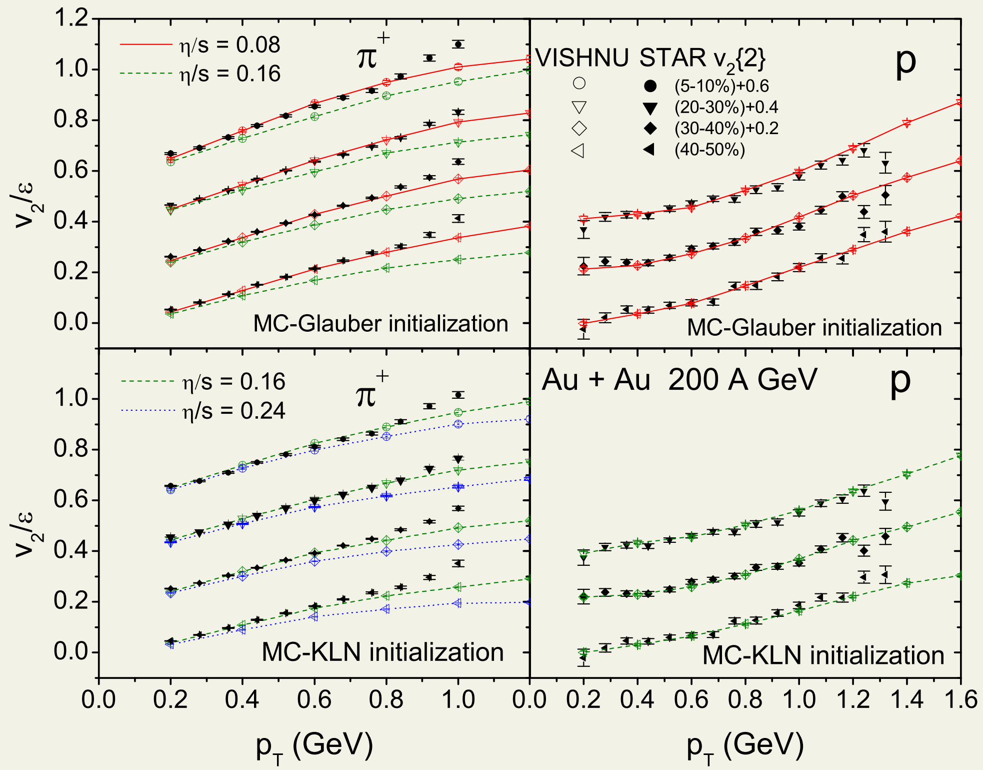
# Success at RHIC?




**include viscosity**

**Schenke *et al.*, PRC 82, 014093**

# Success at RHIC?




**Song *et al.*, PRC 83, 054910**

**viscous hybrid**

# Comparison to transport

PH & Molnar, PRC 79, 014906:

- simplest of all cases - 1D Bjorken (boost invariance)
- $2 \to 2$ transport $\Longleftrightarrow$ conserved particle number
- massless particles $\Longleftrightarrow$ ideal gas EoS, $e = 3p$, $\zeta = 0$

$$\begin{aligned} \dot{p} + \frac{4p}{3\tau} &= -\frac{\pi_L}{3\tau} \\ \dot{\pi}_L + \frac{\pi_L}{\tau}\left(\frac{2K(\tau)}{3C} + \frac{4}{3} + \frac{\pi_L}{3p}\right) &= -\frac{8p}{9\tau}\ , \end{aligned}$$

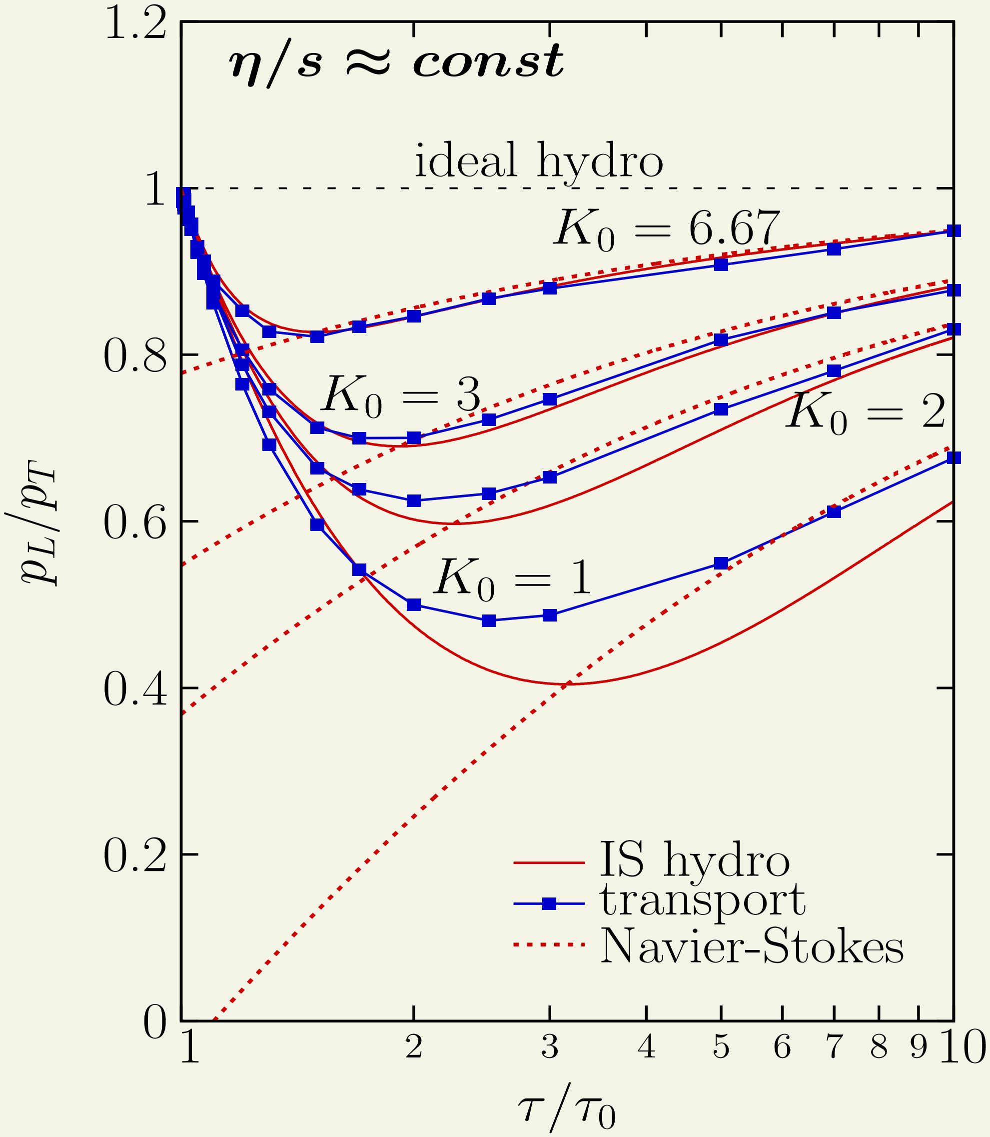
where

$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)} = \frac{\tau_{\rm exp}}{\tau_{\rm scatt}} \approx \frac{6}{5}\frac{\tau_{\rm exp}}{\tau_\pi}$$

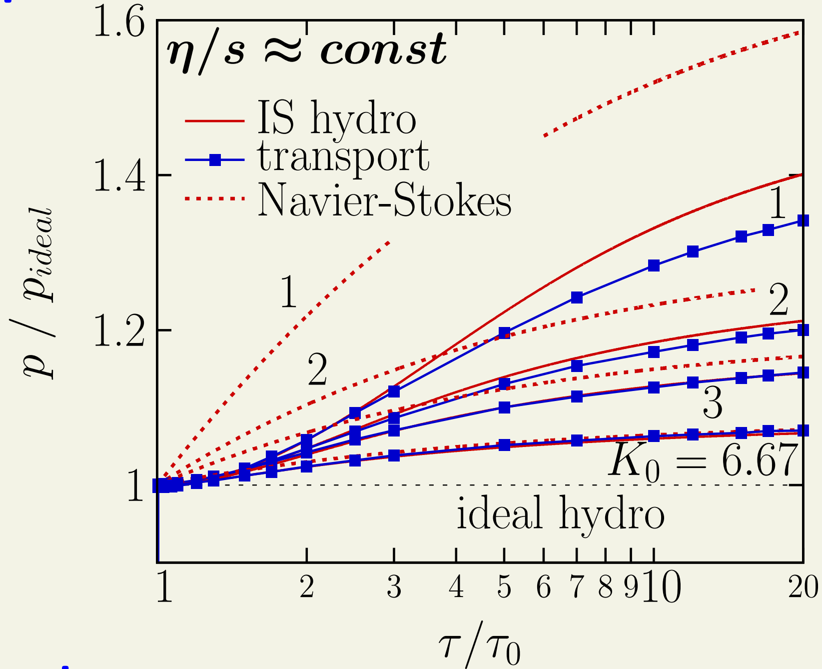
is the inverse Knudsen number and $C \approx 0.8$

# for $\eta/s \approx const$ :

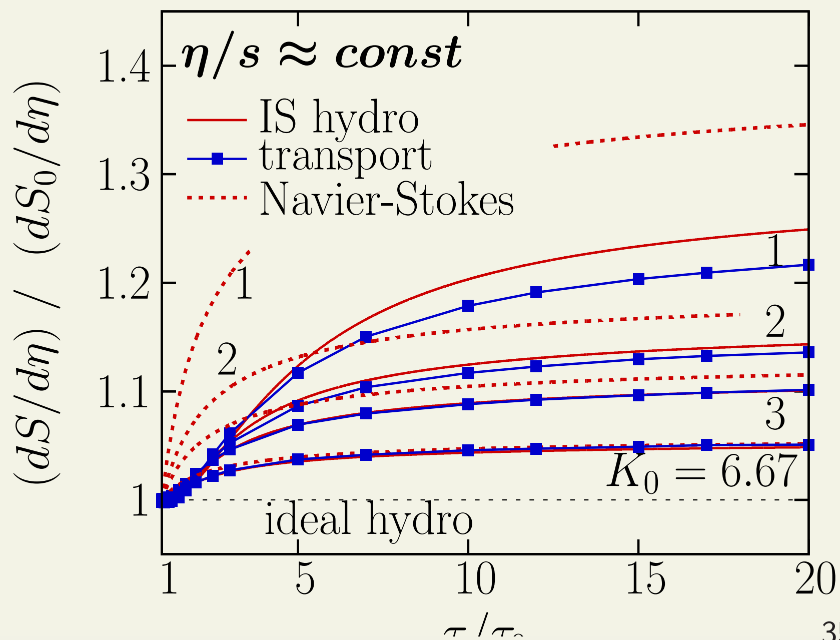
**pressure anisotropy** $T_{zz}/T_{xx}$




**pressure**




**entropy**




**IS hydro 10% accurate when $K_0 \gtrsim 2$**

## Connection to viscosity

$$K = \frac{\tau}{\lambda_{\rm tr}} = \frac{4Tn\tau}{5\eta} \quad \Longrightarrow \quad K_0 = \frac{T_0\tau_0}{5}\frac{s_0}{\eta_0}$$

**For typical RHIC initial conditions $T_0\tau_0 \sim 1$, therefore**

$$K_0 \gtrsim 2 - 3 \quad \Longrightarrow \quad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi}$$

**IS hydro applicable only up to a few times the conjectured lower bound!**

- **The smaller $\tau_0$, the smaller $\eta/s$ must be**
- **At LHC $T_0\tau_0 \gtrsim 1 \Longrightarrow$ applicability is better**

# and at lower energies. . . ?

- **multiplicity estimates:**

$$\sqrt{s_{\rm NN}} = 11\,\mathrm{GeV}: \qquad \frac{\mathrm{d}N}{\mathrm{d}y} \approx 550$$

$$\sqrt{s_{\rm NN}} = 4\,\mathrm{GeV}: \qquad \frac{\mathrm{d}N}{\mathrm{d}y} \approx 250$$

**assuming entropy conservation,** $s \propto T^3$ **and** $s \propto n$**:**

$$T(\tau_0) \approx \sqrt[3]{\frac{\mathrm{d}N/\mathrm{d}y}{\mathrm{d}N_{\rm RHIC}/\mathrm{d}y}}\, T_{\rm RHIC}(\tau_0)$$

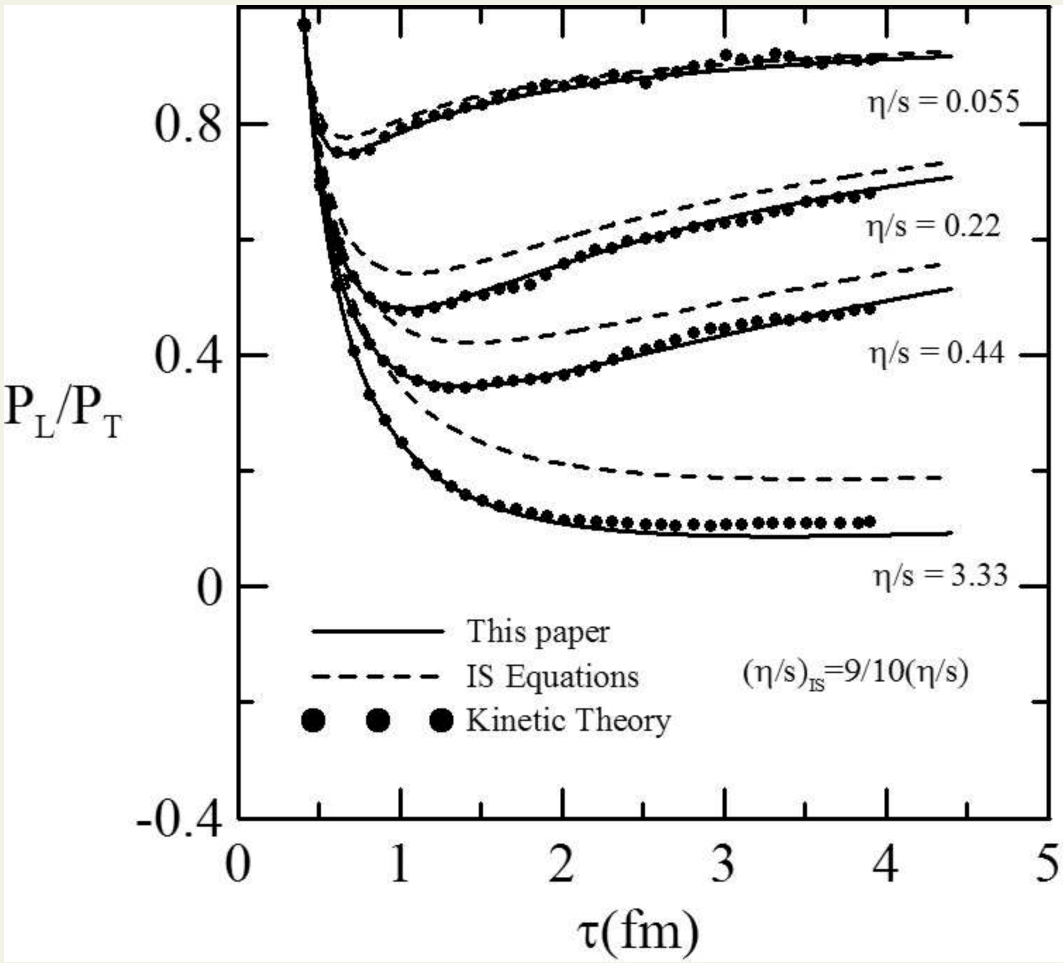
**and** $\tau_0 = \tau_{0,\rm RHIC}$

$$\sqrt{s_{\rm NN}} = 11\,\mathrm{GeV}: \qquad T_0\tau_0 \approx 0.8 \qquad \frac{\eta}{s} \lesssim \frac{1.5}{4\pi}$$

$$\sqrt{s_{\rm NN}} = 4\,\mathrm{GeV}: \qquad T_0\tau_0 \approx 0.6 \qquad \frac{\eta}{s} \lesssim \frac{1}{4\pi}$$

# New derivation of viscous hydro

- 1D Bjorken




Derivation of dissipative relativistic hydro depends on method

- Usual IS: use second moment of Boltzmann equation

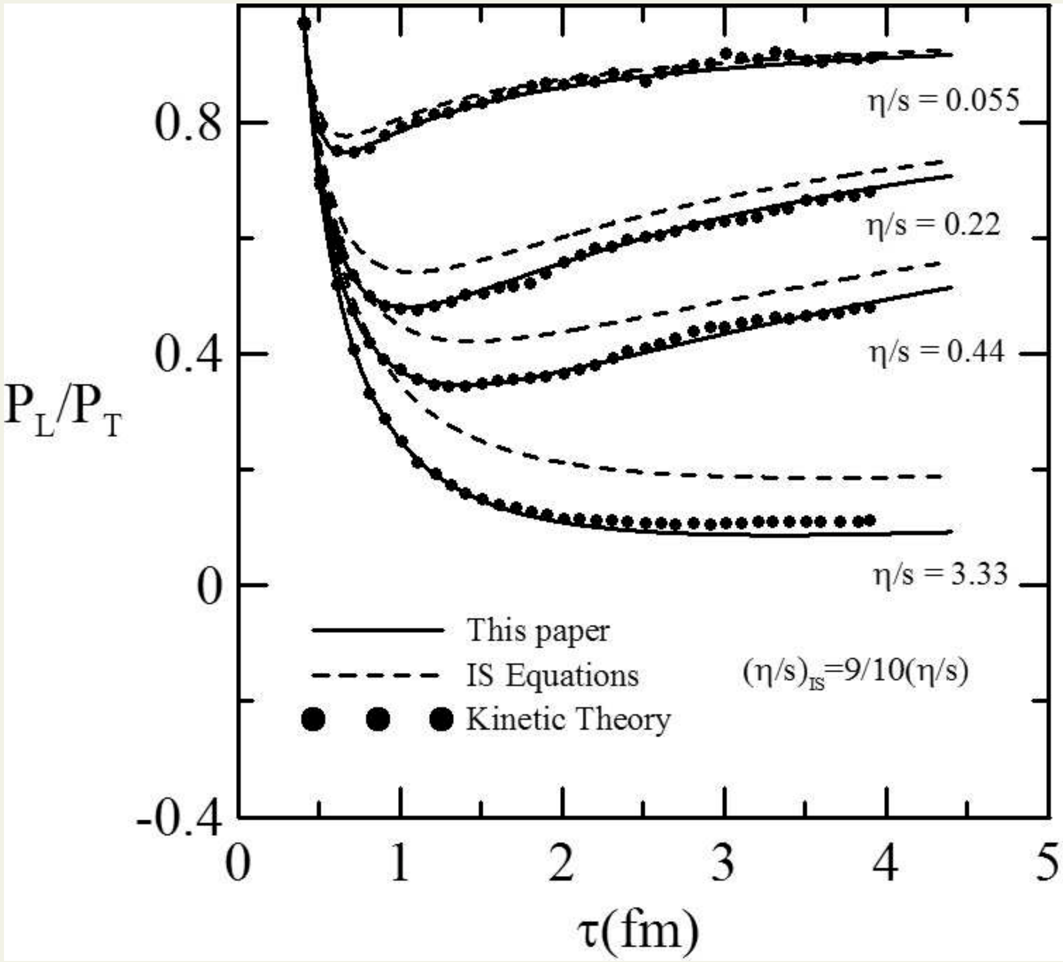
Denicol *et al.*, PRL 105, 162501 and PRD 85, 114047:

- use definition of dissipative currents directly

New equations of same form but different coefficients

# New derivation of viscous hydro

- 1D Bjorken




$$\frac{\eta}{s} = 0.05 \approx \frac{0.6}{4\pi} \quad \Leftrightarrow \quad K \approx 4$$
$$\frac{\eta}{s} = 0.2 \approx \frac{2.5}{4\pi} \quad \Leftrightarrow \quad K \approx 1$$
$$\frac{\eta}{s} = 0.4 \approx \frac{5}{4\pi} \quad \Leftrightarrow \quad K \approx 0.5$$
$$\frac{\eta}{s} = 3.0 \approx \frac{25}{4\pi} \quad \Leftrightarrow \quad K \approx 0.07$$

- significant improvement!
- only $2 \leftrightarrow 2$ processes so far

# 3-fluid dynamics

**Amsden *et al.*, PRC 17, 2080**

**Ivanov & Russkikh, PRC 74, 034904 and PRC 78, 064902**

**3 ideal fluids characterized by separate**

$$N_i^{\mu} \qquad \text{and} \qquad T_i^{\mu\nu}$$

- **each fluid in local equilibrium**
- **NO equilibrium between different fluids**
- **different fluids may coexist at the same space-time point**

# 3-fluid dynamics

**Conservation laws for the whole system**

$$\partial_\mu T^{\mu\nu} = \partial_\mu \sum_{i=1}^{3} T_i^{\mu\nu} = 0$$

**but not for each fluid individually:**

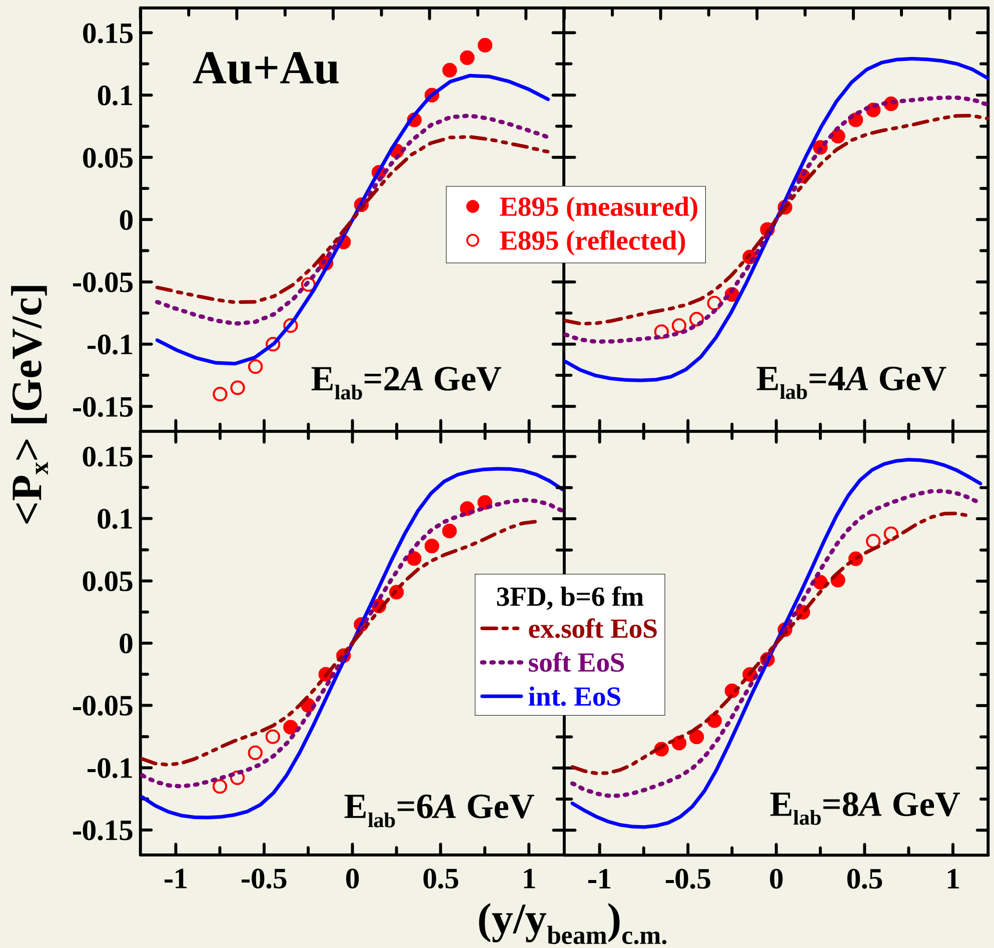
$$\partial_\mu T_i^{\mu\nu} = S_i^\nu$$

**Source terms $S_i^\nu$ describe the interaction of fluids**

- **initial state can be two colliding droplets of fluid**

# 3-fluid dynamics

Ivanov & Russkikh, PRC 74, 034904




directed flow of protons

- works at very low energies

**But**

- what are $S_i^\mu$?
- how are $S_i^\mu$ related to properties of matter?

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**new version of hydro required at FAIR and NICA energies**