Properties of the QCD phase diagram from chemical freeze-out

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GSI Darmstadt, 26.02.2014

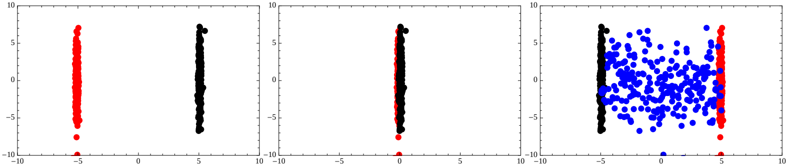
based on

- S. Floerchinger and C. Wetterich *Chiral freeze-out in heavy ion collisions at large baryon densities*, [Nucl. Phys. A, 890-891, 11 (2012)]

see also a related follow-up work

- M. Drews, T. Hell, B. Klein and W. Weise, *Thermodynamic phases and mesonic fluctuations in a chiral nucleon-meson model*, [Phys. Rev. D, 88, 069011 (2013)]

Heavy Ion Collisions



- ions are strongly Lorentz-contracted
- *some* medium is produced after collision
- medium expands in longitudinal direction and gets diluted

Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...
- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down
- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more
- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more

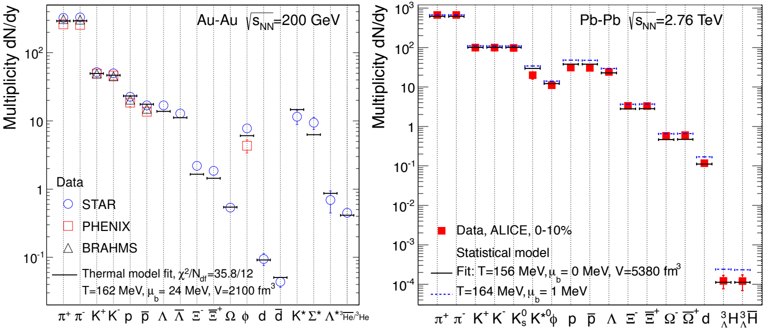
Fluid dynamic regime

- Assumes strong interaction effects leading to local equilibrium.
- Fluid dynamic variables
  - thermodynamic variables: e.g. $T(x)$, $\mu(x)$,
  - fluid velocity $u^\mu(x)$,
  - shear stress tensor $\pi^{\mu\nu}(x)$,
  - bulk viscous pressure $\pi_{\text{Bulk}}(x)$.
- Hydrodynamics is universal: many details of microscopic theory not important.
- Some macroscopic properties are important:
  - ideal hydro: needs equation of state $p = p(T, \mu)$, $n = n(T, \mu)$ from thermodynamics
  - first order hydro: needs also transport coefficients like shear viscosity $\eta = \eta(T, \mu)$ and bulk viscosity $\zeta(T, \mu)$ from linear response theory
  - second order hydro: needs also relaxation times τ_{Shear}, τ_{Bulk} etc.
- Fluid dynamics can be used to probe phase diagram and material properties od QCD!

Experimental facts about chemical freeze-out

Multiplicities of different particle species rather well described by statistical model:

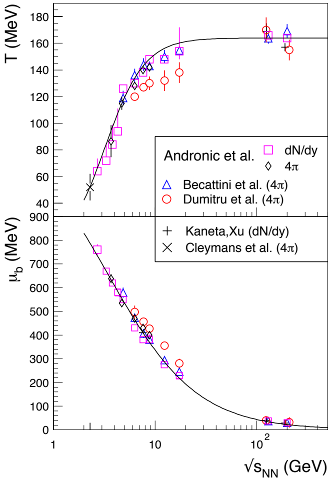
- Non-interacting hadron resonance gas at thermal and chemical equilibrium.
- Includes all hadronic resonances known to the particle data group.
- Fit parameters are temperature T, freeze-out volume V and chemical potentials for baryon number μ_b, isospin, strangeness and charm.



[Andronic, Braun-Munzinger, Redlich, Stachel (2012/2013)]

Chemical freeze-out for different collision energies

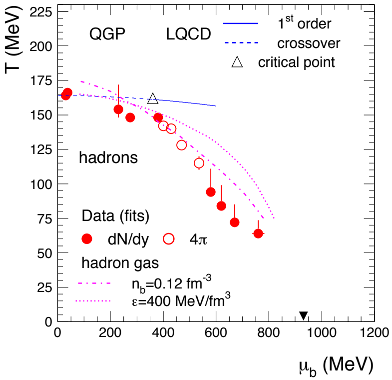
Statistical model fits have been made for heavy ion experiments at various collision energies.



[Andronic, Braun-Munzinger, Stachel (2009)]

A phase diagram from chemical freeze-out?

The fit parameters (T, μ) from different collision energies lead to a suggestive diagram. But what is the physical significance?



[Andronic, Braun-Munzinger, Stachel (2009), LQCD from Fodor, Katz (2004)]

Is the chemical freeze-out universal?

Important question:

- Is the chemical freeze-out dominated by the universal thermodynamic properties of QCD?
- Or do the chemical freeze-out points rather reflect the dynamics of a heavy-ion collision?

Thought experiment 1:

- Consider a universe filled by a quark-gluon plasma with a very slow expansion.
- Temperature decreases only very slowly.
- Chemical equilibrium would be maintained down to very small temperatures.

Thought experiment 2:

- Consider a universe filled by a quark-gluon plasma with a very fast expansion.
- No time for any scatterings.
- Particles would hadronize like jets.

Is chemical freeze-out universal? (2)

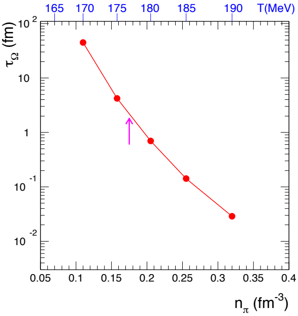
- Thought experiment 1 would lead to thermal particle yields but at very low temperature.
- Thought experiment 2 would lead to non-thermal particle yields but hard to quantify since hadronization is purely understood.
- Experimental findings are in between.
- Although freeze-out temperature inevitably depends on dynamics of the expansion it is conceivable that this dependence is weak for realistic expansions and that freeze-out becomes effectively universal.

Chemical freeze-out and the phase transitions

It has been argued that chemical-freeze out at RHIC energies is essentially on the chiral phase transition / crossover.

[Braun-Munzinger, Stachel, Wetterich, Phys. Lett. B, 596, 61 (2004)]

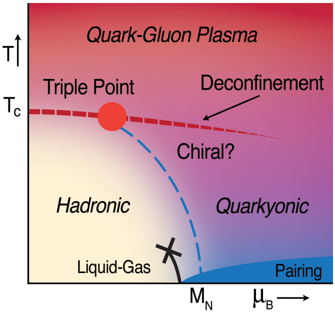
- Expansion rate is approximately $r_T = |\dot{T}/T| \approx 0.13/\text{fm}$
- Rates for $2 \rightarrow 2$-particle scattering with strangeness exchange too small to maintain chemical equilibrium, typically $r_{2\rightarrow 2} \approx 0.018/\text{fm}$
- Multi-hadron strangeness exchange reactions with N_{in} incoming particles have rates that depend strongly on density, $\sim n^{N_{\text{in}}}$.
- Many such processes become important close to T_c.

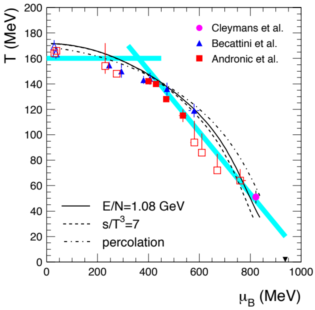


Is chemical freeze-out close to a phase transition everywhere?

It was also proposed that the freeze-out line corresponds to

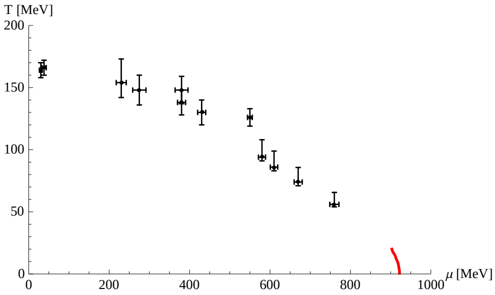
- chiral & deconfinement transition at small μ
- transition to quarkionic matter at small T





[Andronic, Blaschke, Braun-Munzinger, Cleymans, Fukushima, McLerran, Oeschler, Pisarski, Redlich, Sasaki, Satz, Stachel, Nucl. Phys. A 837 (2010) 65.]

Normal nuclear matter and the droplet model



- Normal nuclear matter is sitting on a first order phase transition at $T = 0$, $\mu = \mu_c$.
- For densities $n < n_{\text{nucl}} = 0.153/\text{fm}^3$ one has phase separation: vacuum $(n = 0)$ or nuclear matter $(n = n_{\text{nucl}})$.
- Nuclei are droplets of normal nuclear matter.

The chiral nucleon-meson model

- Model for low-energy degrees of freedom of QCD
  - nucleons (protons, neutrons) $\psi_a = \begin{pmatrix} p \\ n \end{pmatrix}$
  - scalar mesons σ, pseudscalar mesons (pions) π^0, $\pi^\pm$, combined into one isospin matrix field $\phi_{ab} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma + i\pi^0) & i\pi^- \\ i\pi^+ & \frac{1}{\sqrt{2}}(\sigma - i\pi^0) \end{pmatrix}$
  - isospin singlet vector mesons ω_μ

- Chiral transformations are linear rotations in the fields,

$$\psi \to \left(1 + \frac{i}{2}\boldsymbol{\alpha}_V\boldsymbol{\tau} + \frac{i}{2}\boldsymbol{\alpha}_A\boldsymbol{\tau}\gamma_5 + \frac{i}{2}\beta_V + \frac{i}{2}\beta_A\gamma_5\right)\psi,$$

$$\phi \to \phi - \frac{i}{2}\boldsymbol{\alpha}_V[\boldsymbol{\tau}, \phi] - \frac{i}{2}\boldsymbol{\alpha}_A\{\boldsymbol{\tau}, \phi\} + i\beta_A\phi,$$

$$\omega_\mu \to \omega_\mu.$$

The chiral nucleon-meson model (2)

The effective Lagrangian

$$
\begin{aligned}
\mathscr{L} =& \bar{\psi}_a \, i\gamma^{\nu}(\partial_\nu - i\, g\, \omega_\nu - i\, \mu\, \delta_{0\nu})\, \psi_a \\
& + h\sqrt{2}\left[\bar{\psi}_a \left(\tfrac{1+\gamma_5}{2}\right) \phi_{ab}\psi_b + \bar{\psi}_a \left(\tfrac{1-\gamma_5}{2}\right)(\phi^\dagger)_{ab}\psi_b\right] \\
& + \tfrac{1}{2}\phi^*_{ab}(-\partial_\mu\partial^\mu)\phi_{ab} + U_{\mathsf{mic}}(\rho,\sigma) \\
& + \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\,\omega_\mu\omega^\mu .
\end{aligned}
$$

- contains kinetic terms for nucleons ψ_a, scalars ϕ_{ab} and vectors ω_ν
- Yukawa coupling between nucleons and vectors g
- Yukawa coupling between nucleons and scalars h
- Effective potential for scalars

  $$U_{\mathsf{mic}}(\rho,\sigma) = \bar{U}(\rho) - m_\pi^2 f_\pi \sigma$$

  with the chiral invariant combination

  $$\rho = \frac{1}{2}\phi^*_{ab}\phi_{ab} = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2).$$

- Extended and chiral version of Walecka model.
  [Walecka, Ann. Phys. 83, 491 (1974)]

The chiral nucleon-meson model (3)

Note that

- $\mathscr{L}$ does not contain mass terms for nucleons ψ_a, effective masses come from chiral symmetry breaking only.
- $\mathscr{L}$ is invariant under chiral symmetry except for an explicit symmetry breaking term from quark masses which leads to term $\sim m_\pi^2 f_\pi \sigma$ in the effective potential.
- The parameters of the model are
  - pion mass $m_\pi = 135\,\text{MeV}$
  - pion decay constant $f_\pi = 93\,\text{MeV}$
  - vector boson mass $m_\omega = 783\,\text{MeV}$

  as well as

  - vector Yukawa couplings g
  - scalar Yukawa coupling h
  - the form of the effective potential $\bar{U}(\rho)$.

Bosonic field expectation values

Bosonic fields can have non-vanishing macroscopic value. From symmetries it follows that one can have at non-zero density

- time component of vector field $\omega_0 > 0$,
  leads to effective nucleon chemical potential $\mu_{\text{eff}} = \mu + g\,\omega_0$.
- spatial components of vector field vanishes due to rotation symmetry $\omega_j = 0$
- scalar field $\sigma > 0$,
  leads to effective nucleon mass $m_{\text{nucl}} = h\,\sigma$.
- pion field vanishes as well $\pi^0 = \pi^+ = \pi^- = 0$

The thermodynamic and chiral properties follow from the quantum effective potential with includes quantum and thermal fluctuations

$$U(\sigma, \omega_0).$$

Effective potential

Calculation of $U(\sigma, \omega_0)$ from microscopic model is in general difficult. Simplifications come from two points:

- Explicit breaking of chiral symmetry comes only from linear term

  $$U(\sigma, \omega_0) = U(\rho, \omega_0) - m_\pi^2 f_\pi \sigma, \qquad \rho = \sigma^2/2$$

  with chiral invariant potential $U(\rho, \omega_0)$.
- At $T = 0$ and $\mu = \mu_c$ many properties of U are known from nuclear droplet model. Write therefore

  $$U(\sigma, \omega_0; T, \mu) = U(\sigma, \omega_0; 0, \mu_c) + \Delta$$

  where the difference Δ is much easier to compute than the full potential U.

Effective potential (2)

Most important medium modification of U is from nucleon fluctuations

$$U(\sigma,\omega_0;T,\mu) = U_{\mathsf{vac}}(\sigma,\omega_0) - 4\,p_{\mathsf{FG}}(T,\mu+g\omega_0,h\sigma) \\ + \text{boson fluctuations},$$

For the vacuum part use Taylor expansion around minimum

$$\begin{aligned} U_{\mathsf{vac}}(\sigma,\omega_0) = & \tfrac{m_\pi^2}{2}\left(\sigma^2 - f_\pi^2\right) + \tfrac{\lambda}{8}\left(\sigma^2 - f_\pi^2\right)^2 \\ & + \tfrac{\gamma_3}{3f_\pi^2}\left(\sigma^2 - f_\pi^2\right)^3 + \tfrac{\gamma_4}{4f_\pi^4}\left(\sigma^2 - f_\pi^2\right)^4 \\ & - m_\pi^2 f_\pi\left(\sigma - f_\pi\right) - \tfrac{1}{2} m_\omega^2 \omega_0^2. \end{aligned}$$

Fermionic fluctuation part has factor 4 from spin + isospin degeneracy and

$$p_{\mathsf{FG}}(T,\mu,m) = \int \tfrac{d^3p}{(2\pi)^3}\,\tfrac{\vec{p}^2}{3\sqrt{\vec{p}^2+m^2}}\left[\frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}-\mu)}+1} + \frac{1}{e^{\frac{1}{T}(\sqrt{\vec{p}^2+m^2}+\mu)}+1}\right].$$

The bosonic fluctuations can be included by solving functional renormalization group equations [Drews et al. (2013)].

Fixing parameters

In vacuum $T = 0, \mu = 0$ the field equations

$$\frac{\partial}{\partial\sigma}U(\sigma,\omega_0) = \frac{\partial}{\partial\omega_0}U(\sigma,\omega_0) = 0$$

have the solution

$$\sigma = f_\pi, \qquad \omega_0 = 0.$$

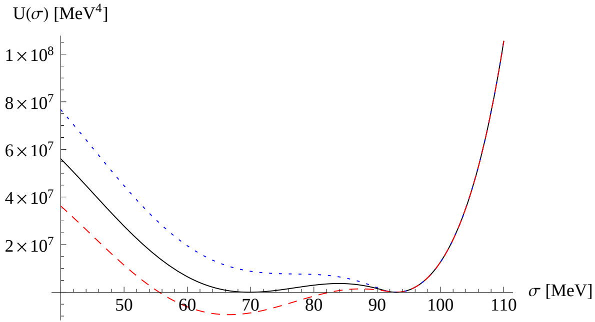
- The linear coefficient in $\boldsymbol{\pi}^2$ determines the pion mass m_π.
- The nucleon mass is given by $m_N = hf_\pi$ which gives $h = 10$.
- The quadratic coefficient in σ gives $m_\sigma^2 = m_\pi^2 + \lambda f_\pi^2$.

First order quantum phase transition

- At vanishing temperature $T = 0$ one has a first-order phase transition at some value μ_c.
- Solve field equation for ω_0:

$$\frac{\partial}{\partial \omega_0} U(\sigma, \omega_0) = 0 \quad \rightarrow \quad U(\sigma) = U(\sigma, \omega_{0,\mathsf{min}})$$

- Effective potential $U(\sigma)$ for $T = 0$:



$\mu = 915$ MeV (dotted), $\mu = 922.7$ MeV (solid), $\mu = 930$ MeV (dashed).

Fixing parameters (2)

- Directly at $\mu = \mu_c$ one has two minima:
  - usual vacuum minimum at $\sigma = f_\pi = 93\,\mathrm{MeV}$
  - nuclear matter minimum at $\sigma = \sigma_{\mathrm{nucl}}$
- The baryon density and vector field condensate have values
  - $n = 0$, $\omega_0 = 0$ at $\sigma = f_\pi$
  - $n = n_{\mathrm{nucl}}$, $\omega_0 = \omega_{0,\mathrm{nucl}}$ at $\sigma = \sigma_{\mathrm{nucl}}$
- From experimental values for
  - nuclear binding energy $\epsilon_{\mathrm{bind}} = -16.3\,\mathrm{MeV}$,
  - nuclear saturation density $n_{\mathrm{nucl}} = 0.153/\mathrm{fm}^3$
  - Landau mass $m_L = \mu_c + g\omega_{0,\mathrm{nucl}} = 0.80\, m_N$

  one can determine
  - the vector Yukawa coupling $g = 9.5$,
  - the critical chemical potential $\mu_c = m_N + \epsilon_{\mathrm{bind}} = 922.7\,\mathrm{MeV}$
  - and the condensates $\sigma_{\mathrm{nucl}} = 69.8\,\mathrm{MeV}$, $\omega_{0,\mathrm{nucl}} = -18\,\mathrm{MeV}$.

Fixing parameters (3)

- Two of the remaining parameters λ, γ_3, γ_4 get fixed by the constraints for a first order phase transition at μ_c

$$U(\sigma_{\mathsf{nucl}}, \omega_{0,\mathsf{nucl}}) = U(f_\pi, 0)$$

and

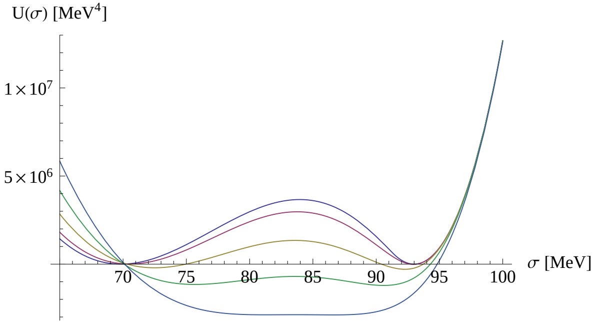
$$\frac{\partial}{\partial\sigma} U(\sigma_{\mathsf{nucl}}, \omega_{0,\mathsf{nucl}}) = 0.$$

- The last parameter can be fixed from other properties of normal nuclear matter.
- The choice $\lambda = 50$, $\gamma_3 = 3$, $\gamma_4 = 50$ gives
  - vacuum mass of σ-meson $m_\sigma = 670\,\mathsf{MeV}$
  - compressibility module $K = \frac{9n}{\partial n/\partial\mu} = 300\,\mathsf{MeV}$
  - surface tension of nuclear droplet $\Sigma = \int_{\sigma_{\mathsf{nucl}}}^{f_\pi} \sqrt{2U(\sigma)}d\sigma = 42000\ \mathsf{MeV}^3$.

  in reasonable agreement with the experimentally established values.

The liquid-gas phase transition

Phase transition can be followed also at $T > 0$. For $\mu = \mu_c(T)$



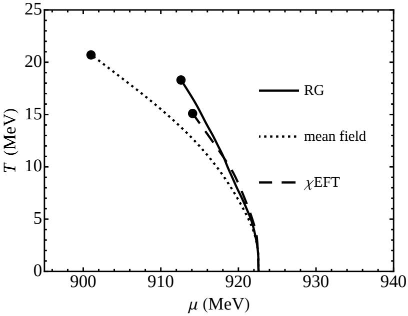
$T = \{0, 5, 10, 15, 20\}$ MeV.

For increasing temperature

- $U(\sigma)$ at minima becomes more negative: pressure p increases
- potential barrier gets smaller: droplet surface tension Σ decreases

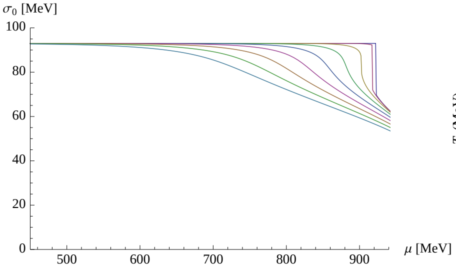
First order line and critical end point

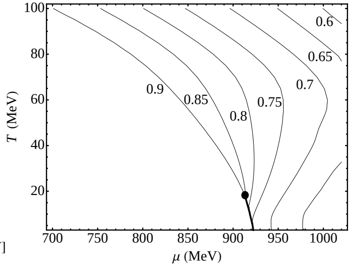
- The first order phase transition line ends in a critical end point at some temperature T_* and chemical potential μ_*.
- The form of the transition line is slightly changed by the effect of bosonic fluctuations.



[Drews, Hell, Klein, Weise, PRD 88, 096011 (2013)]

Chiral order parameter

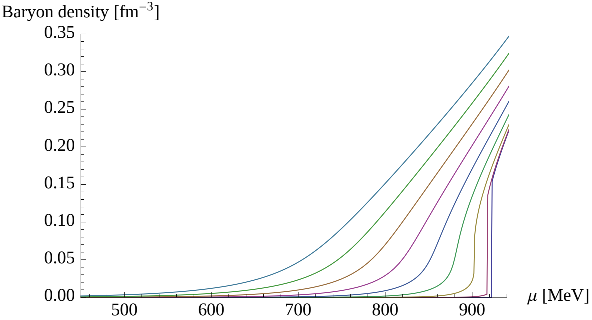




Left: $T = \{0, 10, 20, 30, 40, 50, 60, 70, 80\}$ MeV. Right: Contour plot for σ_0/f_π.

- σ_0 decreases for larger chemical potential and temperature.
- Effective nucleon mass $m_{N,\text{eff}} = h\sigma_0$ gets smaller.
- First order phase transition becomes quickly rather smooth crossover.

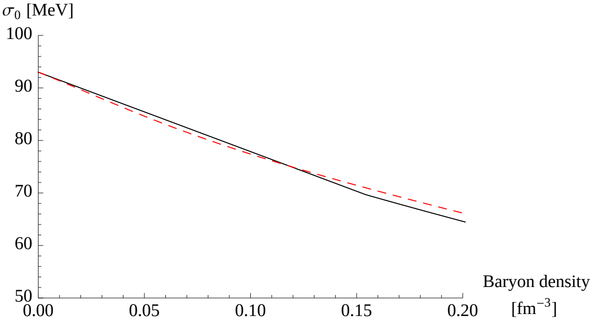
Baryon number density



$T = \{0, 10, 20, 30, 40, 50, 60, 70, 80\}$ MeV.

- n_B increases for larger chemical potential and temperature.
- First order phase transition becomes quickly rather smooth crossover.

Chiral order parameter as a function of density

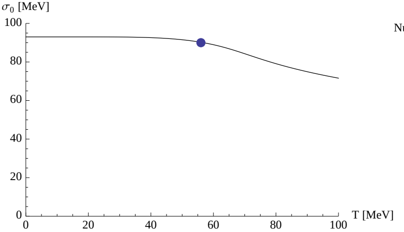


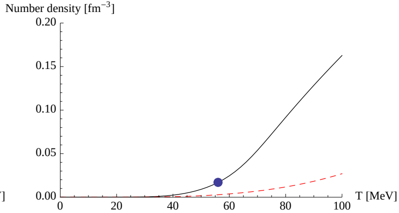
$T = 0$ (solid), $T = 50\,$MeV (dashed).

- Chiral order parameter σ_0 decreases with baryon density.
- Temperature dependence rather small.

Chemical freeze out

- Is there any sign of a phase transition in the region of chemical freeze-out at large μ?
- Consider for example the region around highest baryon density $\mu_{\text{ch}} = 760 \pm 23\,\text{MeV}$, $T_{\text{ch}} = 56^{+9.6}_{-2.0}\,\text{MeV}$.

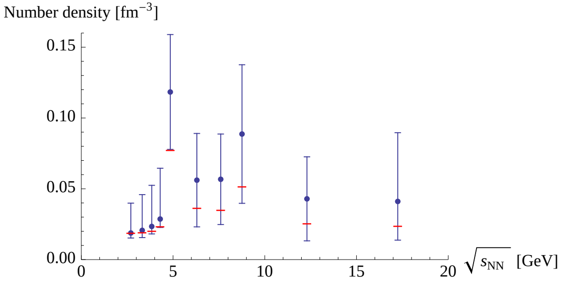




Left: Chiral condensate σ_0 for $\mu = 750\,\text{MeV}$. The dot marks chemical freeze-out.
Right: Baryon number density (solid) and pion number density (dashed).

Chemical freeze-out (2)

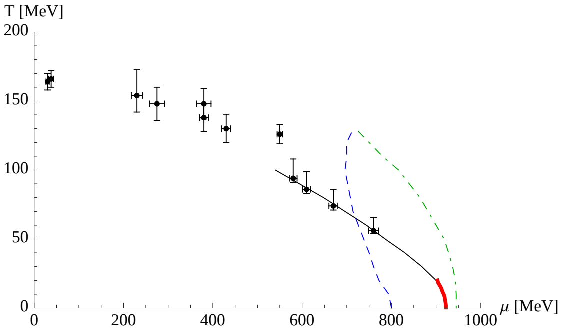
- Freeze-out at large μ does not seem to be related to phase transition or rapid crossover.
- Baryon number density does seem to change quickly there, however.
- Chemical freeze-out at constant number density does make physical sense when rates for strangeness exchange processes depend strongly on density.



Baryon density due to protons, neutrons and Delta baryons in statistical model.

Chemical freeze-out (3)

At small collision energy or high baryon chemical potential chemical freeze-out takes place at constant baryon number density!



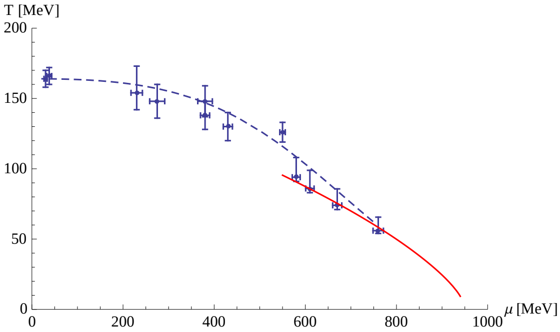
Red line: First order liquid-gas phase transition.
Black line: Constant baryon number density $n_{\text{Baryons}} = 0.15\, n_{\text{nucl}}$.

Conclusions

- "Chemical freeze-out = Phase transition" is too simple.
- No sign of any phase transition or rapid crossover in the region of chemical freeze-out for low-energy experiments.
- Chemical freeze-out seems still related to thermodynamic properties of QCD: $n_{\text{Baryons}} = 0.15\, n_{\text{nucl}}$.
- Chiral nucleon-meson model gives rather detailed and realistic description of nuclear matter as well as thermodynamic properties of QCD at intermediate μ and low temperatures.
- Would also be interesting to determine transport properties i.e. viscosities and conductivities.
- Bottom-up approach to QCD phase diagram might be explored further, could also be extended to smaller chemical potential.
- More information could come in principle from the fluid dynamical evolution of fluctuations in hydrodynamical variables, in particular baryon number density.

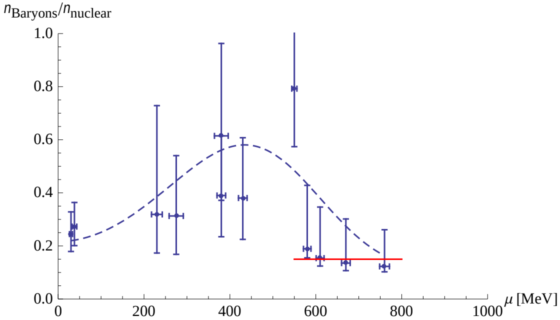
BACKUP

Parametrization in T-μ-plane is dangerous



Parametrization of freeze-out curve seems reasonable with respect to temperature and chemical potential...

Parametrization in T-μ-plane is dangerous (2)



...but is quite far from data points for baryon density.