Phase transitions in fluid dynamical simulations of nuclear collisions

EMMI Nuclear and Quark Matter Seminar 2014

Jan Steinheimer

[1]

17/02/2014

HIC for FAIR Helmholtz International Center

FIAS Frankfurt Institute for Advanced Studies

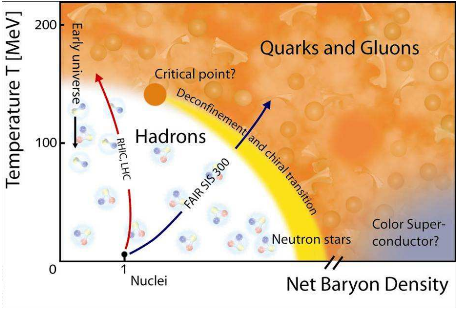
[1] J. Steinheimer and J. Randrup, Phys. Rev. Lett. **109**, 212301 (2012)

1. High Baryon densities: Motivation
   - The Nuclear Liquid-Gas Phase Transition
   - The Hadron-Quark Phase Transition
2. Fluid-Dynamics
   - Formulation and Gradient Term
   - Numerical Tests
3. Results For HIC
   - Moments of the Baryon Density
4. Possible Observables Investigated
5. Summary

Motivation

- Can we learn something about QCD by smashing heavy ions?

Assume a QGP is formed in relativistic HIC at the SPS, RHIC, LHC



Main Questions for HI Physics:

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[2] GSI webpage

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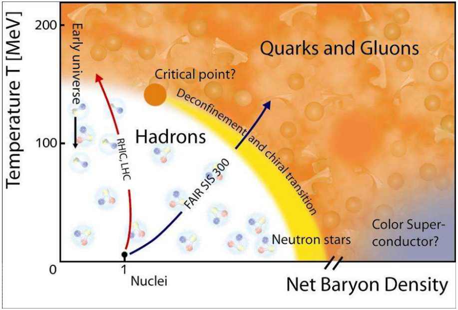

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- Locate the Onset of deconfinement?
- Is it a phase transition or crossover?
- And where is it located?

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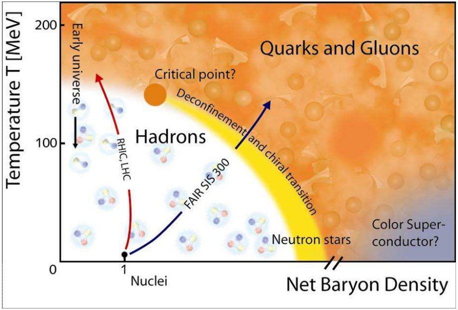

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Main Questions for HI Physics:

- Locate the Onset of deconfinement?
- Is it a phase transition or crossover?
- And where is it located?

If it is a phase transition

What is different in terms of modeling?
E.g. fluid-dynamics.

[2] GSI webpage

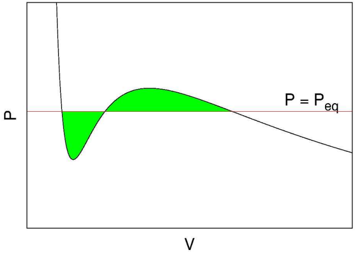
A Phase Transition in Fluid Dynamics

Softening of the EoS

Usually, phase transitions are implemented in fluid dynamics by use of a Maxwell construction.

"In thermodynamic equilibrium,
a necessary condition for stability is that pressure P does not increase with volume V."
" The Maxwell construction is a way of correcting this deficiency."

Basic assumption is **Phase Equilibrium** .
Conditions:
$P_I = P_{II}$, $T_I = T_{II}$, $\mu_I = \mu_{II}$



Isothermal speed of sound vanishes!
Isentropic speed of sound is small!

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Applicability depends on timescales: expansion time vs. phase equilibration time!
OK for general effects of softening.

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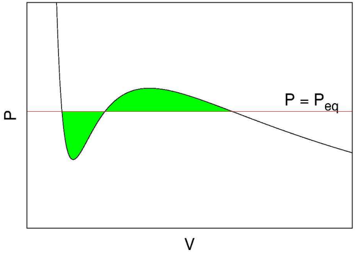

Isothermal speed of sound vanishes!
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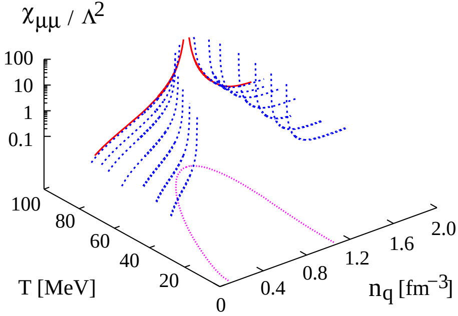
Signals for the Softening

Over the last decades the "softening" of the EoS was the main road pursued in search of the phase transition.

A Phase Transition in Fluid Dynamics

- In a dynamical scenario, locally the system may not be in phase eq.
- Phase separation occurs.
- What about the surface tension; Can be as important as viscosity.



C. Sasaki, B. Friman and K. Redlich, Phys. Rev. Lett. **99**, 232301 (2007)

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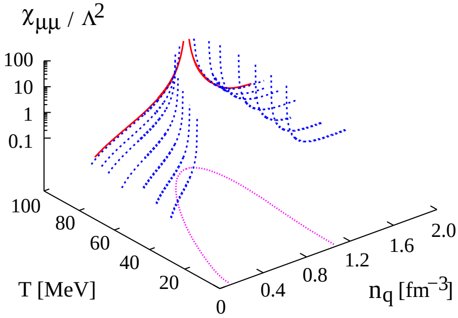


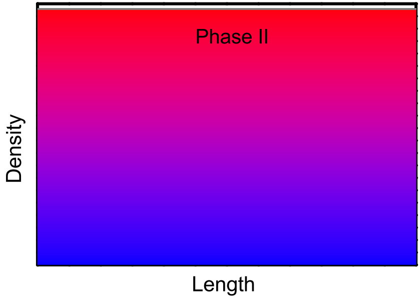
- Susceptibilities diverge due to mechanically unstable phase.
- Separation of the two phases: Spinodal Instabilities.
- It's not the amplitude of the density fluctuation which diverges!
- Is there a straight forward way to describe this with fluid dynamics?

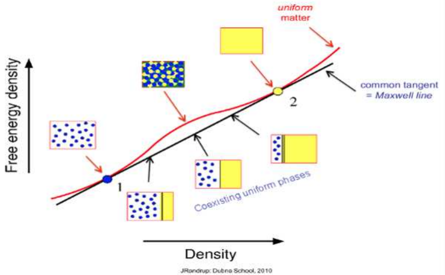
C. Sasaki, B. Friman and K. Redlich, Phys. Rev. Lett. **99**, 232301 (2007)

Non-Equilibrium Phase Transition

Equilibrium Phase Transition (Maxwell construction)

As the system dilutes, the phases are always well separated





Free energy for the different scenarios.

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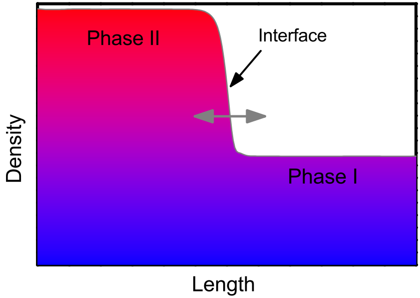

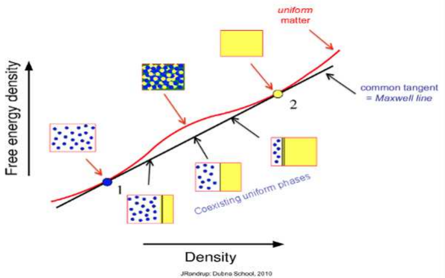

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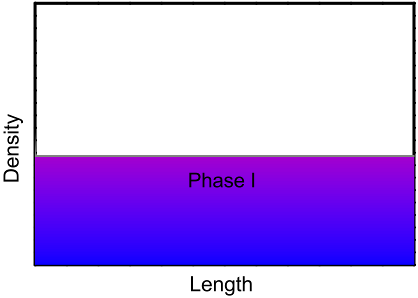

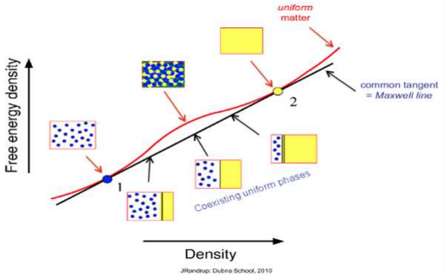

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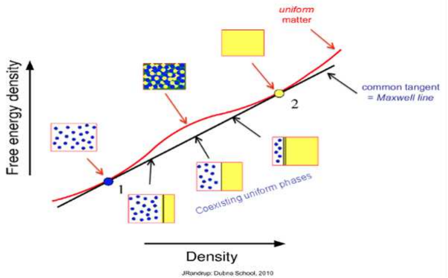

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Non-Equilibrium Phase Transition

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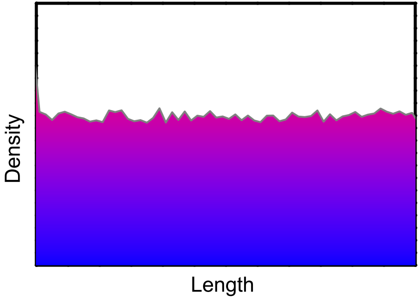

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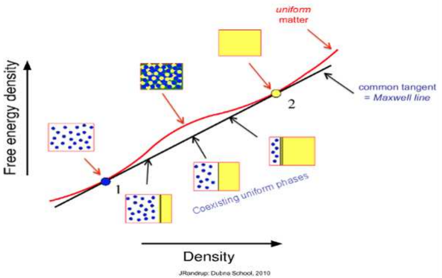

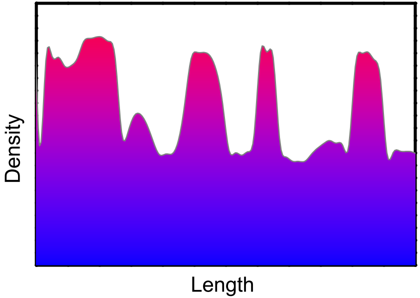

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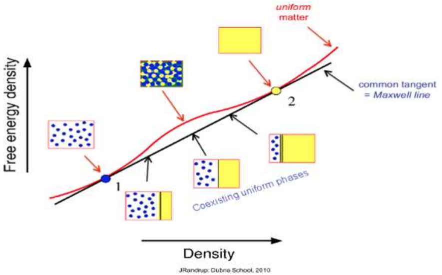
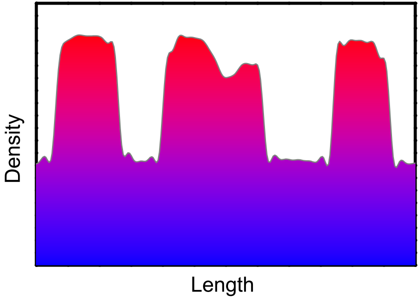


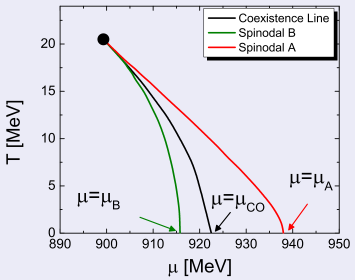


Free energy for the different scenarios.

The Phase Diagram of QCD

How can we describe a first order phase transition in QCD and find observables related to it?

The nuclear Liquid-Gas phase-transition:



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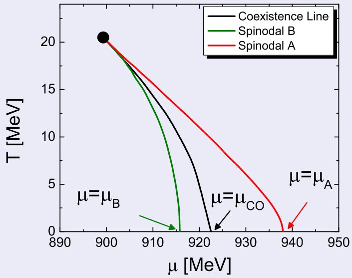

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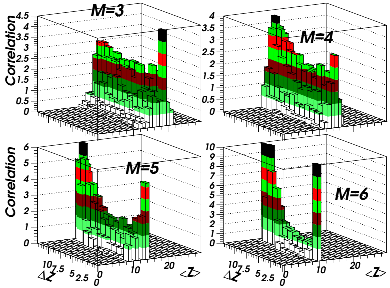


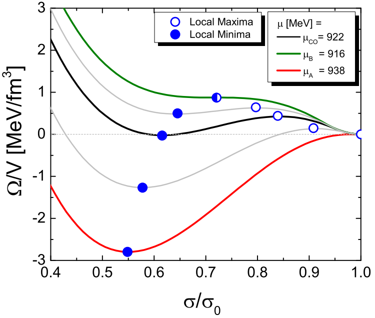
- Study higher-order charge correlations for fragments.
- Observe an enhancement of events with nearly equal-sized fragments.
- Spinodal decomposition in the nuclear Liquid-Gas phase-transition!

[3] B. Borderie *et al.* [INDRA Collaboration], Phys. Rev. Lett. **86**, 3252 (2001)

A Model for the Liquid-Gas P.T.

Take a mean field model: Walecka or hadronic σ-model.

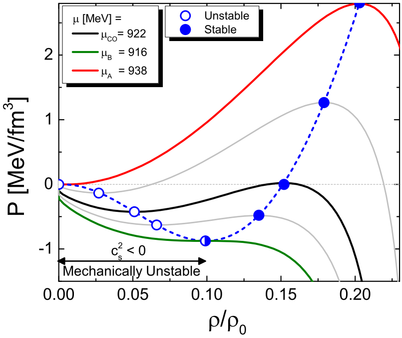
- σ serves as the order parameter.
- Calculate the Grand Canonical potential Ω/V for fixed T and μ as function of σ



- At $T = 0$ and $\mu = \mu_{CO}$, Ω/V has two, equally deep, minima.
- Within the spinodal region always two minima!
- Each corresponding to a meta-stable phase.
- Maxima: unstable!

A Model for the Liquid-Gas P.T.

$P = -\Omega/V$ as a function of density at $T = 0$

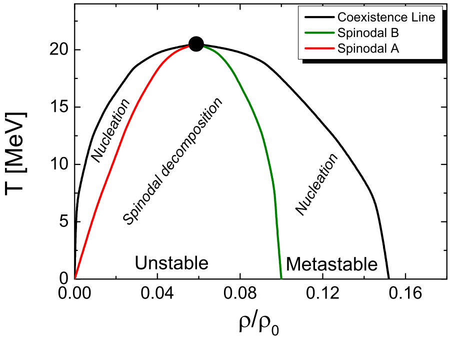


- Curve through the points gives the nuclear EoS.
- Mechanically unstable region defined by imaginary speed of sound

$$\frac{\partial P}{\partial \epsilon}\Big|_{T=0} = c_s^2 < 0$$

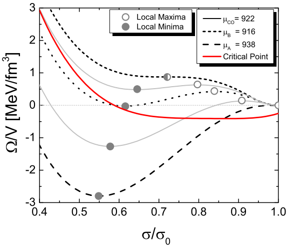
The Phase Diagram in Density

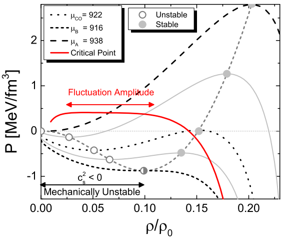
A More convenient representation of the phase diagram is in T and ρ.



- Metastable and Unstable regions visible.
- Note: Order of Spinodals is reversed
- Supercooled and Overheated phases.

The Critical Point





Phase Transition:

- Density domains created dynamically inside the unstable and metastable region $\rightarrow$ Driven by pressure gradient: Timescales similar to expansion times.
- Amplitude of fluctuations favor certain values.
- Thermal fluctuations add but are not necessary

Critical Point:

- Density domains created only at the critical point $\rightarrow$ In equilibrium, the critical point is reached only by precise tuning.
- Amplitudes of fluctuation are not driven by pressure gradient: Timescales?
- Fluctuation on all possible scales
- No creation of fluctuations by the EoS. Driven by field dynamics.

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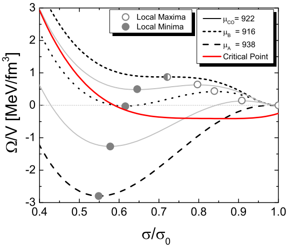

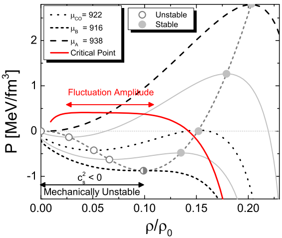

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M.Nahrgang, S.Leupold, C.Herold and M.Bleicher, Phys. Rev. C **84, 024912 (2011)**

Differences to the Liquid-Gas P.T.

The nuclear Liquid-Gas phase transition

- $0-1$ times nuclear ground state density $\rho_0 \approx 0.15-0.16\ fm^{-3}$
- Nuclear ground state stable w.r.t. vacuum
- Several experimental constraints, e.g. binding energy, compressibility.
- Droplets/Fragments can be observed in detector

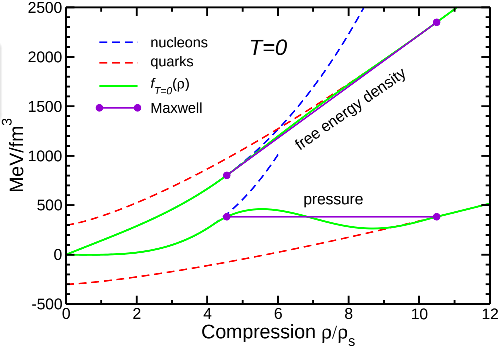
The hadron-quark phase transition

- $2-10?$ times nuclear ground state density
- Quark droplets/domains/clusters stable in dense hadronic medium but unstable w.r.t. vacuum
- Almost no theoretical constraints. Lattice QCD not conclusive (yet?).
- How to observe evidence for droplets/fragments in detector?

Constructing an Effective EoS

Obtain the free energy density $f_T(\rho) = \epsilon_T(\rho) - T s_T(\rho)$ by a spline between a Gas of int. nucleons+pions and a QGP.

$\partial_\rho^2 f_T(\rho) > 0$: (Meta-)Stable
$\partial_\rho^2 f_T(\rho) < 0$: Unstable



Alternatively: Do a Maxwell construction.

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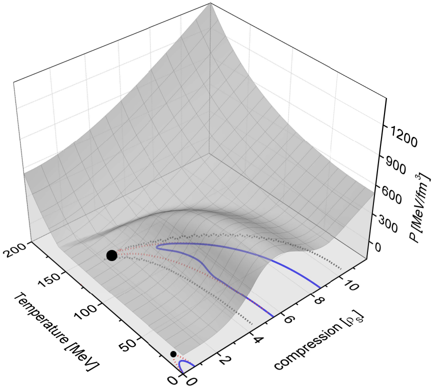
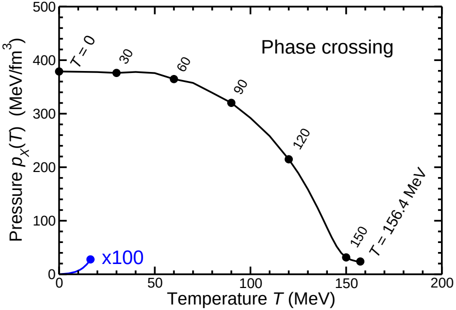


Fluid evolution: $T \neq$ const. but $S/A =$ const.

$c_s^2\big|_T < 0$: Isothermal spinodal
$c_s^2\big|_{S/A} < 0$: Isentropic spinodal

Alternatively: Do a Maxwell construction.

Differences to the Liquid-Gas type P.T.



- Phase diagram in pressure and temperature reveals that they are of different type.
- While the L.-G.- Phase transition is driven by the density, with no change of degrees of freedom
- The Hadron-Quark transition is driven by the entropy, with significant change of degrees of freedom
- Still, both can be a 'true' first order transition including a latent heat and a 'mixed phase region'.

Solving Fluid-Dynamics

The equations for relativistic fluid-dynamics are solved numerically on a grid of cell size $\Delta x = 0.2$ fm:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu N^\mu = 0$$

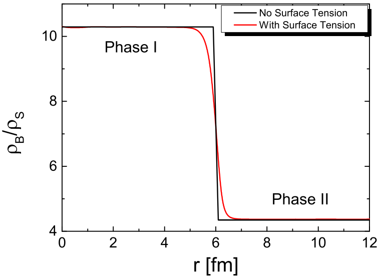
$T^{\mu\nu}$ is the relativistic energy momentum tensor and N^μ the baryon four-current. In ideal fluid-dynamics these can be written as:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \quad \text{and} \quad N^\mu = nu^\mu$$

To close this system of equations one needs the equation of state, of the form $p = p(\epsilon, n)$, as an additional input.
For the numerical solution in 3+1D we will use the SHASTA.

The Gradient Term

A proper description of spinodal decomposition requires that finite-range effects be incorporated.



We rewrite the local pressure as

$$p(\boldsymbol{r}) = p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - a^2 \frac{\varepsilon_s}{\rho_s^2} \rho(\boldsymbol{r}) \boldsymbol{\nabla}^2 \rho(\boldsymbol{r})$$

The gradient term will cause a diffuse interface to develop when matter of two coexisting phases are brought into physical contact.

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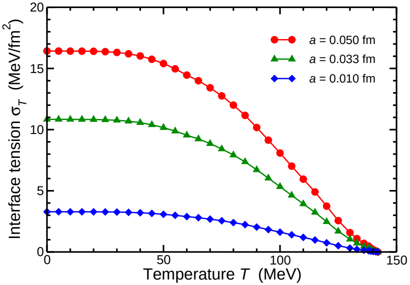

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The associated interface tension can be determined from the EoS,

$$\sigma_T \;=\; a \int_{c_1(T)}^{c_2(T)} \left[2\varepsilon_s \Delta f_T(c)\right]^{1/2} dc$$

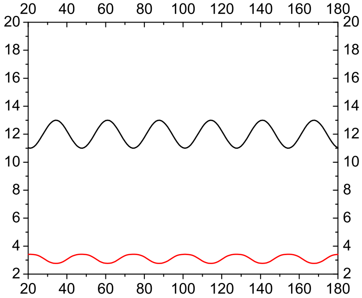
Growth Rates in Numerical Fluid-Dynamics

Calculation in a box with periodic boundaries

The amplitude of a density undulation should grow exponentially within the unstable region

$$A(t) = A_{t=0}(e^{\gamma_k t} + e^{-\gamma_k t})$$

In the stable region the solution is a standing wave.



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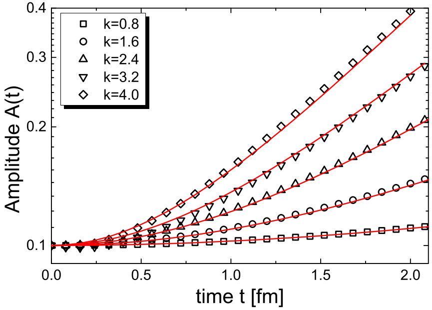

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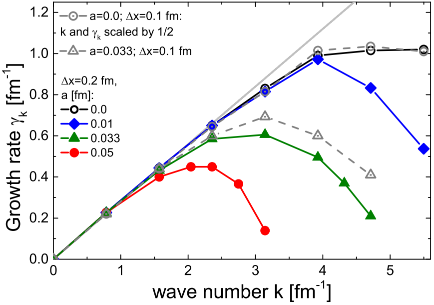


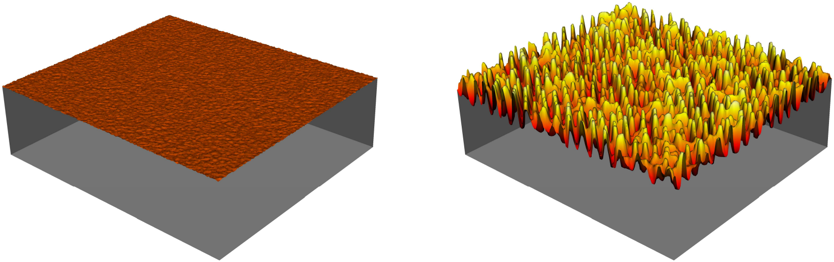
- Numerical viscosity cuts off large wave number growth
- The gradient term modifies the growth rates:

$$\gamma_k^2 = |v_s|^2 k^2 - a^2(\varepsilon_s/h)(\rho/\rho_s)^2 k^4$$

- Depending on a certain wave numbers are favored

Show Animation I

Initialize Random noise in the unstable region and let it evolve.



Show Animation I

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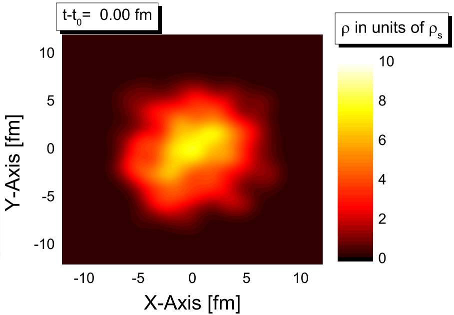
Initial State And Setup

We apply the UrQMD transport model for the initial, non-equilibrium, part of the collision.

- When contracted nuclei have passed through each other,
- energy-, momentum- and baryon densities are mapped onto the computational grid.

$$t_{start} = \frac{2R}{\gamma v} = \frac{2R}{\sqrt{\gamma^2 - 1}} = 2R\sqrt{\frac{2m_N}{E_{\mathrm{lab}}}}$$

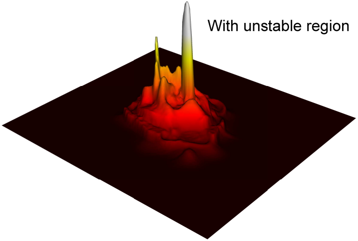


- σ_{ini} controls width of initial fluctuations

$$\epsilon_{\mathrm{cf}}(x,y,z) = Ne^{-\frac{(x-x_p)^2+(y-y_p)^2+(\gamma_z(z-z_p))^2}{2\sigma_{ini}^2}}$$

Evolution in Fluid-Dynamics

EoS with unstable phase:



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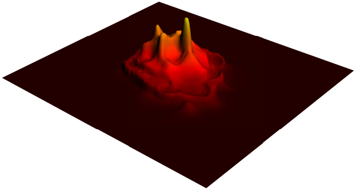

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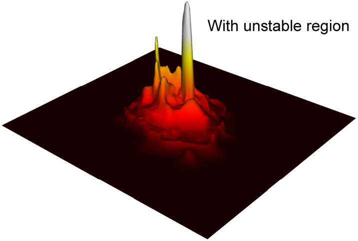

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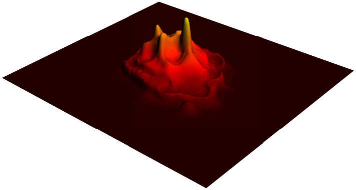
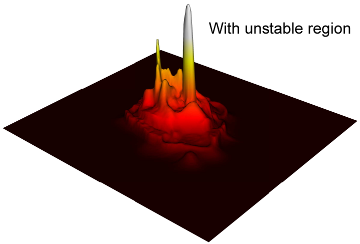
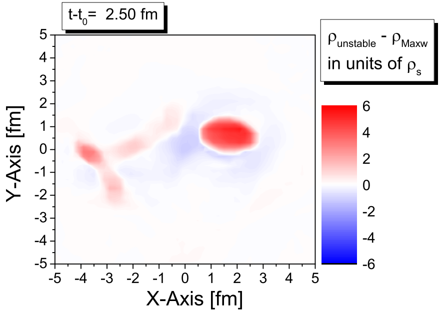


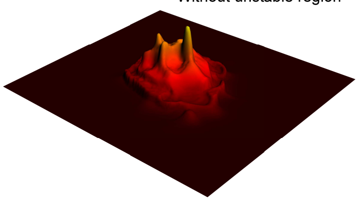
Evolution in Fluid-Dynamics

EoS with unstable phase:





EoS with Maxwell construction:



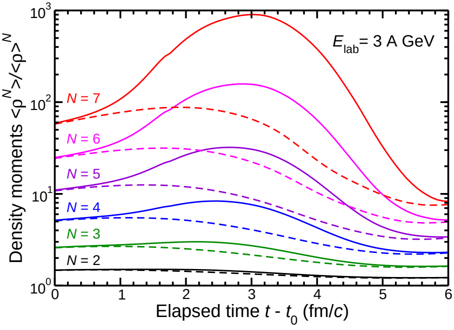
Notable difference observed.
Unstable phase leads to clustering of baryonnumber!

Moments of the Baryon Density

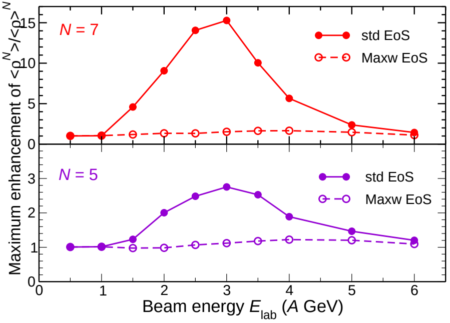
Let's be more quantitative

Define Moments of the net baryon density distribution:

$$\langle \rho^N \rangle \equiv \frac{1}{A} \int \rho(\boldsymbol{r})^N \rho(\mathbf{r})\, d^3\boldsymbol{r}$$



As a function of time



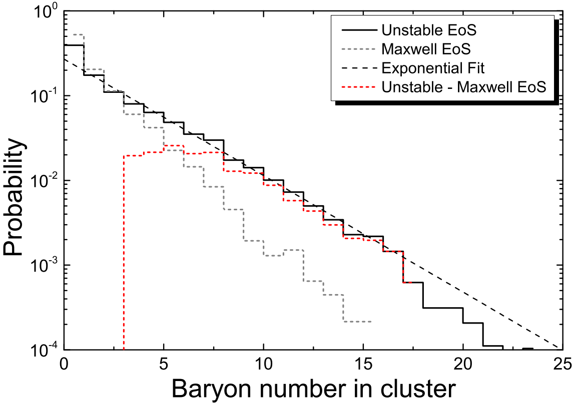
As a function of beam energy

Cluster Distribution

Let's be more quantitative

Define Clusters: A coherent chunk of baryon number with density above

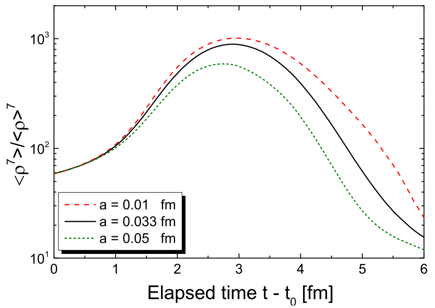
$$\rho > 7\ \rho_S$$



- Probability distribution for cluster size is exponential (at time $t - t_0 \approx 3$ fm)
- Probability to find a large cluster drastically enhanced with unstable phase!

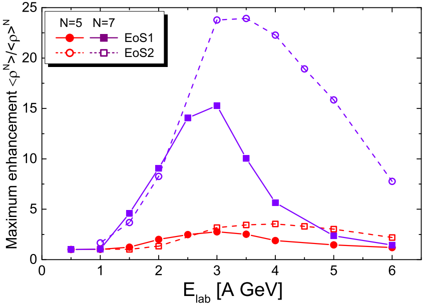
Changing Parameters

We change the value of the surface tension: $0.01 < a < 0.05$.



Quantitative results depend on choice of a.

We change the coexistence densities for the EoS by a factor of 1.3



Beam energy increases slightly.
Robust prediction on energy region.

Nuclear Clusters

Nuclei from Coalescence

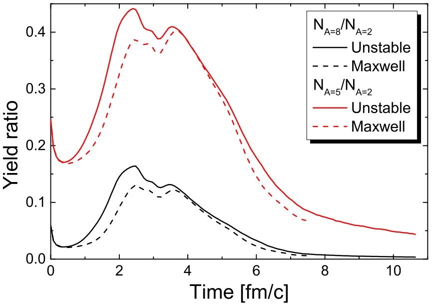
$$E_A \frac{dN_A}{d^3P_A} \propto F_A \left(\left[E \frac{dn}{d^3p} \right]_{p=P_A/A} \right)^A$$

- One might naively expect that the composite production should be enhanced.

Nuclei from Thermal Production

$$N_A = \int d^3p\, d^3r\ f_A(r,p)\ , \tag{1}$$

- One might naively expect that the composite production should be enhanced.
- However, as the local baryon density is being enhanced also the local excitation energy per baryon is increased almost compensating the increase.

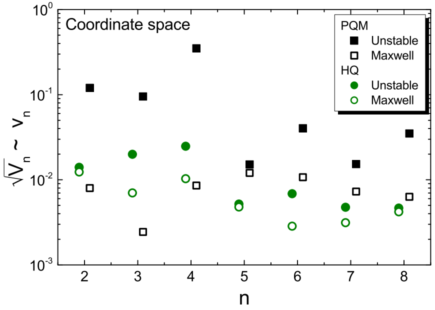


Angular Correlations

Strong irregularities in the angular distribution

Will they lead to large higher moments of the spacial angular distribution of the baryon number?

In position space these can be computed directly from the fluid dynamical simulations.



$$V_n^{\mathrm{pos}} = \frac{1}{N^2} \sum_{i,j=1}^{N} \rho_i \, \rho_j \, \cos(n(\phi_i^{\mathrm{pos}} - \phi_j^{\mathrm{pos}}))$$

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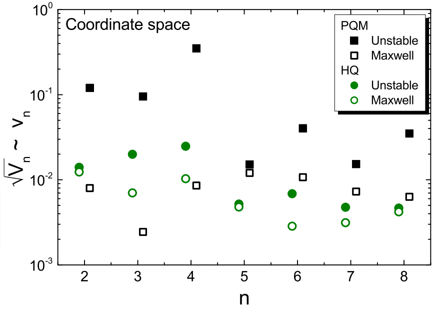
Large enhancement in spacial correlations in the PQM EoS.



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Angular Correlations

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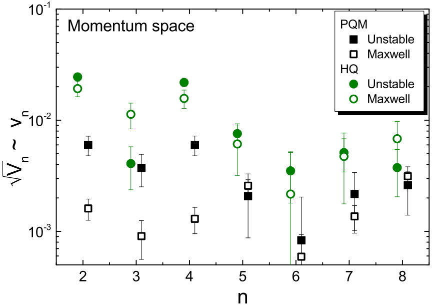
Will they lead to large higher moments of the angular momentum distribution of the baryon number?

In momentum space these can be computed from the fluid dynamical simulations by sampling the Cooper-Frye equation with the local values of four velocity u_ν, temperature T, and chemical potential μ.



$$V_n^{\mathrm{mom}} = \frac{1}{N_{ij}} \sum_{ij} \rho_i \, \rho_j \, \cos(n \Delta \Phi_{ij}^{\mathrm{mom}})$$

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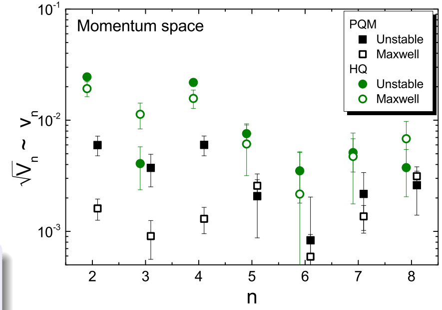


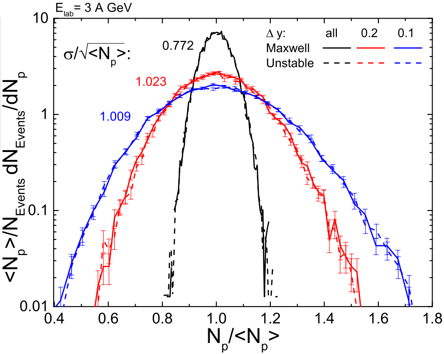
NO large enhancement in momentum space correlations.

$$V_n^{\mathrm{mom}} = \frac{1}{N_{ij}} \sum_{ij} \rho_i \, \rho_j \, \cos(n\Delta\Phi_{ij}^{\mathrm{mom}})$$

Proton Number Fluctuations

Propability distribution for the number of protons in a fixed rapidity interval, around mid rapidity:
$-\Delta y < y < \Delta y$

- We simulated 15000 Pb+Pb collisions with $b < 3.4$, and no special selection on the number of participants.
- We make a iso-energy density transition via Cooper-Frye after which the hadronic state is simulated using UrQMD as afterburner.



Within the limited set of statistics no difference between the Maxwell and unstable EoS is observed.

Summary

- We have obtained a transport model that is suitable for simulating nuclear collisions in the presence of a first-order phase transition.
- We have found that the associated instabilities may cause significant amplification of initial density irregularities.
- The present study demonstrates that the phase structure does affect the character of the density evolution.
- We hope that these results will stimulate efforts to develop analysis techniques for extracting the related observables from experimental data.

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- The present study demonstrates that the phase structure does affect the character of the density evolution.
- We hope that these results will stimulate efforts to develop analysis techniques for extracting the related observables from experimental data.

- So far we are still trying to identify observables sensitive on the enhanced fluctuations.
- Nuclei production and angular correlations show only a very weak signal of the unstable dynamics of the phase transition.

Open Issues

Not necessarily in this order

- Investigate effects of viscosity.
- Include strangeness: Dynamical distillation of strangeness!
- Include final hadronic state.
- Investigate effects of "critical slowing down".
- Thermal fluctuations.
- Different/Interesting equations of state.
- Observables, e.g. dileptons etc.
- ...