UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Heavy Quarkonium from the Lattice

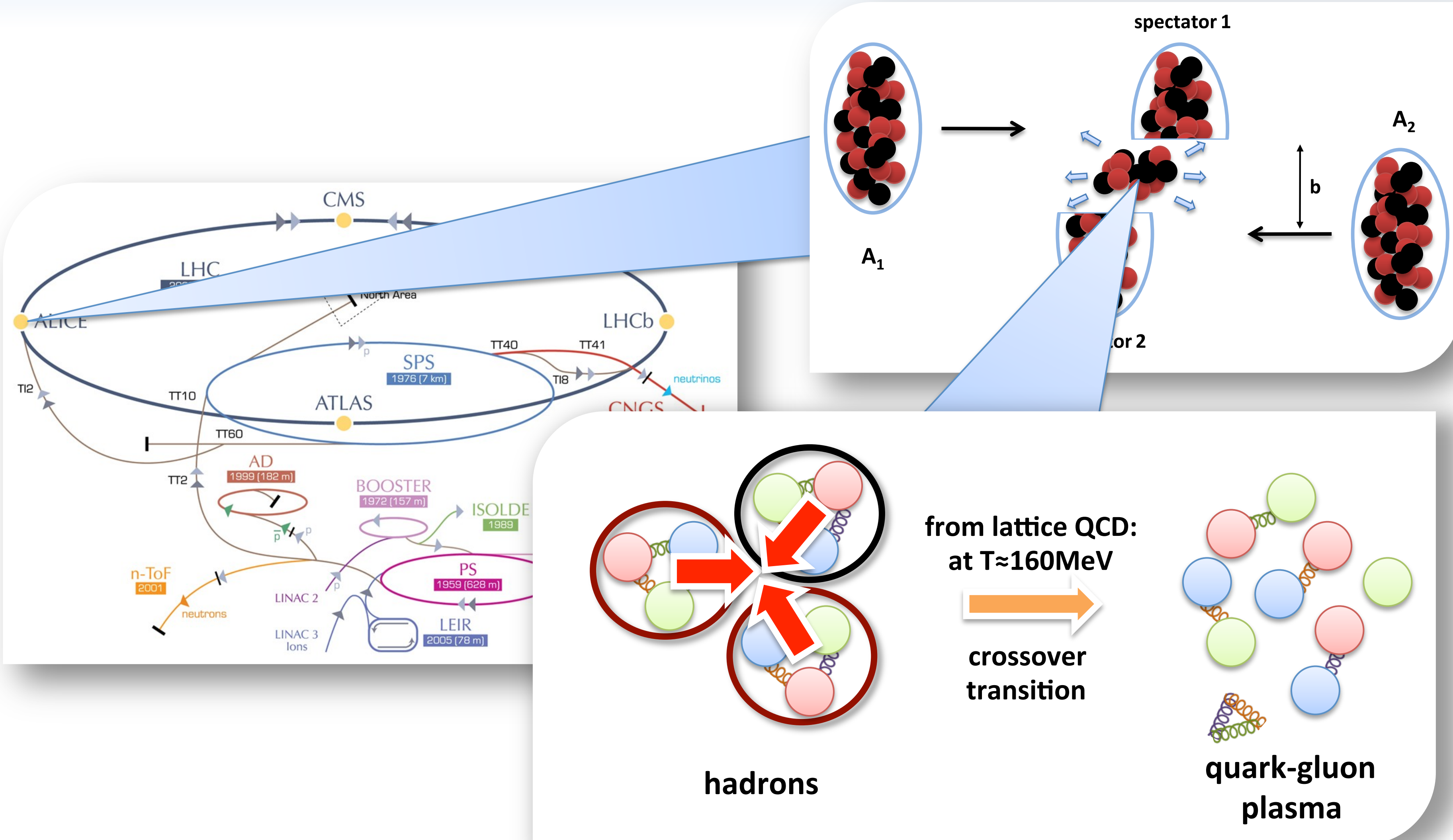
Alexander Rothkopf
Insitute for Theoretical Physics
Heidelberg University

2014/04/09 EMMI Seminar @ GSI

Deutsche
Forschungsgemeinschaft

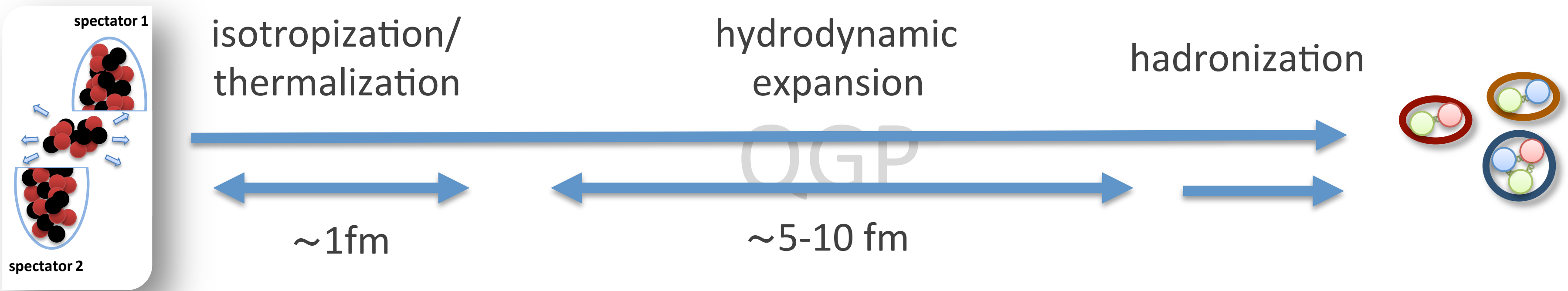
Physics Motivation: Heavy Ion Collisions

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386



A tale of several scales

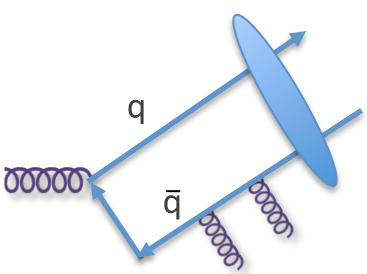
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



- Probes that are susceptible to medium but distinguishable from it: $Q_{probe} >> T_{med}$

Bound states of $c\bar{c}$ or $b\bar{b}$:	**Heavy quarkonium**	$m_Q >> T_{med}$

expectation: $\tau_{form} \sim 1/\langle E_b \rangle$ $\tau_{therm} \sim m_Q/T\ \tau^{QGP}_{therm}$

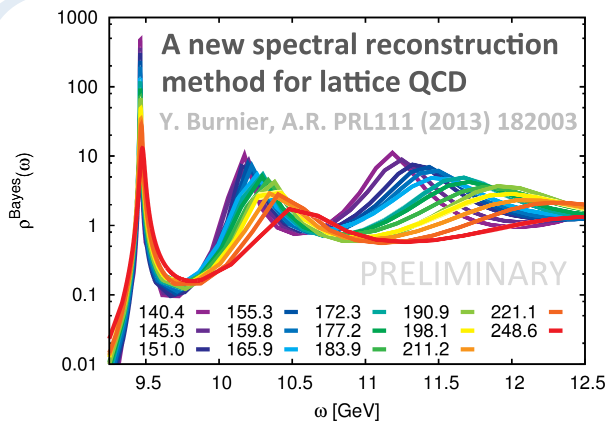


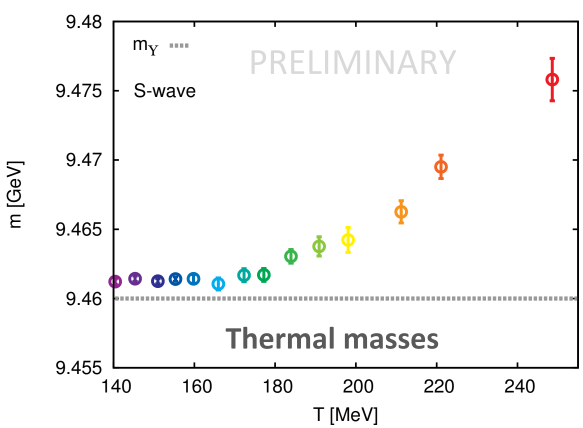
$m^{J/\psi}=3.1$ GeV $\tau^{J/\psi}_{form} \approx 0.4$ fm ⇨ Slow formation, fast thermalization

$m^{\Upsilon}=9.46$GeV $\tau^{\Upsilon}_{form} \approx 0.18$ fm ⇨ Fast formation, slow thermalization

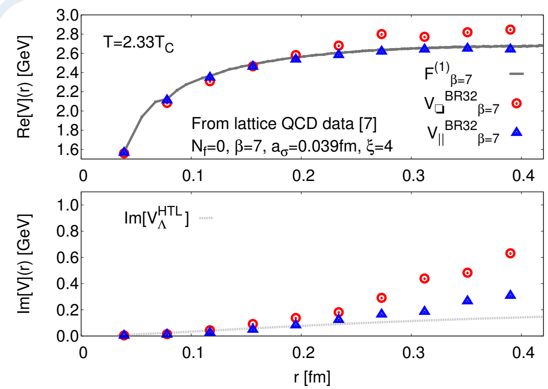
A selection of recent developments

- Towards a dependable in-medium heavy quarkonium picture from the lattice:



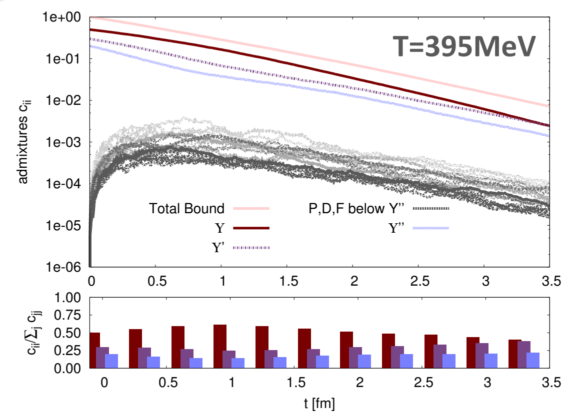


S.Kim, P.Petreczky, A.R. POSLAT 2013 and work in progress



Lattice QCD based complex real-time potentials

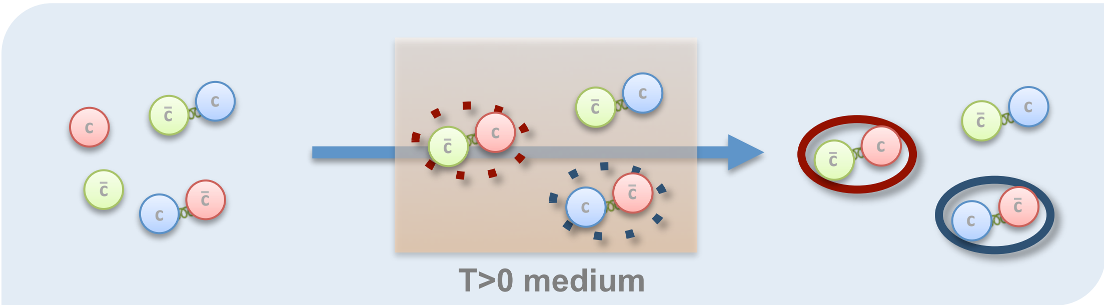
A.R., T. Hatsuda, S.Sasaki Phys.Rev.Lett. 108 (2012) 162001
Y. Burnier, A.R. Phys.Rev. D87 (2013) 11, 114019



Quarkonium real-time evolution in the presence of a complex potential

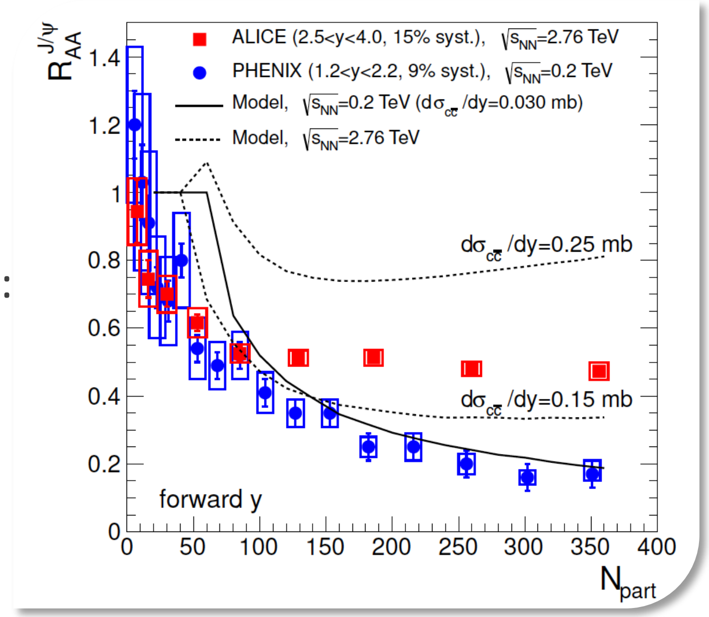
Y. Akamatsu, A.R. PRD85 (2012), A.R. arXiv:1312.3246 t.b.p. in JHEP

Limit 1: Thermal Charm



- Charm quarks become kinetically thermalized
- Question of J/Psi formation Matsui & Satz 1986
- Statistical model reproduces suppression pattern: recombination of J/Psi at QGP phase boundary
- Invites the study of thermal ccbar properties

bound states in spectral functions ...



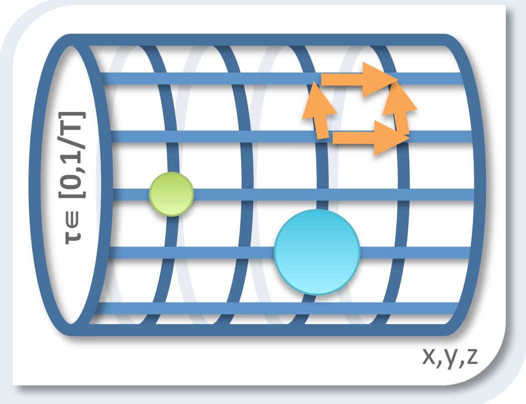
Andronic, Braun-Munzinger, Redlich, Stachel Nucl.Phys.A904-905 2013

Charmonium on the lattice

- Direct simulation feasible since separation of scales not pronounced
  - ccbar and thermal medium: $a < \frac{1}{2m_c} \approx 0.06\mathrm{fm}$ $\quad \frac{1}{T} = N_\tau a \sim 1\mathrm{fm}$

$$S^E_{LQCD} = \sum_x \sum_{\mu<\nu} \frac{2}{g^2} \mathrm{ReTr}[1 - U_{\mu\nu}(x)] \qquad U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x)$$

$$+ \sum_{x,y} \Delta x^4 \bar{Q}(x) \Big((m_Q + \frac{4}{\Delta x}) \delta_{xy} - \frac{1}{2\Delta x} \sum_{\mu=0}^{\pm 4} (1-\gamma_\mu) U(x) \delta_{x+\nu,y} \Big) Q(y) + \ldots$$



  - Gauge fields as links: $U_\mu(x)=\exp[\, i\, g\, \Delta x_\mu\, A_\mu(x)\,]$

- Monte Carlo simulation Gattringer, Lang, Lect.Notes Phys. 788 (2010) 1-343
  - Generate collection of $U^{(k)}$ with $P[U]=\exp[-S^E_{LQCD}(U)]$

$$\langle O[U] \rangle = \int \mathcal{D}U\, O(U)\, e^{-S^E_{LQCD}} = \lim_{n->\infty} \frac{1}{n} \sum_{k=0}^{n-1} O(U^{(k)})$$

+ non-perturbative, applicable at any T

− heavy quarks costly, only Euclidean time

Heavy quarkonium spectral functions

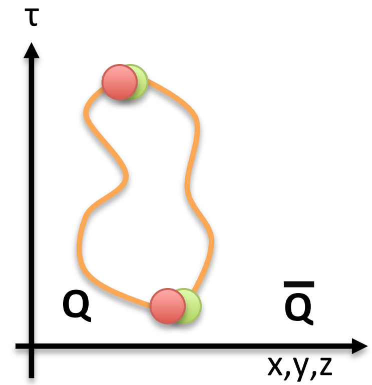
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- What are the available bound states for thermal ccbar pairs?
  - Encoded in the current correlator: $J(x,\tau)=\bar{Q}(x,\tau)\,\Gamma\,Q(x,\tau)$

$$G^E(\mathbf{x},\tau)=\sum_{\mathbf{x}_0}\langle J(\mathbf{x},\tau)J^\dagger(\mathbf{x}_0,0)\rangle$$

  - Spectral function $\rho(\omega)$:

$$G^E(\tau)=\int_0^\infty \frac{\cosh[w(\tau-\frac{1}{2}\beta)]}{\sinh[\frac{1}{2}\omega\beta]}\,\rho(\omega)\,d\omega$$



- An ill-defined inverse problem:

$$N_\tau \sim \mathcal{O}(10-100) \quad \ll \quad N_\omega \sim \mathcal{O}(10^3-10^4)$$

$$G^E(\tau_i)=\frac{1}{n}\sum_{k=1}^{n} G^E_k(\tau_i)$$

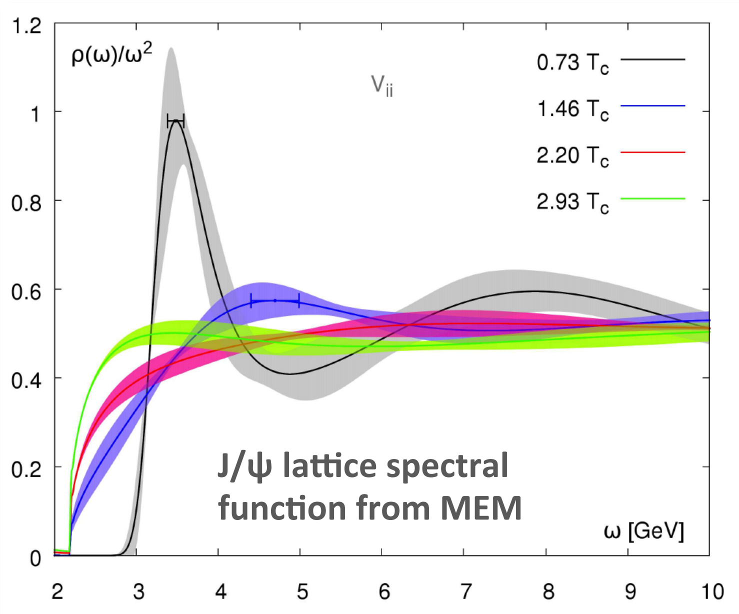
  - Use prior knowledge to regularize χ^2 fitting
  - Maximum Entropy Method: a particular choice of regularization functional

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459-508

Charmonium lattice spectra

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

H.T. Ding, et.al, Phys.Rev. D86 (2012) 014509



- J/Psi in a gluonic medium
  - $a \approx 0.01$ fm $a^{-1} \approx 19$ GeV >> $m_c \approx 1.3$ GeV
  - Nx=128 L = Na > 1fm
  - Bound state peak disappears for $T > 1.5T_C$
  - systematic uncertainties from MEM since T changed by N_τ=96, 48, 32, 24

+ first principles QCD information, allows comparison to dilepton rates

− comparison to experiment only in thermal equilibrium, requires high precision correlator data, dependence on prior information

A recent improvement on the MEM

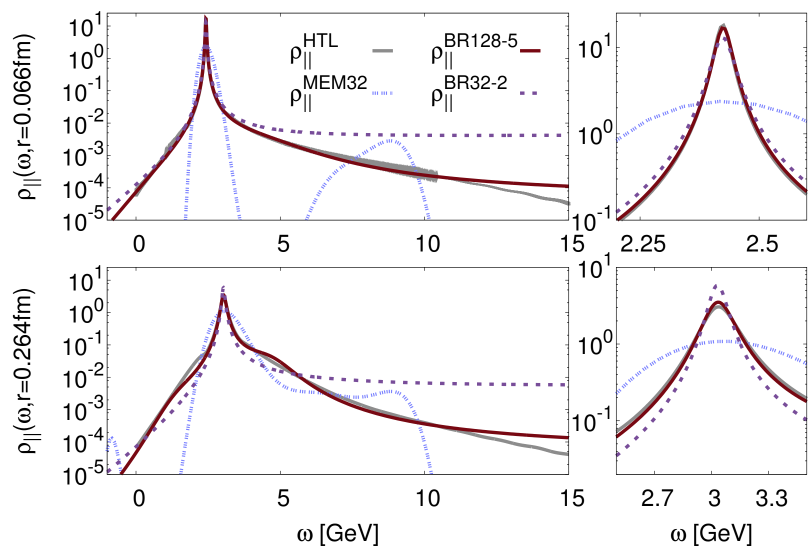
UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Limitations of the standard MEM:
  - N_τ artificially limits resolution & regularization functional has flat directions
- New Bayesian prescription that assumes only: ρ>0 and smoothness

Y. Burnier, A.R. Phys. Ref. Lett. 111 (2013) 182003

Mock testing

- Generate Euclidean datapoints from known ρ (gray)
- Add Gaussian noise and feed to reconstruction algorithm
- Compare outcome to mock spectrum

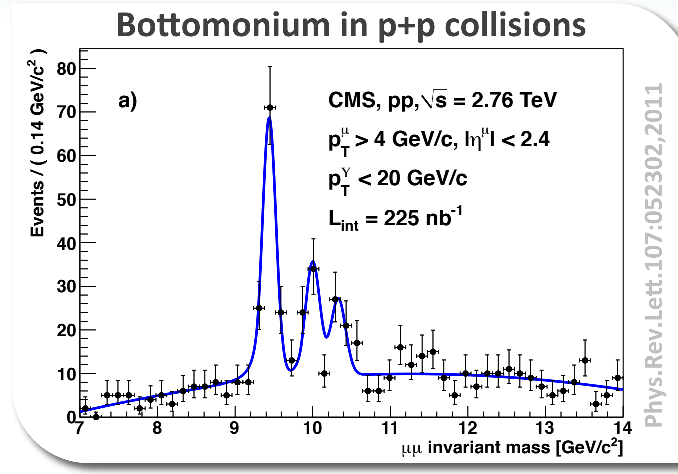


➕ Consistently outperforms MEM in mock analyses

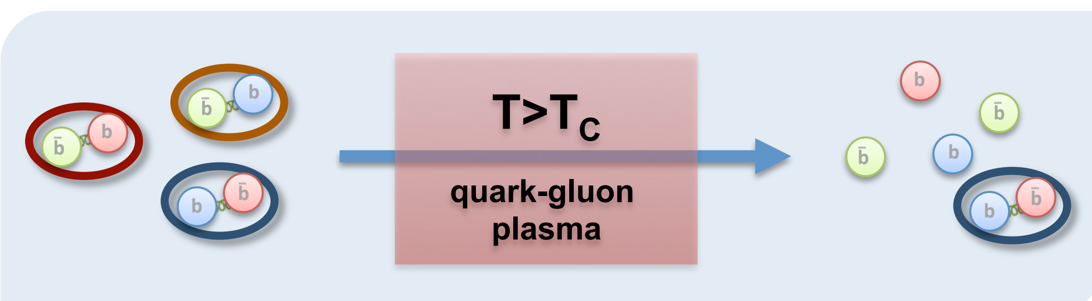
➖ Does not cope well with kinks in ρ

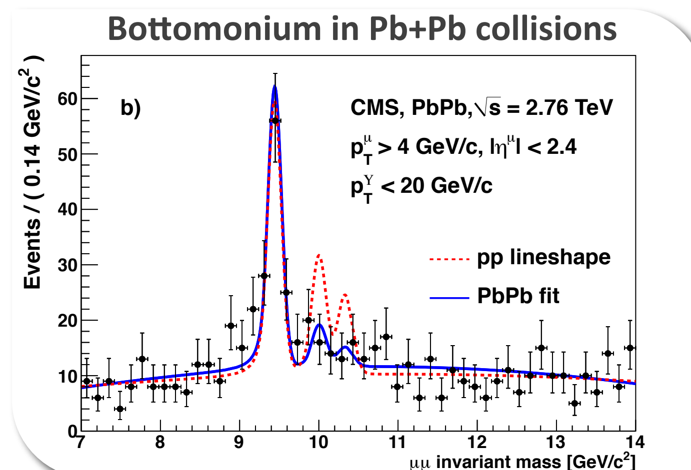
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Limit 2: Bottomonium as a test particle



- Non-thermal bound states traverse the bulk as probe
- Long lifetime: decay outside the medium (OZI rule)





- **Goal: Real-time in-medium evolution of Bottomonium**

Bottomonium and the lattice

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Direct simulation too costly since separation of scales much more pronounced
  - bbar and thermal medium: $a < \frac{1}{2m_b} \approx 0.02\mathrm{fm}$ $\qquad \frac{1}{T} = N_\tau a \sim 1\mathrm{fm}$
- **Goal**: use separation of scales to treat heavy quarks differently $\qquad \frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$
  - **Tools**: Effective field theory and Lattice QCD

$$\mathcal{S}_{QCD} = \int d^4x \left[\bar{Q}(x)\Big(i\gamma^\mu D_\mu(x;A) - m_Q\Big)Q(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right]$$

- Foldy-Tani transformation: expand heavy part in powers of $1/m_Q$ $\qquad Q = \begin{pmatrix} \xi \\ \chi \end{pmatrix}$

$$\mathcal{S}_{NRQCD} = \int d^4x \left[\xi^\dagger(x)\Big(i\partial_t - gA^0(x) + \frac{\Delta}{2m_Q} - \frac{g}{2m_Q}\sigma\cdot\mathbf{B} - m_Q\Big)\xi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \chi^\dagger(..)^\dagger\chi \right]$$

Deriving a potential for heavy quarkonia

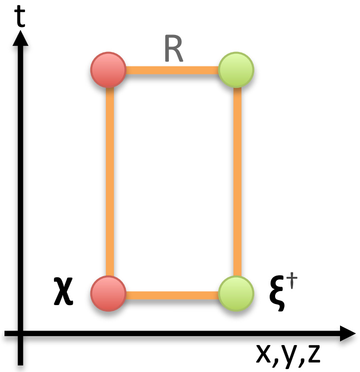
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Real-time evolution of two test quarks:
  - Use heavy meson operators: $M(x,y,t)= \chi^{\dagger}(x,t)\, \sigma^i\, U(x,y)\, \xi(y,t)$

$$D^{>}(R,t) = \langle M(x,y,t)\, M^{\dagger}(x,y,0)\rangle_{\text{med}}$$

  - In the static limit: $D^{>}$ becomes the **real-time Wilson loop**

$$D^{>}(R,t) \overset{m\to\infty}{=} W_{\square}(R,t) = \langle \mathrm{Tr}\Big(\exp\Big[-ig\int_{\square} dz^{\mu} A_{\mu}(z)\Big]\Big)\rangle$$



see e.g.: Barchielli et. al. Nucl.Phys. B296 (1988) 625

  - Potential emerges at late times: qqbar timescale much slower than gluons

$$i\partial_t W_{\square}(R,t) \overset{t\to\infty}{=} V^{QCD}(R) W_{\square}(R,t)$$

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_{\square}(R,t)}{W_{\square}(R,t)}$$

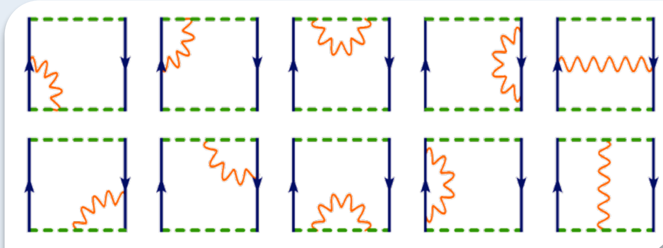
UNIVERSITÄT HEIDELBERG
ZUKUNFT
SEIT 1386

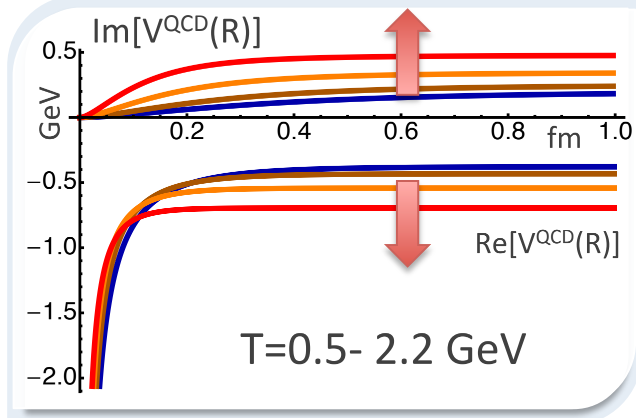
The high temperature potential

- $T \gg T_C$: Asymptotic freedom of QCD allows weak coupling evaluation

Laine et al. JHEP03 (2007) 054;
Beraudo et. al. NPA 806:312,2008

$$W_{\square}(R,t) =$$





$$V^{QCD}_{HTL}(R) = -\frac{gC_F}{4\pi}\left[m_D + \frac{e^{-m_D R}}{R}\right] - i\frac{g^2TC_F}{4\pi}\phi(m_D R)$$

$$\phi(x) = \int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin[zx]}{zx}\right]$$

Debye screening **Landau damping**

  - Real part from Debye screening: presence of deconfined color charges
  - Imaginary part from Landau damping: Scattering of medium partons with the QQbar

- Presence of Im[V] is a QCD result not a model assumption

Extracting V^{QCD} from lattice QCD

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Real-time not directly accessible!
- How to connect to the Euclidean domain: **spectral functions**

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau}\rho_\square(R,\omega)$$

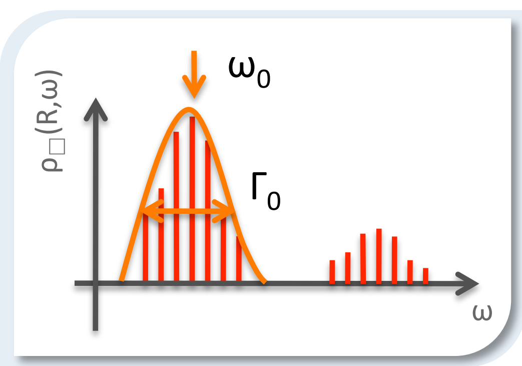
A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{i\partial_t W_\square(R,t)}{W_\square(R,t)} = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t}\rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\rho_\square(R,\omega)}$$

Bayesian spectral analysis

Y.Burnier, A.R. PRL 111 (2013) 182003

- Intuitive relation between spectrum and potential



$$\rho_\square(R,\omega) \propto \frac{\Gamma_0(R)}{(\omega_0(R)-\omega)^2 + \Gamma_0^2(R)}$$

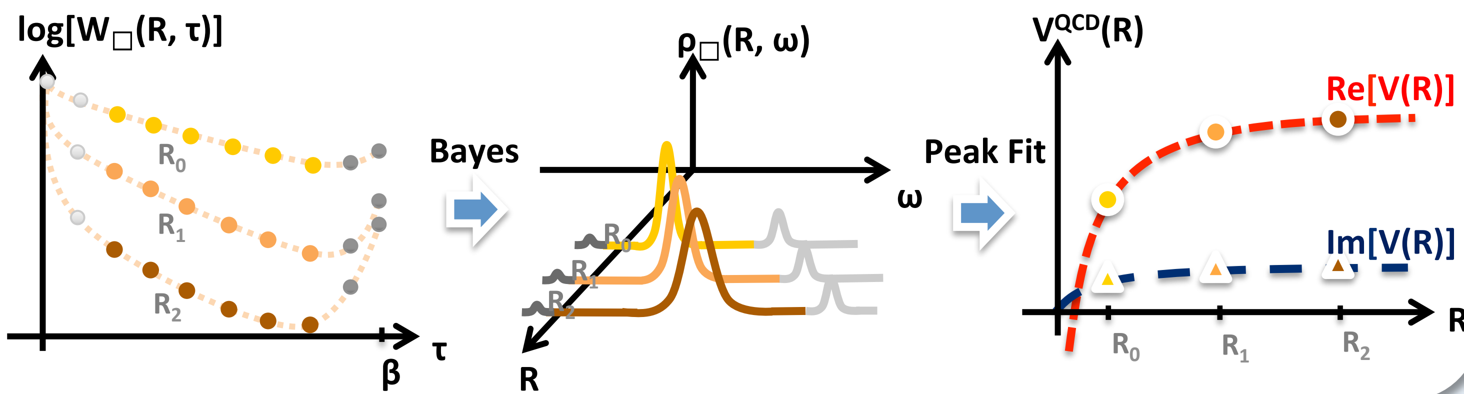
$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

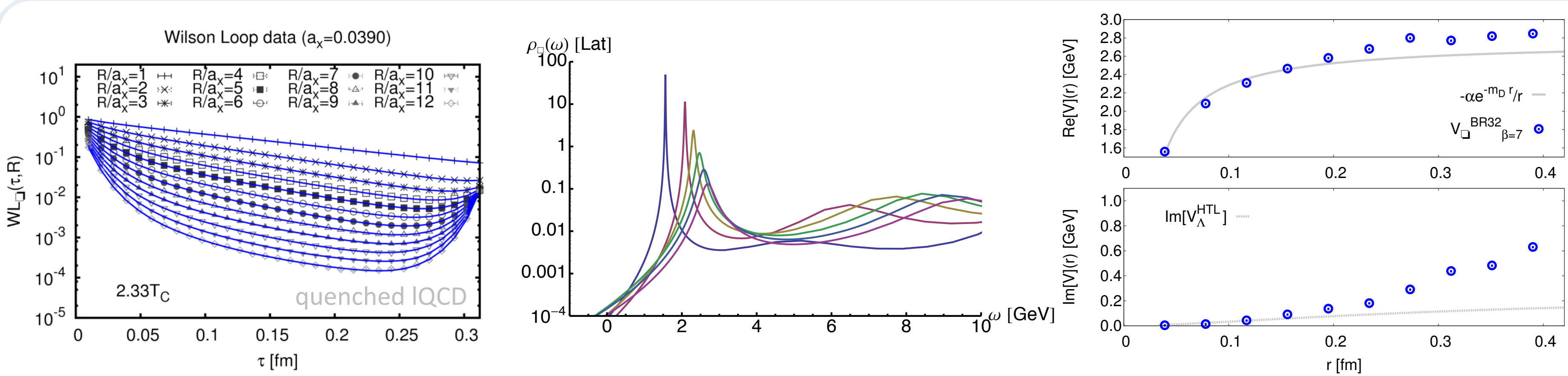
technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

The extraction strategy

- From lattice QCD Euclidean Wilson loops to the complex heavy quark potential

A.R. Mod. Phys. Lett. A, 28, 1330005 (2013)



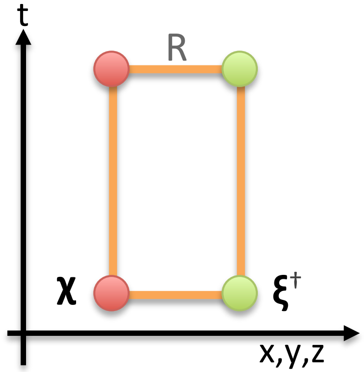


Y.Burnier, A.R. PRL 111 (2013) 182003

An interpretation for the imaginary part

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- What is the meaning of the imaginary part?



$$i\partial_t D^>(R,t) = V^{QCD}(R) D^>(R,t)$$

$$i\partial_t \langle \psi_{Q\bar{Q}}(R,t)\psi^*_{Q\bar{Q}}(R,0)\rangle = V^{QCD}(R)\ \langle \psi_{Q\bar{Q}}(R,t)\psi^*_{Q\bar{Q}}(R,0)\rangle$$

- Evolved object is not the wave-function itself, it is a thermally averaged correlation function
- Heavy quarks cannot annihilate in the non-relativistic approximation
- Correlations with initial state decay over time: **decoherence**

Open Quantum Systems

- Heavy Quarkonium coupled to a thermal medium

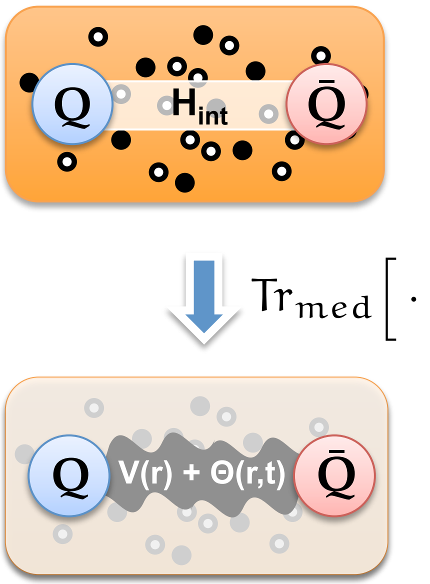
see e.g. H.-P. Breuer, F. Petruccione, Theory of Open Quantum Systems

  - Overall system is closed, hermitian Hamiltonian

$$H = H_{Q\bar{Q}} \otimes I_{med} + I_{Q\bar{Q}} \otimes H_{med} + H_{int}$$

$$\frac{d}{dt}\sigma(t) = \frac{1}{i\hbar}[H, \sigma(t)]$$

unitary evolution: $H=H^{\dagger}$



  - Interested in the dynamics of the QQbar system only

$$\sigma_{Q\bar{Q}}(t, \mathbf{R}, \mathbf{R}') = \mathrm{Tr}_{med}\Big[\sigma(t, \mathbf{R}, \mathbf{R}')\Big]$$

$$\frac{d}{dt}\sigma_{Q\bar{Q}}(t) = \mathcal{L}\sigma_{Q\bar{Q}}(t)$$

dynamical map L

  - L can be implemented (unraveled) into stochastic dynamics of the wave function (similar concepts are quantum state diffusion, quantum jumps, etc..)

see also: N.Borghini, C.Gombeaud: Eur.Phys.J. C72 (2012) 2000; 1103.2945

Time evolution of the wavefunction

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

- Construct unitary stochastic time evolution (neglects back reaction on medium)

$$\psi_{Q\bar{Q}}(\mathbf{R},t) = \mathcal{T}\exp\Big[-\frac{i}{\hbar}\int_0^t\Big\{-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{R})+\Theta(\mathbf{R},t)\Big\}\Big]\psi_{Q\bar{Q}}(\mathbf{R},0)$$

Y. Akamatsu, A.R. Phys.Rev. D85 (2012) 105011

$$\langle\Theta(\mathbf{R},t)\rangle = 0, \quad \langle\Theta(\mathbf{R},t)\Theta(\mathbf{R}',t')\rangle = \hbar\Gamma(\mathbf{R},\mathbf{R}')\delta_{tt'}/\Delta t$$

- Use Ito stochastic calculus to write down the Schrödinger equation

$$i\frac{d}{dt}\psi_{Q\bar{Q}}(\mathbf{R},t) = \Big(-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{R})+\Theta(\mathbf{R},t)-i\frac{\Delta t}{2}\Theta^2(\mathbf{R},t)\Big)\psi_{Q\bar{Q}}(\mathbf{R},t)$$

  - Average wavefunction only depends on diagonal correlations

$$i\frac{d}{dt}\langle\psi_{Q\bar{Q}}(\mathbf{R},t)\rangle_\Theta = \Big(-\frac{\nabla^2}{m_Q}+V_{Q\bar{Q}}(\mathbf{R})-\frac{i}{2}\Gamma(\mathbf{R},\mathbf{R})\Big)\langle\psi_{Q\bar{Q}}(\mathbf{R},t)\rangle_\Theta$$

- Since $D^> = \langle\psi(t)\psi^*(0)\rangle$: $\mathbf{V_{QQ}(R)=Re[V^{QCD}(R)]}$ and $\mathbf{\Gamma(R,R)=Im[V^{QCD}(R)]}$

How to quantify excited states melting

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

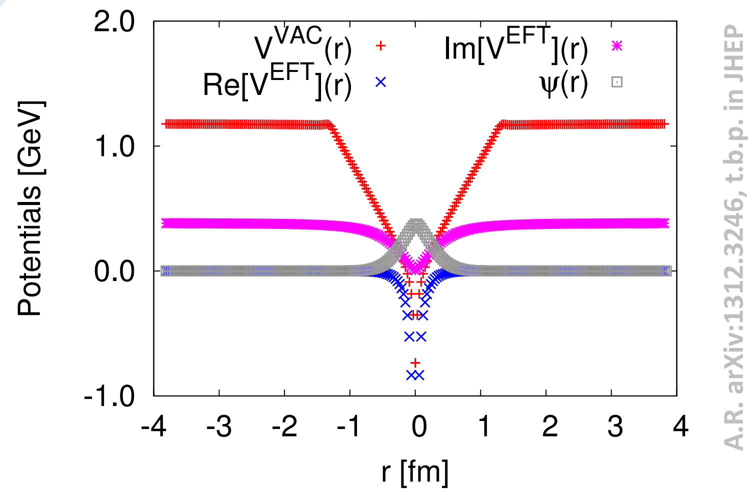
- Survival probability of Heavy Quarkonia (Vacuum: H^{vac} QGP: H)
  - Admixture $|c_{nn}|^2$ of initial bound eigenstates Φ_n of H^{vac} at the current time

$$c_{nn}(t) = \int d\mathbf{R} d\mathbf{R}' \; \Phi^*_n(\mathbf{R}) \; \langle \psi_{Q\bar{Q}}(\mathbf{R}, t) \psi^*_{Q\bar{Q}}(\mathbf{R}', t) \rangle_\Theta \; \Phi_n(\mathbf{R}')$$

  - $\langle\psi(R,t)\psi^*(R',t)\rangle = \sigma(t)$ density matrix of states
  - c_{nn} depends on off-diagonal noise correlations $\Gamma(R,R')$

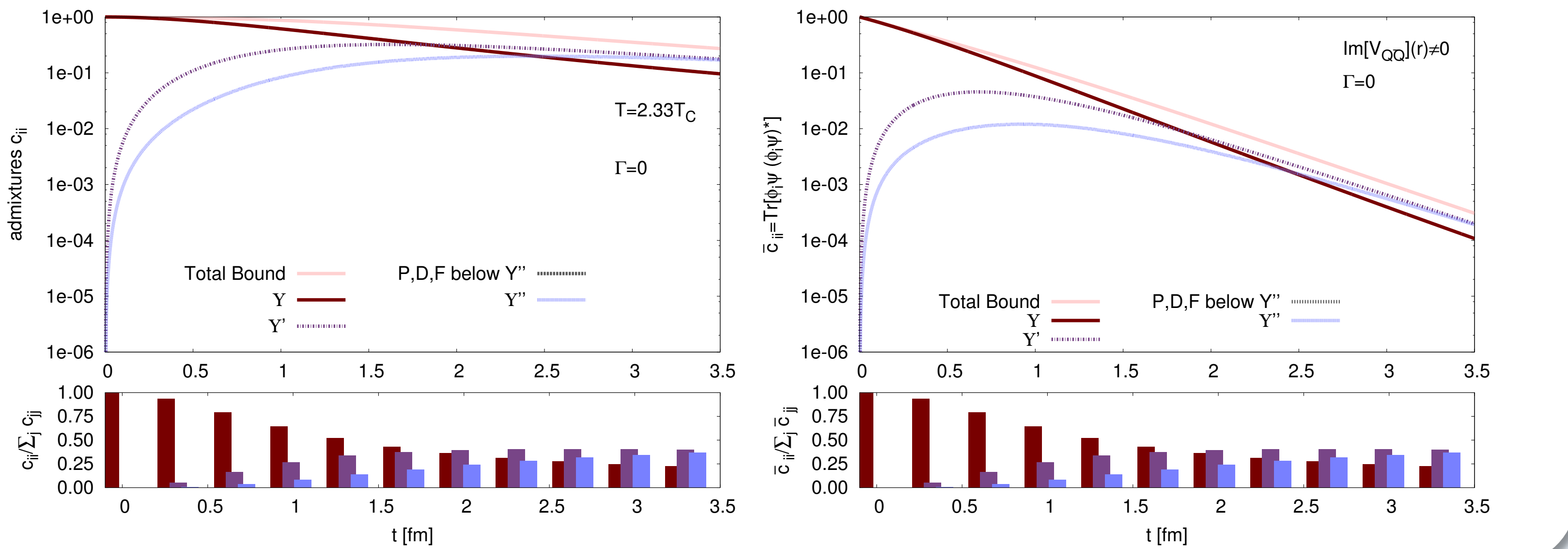
A 3 dimensional numerical simulation

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386



- Vaccuum: Cornell + string breaking (3 bound S)
- Medium: $V_{Q\bar{Q}}(R)$ and $\Gamma(R,R)$ from perturbative potential V^{QCD} at **$T=2.33T_C$**
- Off-digagonal noise: $\Gamma(R,R') \propto \exp[-(R-R')^2/\lambda^2]$ $\lambda=1/T$
- N=256, Crank-Nicolson algorithm: $|\psi(R,t)|=1$

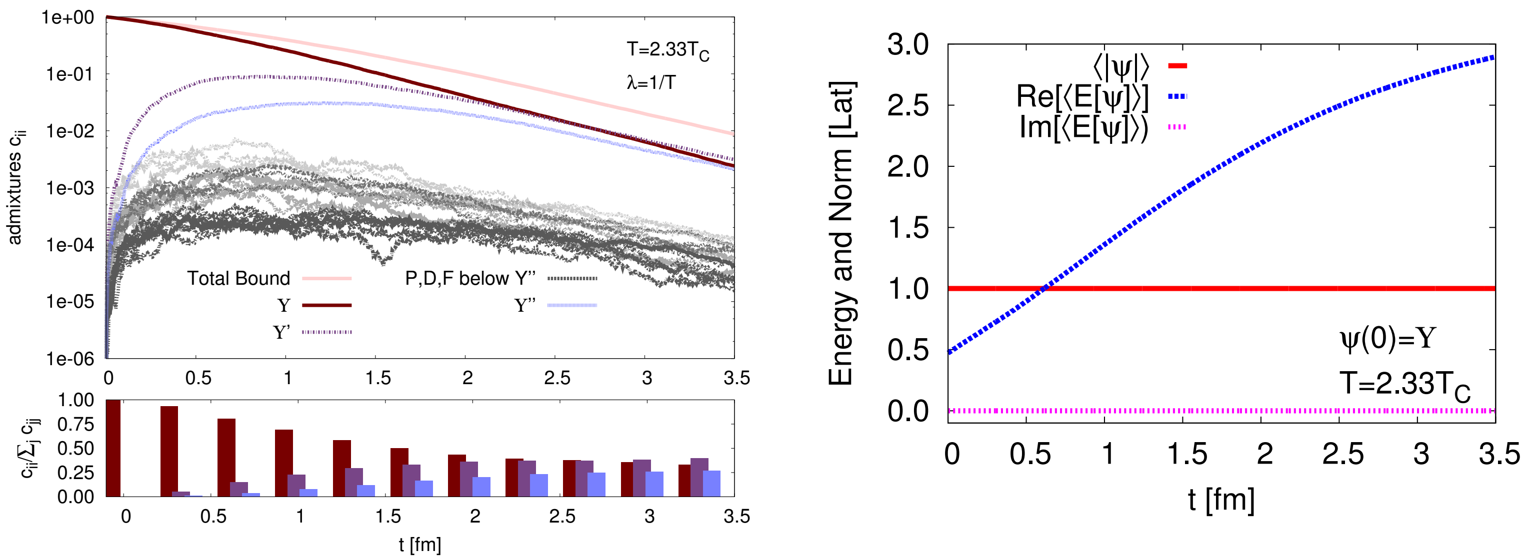
Previous attempts at Upsilon evolution



- Debye screened potential $V_{Q\bar{Q}}(R)=\mathrm{Re}V^{QCD}(R)$ leads to state mixing
  see also.: J. Casalderrey-Solana, JHEP 1303 (2013) 091
- Explicit imaginary part leads to damping of all admixtures: no thermalization
- Im[V]=0 and naïve Im[V]≠0 lead to similar relative abundances
- For naïve Im[V]≠0: energies are complex, contradicts m->∞ quark case

Open Quantum System: Upsilon evolution

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

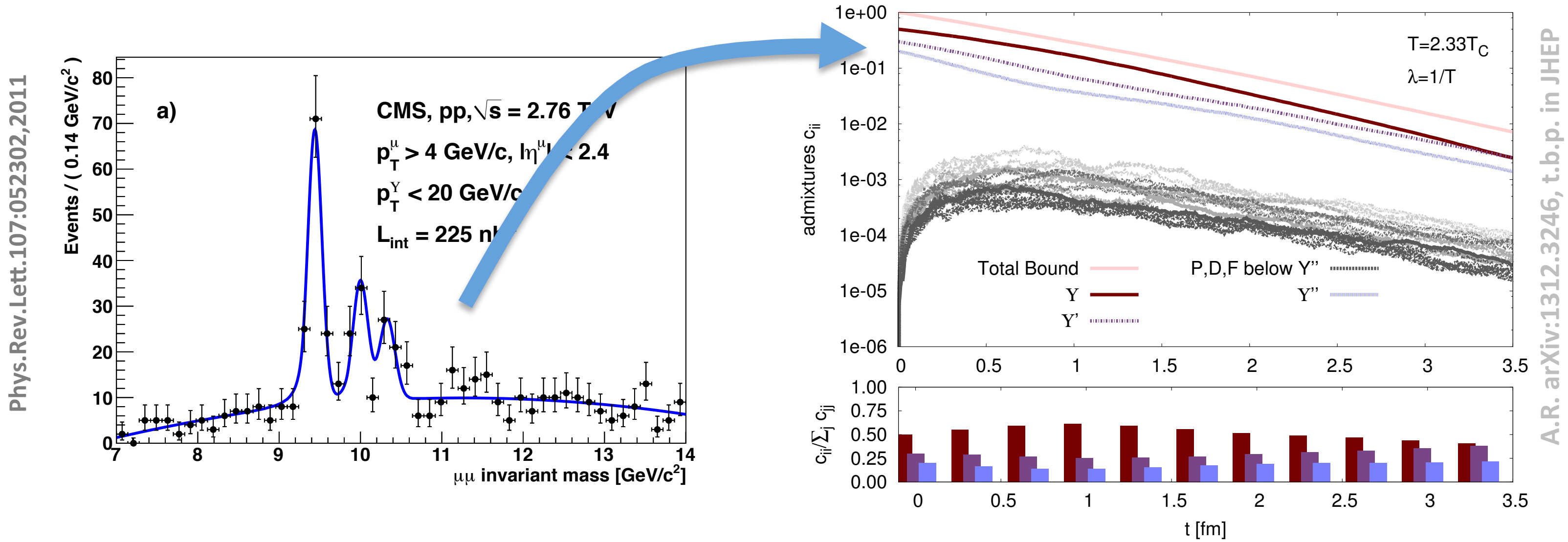


A.R. arXiv:1312.3246, t.b.p. in JHEP

- In the presence of noise: state mixing still important
- Thermal fluctuations : $\Gamma(R,R)=\mathrm{Im}V^{QCD}(R)$ induces additional (de)excitations
- Due to unitary time evolution: all energies remain real
- Distinct difference: ground state is strengthened relative to excited states

Non-thermal ensemble evolution

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



- As a first try: use p+p ratios as initial mixture (ϒ:ϒ':ϒ'' = 5:3:2)
- At early times, indeed suppression of excited states observed
- Regeneration of excited states beyond t>2fm
- Many effects are not yet included: temperature evolution from hydro, …

Conclusions

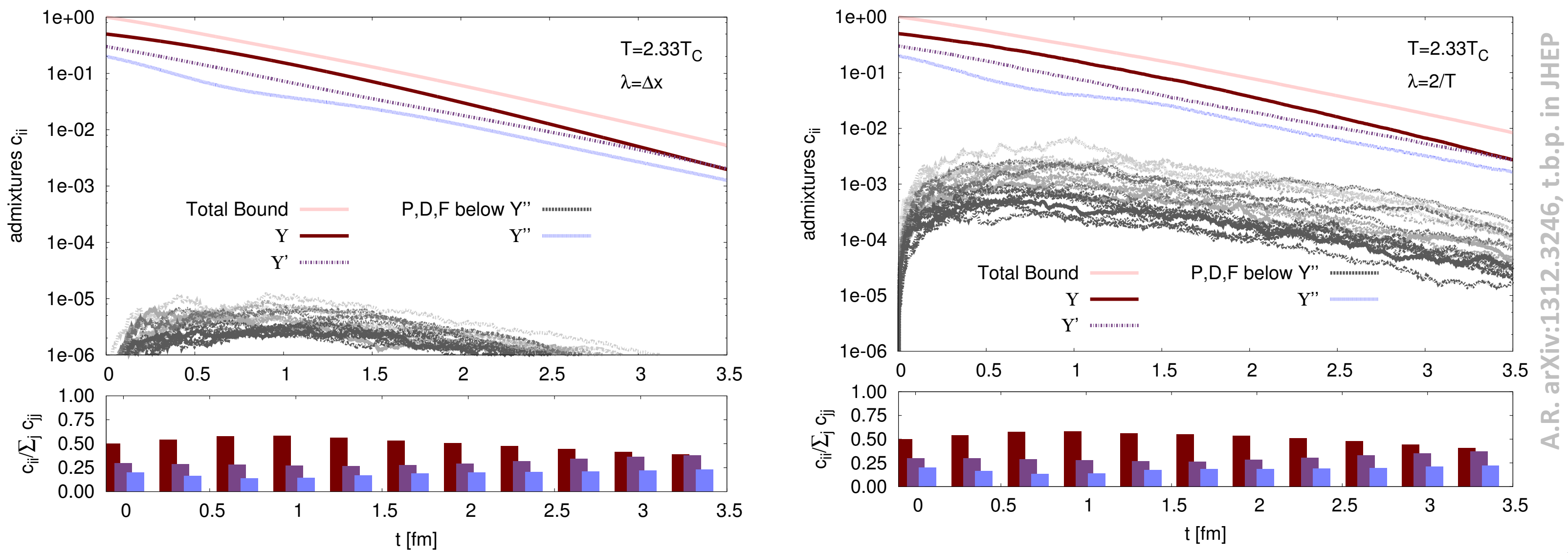
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

- Lattice studies of heavy quarkonium provide non-perturbative physics insight
- Thermal charm: Spectral functions allow the study of **J/Psi formation**
  - New Bayesian approach **improves reconstruction** from lattice QCD
- Bottomonium as test particle: **Potential** based **real-time** description
  - The potential can be derived directly from QCD: at finite T it is **complex valued**
  - Its values can be extracted from the spectral structure of the Euclidean Wilson loop
- Imaginary part in the potential related to **thermal decoherence**
- In **open-quantum systems** approach: thermal fluctuations -> Im[V^{QCD}]
  - Combination of mixing and (de-)excitation: **relative ground state enhancement**

Vielen Dank für Ihre Aufmerksamkeit - Thank you for your attention

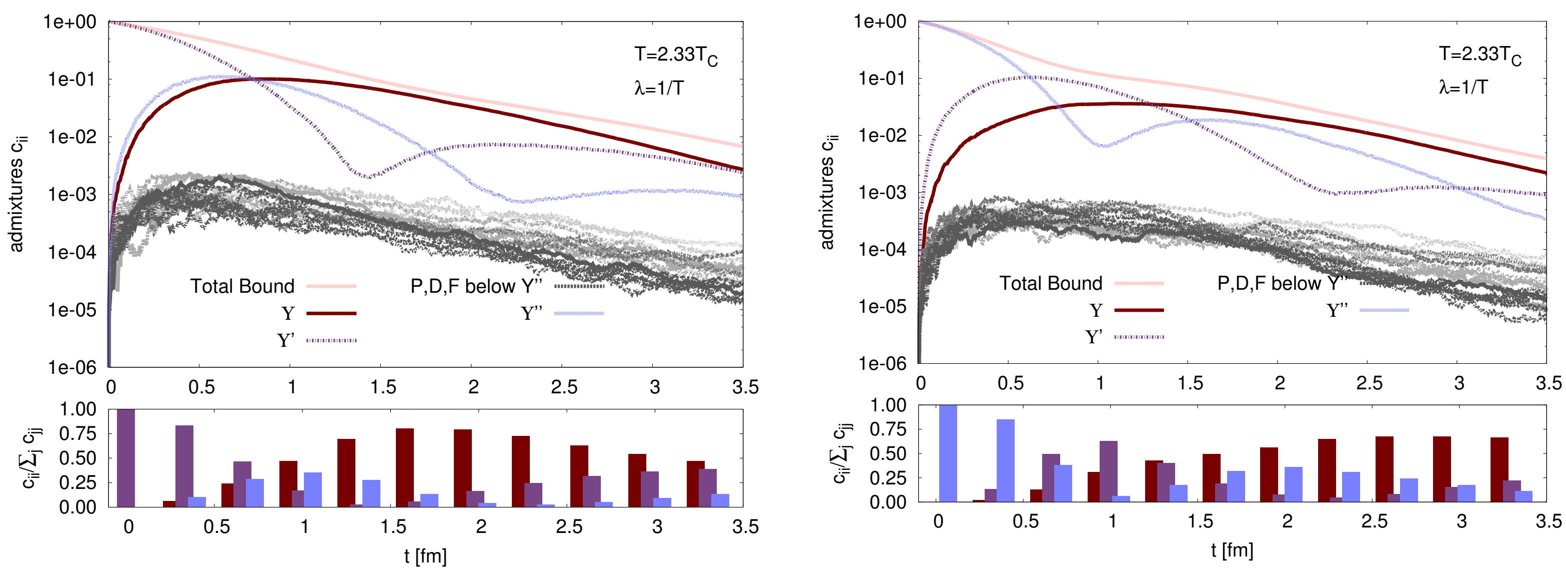
Dependence on the off-diagonal noise

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386



- At T=2.33TC: $\mathrm{Im}V^{QCD}(R)$ is relatively small, change in λ is weak
- Interesting: choice λ=1/T seems to have highest survival of ϒ at short times
- At late times: longer correlations slightly favor ground state survival
- Need to devise a lattice QCD observable from which to extract Γ(R,R')

Excited S-wave states evolution



A.R. arXiv:1312.3246, t.b.p. in JHEP

- Excited states show faster decay time scales than ground state
- Intermediate times dominated by ground state admixture