

What can continuum QCD tell us about heavy ion collisions and the hadron spectrum?

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Universität Heidelberg & ExtreMe Matter Institute

GSI, April 15th 2015



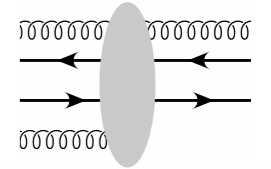
Outline

- **Vacuum QCD & the hadron spectrum**
- **Phase structure of QCD**
- **Spectral Functions & Transport Coefficients**
- **Outlook**

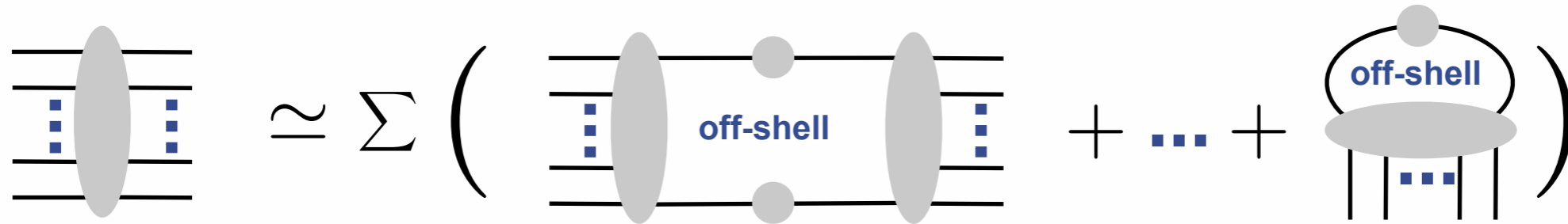
Continuum Methods for QCD

quark-gluon correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) \rangle$$



functional relations

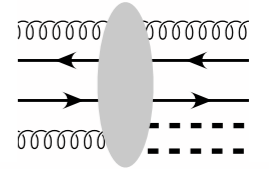


scattering amplitude/
vertex functions

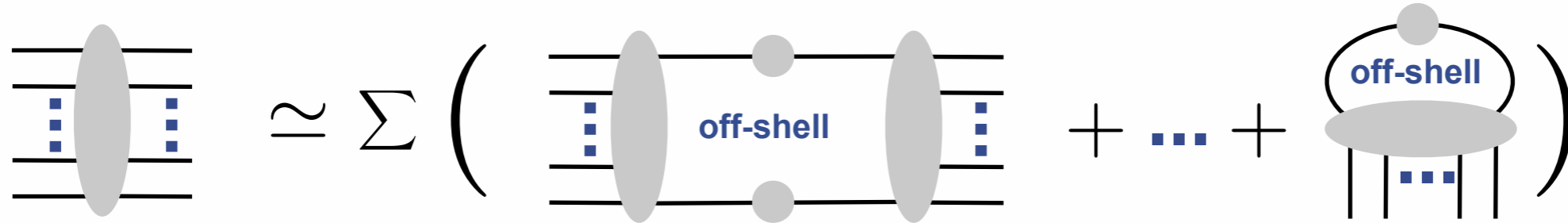
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



functional relations



scattering amplitude/
vertex functions

Functional renormalisation group equations

Dyson-Schwinger equations

2PI/nPI hierarchies

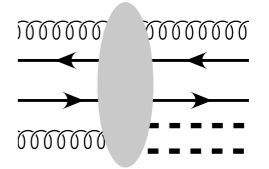
Bethe-Salpeter equations

⋮

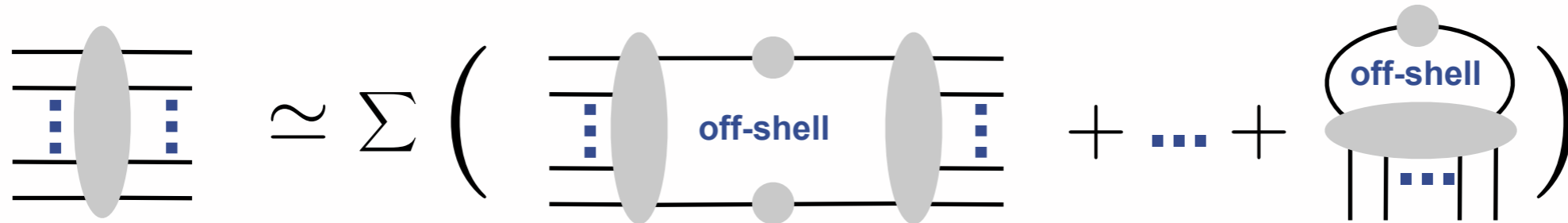
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functional relations



scattering amplitude/
vertex functions

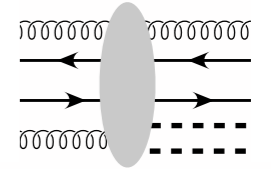
properties

- access to physics mechanisms 
- numerically tractable
no sign problem
systematic error control via closed form
- low energy models naturally incorporated

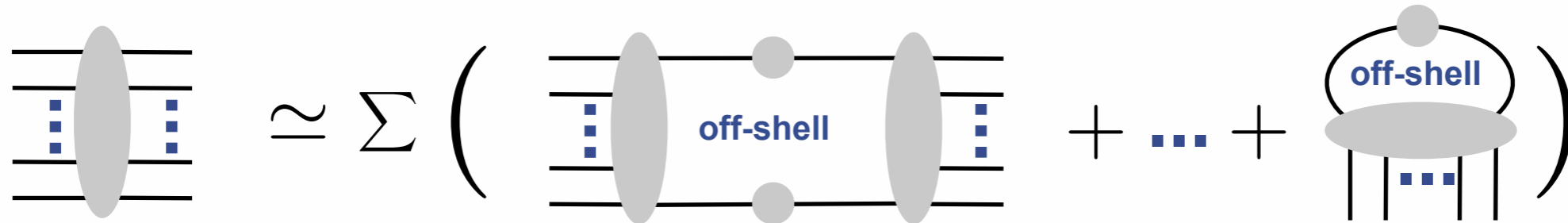
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functional relations



scattering amplitude/
vertex functions

properties

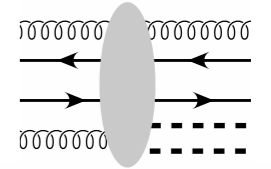
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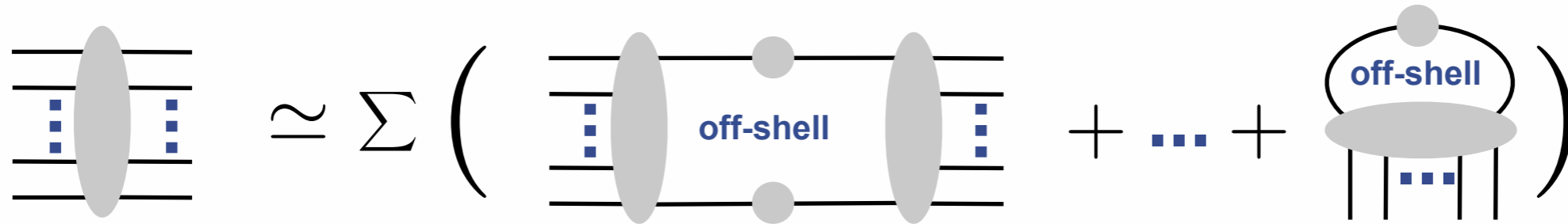
Continuum Methods for QCD

quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



functional relations




scattering amplitude/
vertex functions

e.g. lattice input on rhs

e.g. volume flucs., finite density,
dynamics, ...

properties

- access to physics mechanisms 
- numerically tractable
no sign problem
systematic error control via closed form
- low energy models naturally incorporated

FunMethods complementary to lattice



'Local' expertise in functional continuum methods



J. Berges (Heidelberg)
J. Braun (Darmstadt)
M. Buballa (Darmstadt)
C.S. Fischer (Gießen)
B. Friman (GSI)
M. Lutz (GSI)
JMP (Heidelberg)
D. Rischke (Frankfurt)
B.-J. Schaefer (Gießen)
L. von Smekal (Darmstadt)
J. Wambach (Darmstadt)
C. Wetterich (Heidelberg)

N. Christiansen
A. Cyrol
N. Khan
N. Müller
F. Rennecke

Young guns (PostDocs)

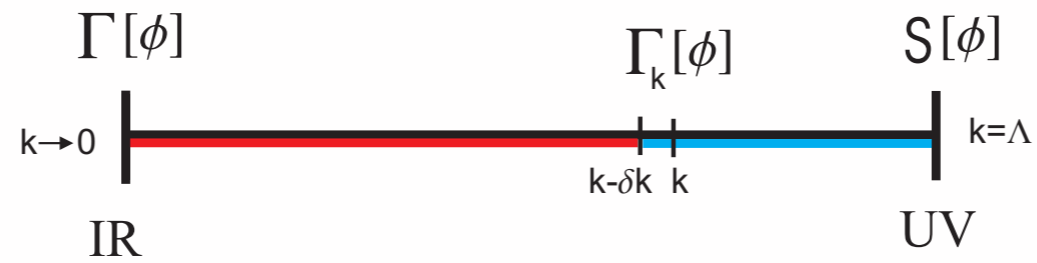
T. Herbst (Heidelberg)
G. Eichmann (Giessen)
W.-j. Fu (Heidelberg)
M. Mitter (Heidelberg)
S. Rechenberger (Darmstadt)
H. Sanchis-Alepuz (Gießen)
N. Strodthoff (Heidelberg)
R. Stiele (Gießen)
R. Williams (Gießen)

unique concentration

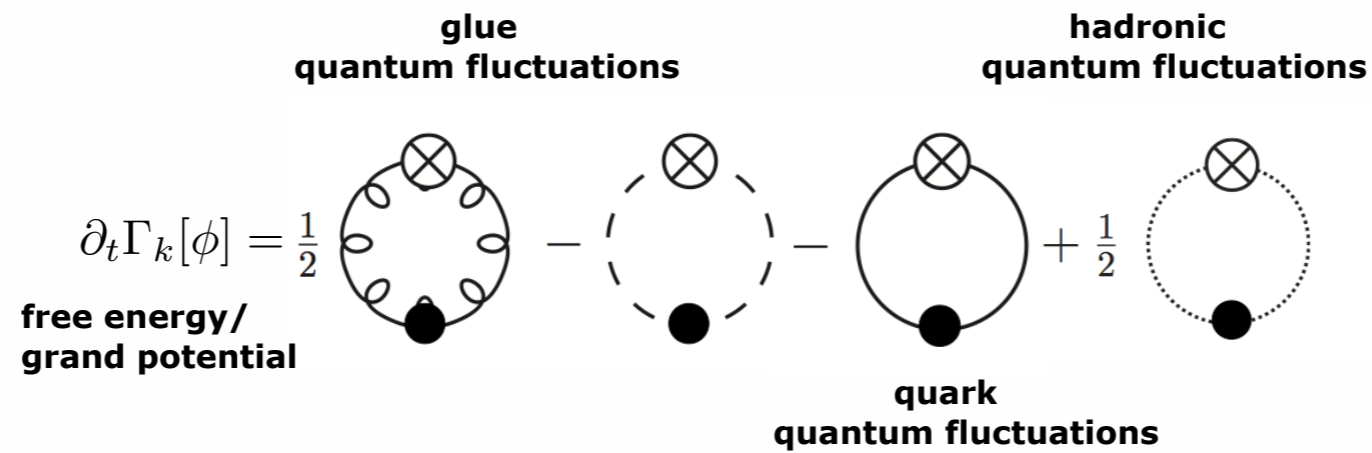
Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113-124

free energy at momentum scale k



ab initio

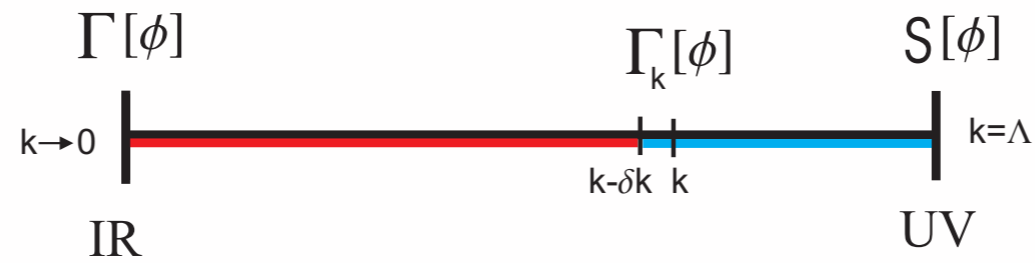


RG-scale k : $t = \ln k$

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113-124

free energy at momentum scale k

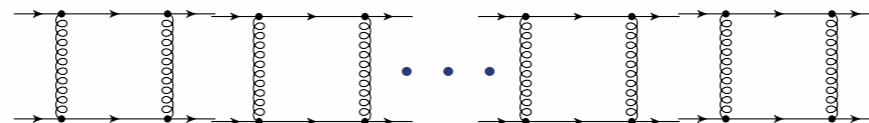


ab initio

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue quantum fluctuations} - \text{quark quantum fluctuations} + \frac{1}{2} \text{hadronic quantum fluctuations} \right)$$

free energy/
grand potential

RG-scale k : $t = \ln k$



Dynamical hadronisation \rightarrow **dynamical**
 Gies, Wetterich '01
 JMP '05
 Flörchinger, Wetterich '09

Functional RG for QCD

fQCD collaboration: J. Braun, A. Cyrol, L. Fister, W.-j. Fu, T.K. Herbst, M. Mitter, N. Mueller, JMP, S. Rechenberger, F. Rennecke, N. Strodthoff

TARDIS, ERGE, DoFun2.0

DoFun

Braun, Huber, Comput.Phys.Commun. 183 (2012) 1290-1320

Mitter, JMP, Strodthoff, Phys.Rev. D91 (2015) 054035

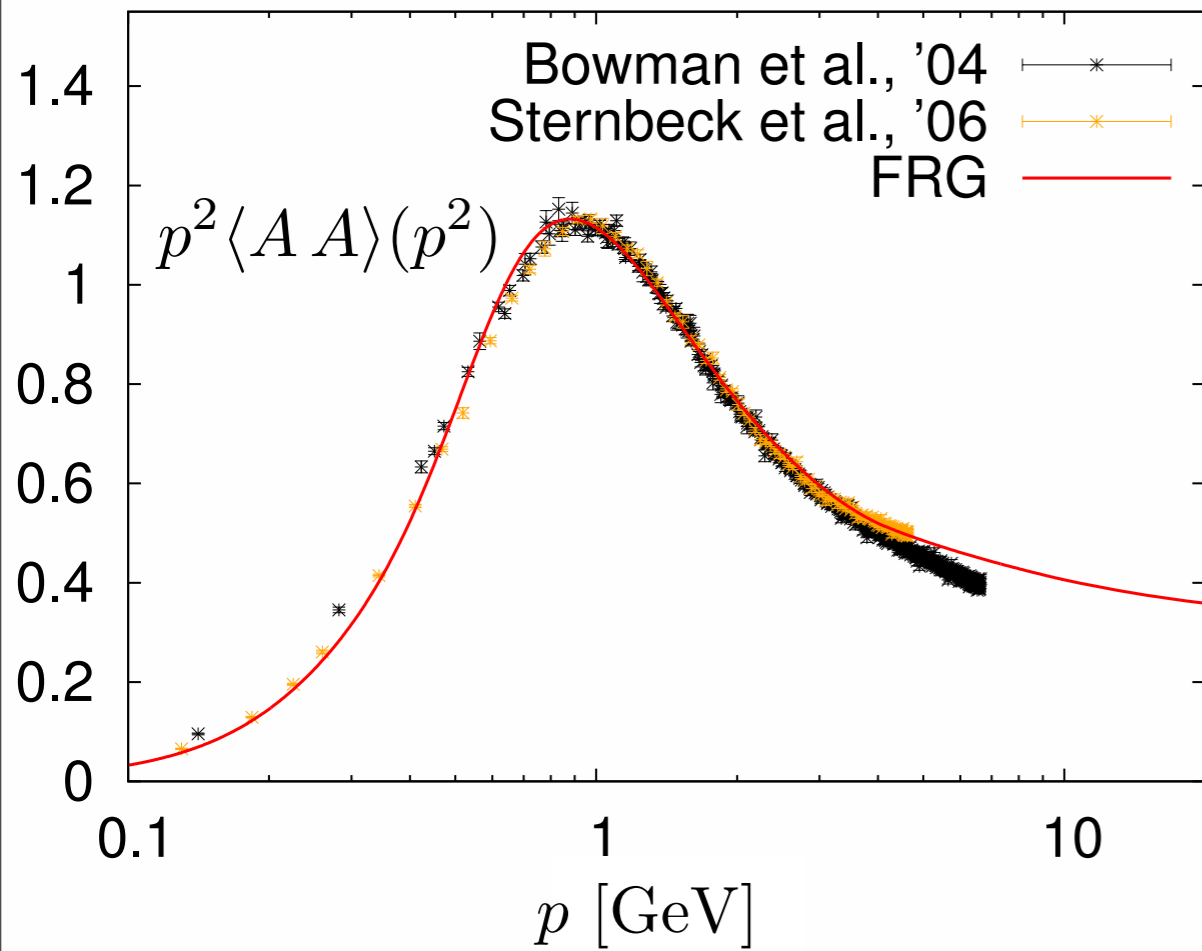
Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

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Glue sector

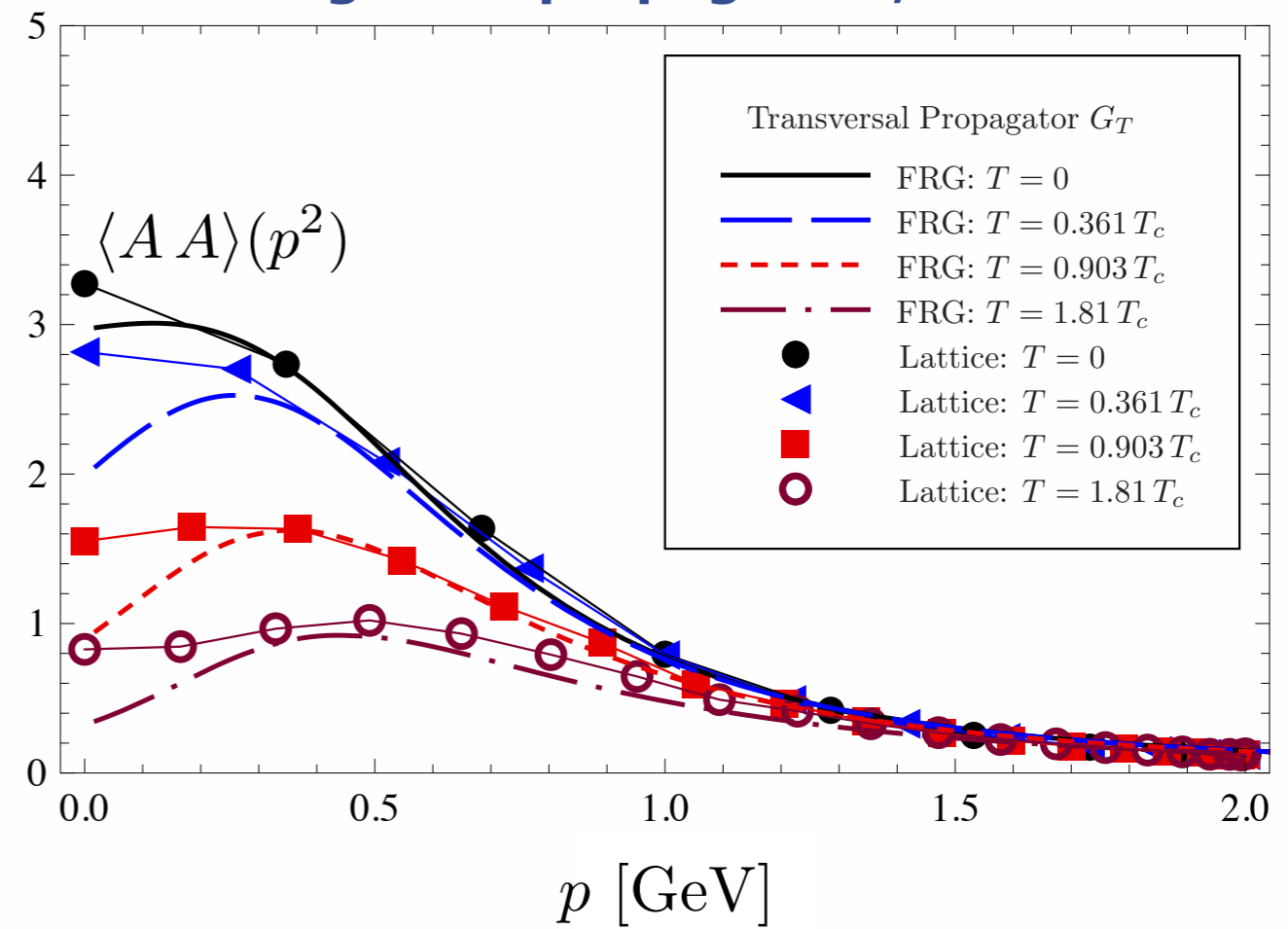
Yang-Mills propagators, $T=0$



Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

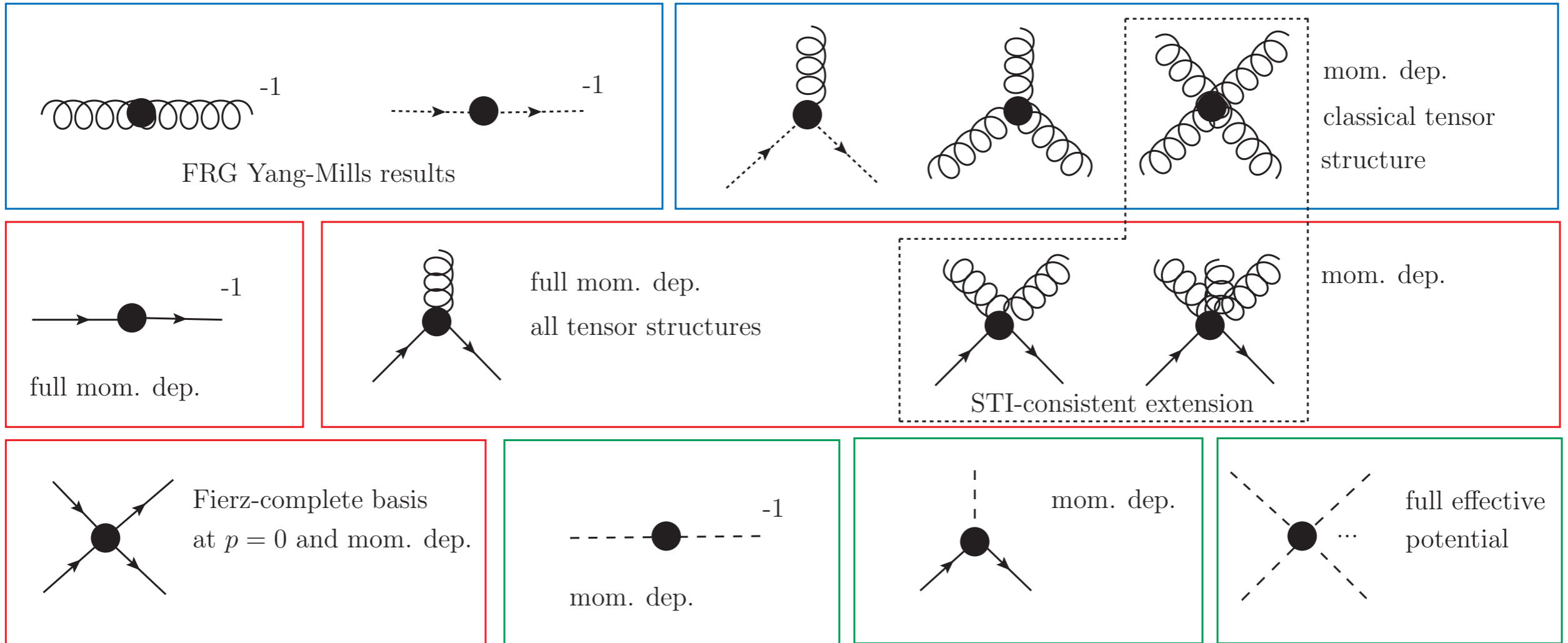
Fister, JMP '14

Yang-Mills propagators, finite T



Fister, JMP, arXiv:1112.5440

Chiral symmetry breaking



fQCD

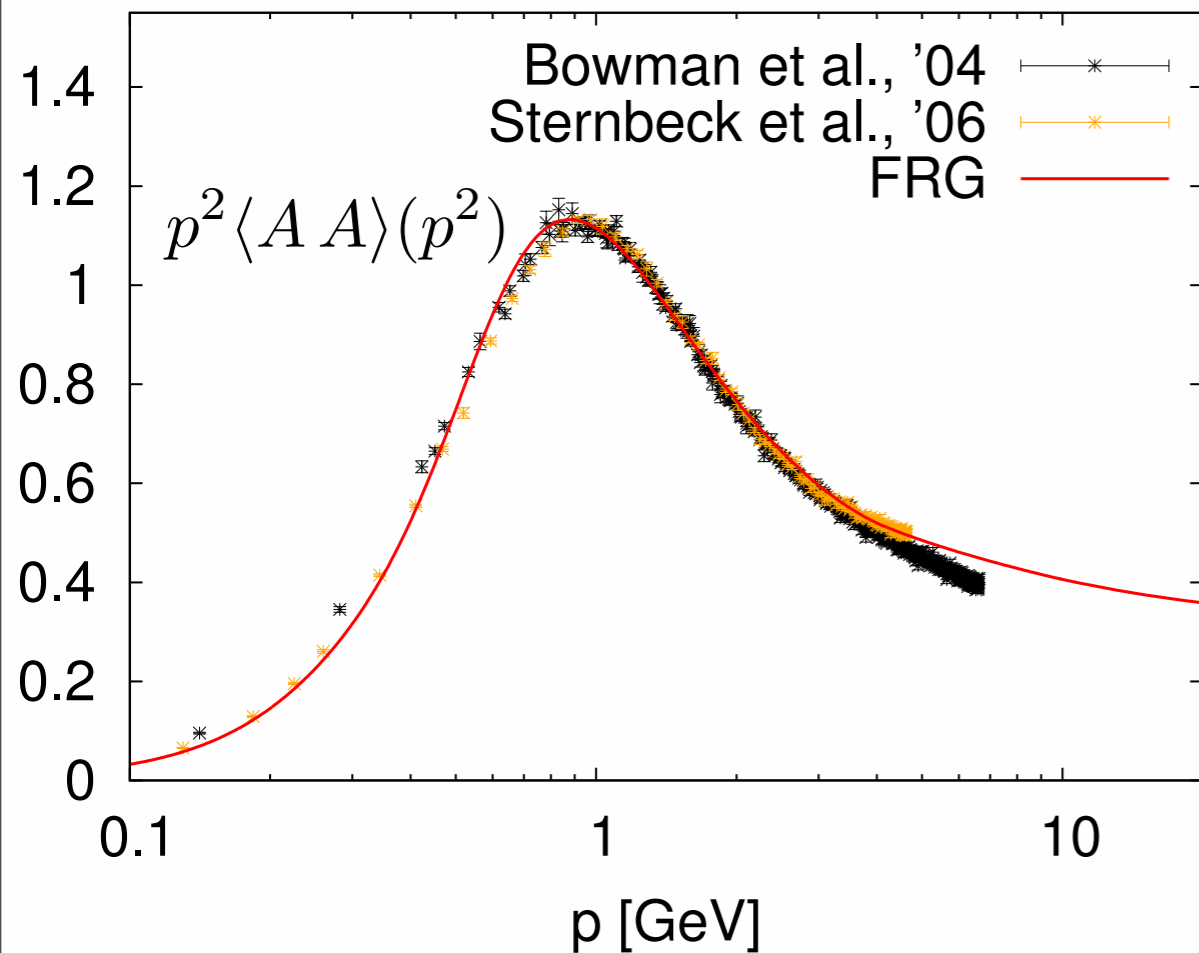
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

see also Williams, arXiv:1404.2545

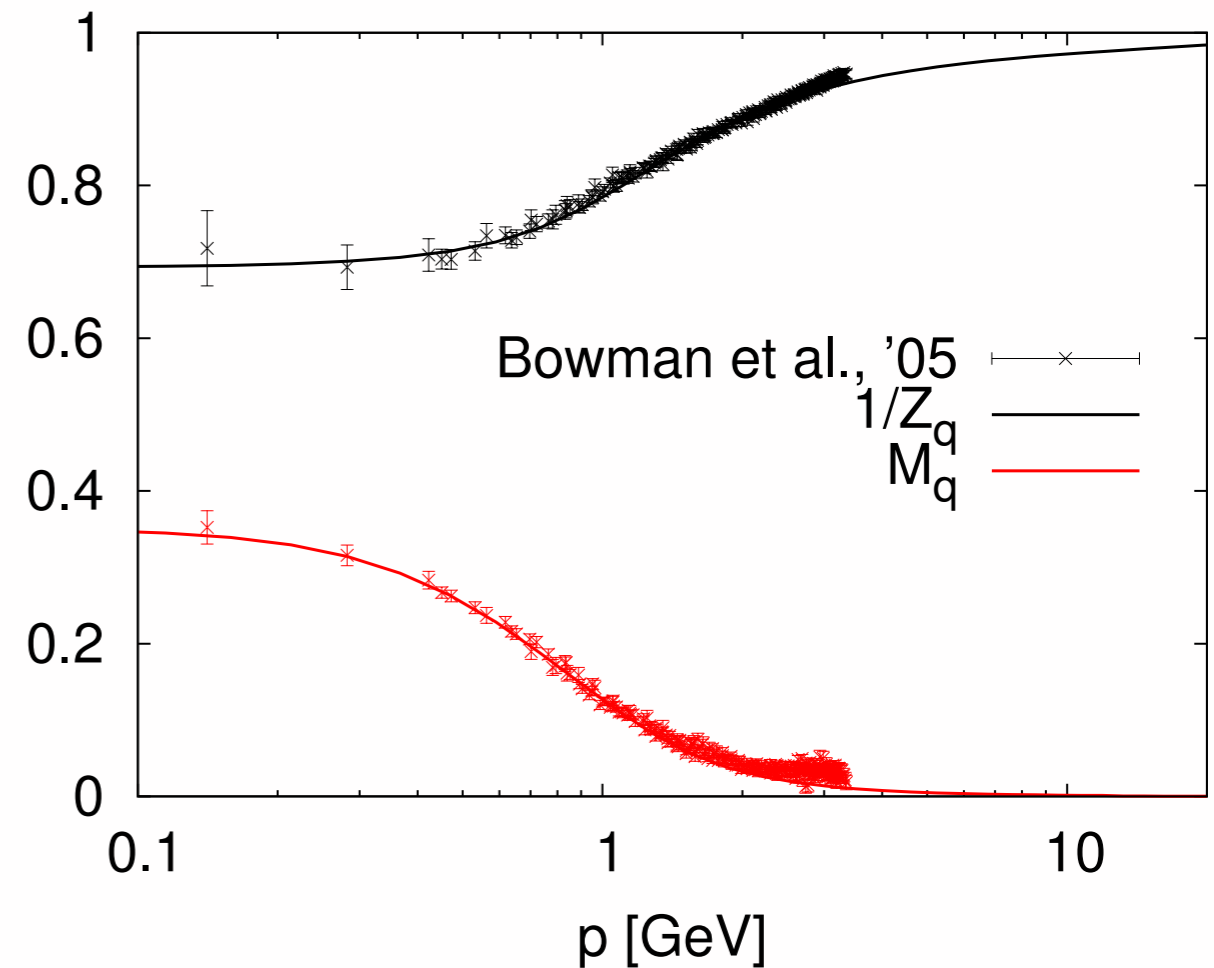
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenched QCD

quenched gluon dressing



quark propagator



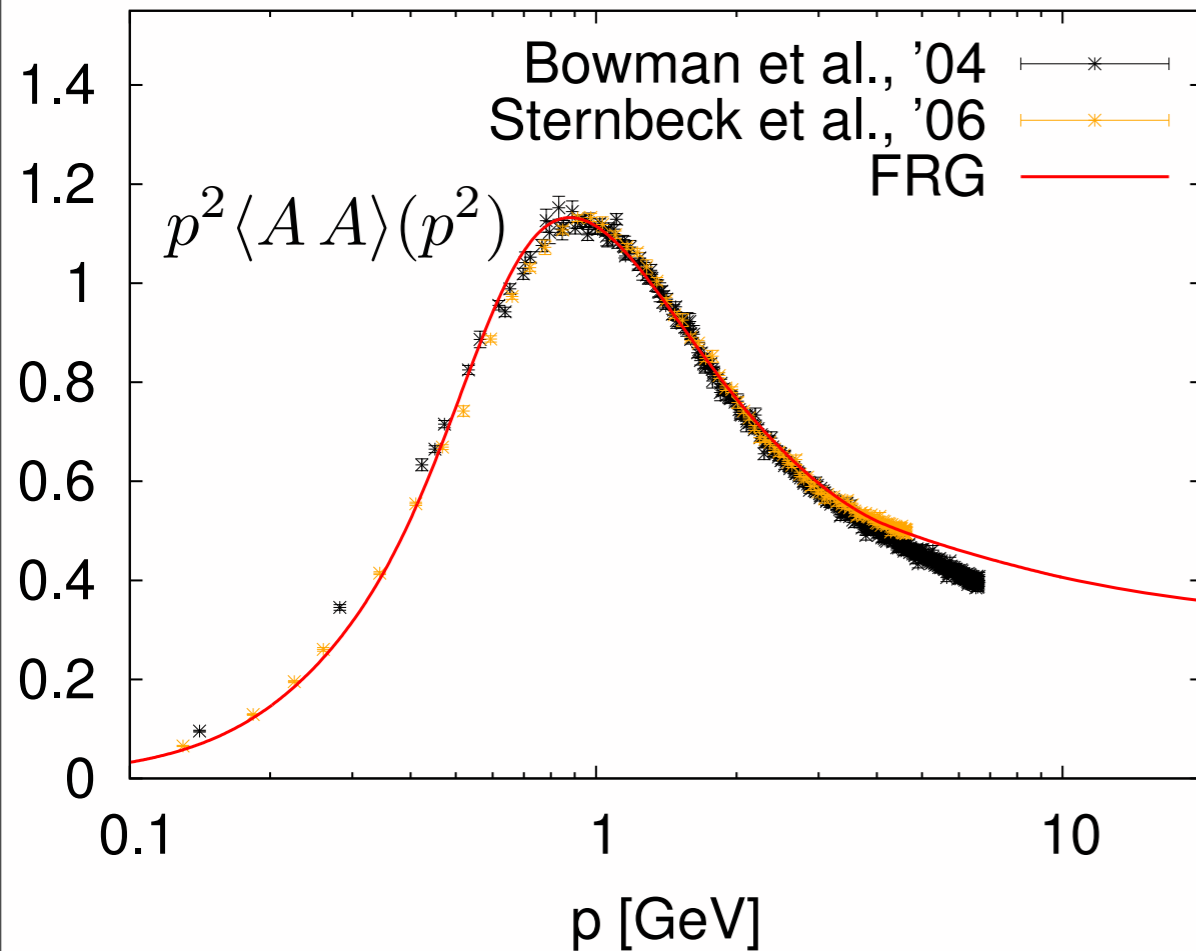
$N_f = 2$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

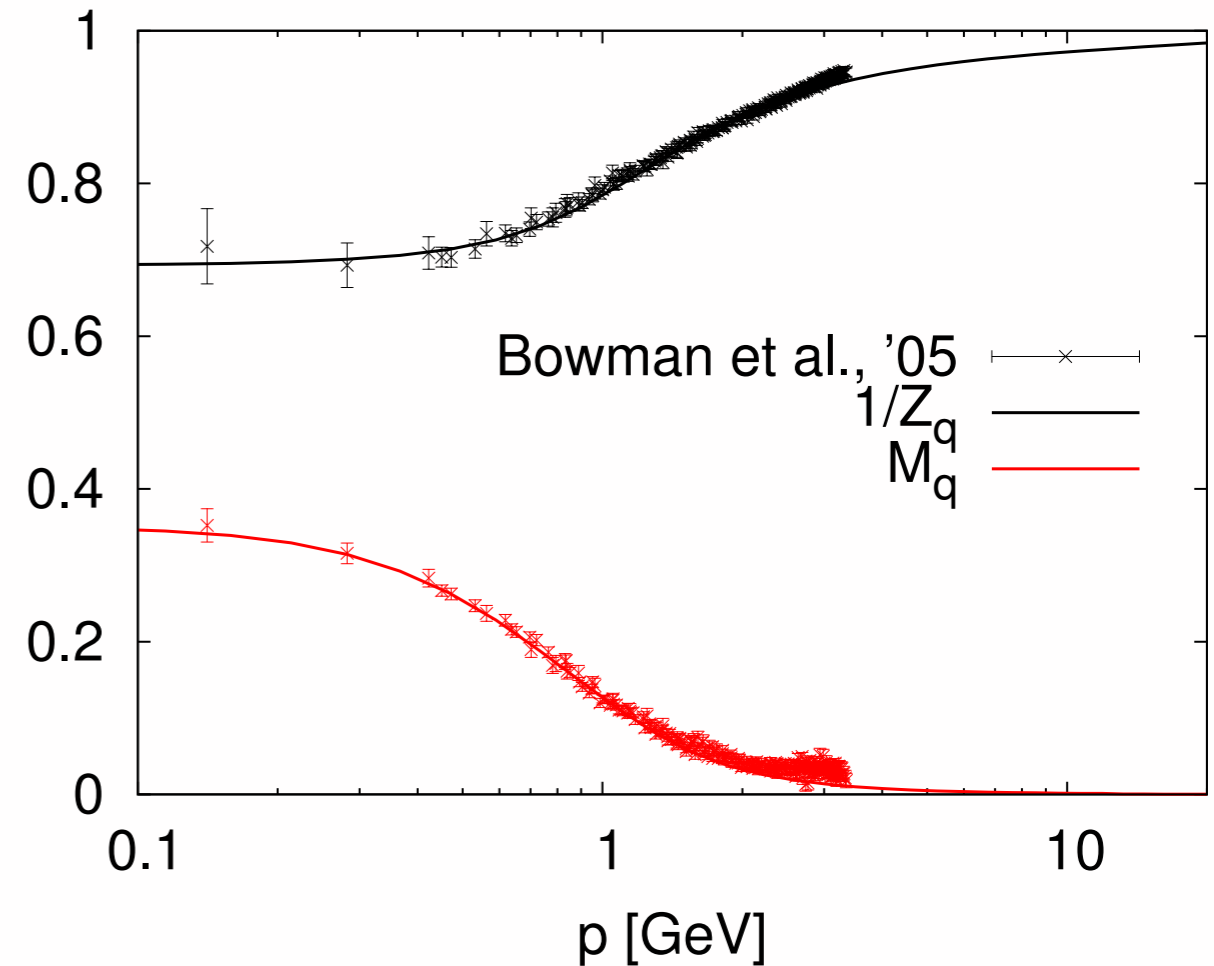
Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenched QCD

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JMP, Rennecke, PRD 90, 076002

Helmboldt, JMP, Strodthoff, PRD 91 (2015) 5, 054010

Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

$N_f = 2$

systematic error estimate: $\sim 10\%$

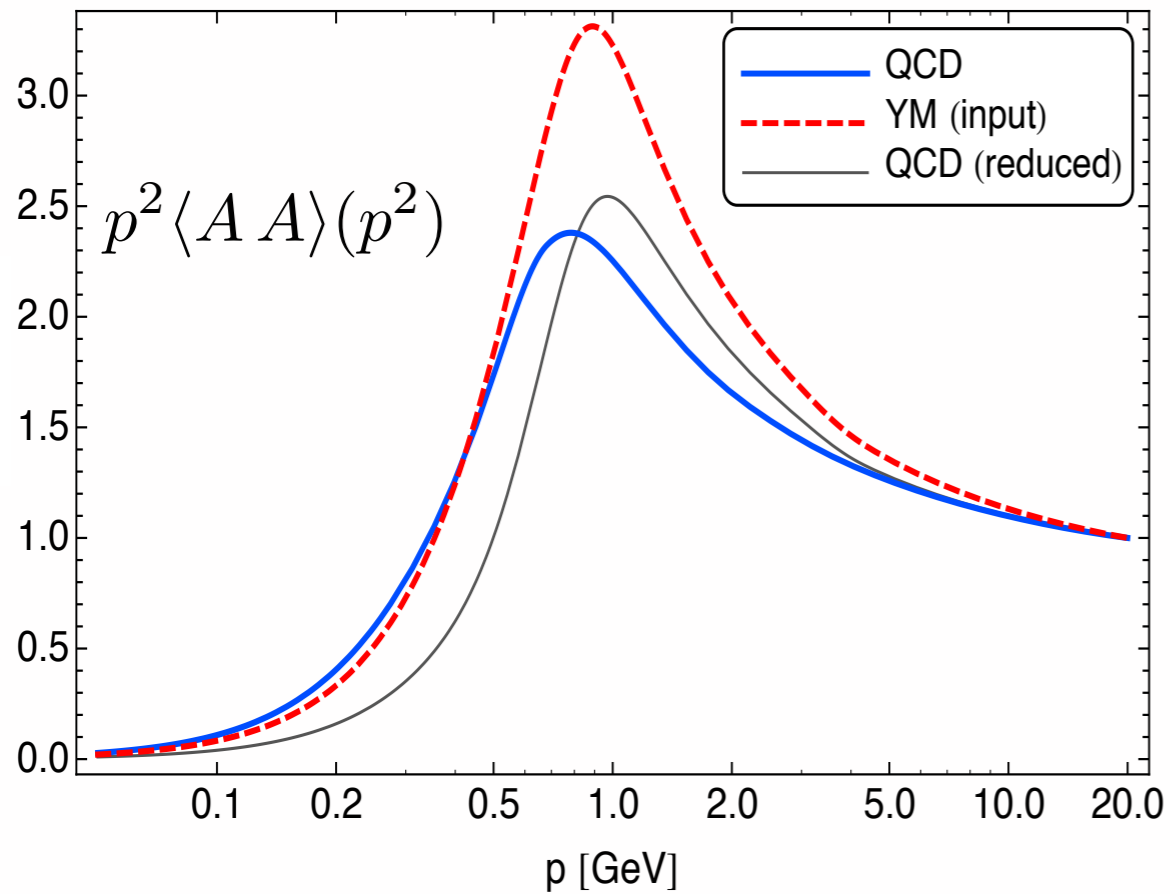
JMP

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Chiral symmetry breaking

FRG-quenched QCD vs lattice-quenched QCD

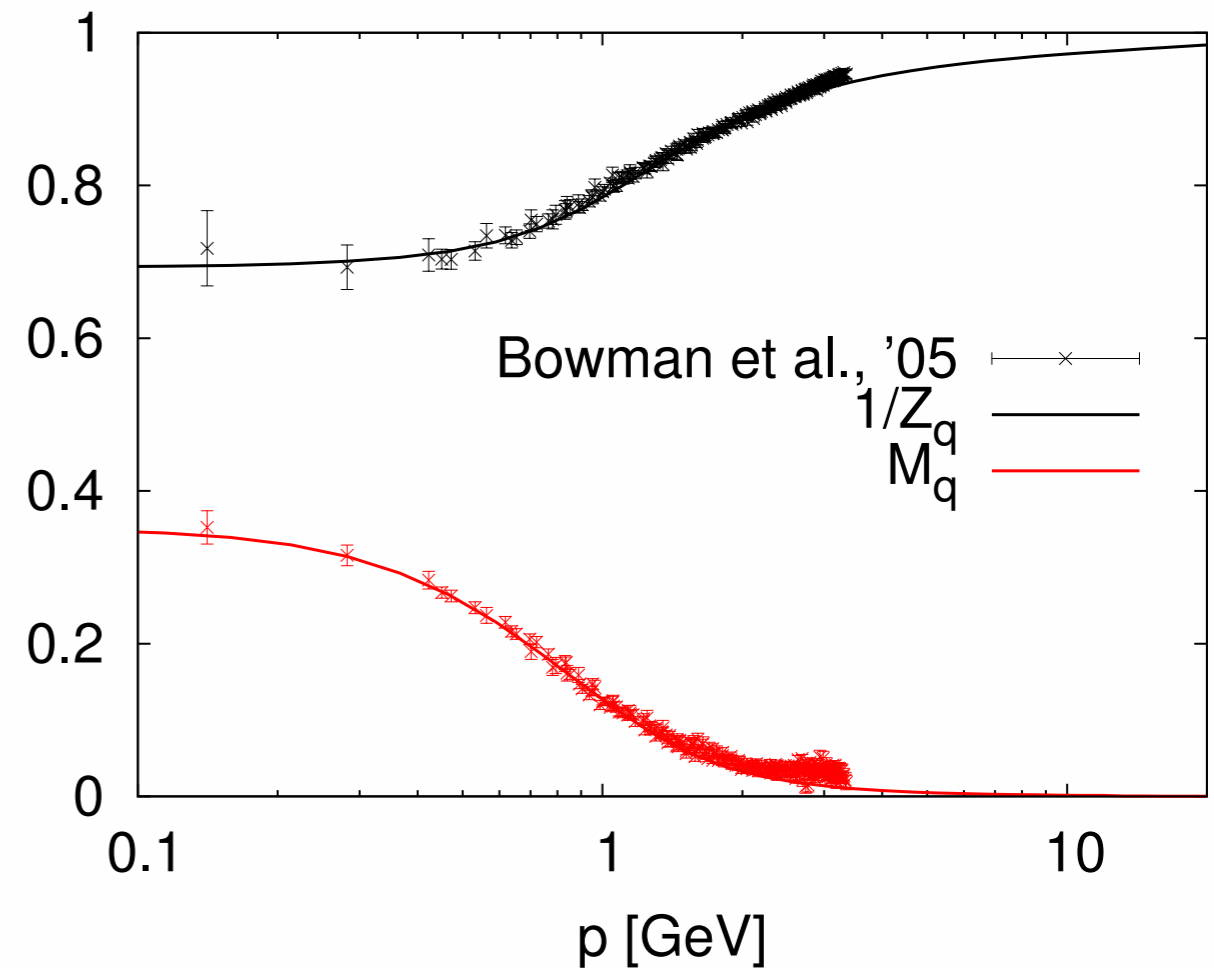
unquenched gluon dressing



Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

Rennecke, arXiv:1504.03585

quark propagator



$N_f = 2$

systematic error estimate: $\sim 10\%$

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

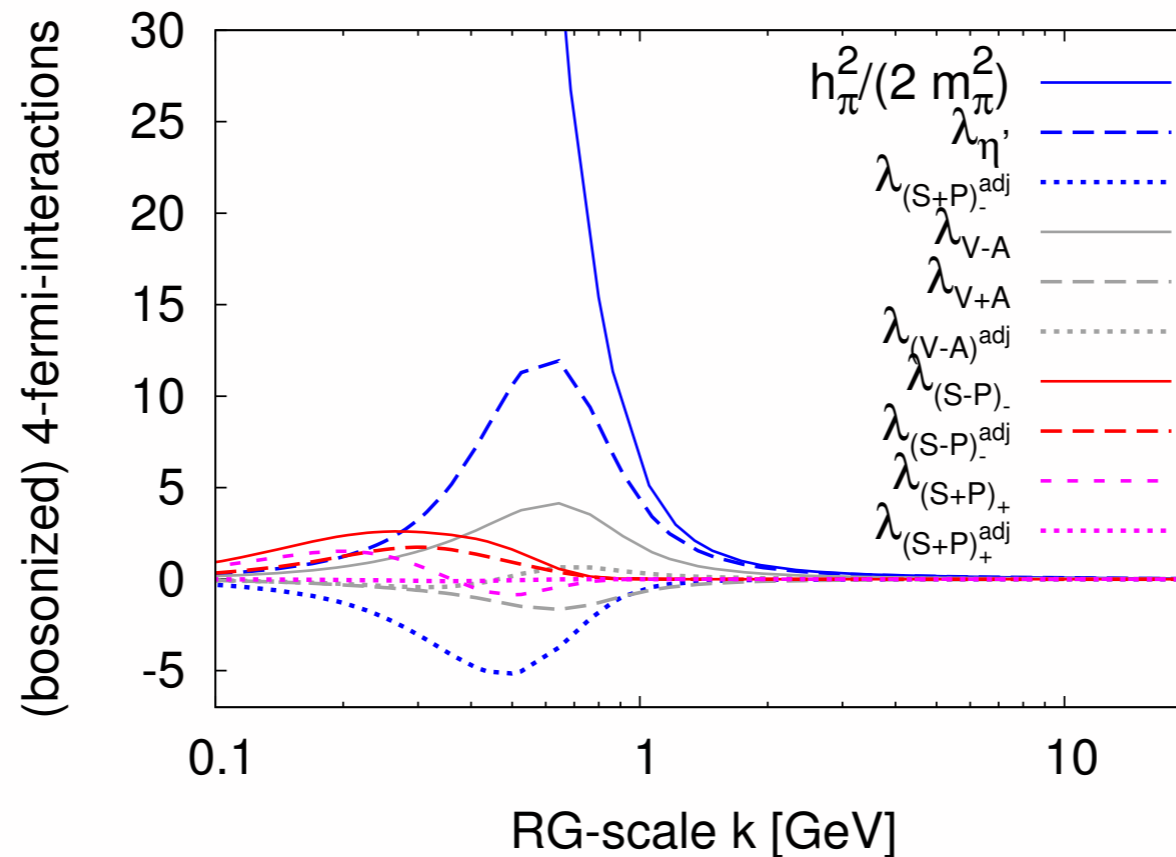
A glimpse at the hadron spectrum

quenched QCD

four-fermi scattering amplitude at pion pole

$$\langle \bar{q} \vec{\sigma} \gamma_5 q(p) \quad \bar{q} \vec{\sigma} \gamma_5 q(-p) \rangle \rightarrow \frac{\chi_{\bar{q}\pi q} \bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \text{finite terms}$$

$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$



Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

quenched QCD

four-fermi scattering amplitude at pion pole

$$\begin{array}{ccc} \langle \bar{q} \vec{\sigma} \gamma_5 q(p) \quad \bar{q} \vec{\sigma} \gamma_5 q(-p) \rangle & \rightarrow & \frac{\chi_{\bar{q}\pi q} \bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \text{finite terms} \\ \downarrow & & \downarrow \\ \Gamma_{(\bar{q} \gamma_5 \vec{\sigma} q)^2}^{(4)}(p, p, -p, -p) & & \frac{\Gamma_{\bar{q}\pi q}^{(3)} \Gamma_{\bar{q}\pi q}^{(3)}}{p^2 - m_\pi^2} \end{array}$$

A glimpse at the hadron spectrum

quenched QCD

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The diagram consists of two boxes connected by a right-pointing arrow. The left box contains the four-fermi scattering amplitude $\langle \bar{q} \vec{\sigma} \gamma_5 q(p) \quad \bar{q} \vec{\sigma} \gamma_5 q(-p) \rangle$ and below it, a large red arrow pointing down to the expression $\Gamma_{(\bar{q} \gamma_5 \vec{\sigma} q)^2}^{(4)}(p, p, -p, -p)$. The right box contains the pion pole contribution $\frac{\chi_{\bar{q}\pi q} \bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$ and below it, another large red arrow pointing down to the expression $\frac{\Gamma_{\bar{q}\pi q}^{(3)} \Gamma_{\bar{q}\pi q}^{(3)}}{p^2 - m_\pi^2}$. To the right of the right box is the text '+ finite terms'.

pion decay constant f_π via normalisation of $\Gamma_{\bar{q}\pi q}^{(3)}$

aka BSE wave function

recent mini-review on DSE-BSE
Sanchis-Alepuz, Williams, arXiv:1503.05896


Hadron DSE-BSE center Gießen

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

quenched QCD

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$$\Gamma_{(\bar{q}\gamma_5 \vec{\sigma} q)^2}^{(4)}(p, p, -p, -p) \quad \frac{\Gamma_{\bar{q}\pi q}^{(3)} \Gamma_{\bar{q}\pi q}^{(3)}}{p^2 - m_\pi^2}$$

pion decay constant f_π via normalisation of $\Gamma_{\bar{q}\pi q}^{(3)}$

$$f_\pi = 88 \text{ MeV}$$

continuum QCD

$$f_\pi = 89 \text{ MeV}$$

lattice Aoki et al, PRD 62 (2000) 094501

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

quenched QCD

four-fermi scattering amplitude at pion pole

$$\begin{array}{ccc}
 \langle \bar{q} \vec{\sigma} \gamma_5 q(p) \quad \bar{q} \vec{\sigma} \gamma_5 q(-p) \rangle & \rightarrow & \frac{\chi_{\bar{q}\pi q} \bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} + \text{finite terms} \\
 \downarrow & & \downarrow \\
 \Gamma_{(\bar{q} \gamma_5 \vec{\sigma} q)^2}^{(4)}(p, p, -p, -p) & & \frac{\Gamma_{\bar{q}\pi q}^{(3)} \Gamma_{\bar{q}\pi q}^{(3)}}{p^2 - m_\pi^2}
 \end{array}$$

pion decay constant f_π via normalisation of $\Gamma_{\bar{q}\pi q}^{(3)}$

real-time:

- Spectral Fcts & Transport Coeffs

$$f_\pi = 88 \text{ MeV}$$

continuum QCD

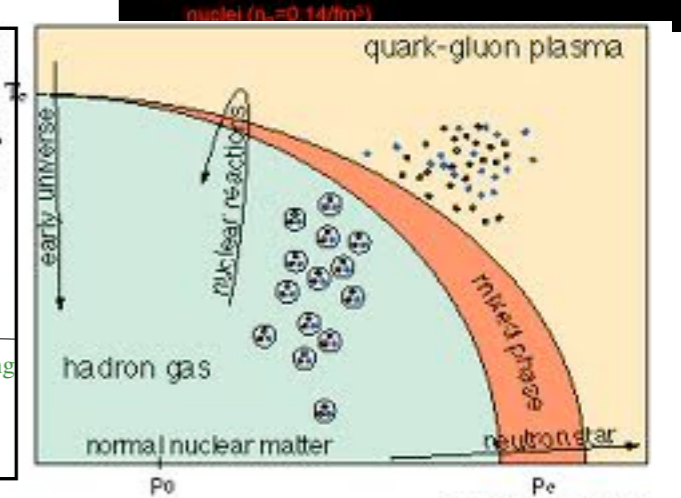
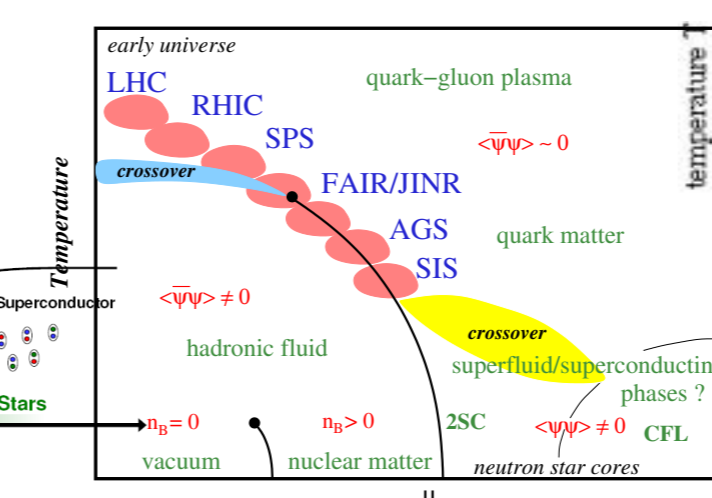
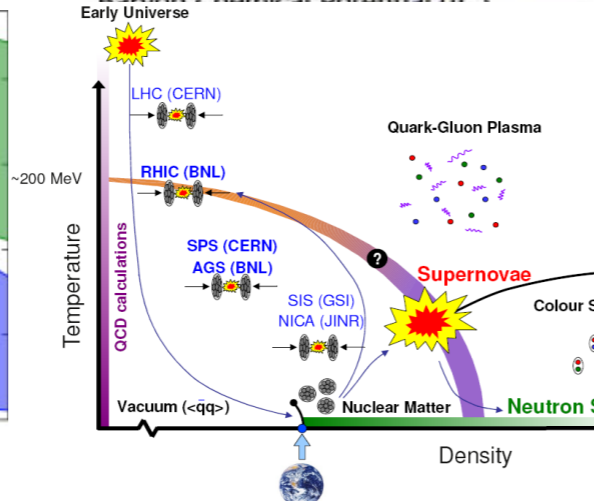
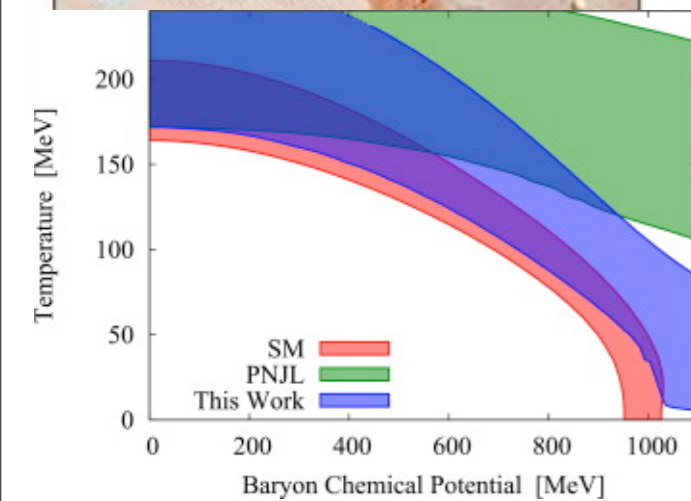
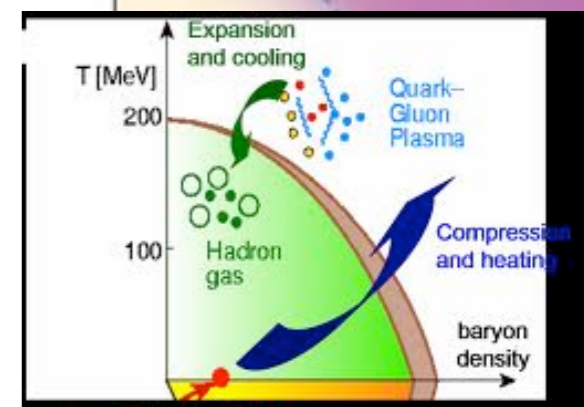
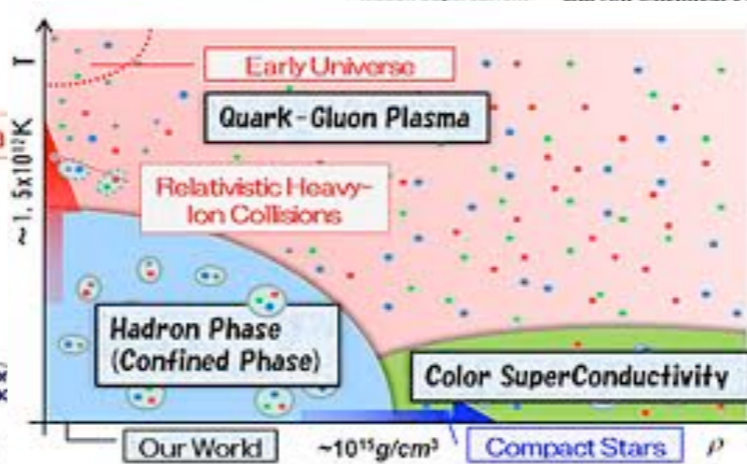
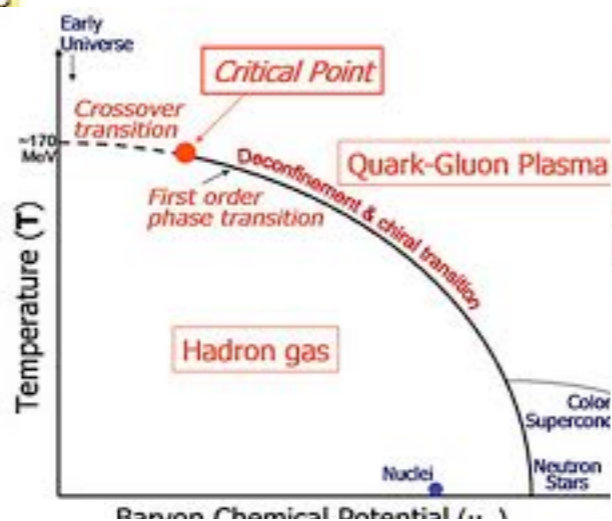
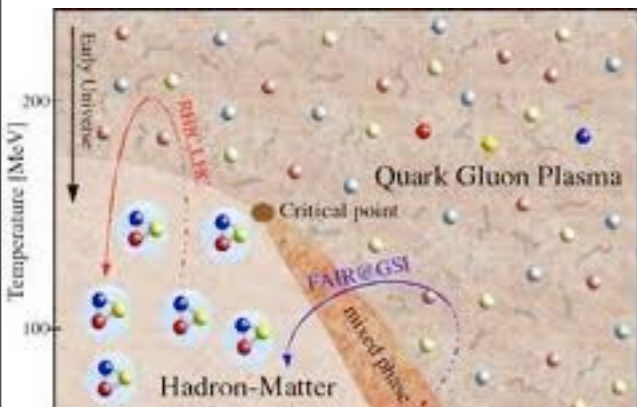
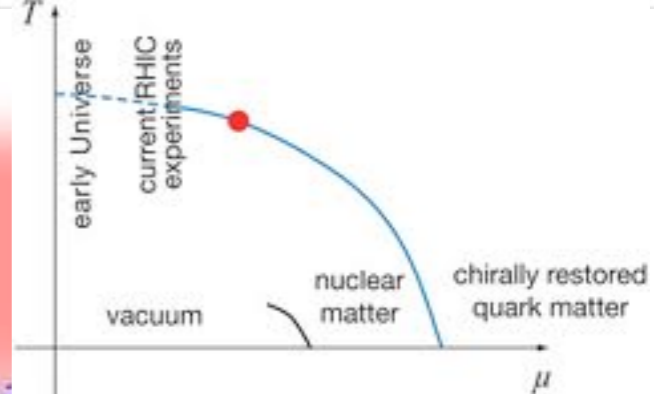
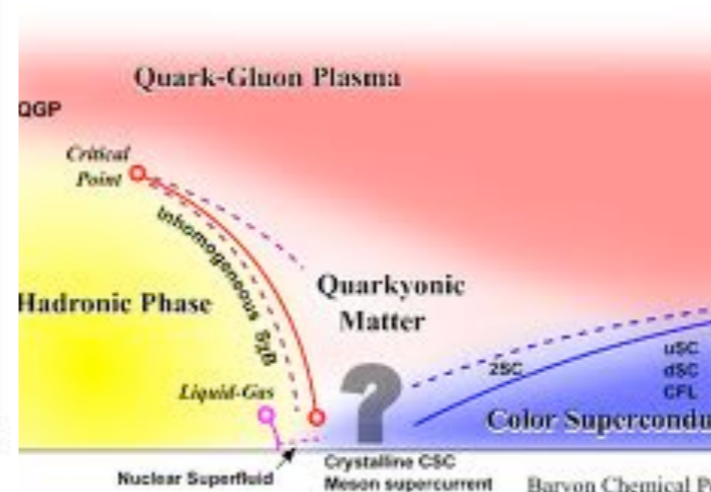
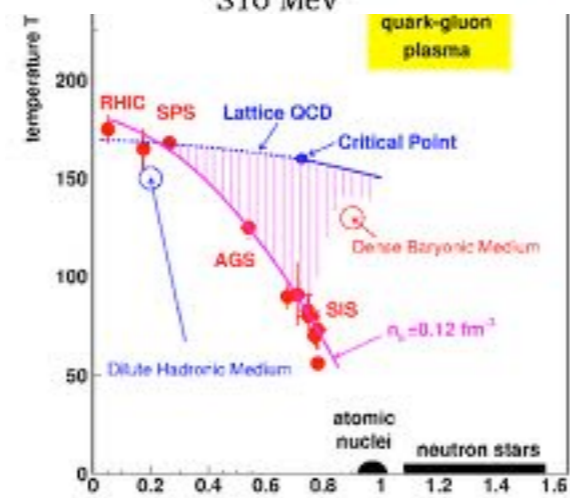
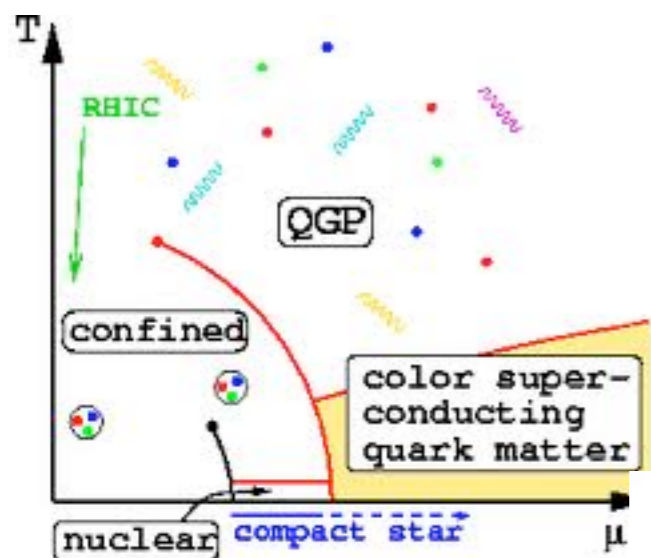
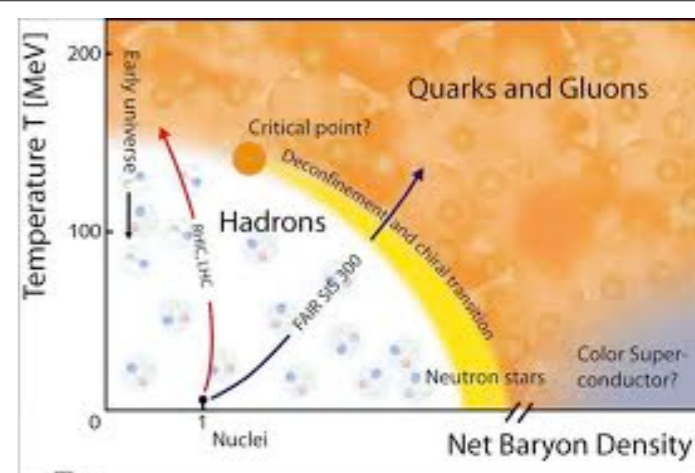
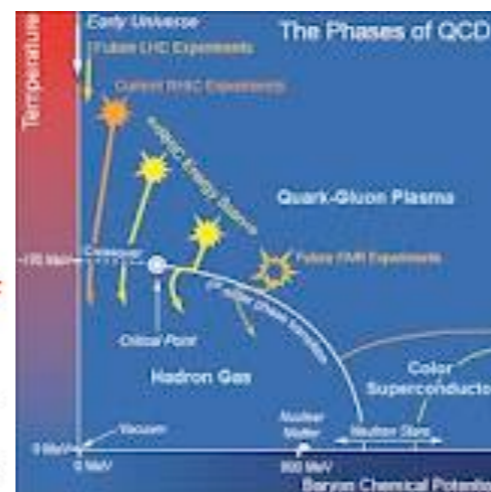
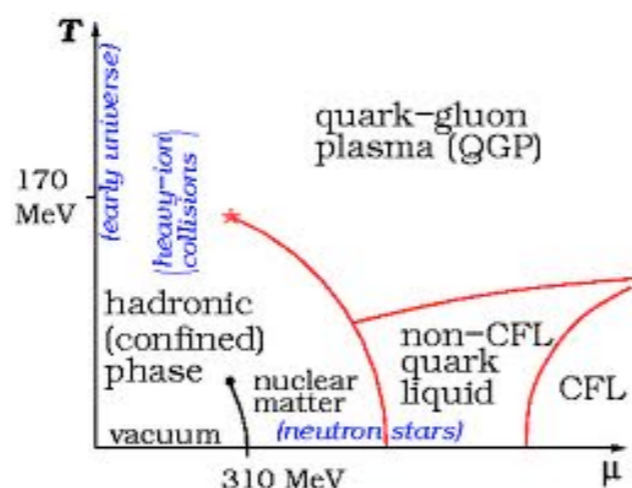
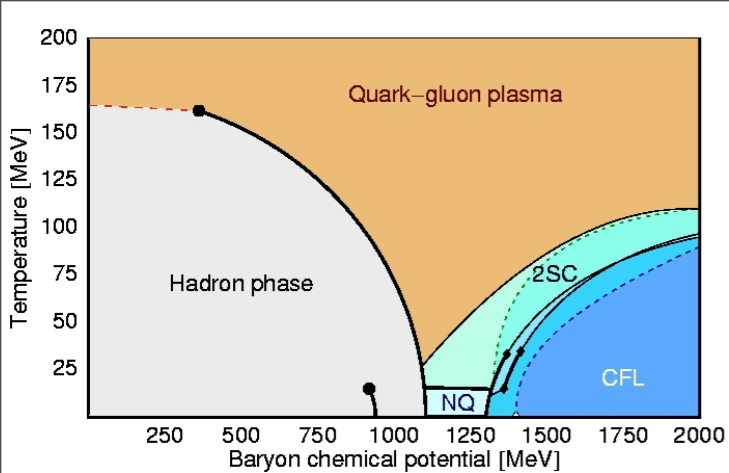
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lattice Aoki et al, PRD 62 (2000) 094501

Mitter, JMP, Strodthoff, in preparation

Outline

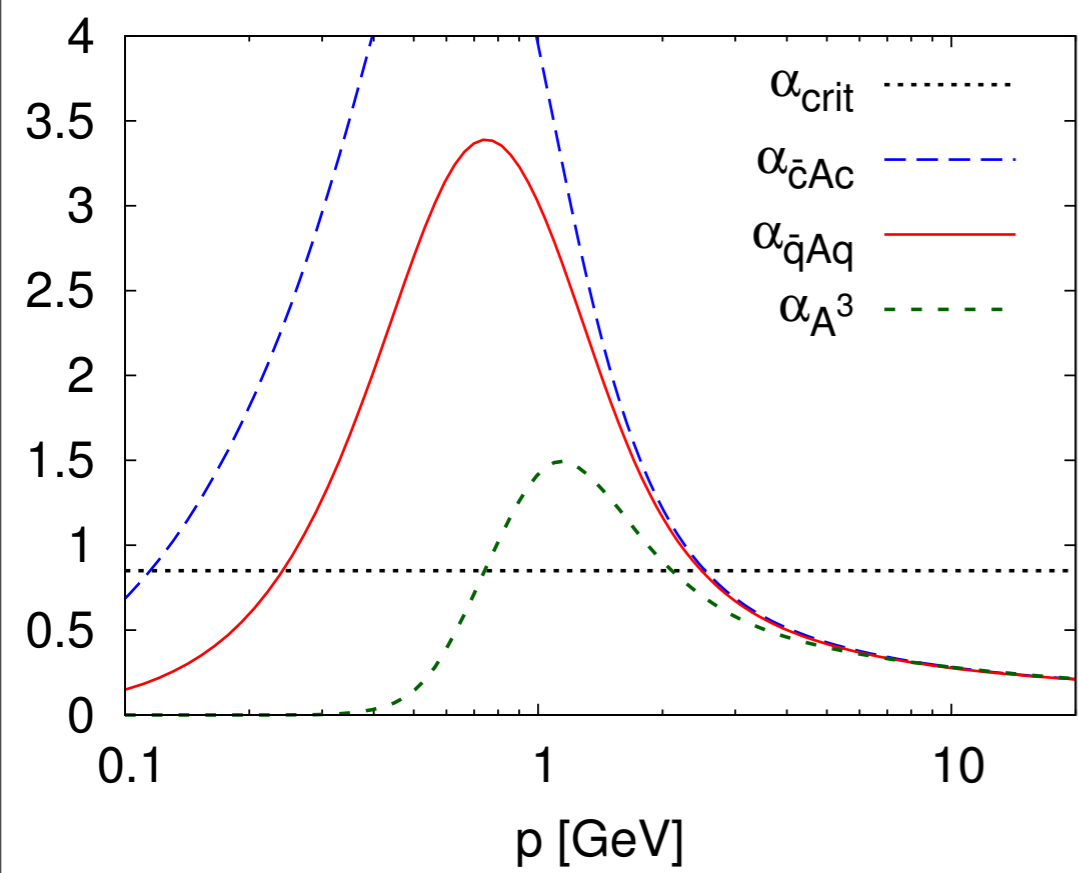
- Vacuum QCD & the hadron spectrum
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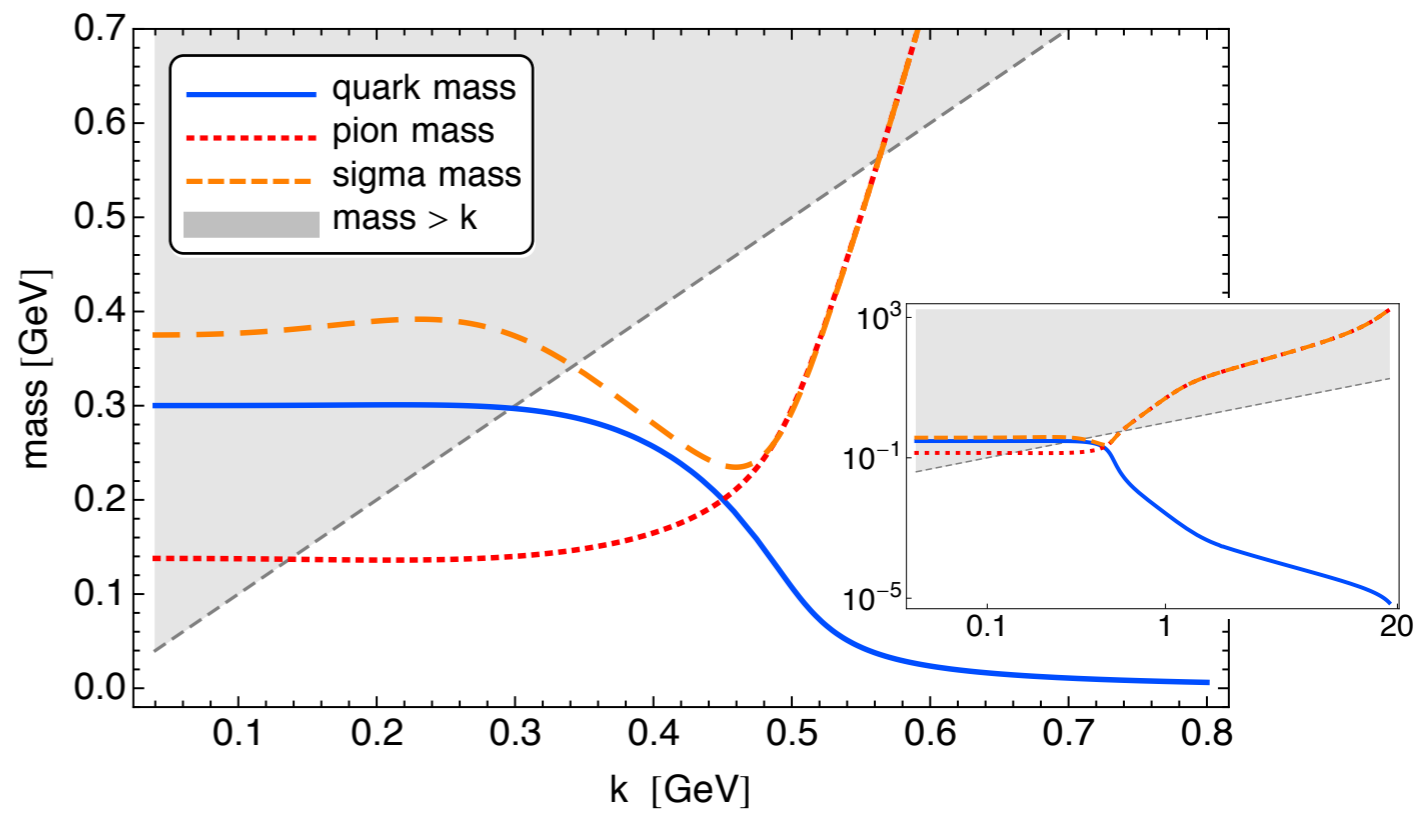
QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{diagram 1} - \text{diagram 2} - \text{diagram 3} + \frac{1}{2} \text{diagram 4} \right)$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

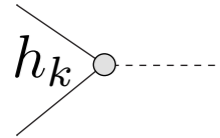


Braun, Fister, Haas, JMP, Rennecke, arXiv:1412.1045

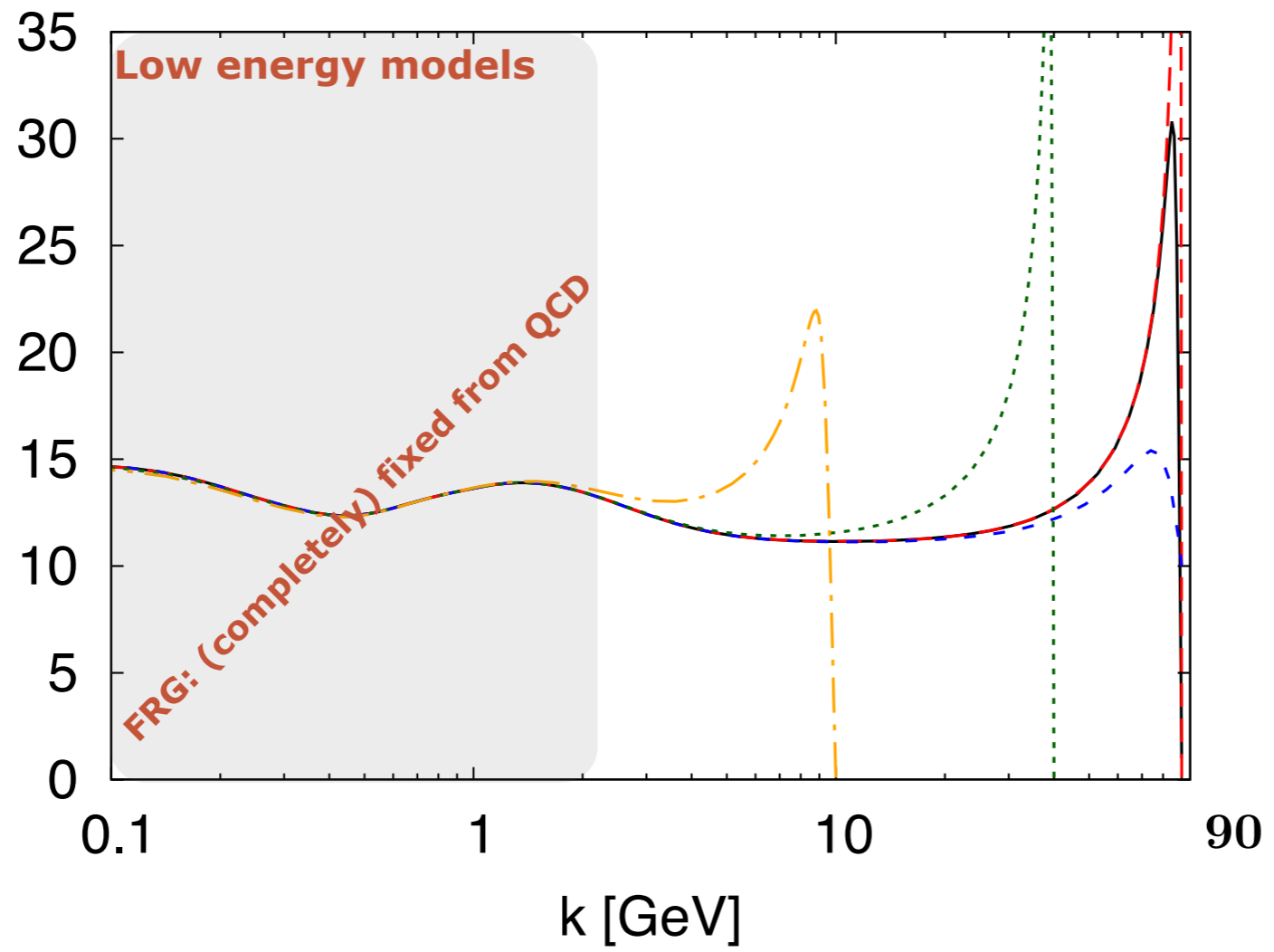
Rennecke, arXiv:1504.03585

QCD

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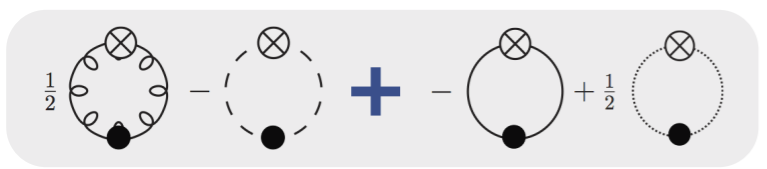


quark-meson coupling



fQCD

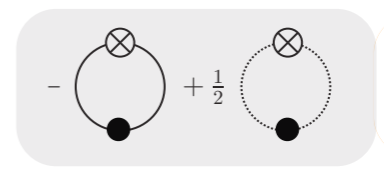
PQM-model



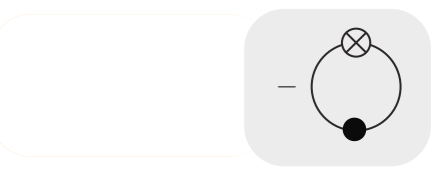
PNJL-model



QM-model

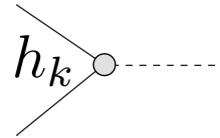


NJL-model

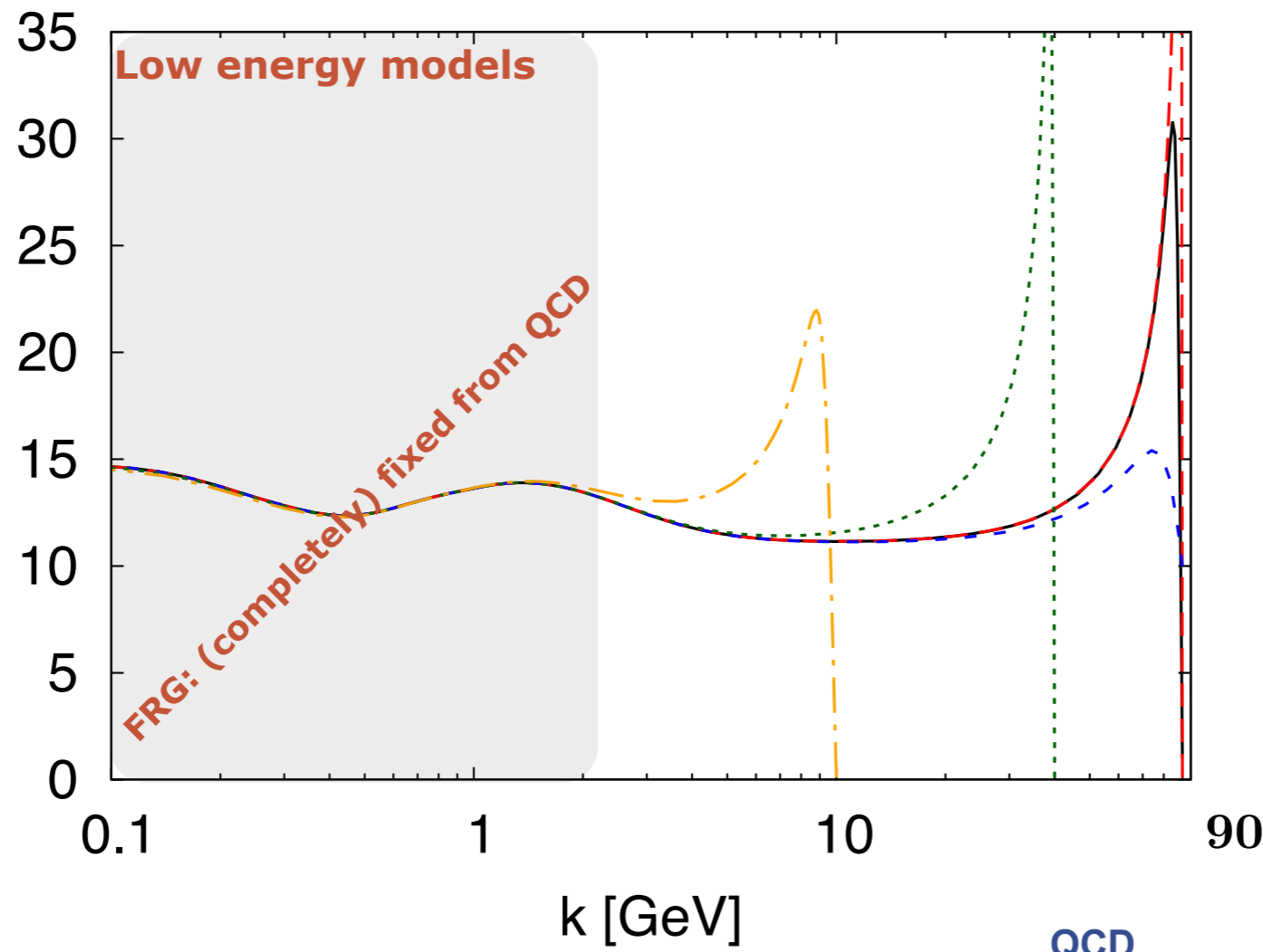


QCD

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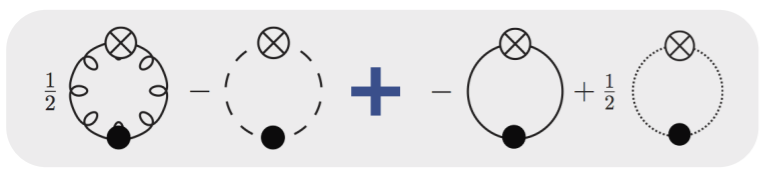
quark-meson coupling



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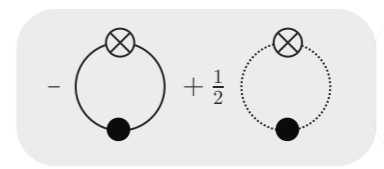
PQM-model



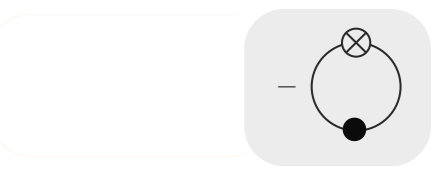
PNJL-model



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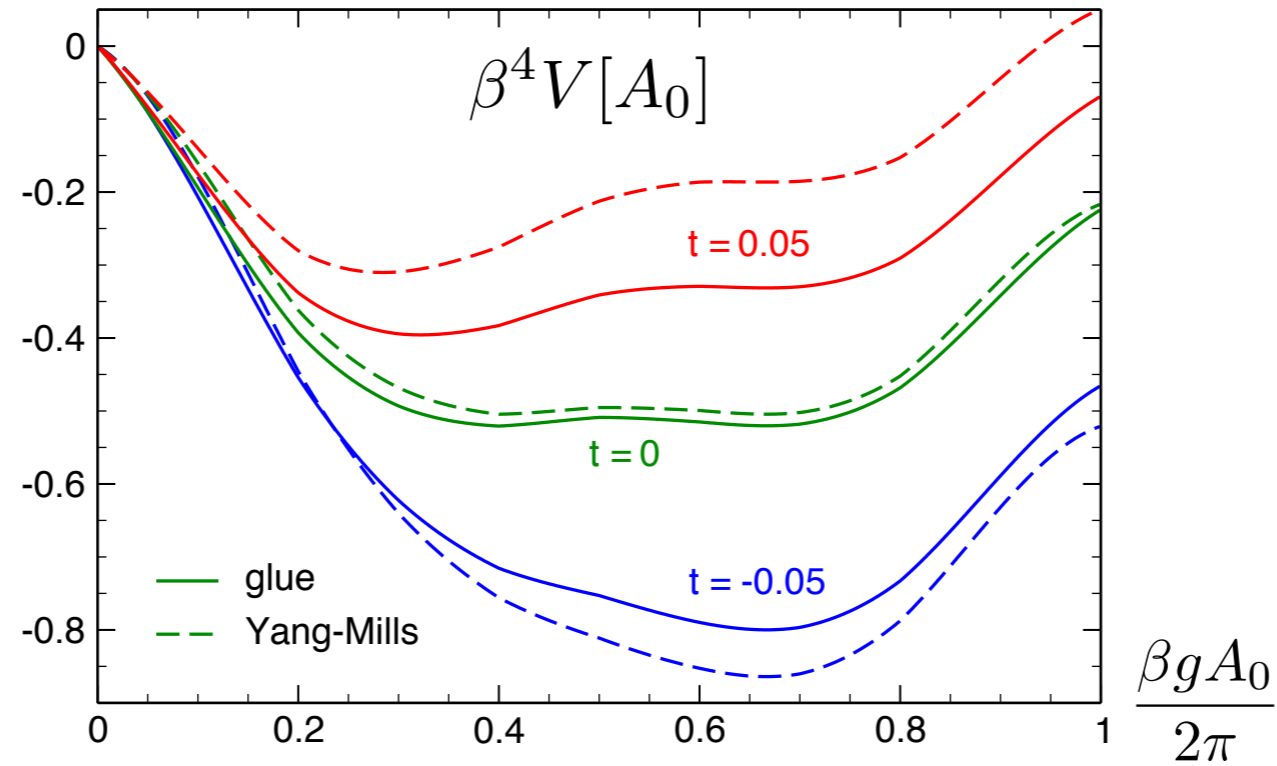
NJL-model



QCD

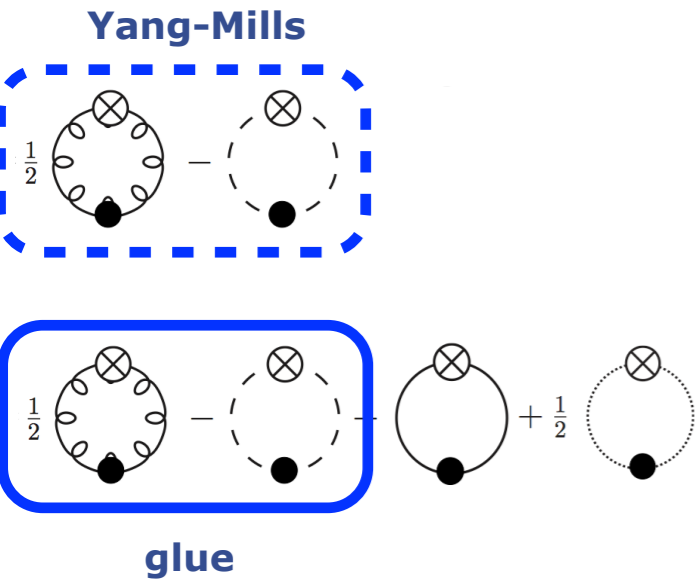
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{glue loop} - \text{Yang-Mills loop} - \text{glue loop} + \frac{1}{2} \text{Yang-Mills loop} \right)$$

Polyakov loop potential in full QCD

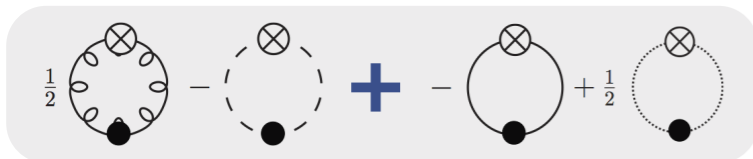


JMP, AIP Conf.Proc. 1343 (2011)

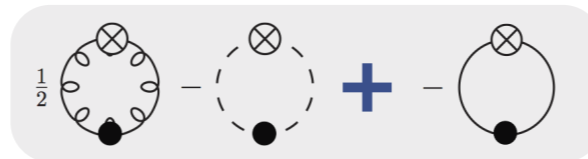
Haas, Stiele et al, PRD 87 (2013) 076004



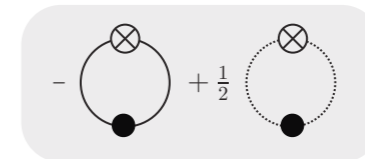
PQM-model



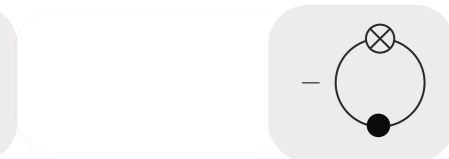
PNJL-model



QM-model

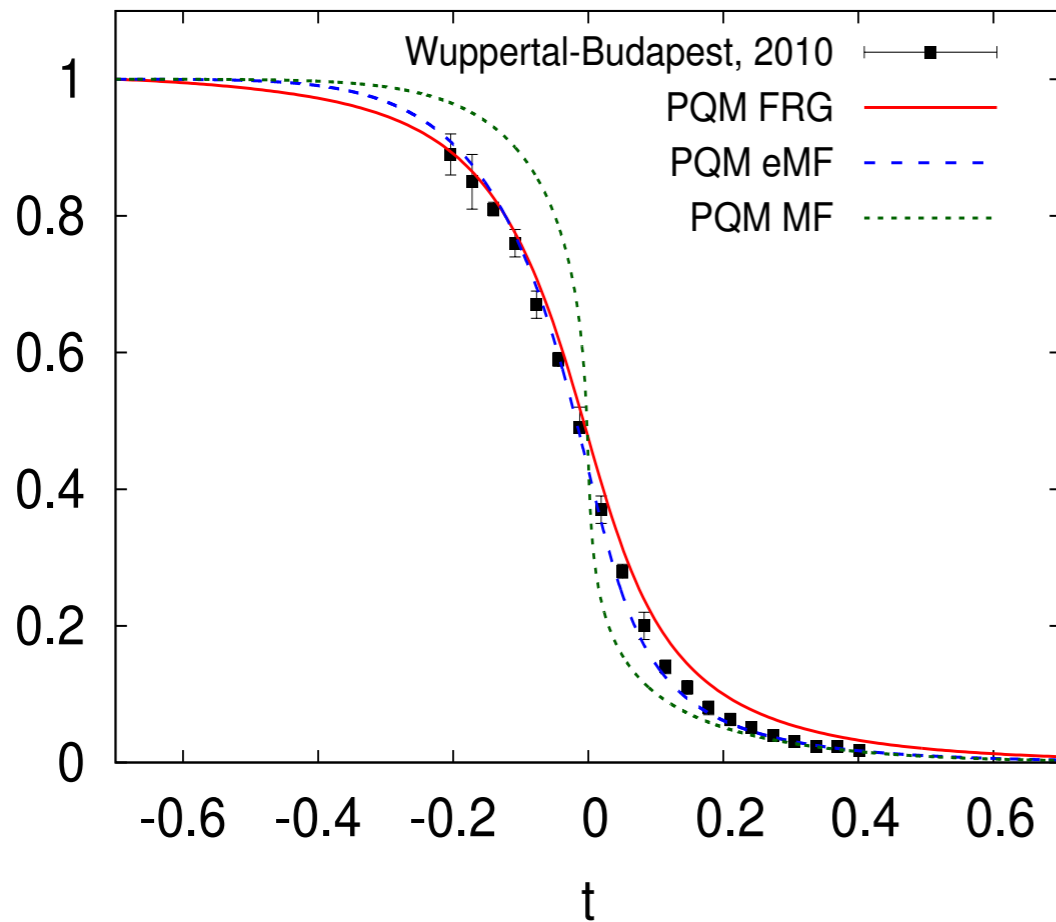


NJL-model



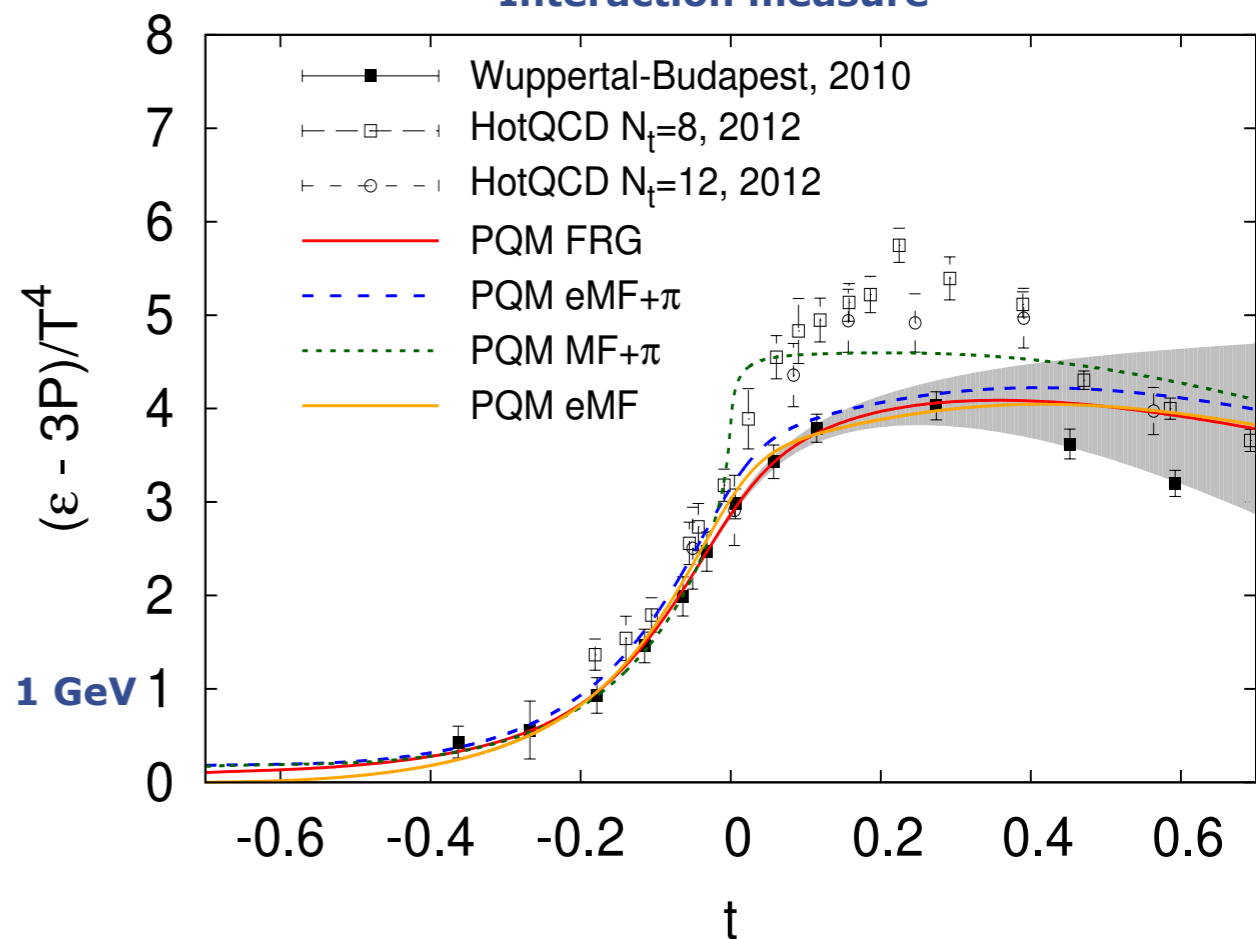
Thermodynamics

reduced chiral condensate

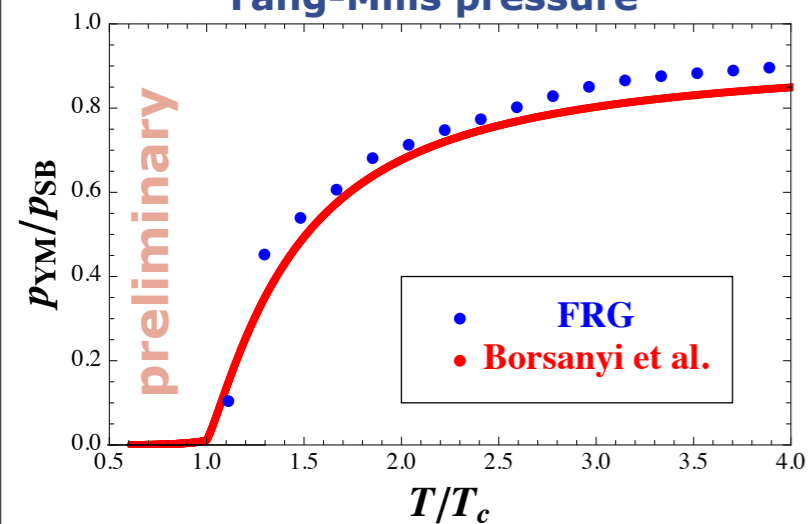


2+1 flavor QCD - enhanced PQM-model

Interaction measure



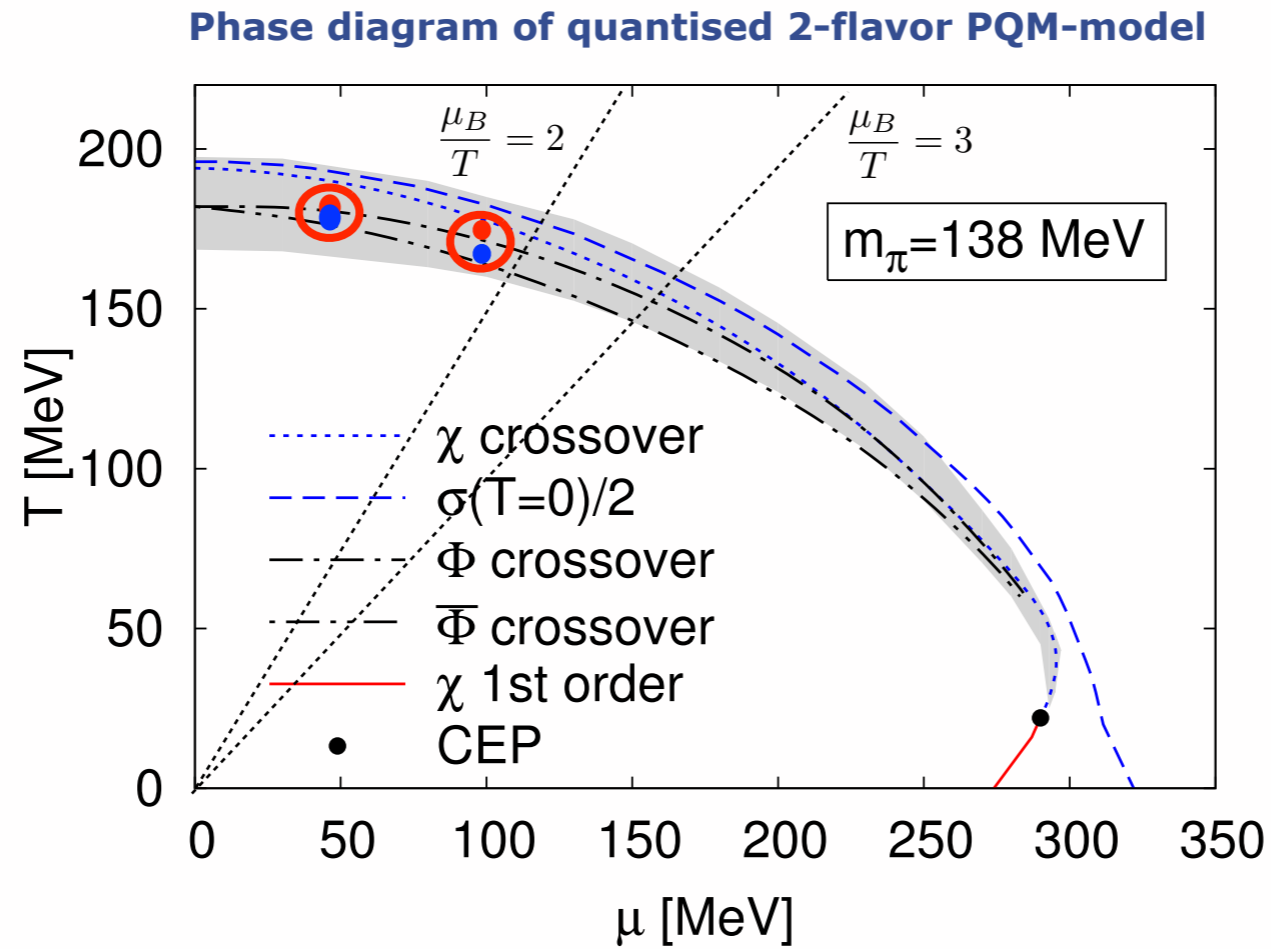
Yang-Mills pressure



Fister, JMP '11

Herbst, Mitter et al, PLB 731 (2014) 248-256

Phase structure at finite density



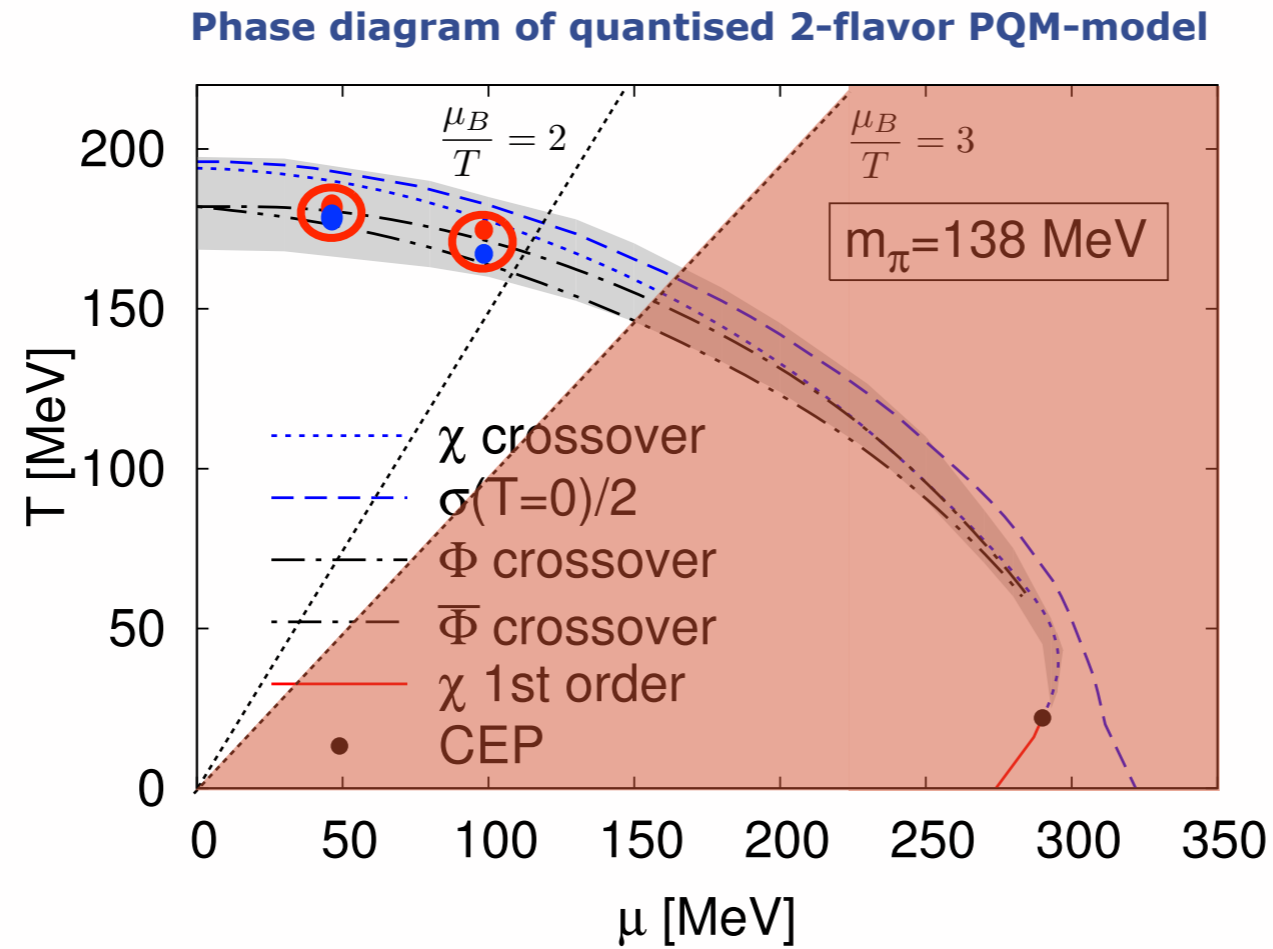
Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Phase structure at finite density



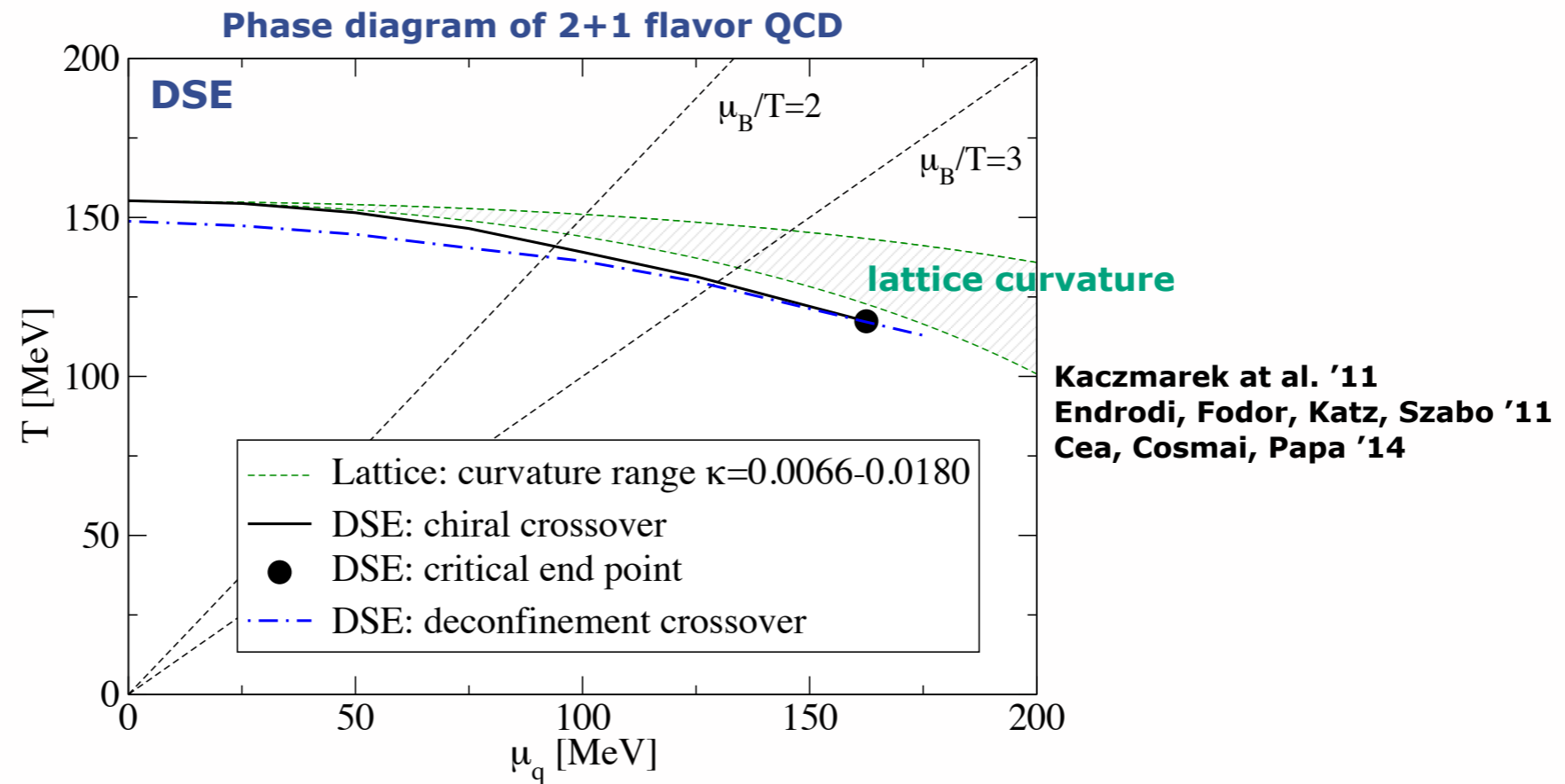
Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Phase structure at finite density



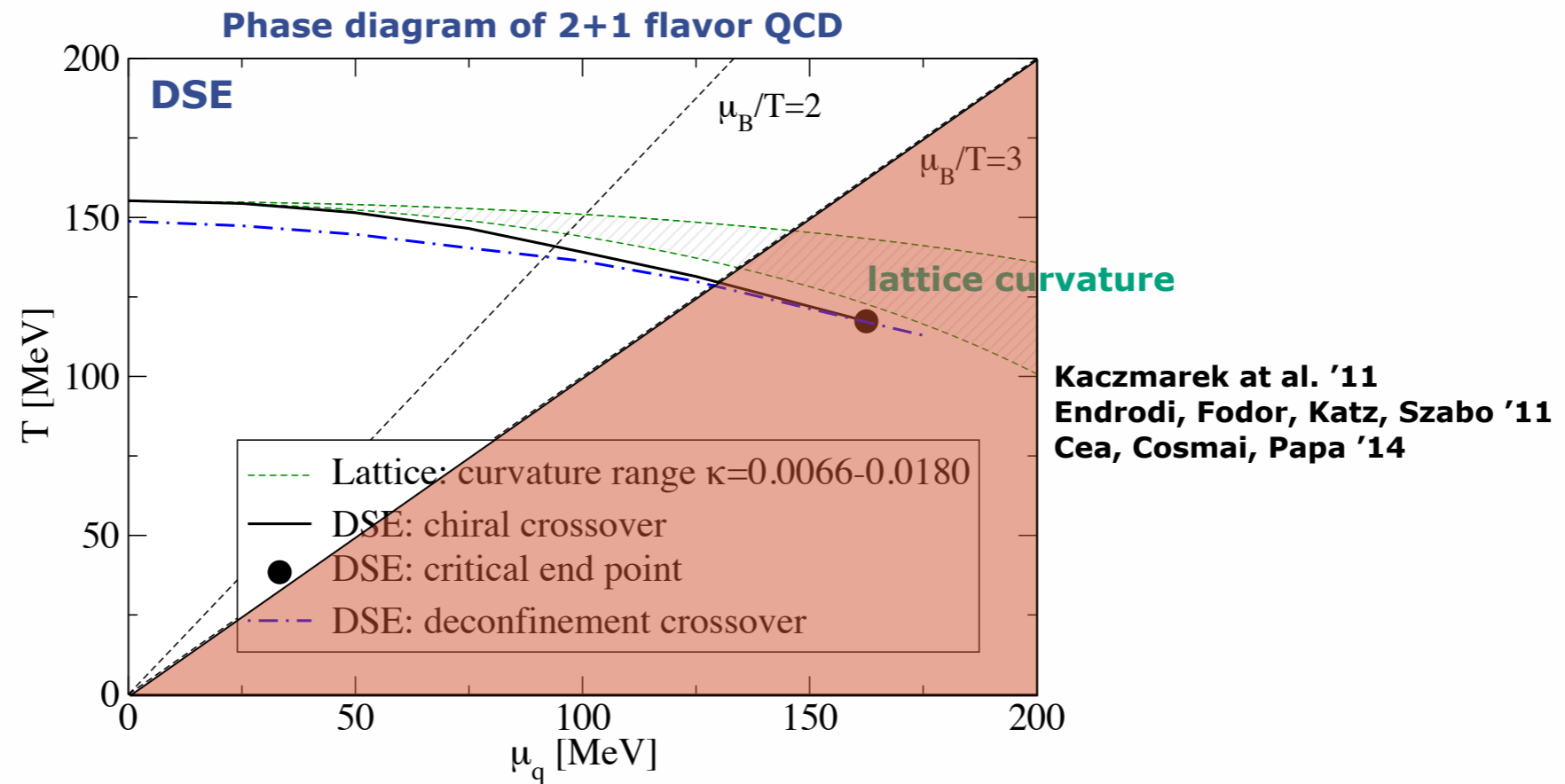
Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \left[\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} - \frac{1}{6} \text{Diagram 4} + \text{Diagram 5} \right]$$

Fister, JMP, PRD 88 (2013) 045010

Phase structure at finite density



Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

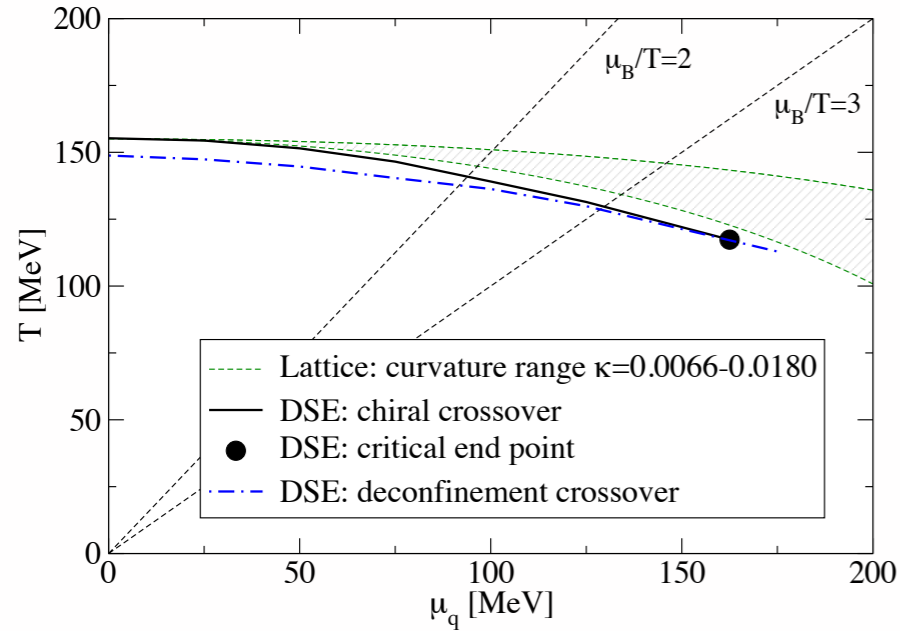
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \left[\text{diagram 1} - \text{diagram 2} - \text{diagram 3} - \frac{1}{6} \text{diagram 4} + \text{diagram 5} \right]$$

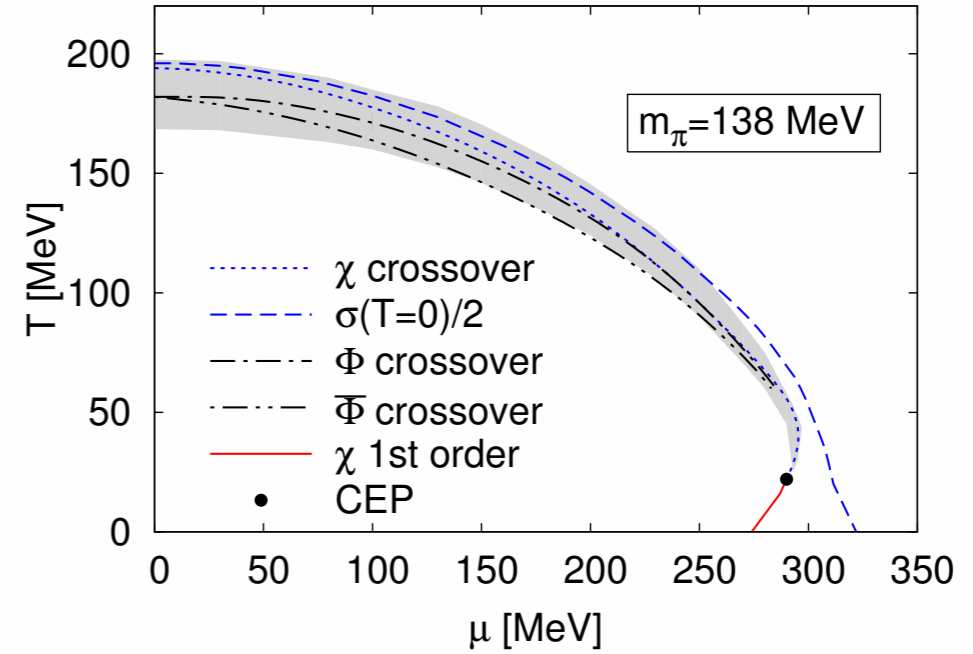
Fister, JMP, PRD 88 (2013) 045010

Phase structure at finite density

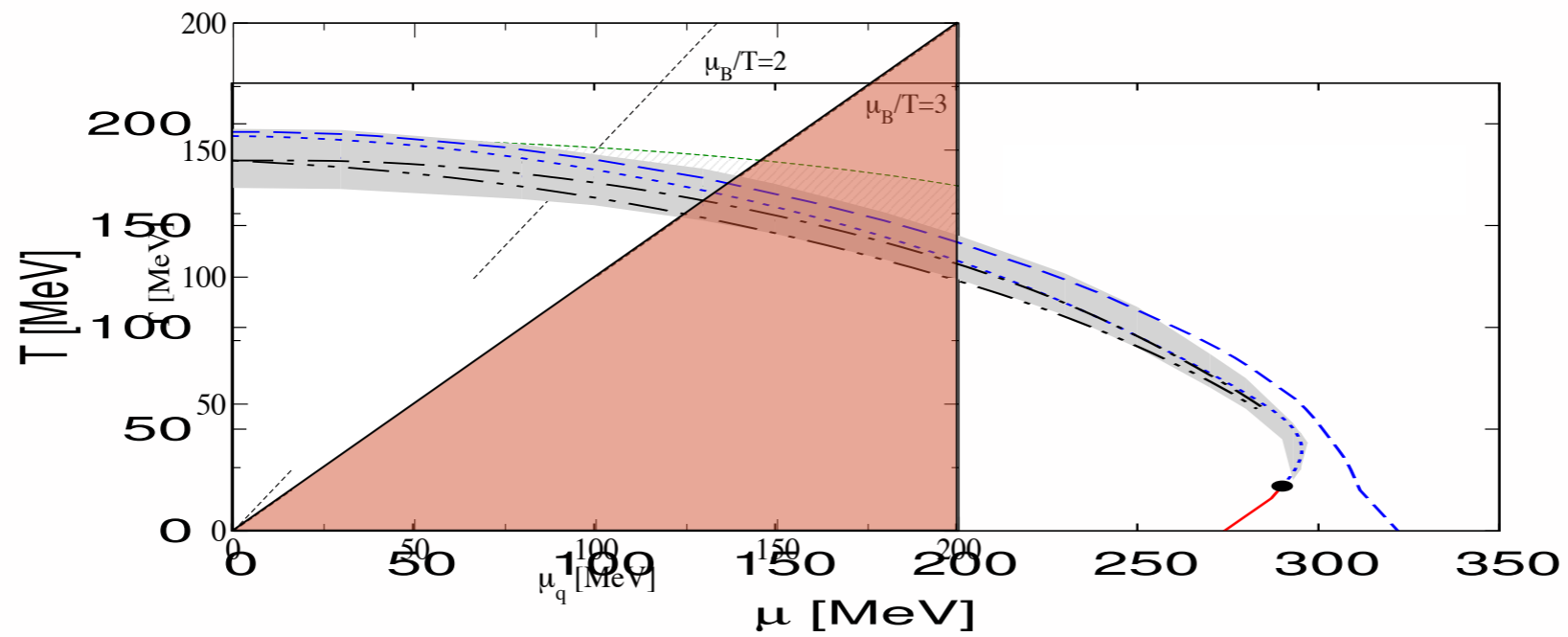
Phase diagram of 2+1 flavor QCD



Phase diagram of quantised 2-flavor PQM-model

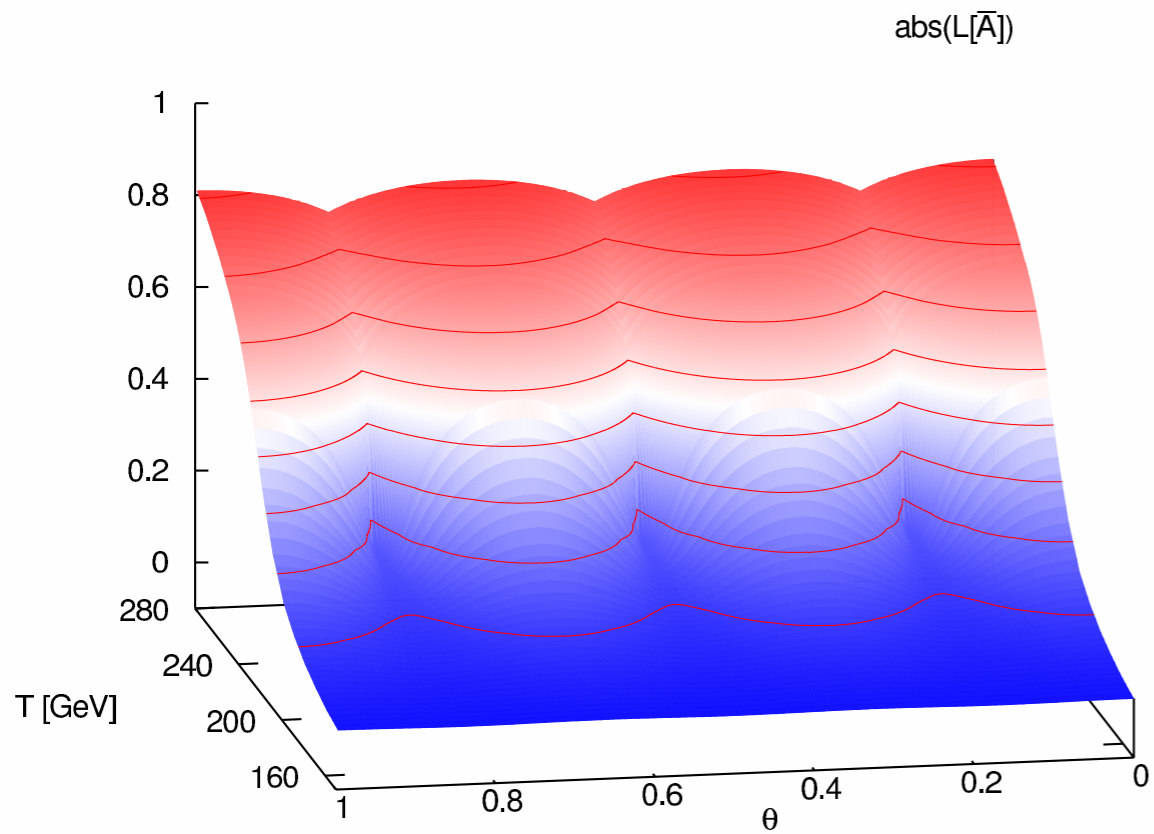


Comparison with 2 flavor vs 2+1 flavor scale matching of T_c

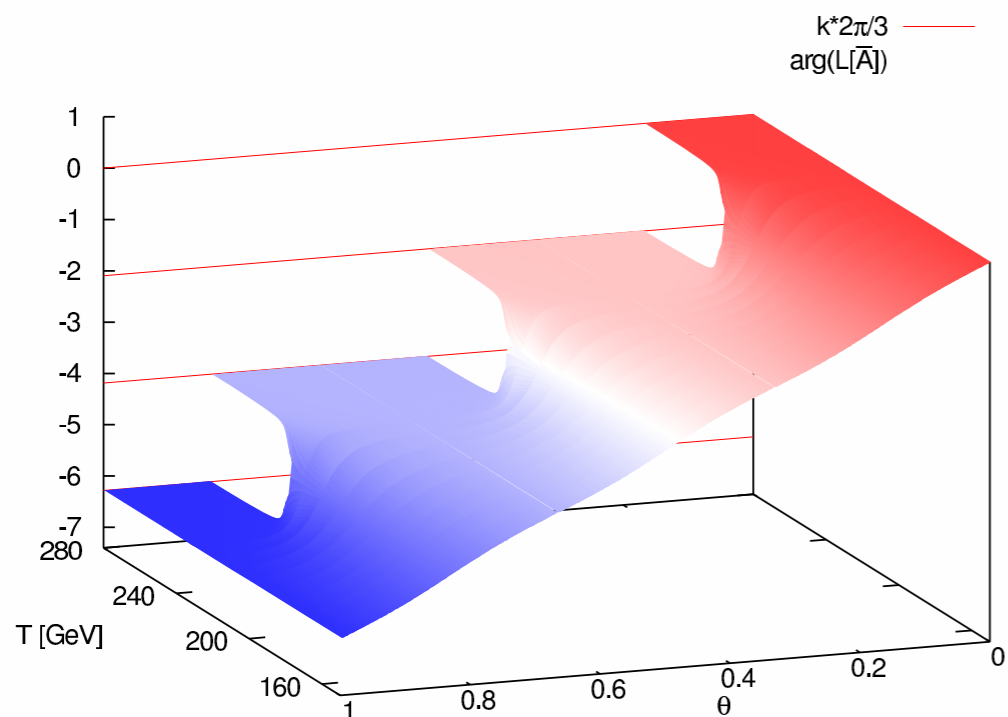
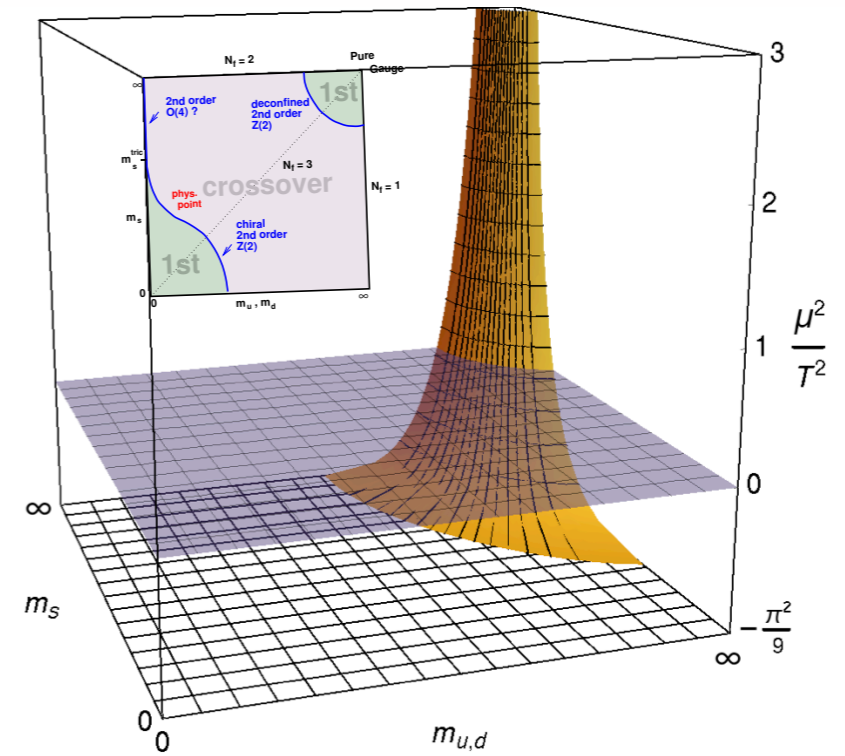


Phase structure at finite density for heavy quarks

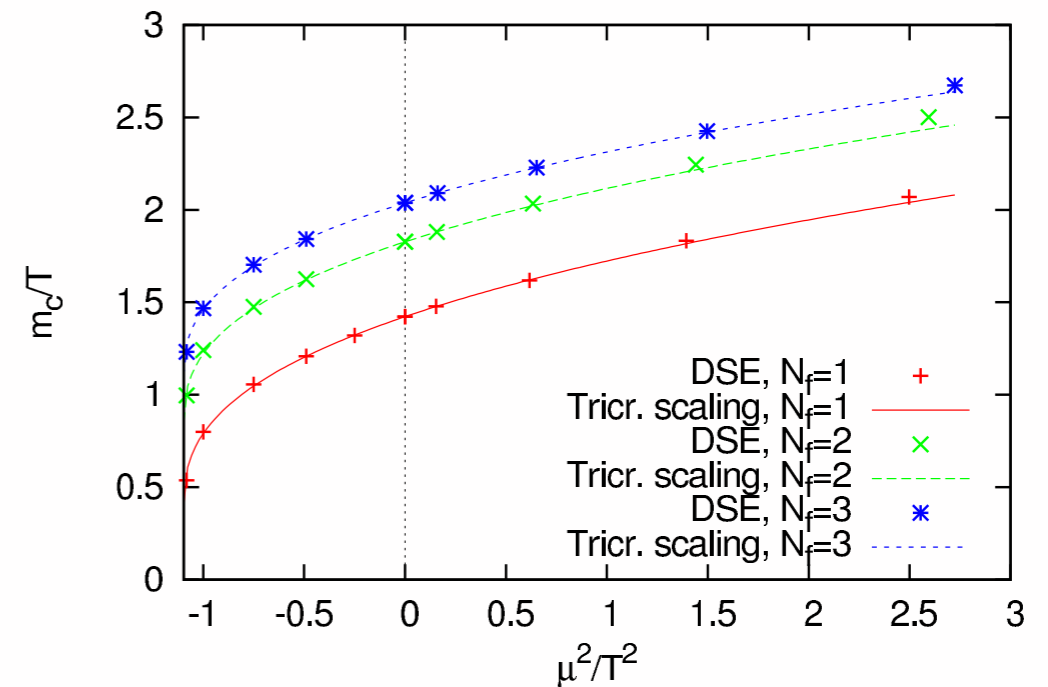
Polyakov loop at finite density



Critical surface at large masses



Tricritical scaling



Outline

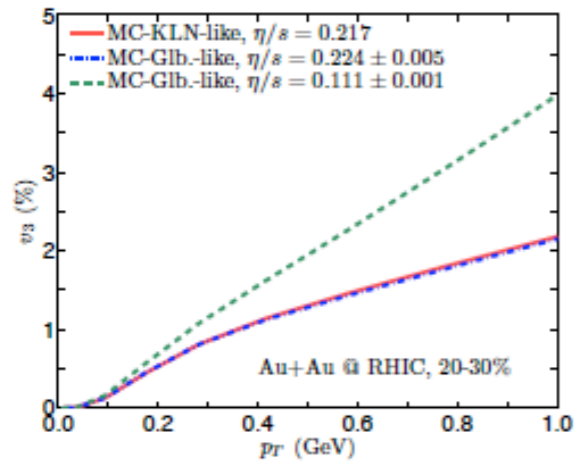
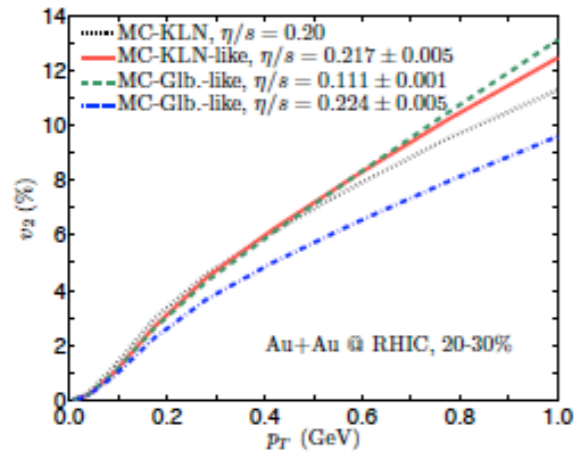
- Vacuum QCD & the hadron spectrum
- Phase structure of QCD
- Spectral Functions & Transport Coefficients
- Outlook

Dynamics

Shooting the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



- Take ensemble of sum of deformed Gaussian profiles, $s(\mathbf{r}_\perp) = s_2(\mathbf{r}_\perp; \tilde{\epsilon}_2, \psi_2) + s_3(\mathbf{r}_\perp; \tilde{\epsilon}_3, \psi_3)$, with
 1. equal Gaussian radii $R_2^2 = R_3^2 = 8 \text{ fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
 2. $\tilde{\epsilon}_2$ and $\tilde{\epsilon}_3$ adjusted such that
 - $\tilde{\epsilon}_{2,3} = \langle \epsilon_{2,3} \rangle_{\text{KLN}}^{20-30\%}$ ("MC-KLN-like")
 - $\tilde{\epsilon}_{2,3} = \langle \epsilon_{2,3} \rangle_{\text{G1}}^{20-30\%}$ ("MC-Glauber-like")
 3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
 - $\Rightarrow (\eta/s)_{\text{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
 - $(\eta/s)_{\text{G1}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\text{G1}}=0.217$ and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
 - $\Rightarrow (\eta/s)_{\text{G1}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\text{G1}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data \Rightarrow clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields **the same η/s** (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and **only one** (more precisely: at most one!) of the models **will fit the corresponding $v_2(p_T)$ data**.

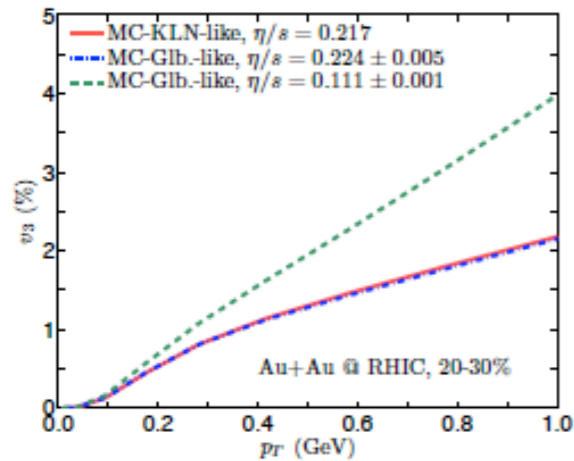
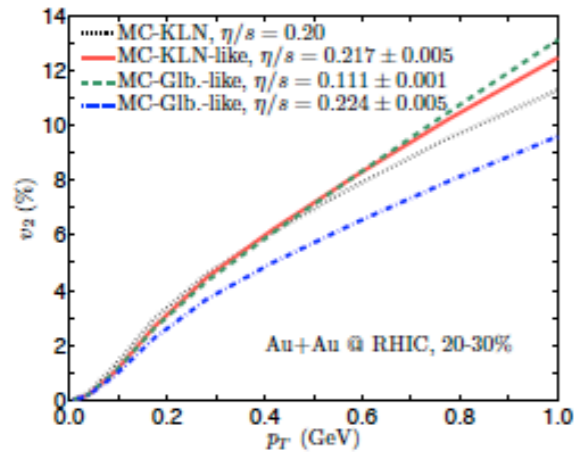
U. Heinz, talk at RETUNE '12

Dynamics

Computing the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



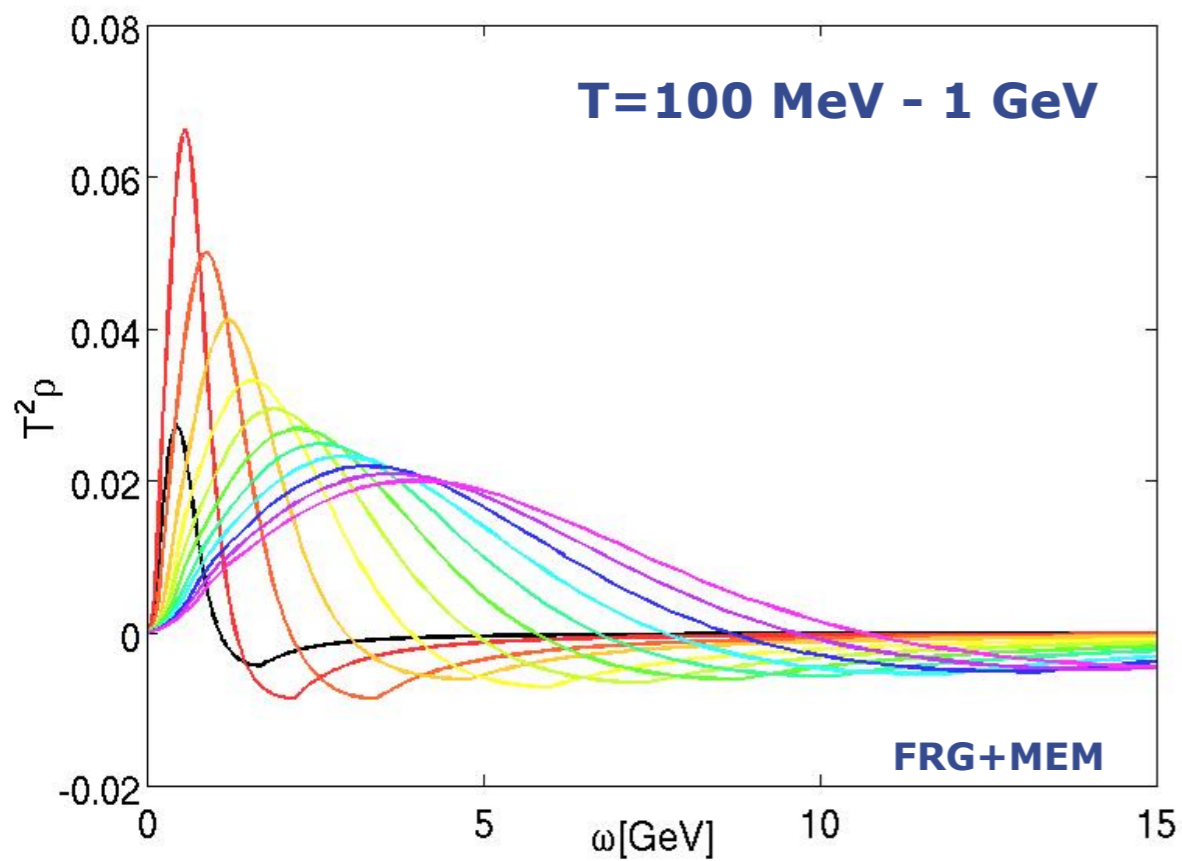
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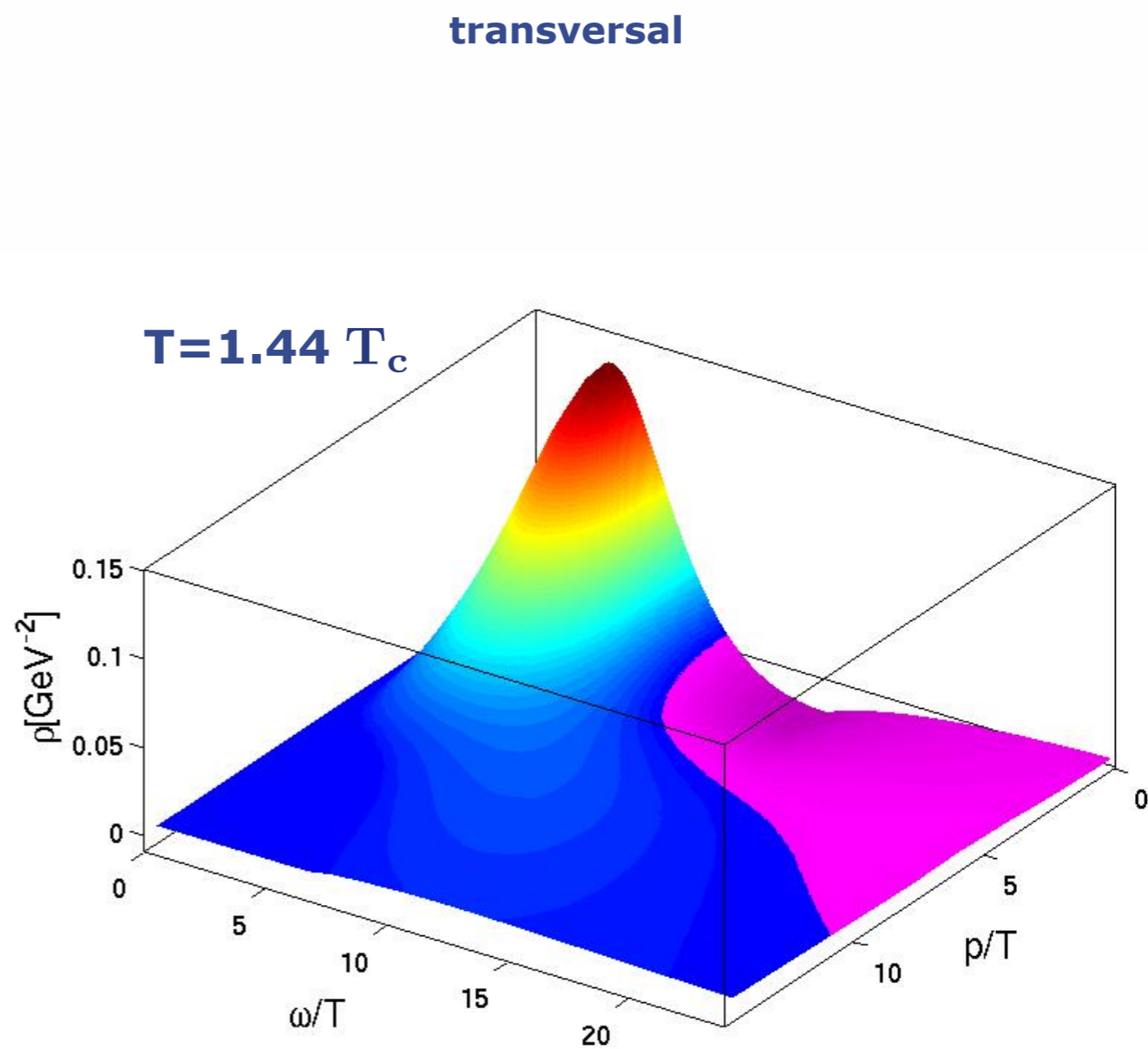
computing transport coefficients

Dynamics

gluon spectral function at finite T

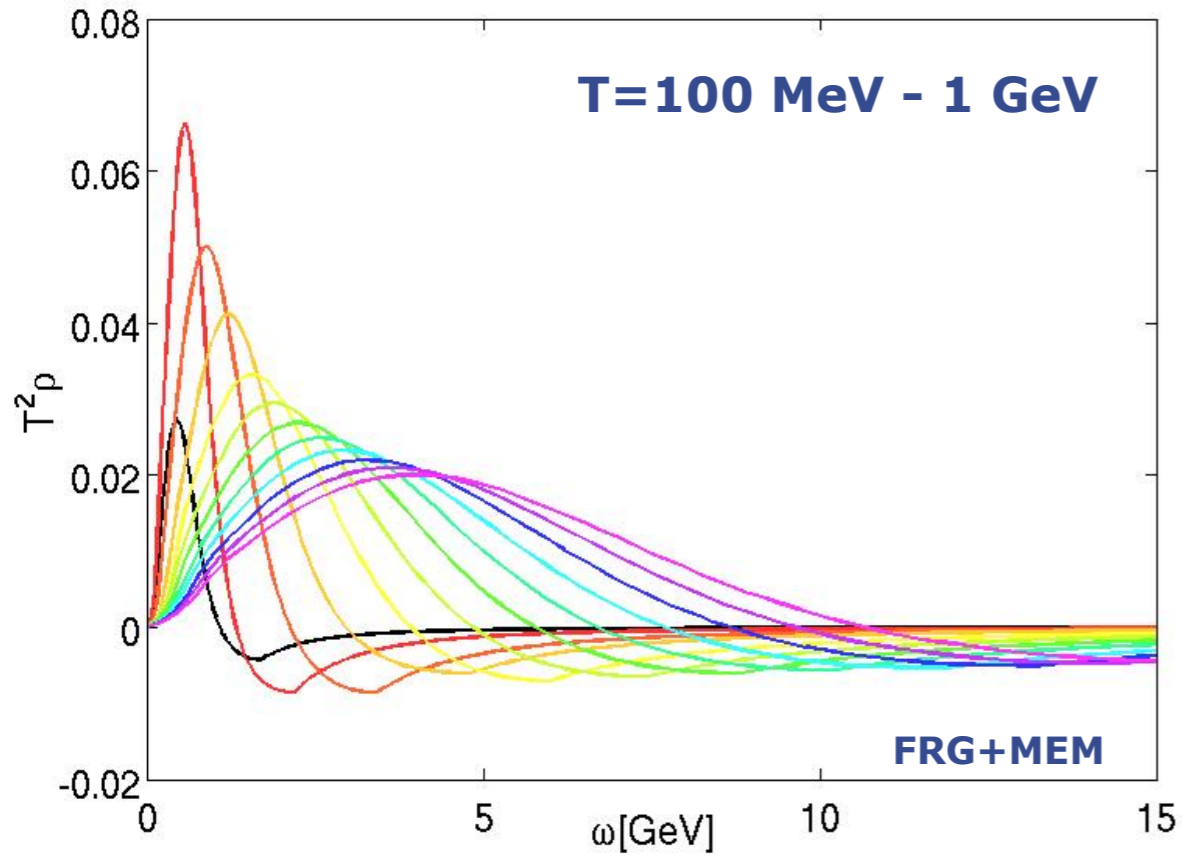


MEM

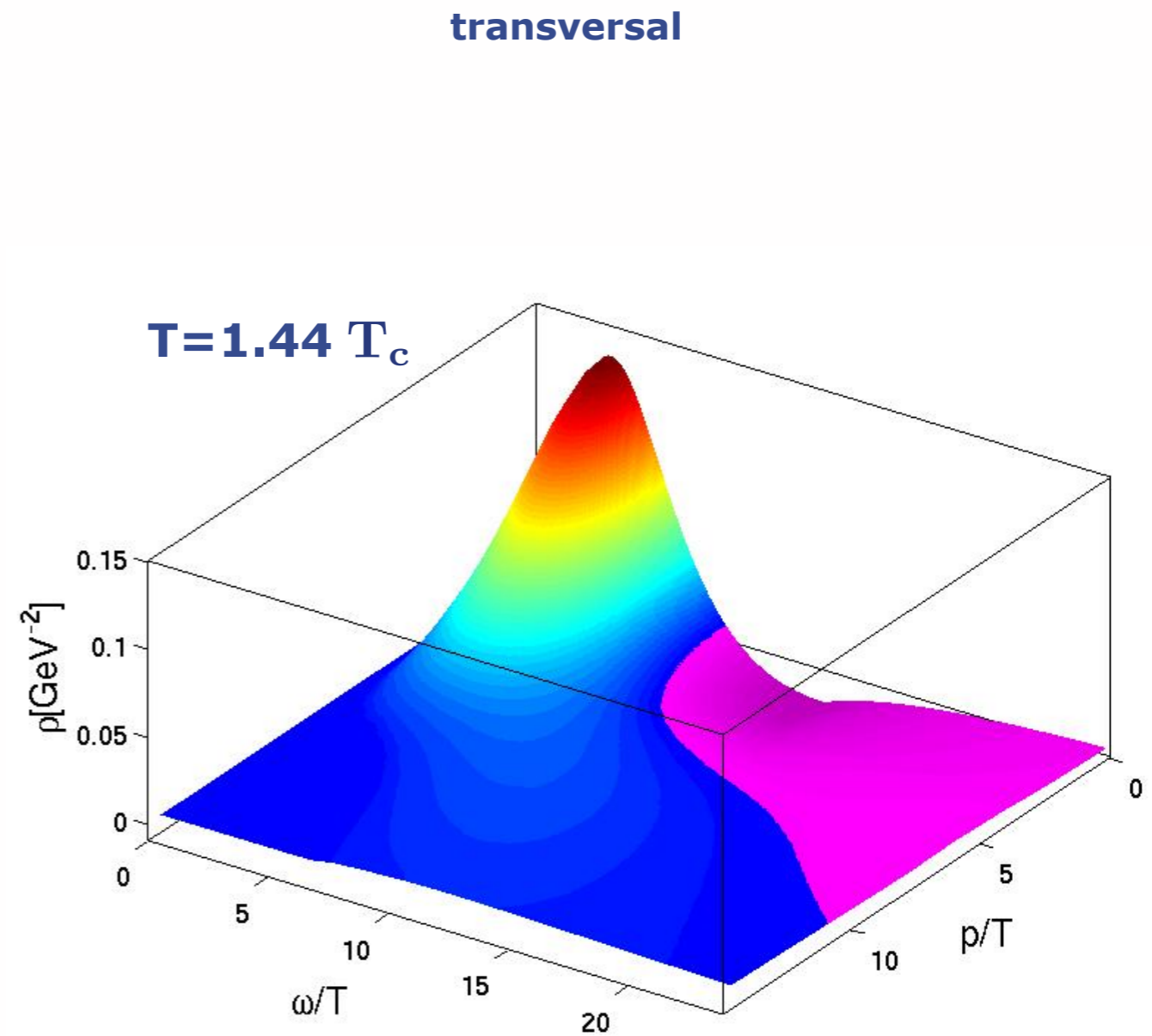


Dynamics

gluon spectral function at finite T



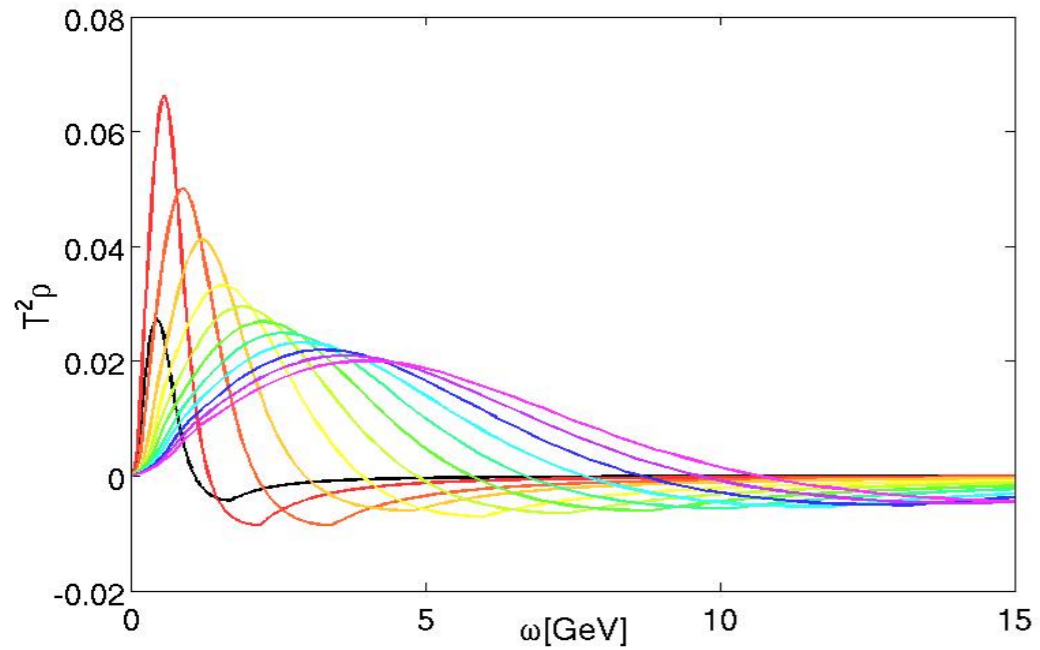
'Those are my methods (principles), and if you don't like them...well, I have others'
direct computation Groucho Marx



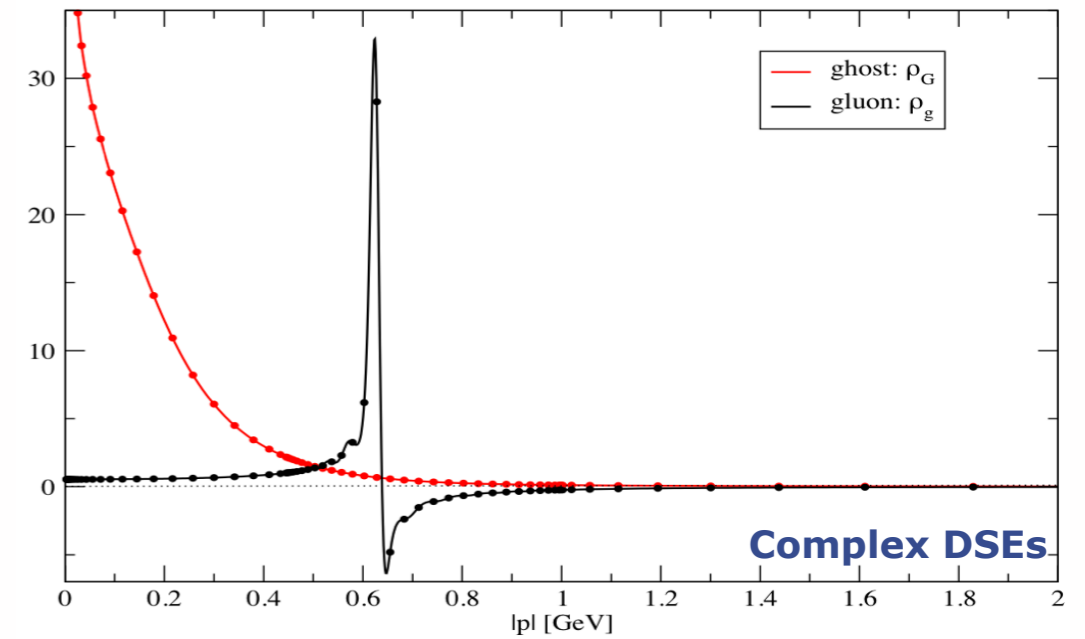
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Dynamics

gluon spectral functions



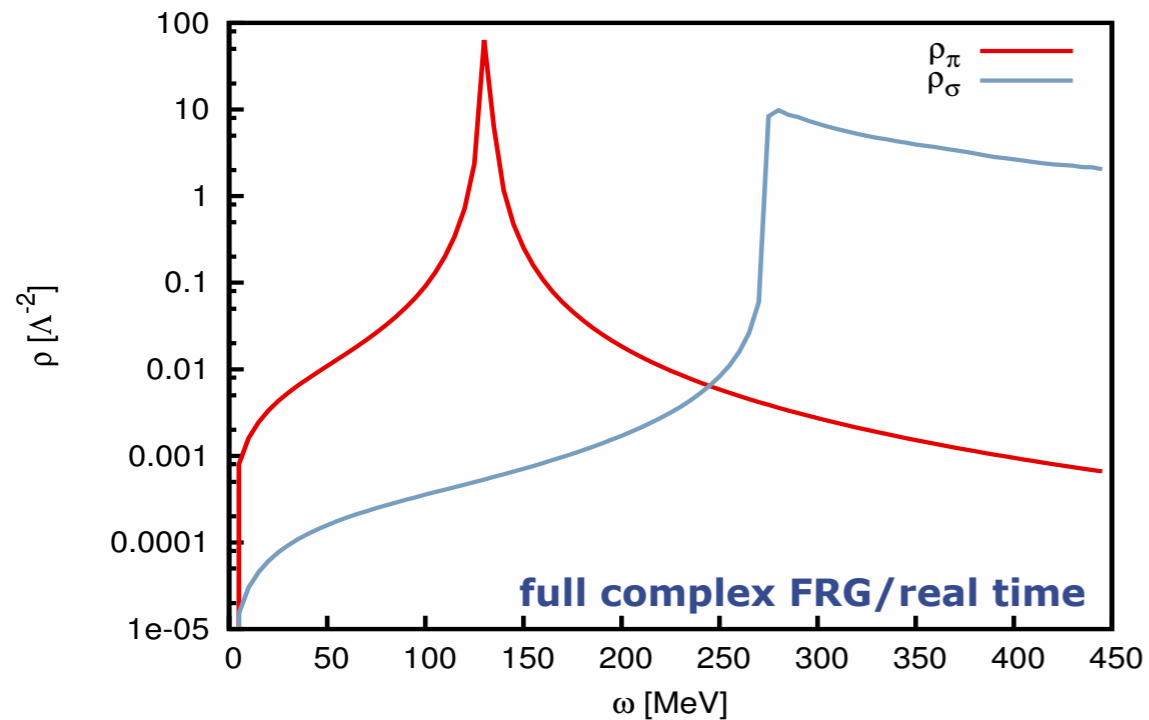
Haas, Fister, JMP, PRD 90 (2014) 9, 091501



Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions

4d N=2 exponential regulator, $\epsilon=0.1$ MeV



full complex FRG/real time

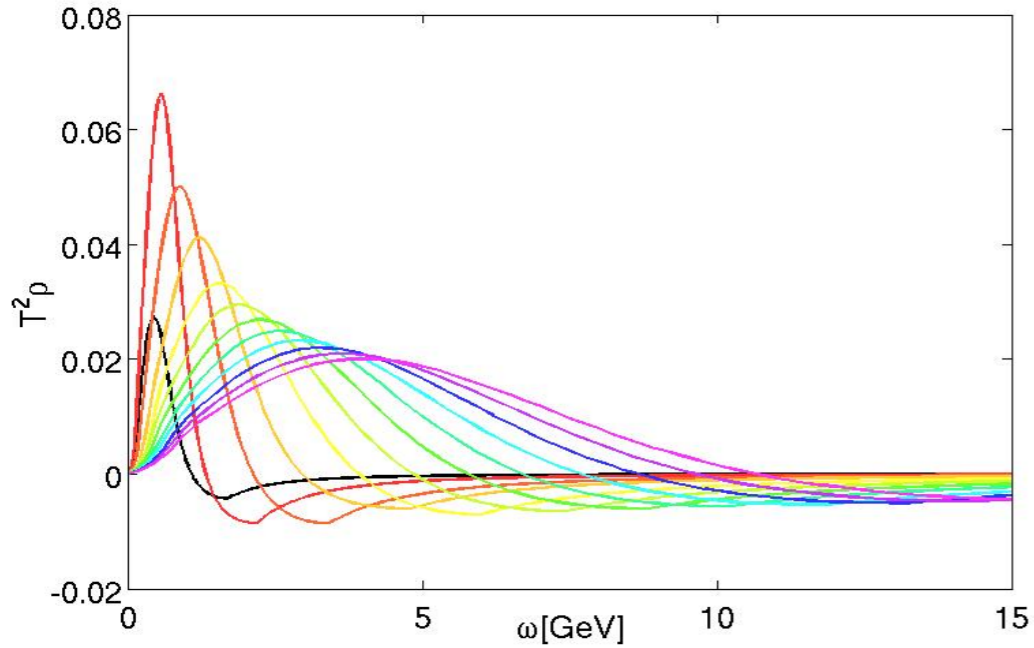
JMP, Strodtthoff, in preparation

analytic complex FRG

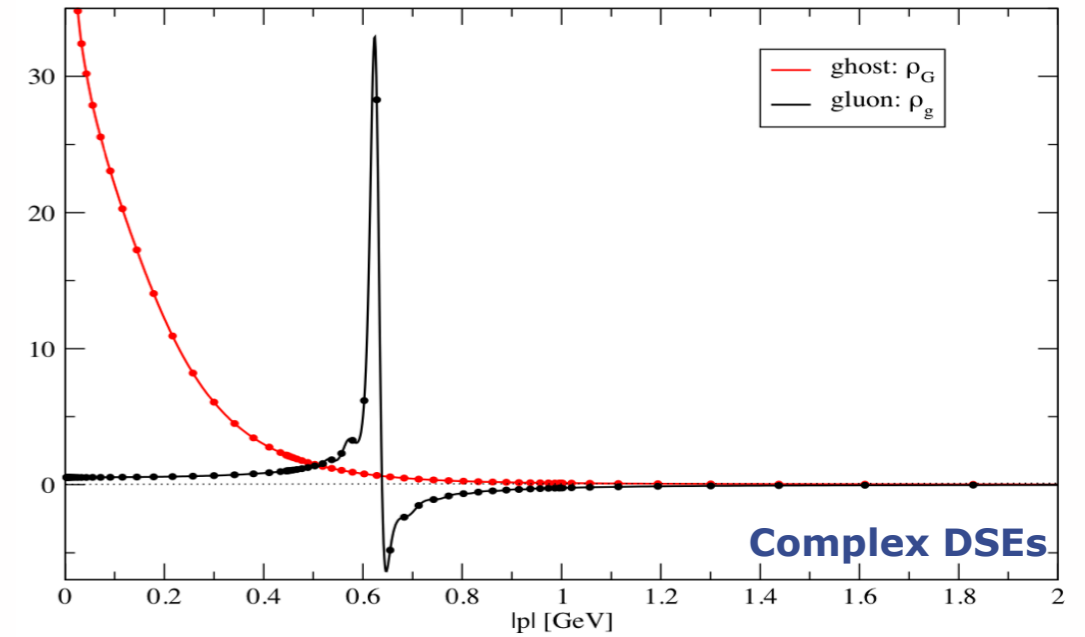
Tripolt, Strodtthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodtthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Dynamics

gluon spectral functions



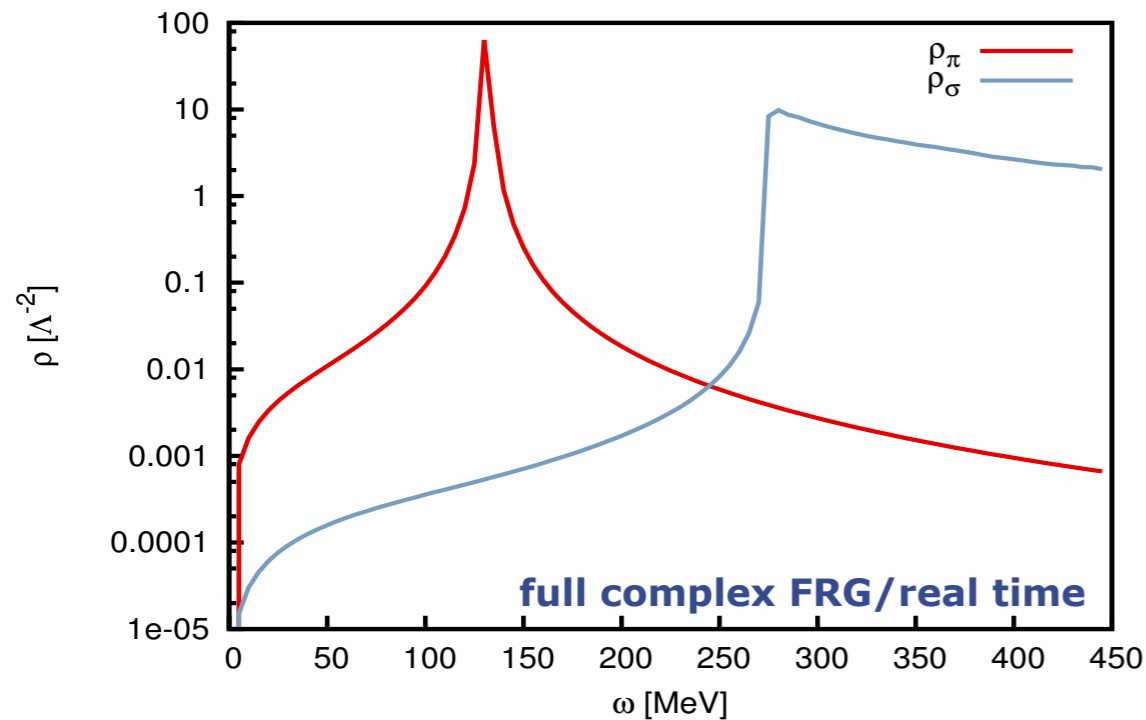
Haas, Fister, JMP, PRD 90 (2014) 9, 091501



Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

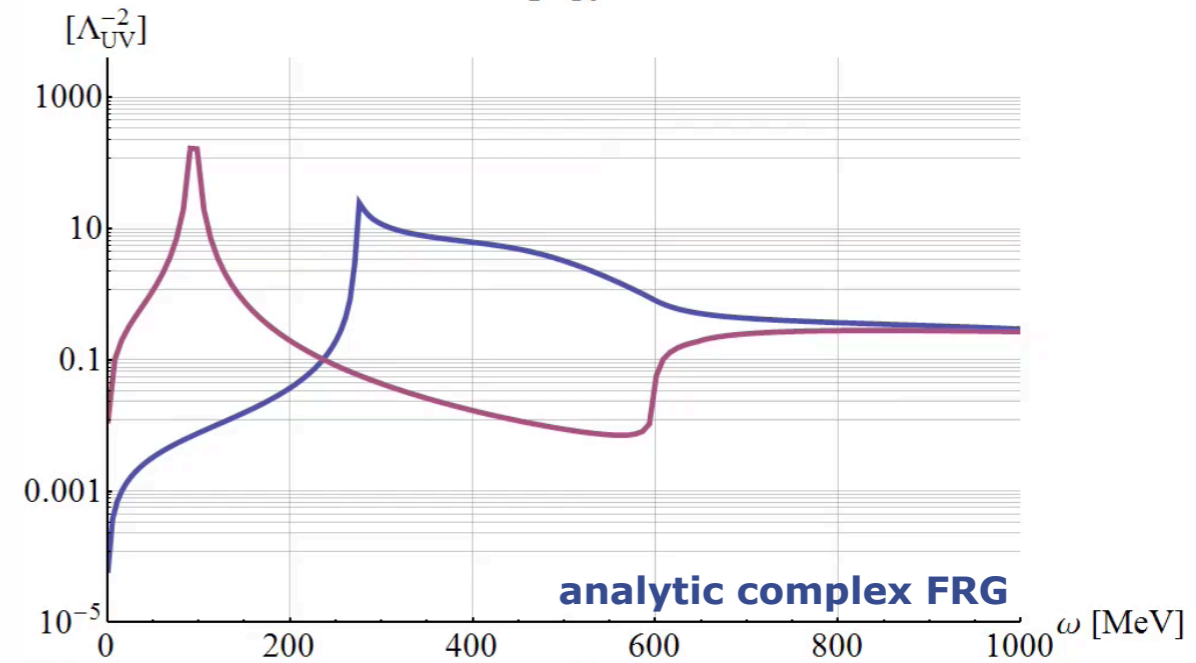
pion and sigma spectral functions

4d N=2 exponential regulator, $\epsilon=0.1$ MeV



JMP, Strodtthoff, in preparation

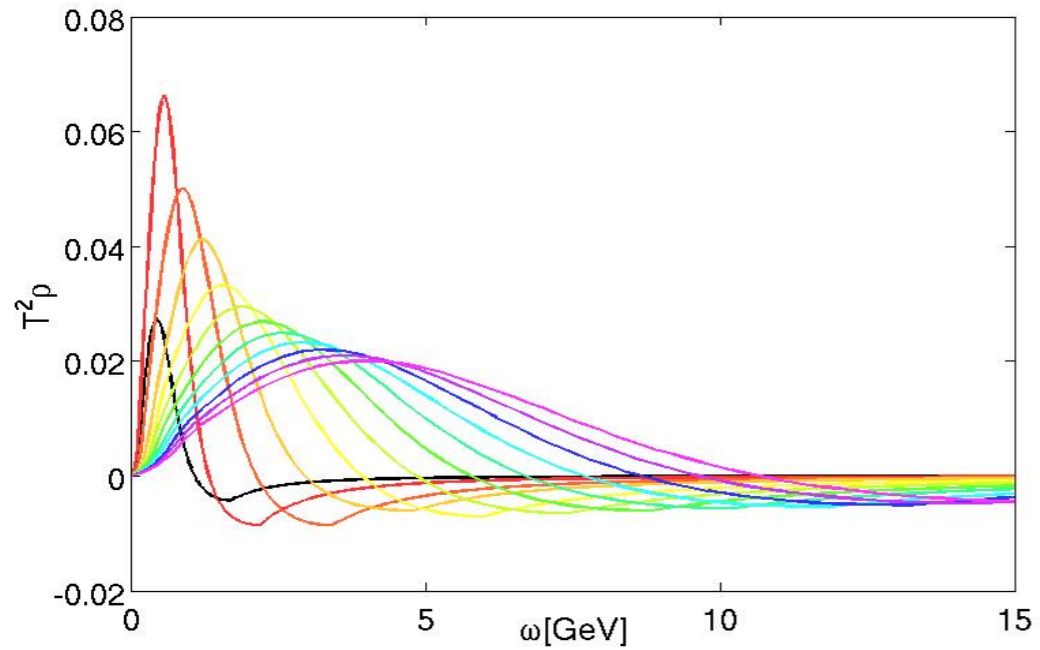
T=10



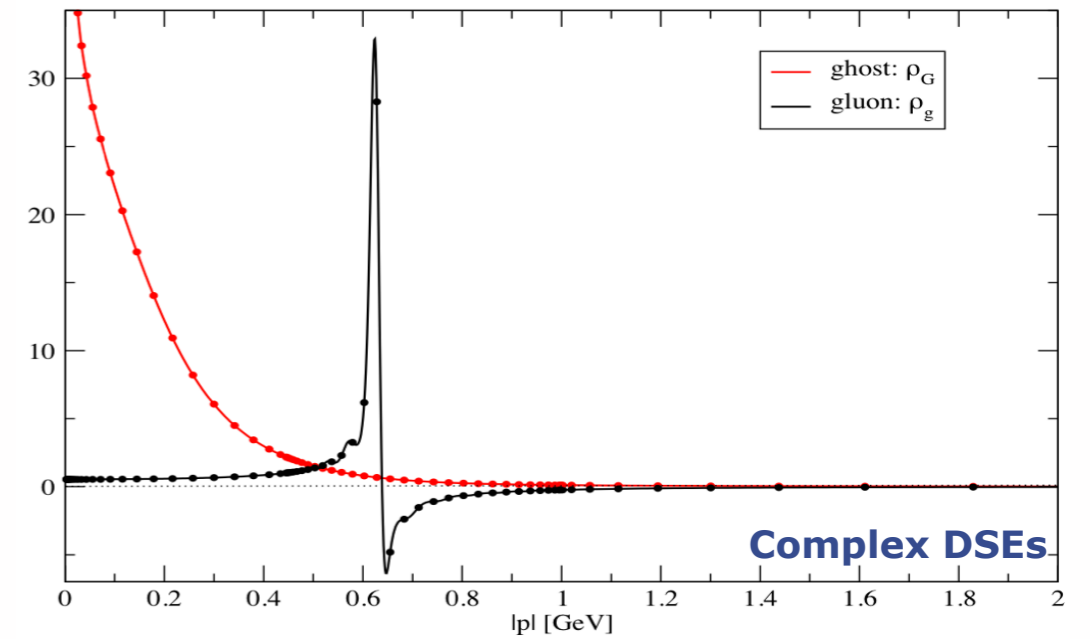
Tripolt, Strodtthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodtthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Dynamics

gluon spectral functions

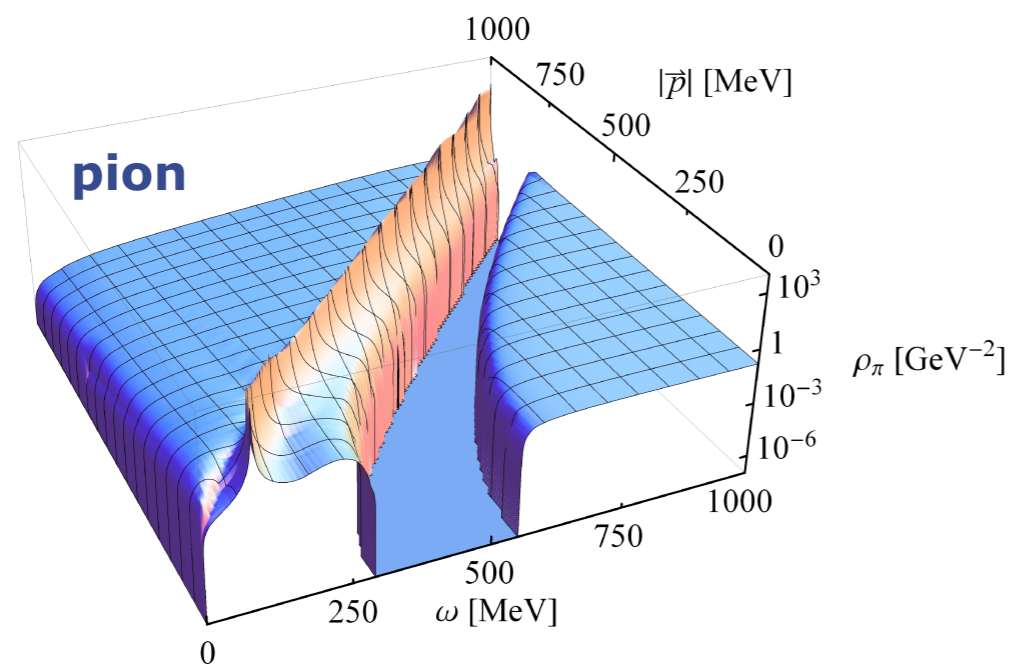


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

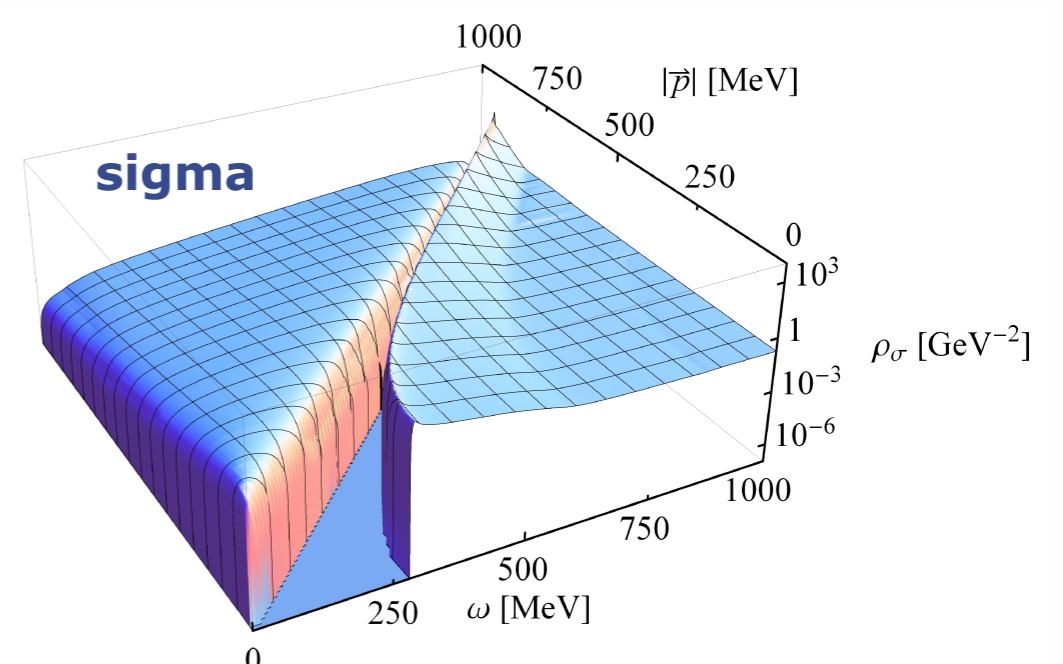


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions



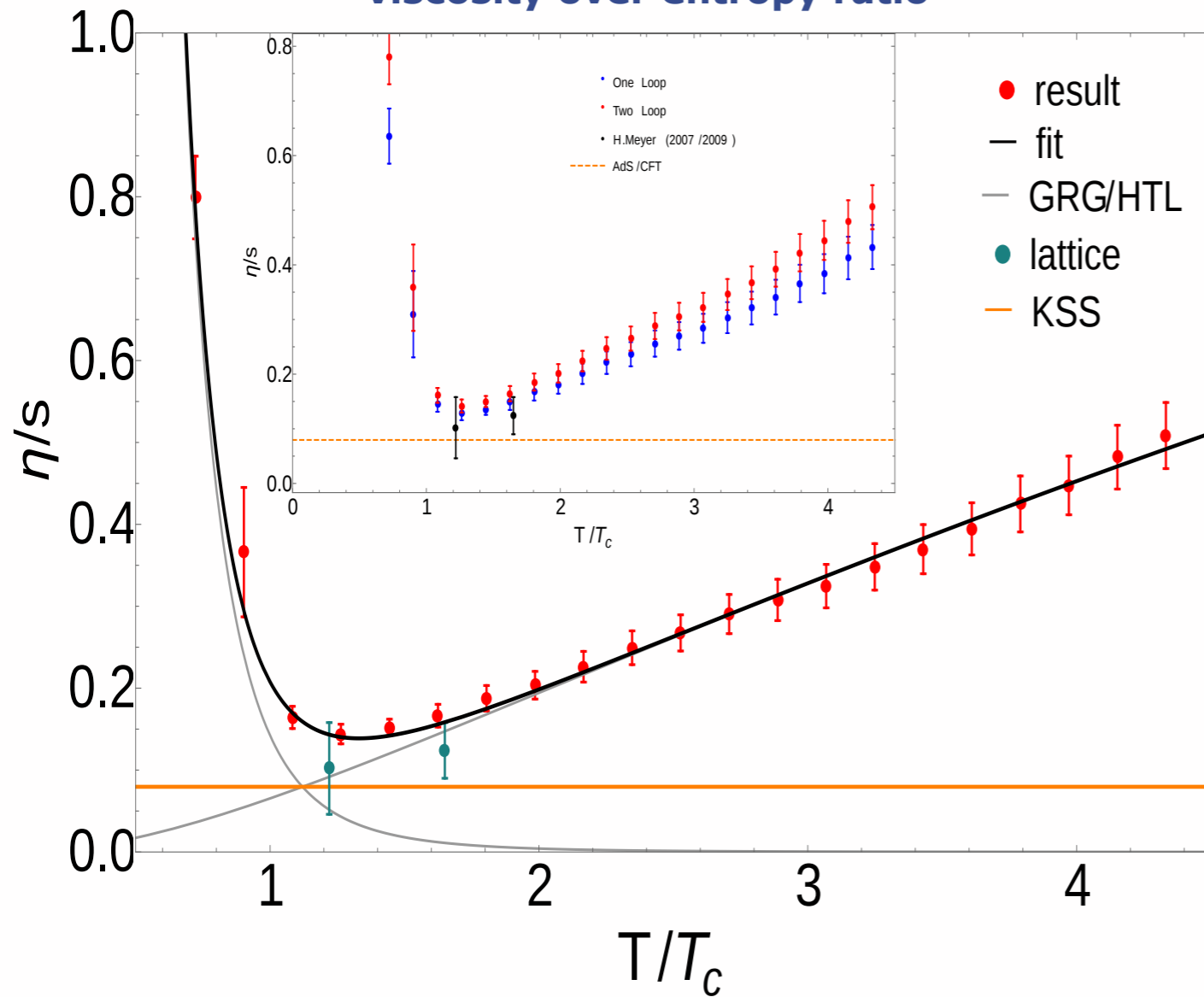
Tripolt, von Smekal, Wambach, Phys.Rev. D90 (2014) 7, 074031
 Tripolt, PhD-thesis



Dynamics

transport coefficients

viscosity over entropy ratio

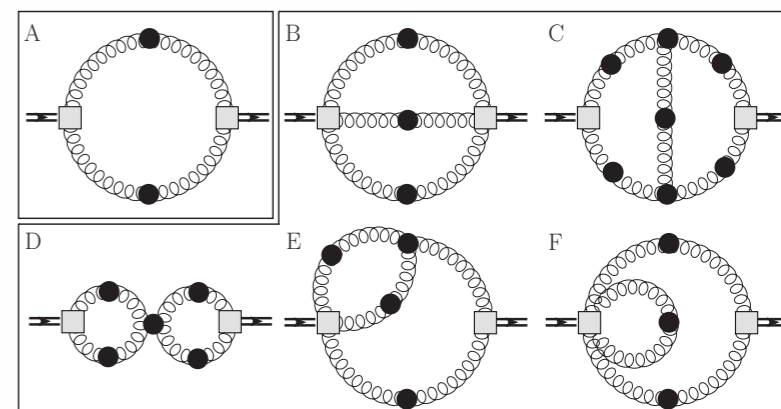


Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

'3-loop' exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



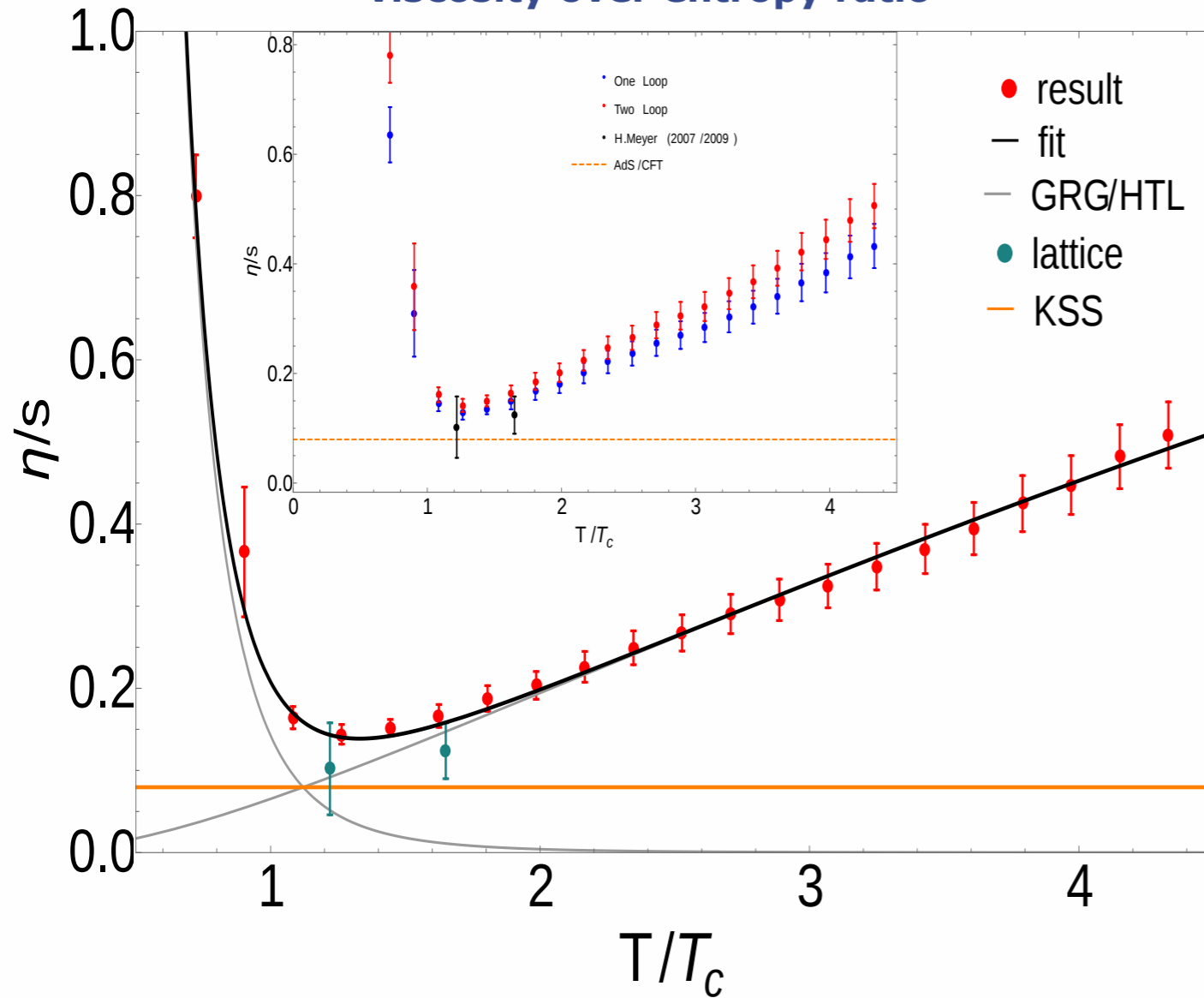
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, arXiv:1411.7986

Dynamics

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\gamma_{\text{grg}} \approx 5$$

$$\gamma_{\text{qgp}} \approx 1.6$$

pure glue

$$a_{\text{qgp}} \approx 0.15$$

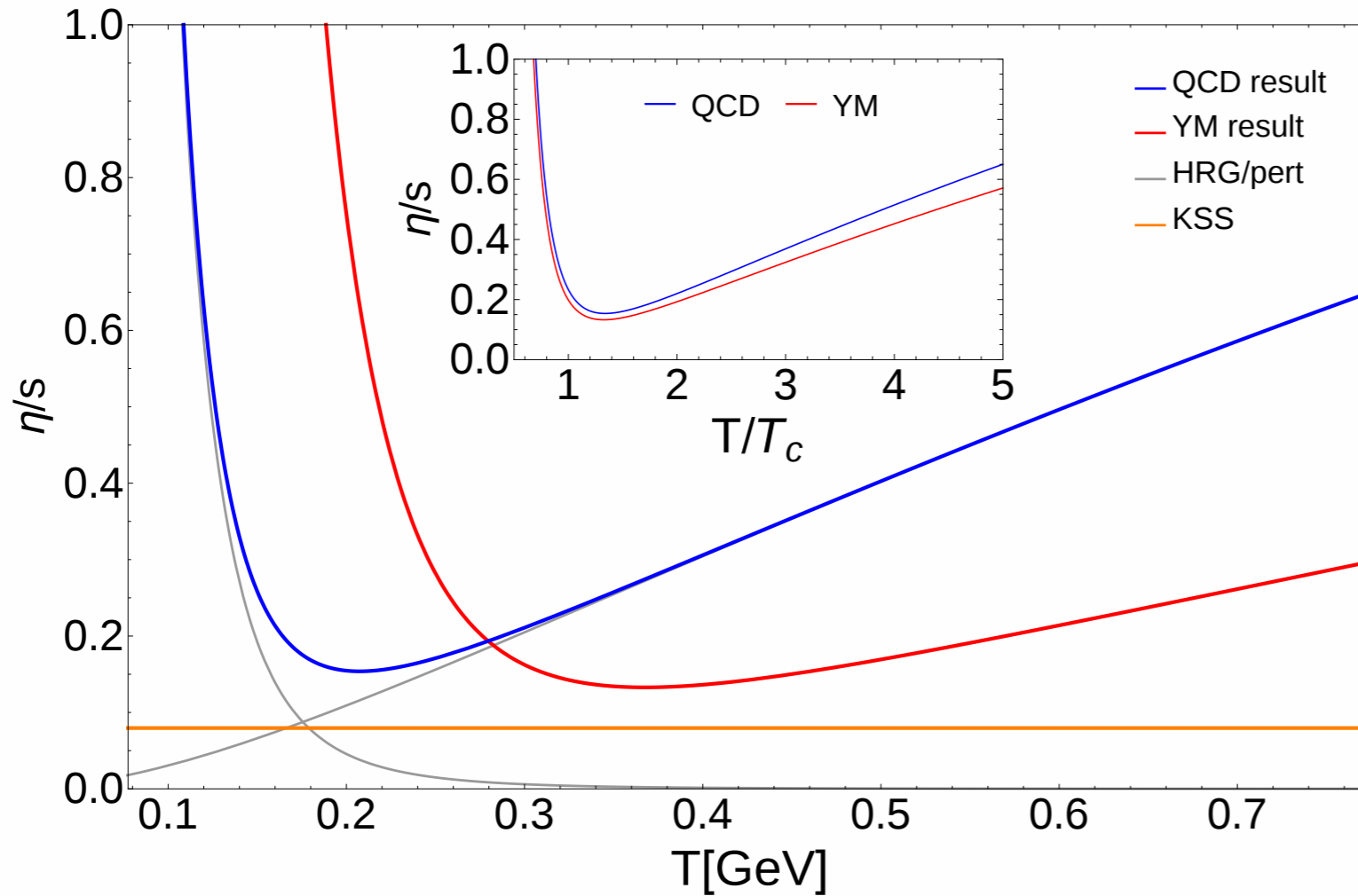
$$a_{\text{hrg}} \approx 0.14$$

$$c \approx 0.66$$

$$\frac{\eta}{s}(T) = \frac{a_{\text{qgp}}}{\alpha_s^{\gamma_{\text{qgp}}}(cT/T_c)} + \frac{a_{\text{grg}}}{(T/T_c)^{\gamma_{\text{grg}}}}$$

Dynamics

QCD - estimate for viscosity over entropy ratio



$$a_{\text{qgp}} \approx 0.2$$

$$a_{\text{hrg}} \approx 0.16$$

$$c \approx 0.79$$

QCD

$$\gamma_{\text{grg}} \approx 5$$

$$\gamma_{\text{qgp}} \approx 1.6$$

pure glue

$$a_{\text{qgp}} \approx 0.15$$

$$a_{\text{hrg}} \approx 0.14$$

$$c \approx 0.66$$

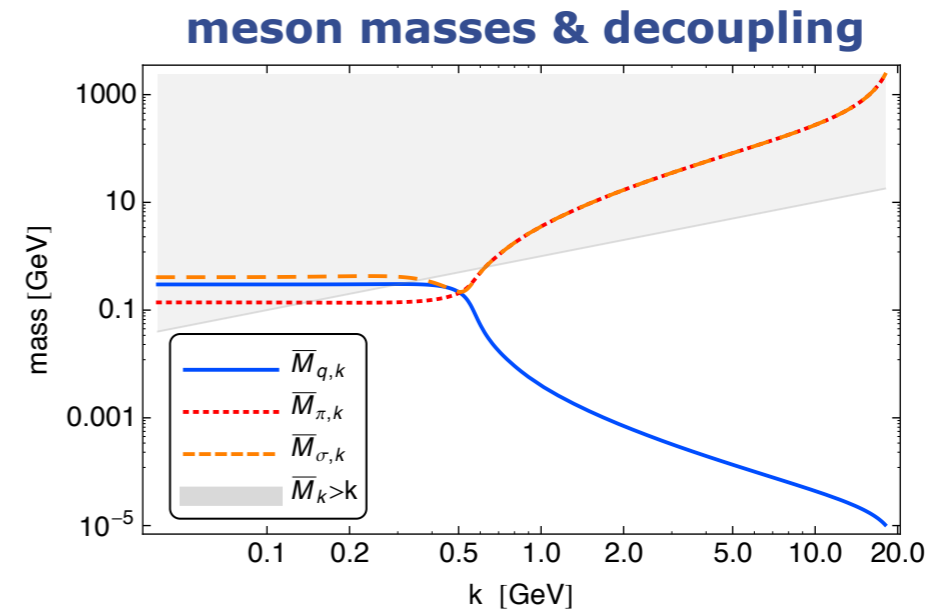
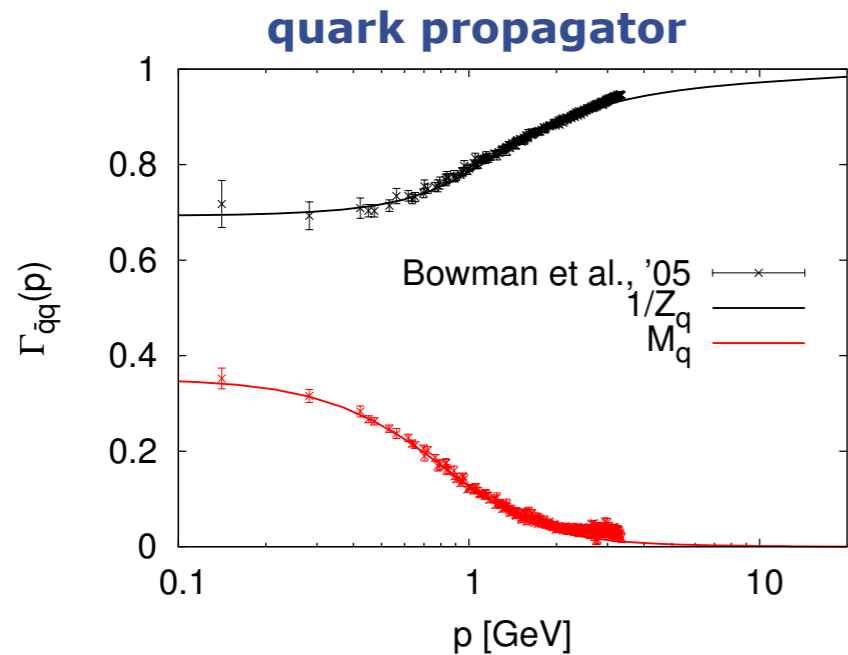
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Summary & Outlook

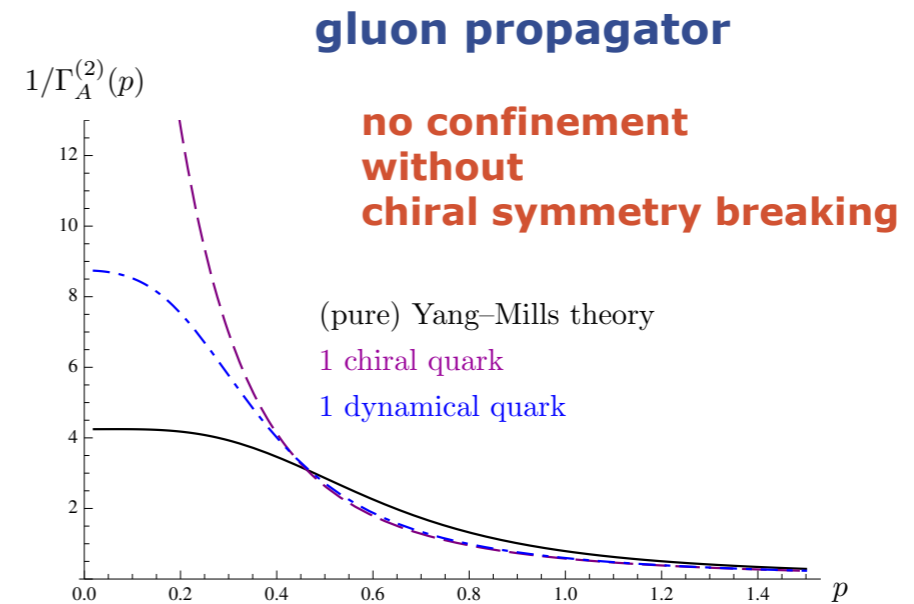
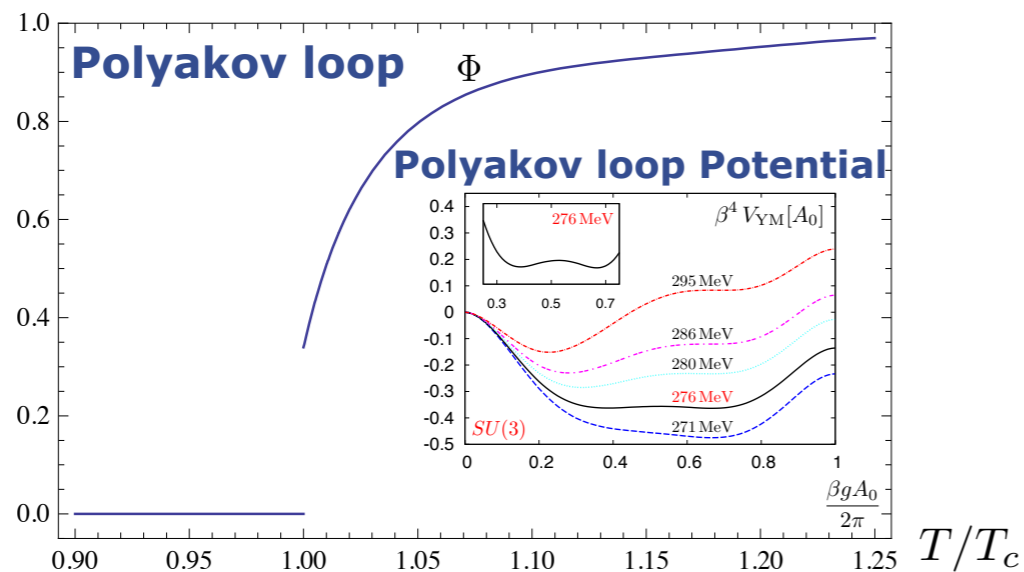
Summary & Outlook

Chiral Symmetry Breaking and Confinement

$$\frac{f_{\pi, \text{FRG}}}{f_{\pi, \text{lattice}}} = 0.99$$

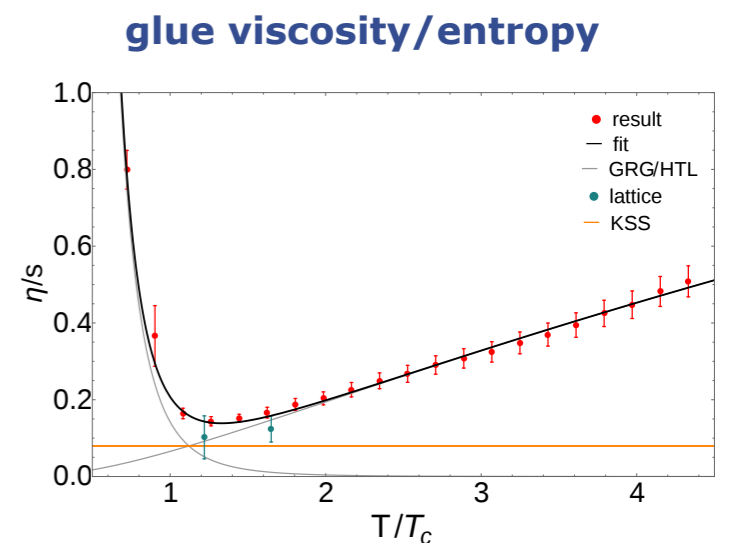
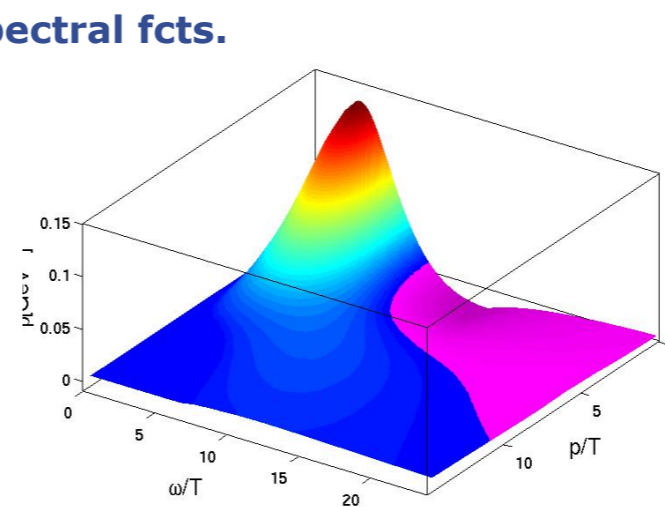
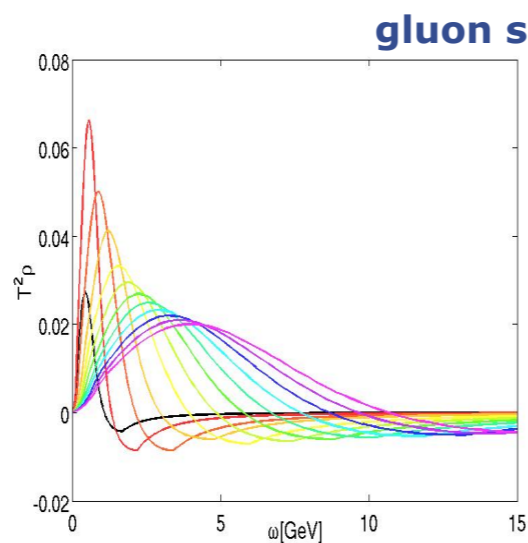
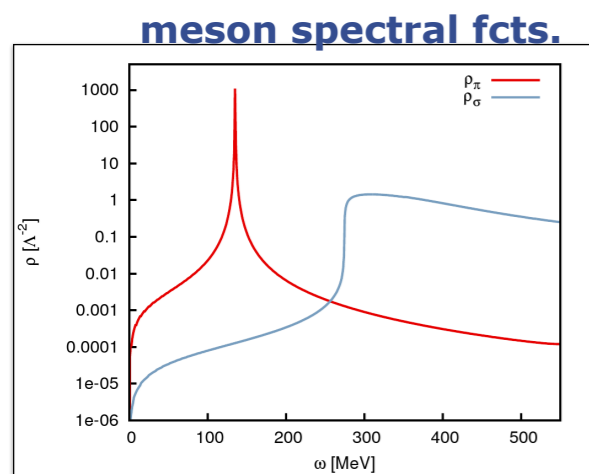
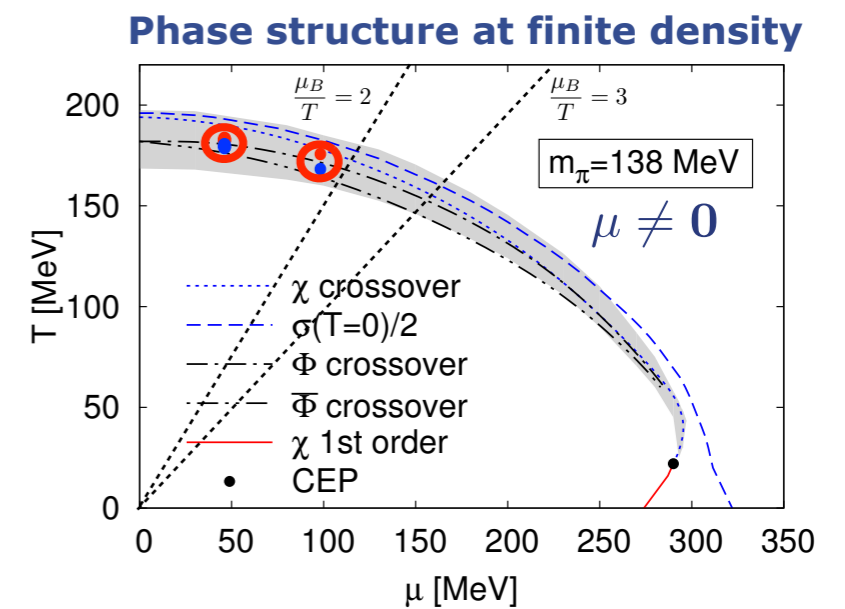
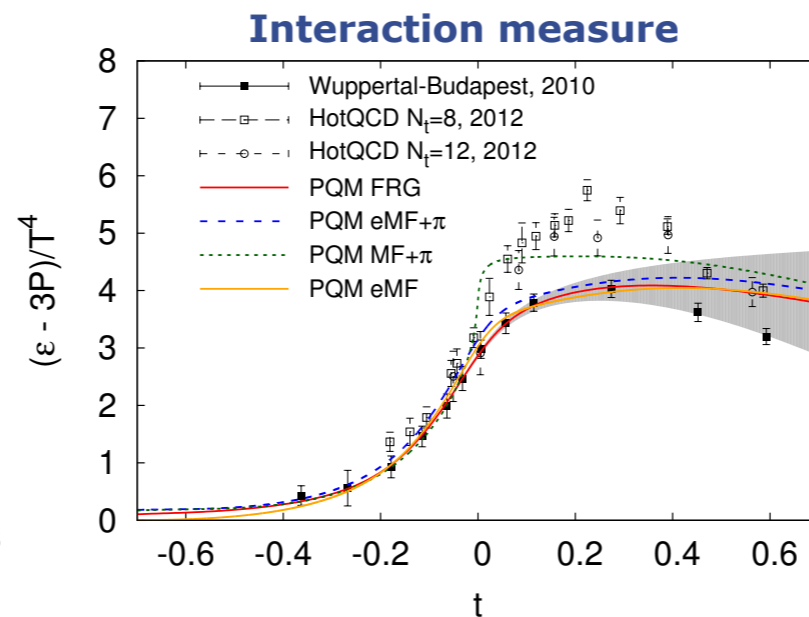
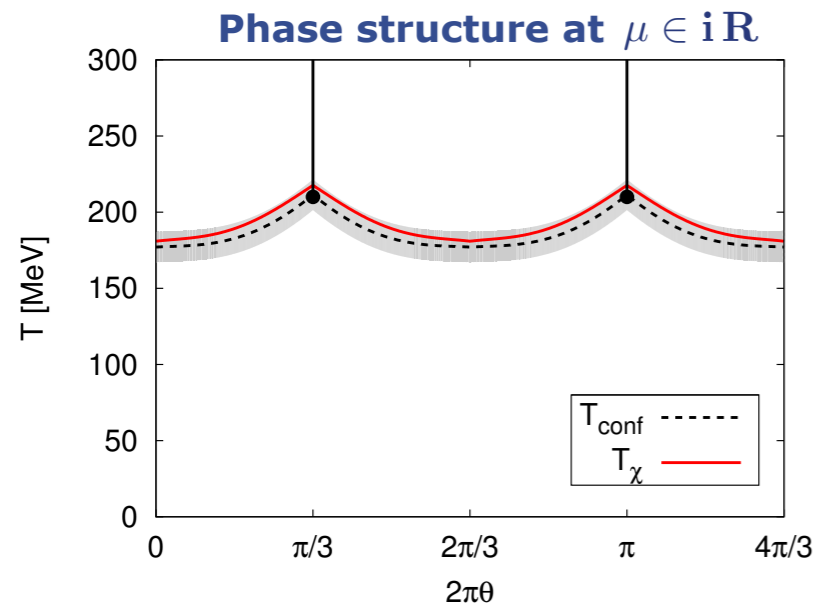


fQCD



Summary & Outlook

Phase structure and Transport

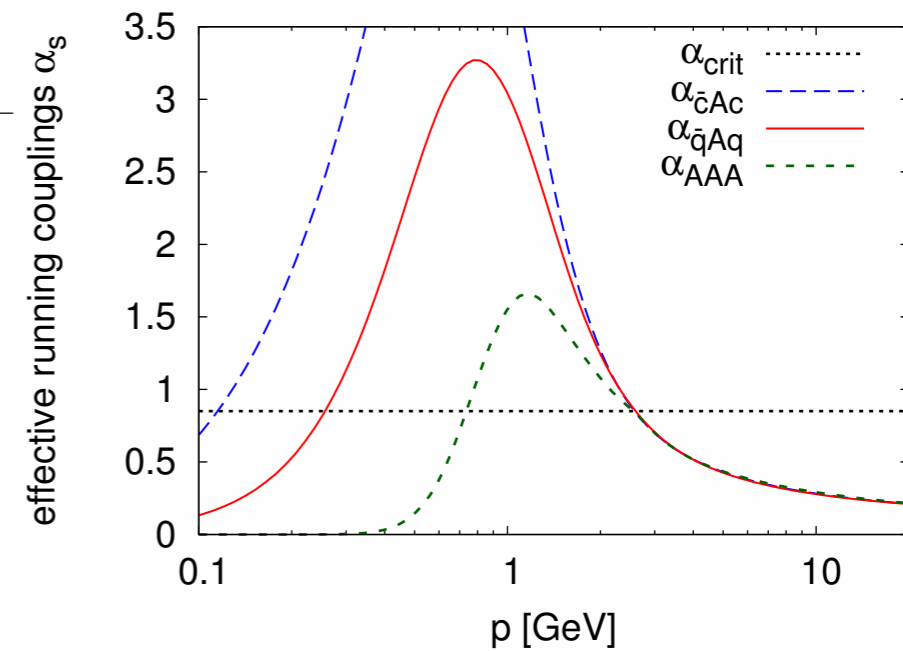
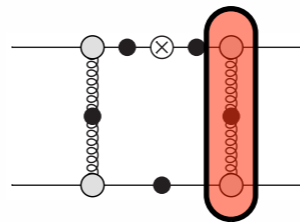
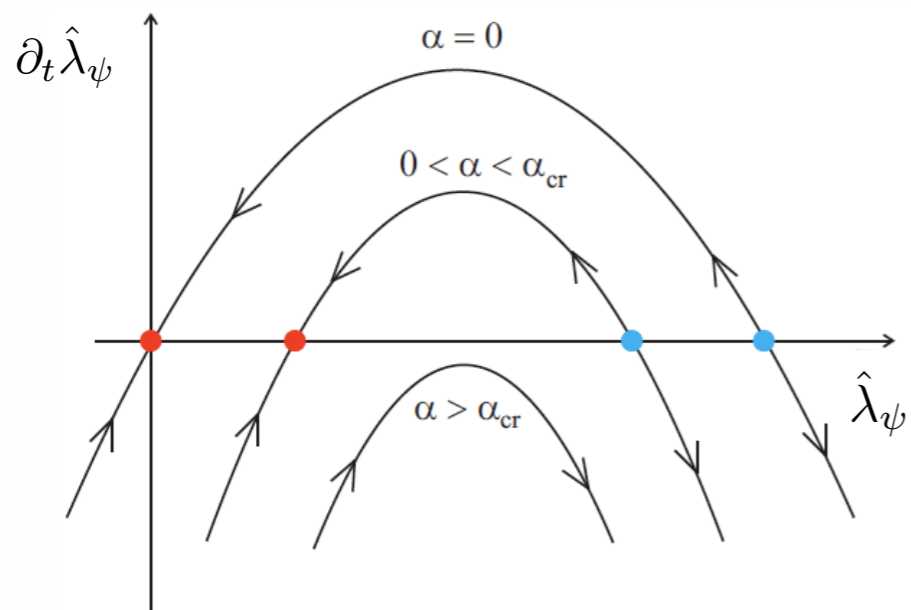


Summary & Outlook

- **Chiral Symmetry Breaking and Confinement**
- **Phase Structure and Transport**
- **Towards quantitative precision**
- **Baryons, high density regime & CEP, dynamics**
- **Hadronic properties**
 - **hadron spectrum & in medium modifications**
 - **low energy constants**

Additional material

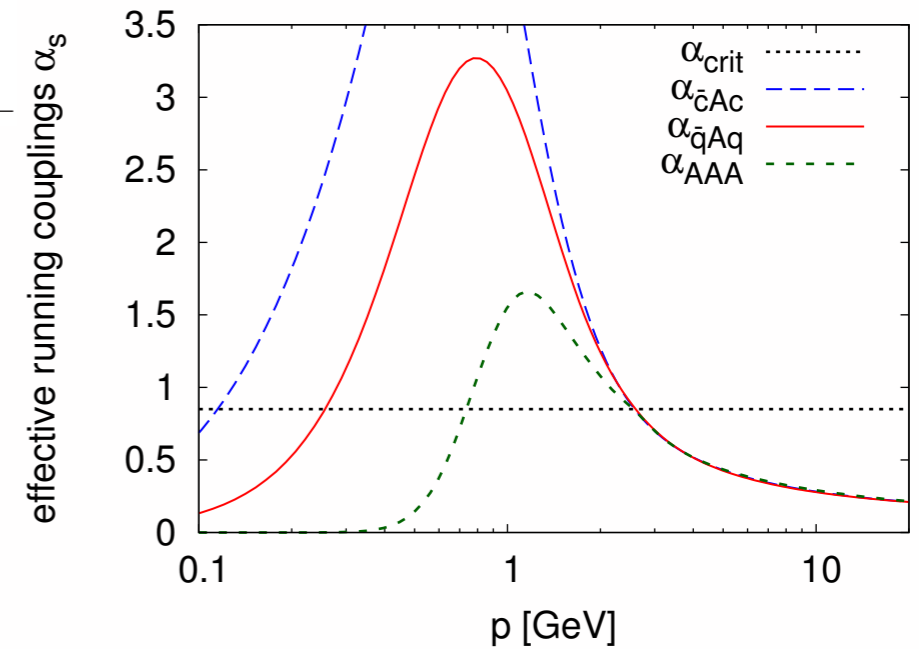
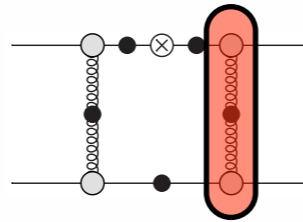
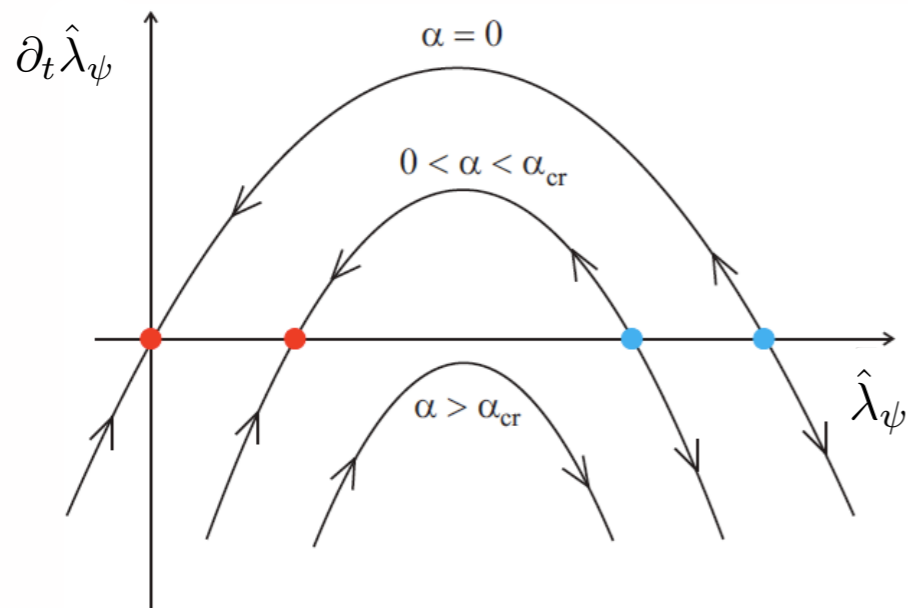
Confinement & symmetry breaking



Mitter, JMP, Strodthoff '14

Braun, Fister, Haas, JMP, Rennecke '14

Confinement & symmetry breaking



Mitter, JMP, Strodthoff '14

Braun, Fister, Haas, JMP, Rennecke '14

**dynamical correlation of confinement
and
chiral symmetry breaking**

confinement

**gluon propagator
gapped relative to
ghost propagator**

chiral symmetry breaking

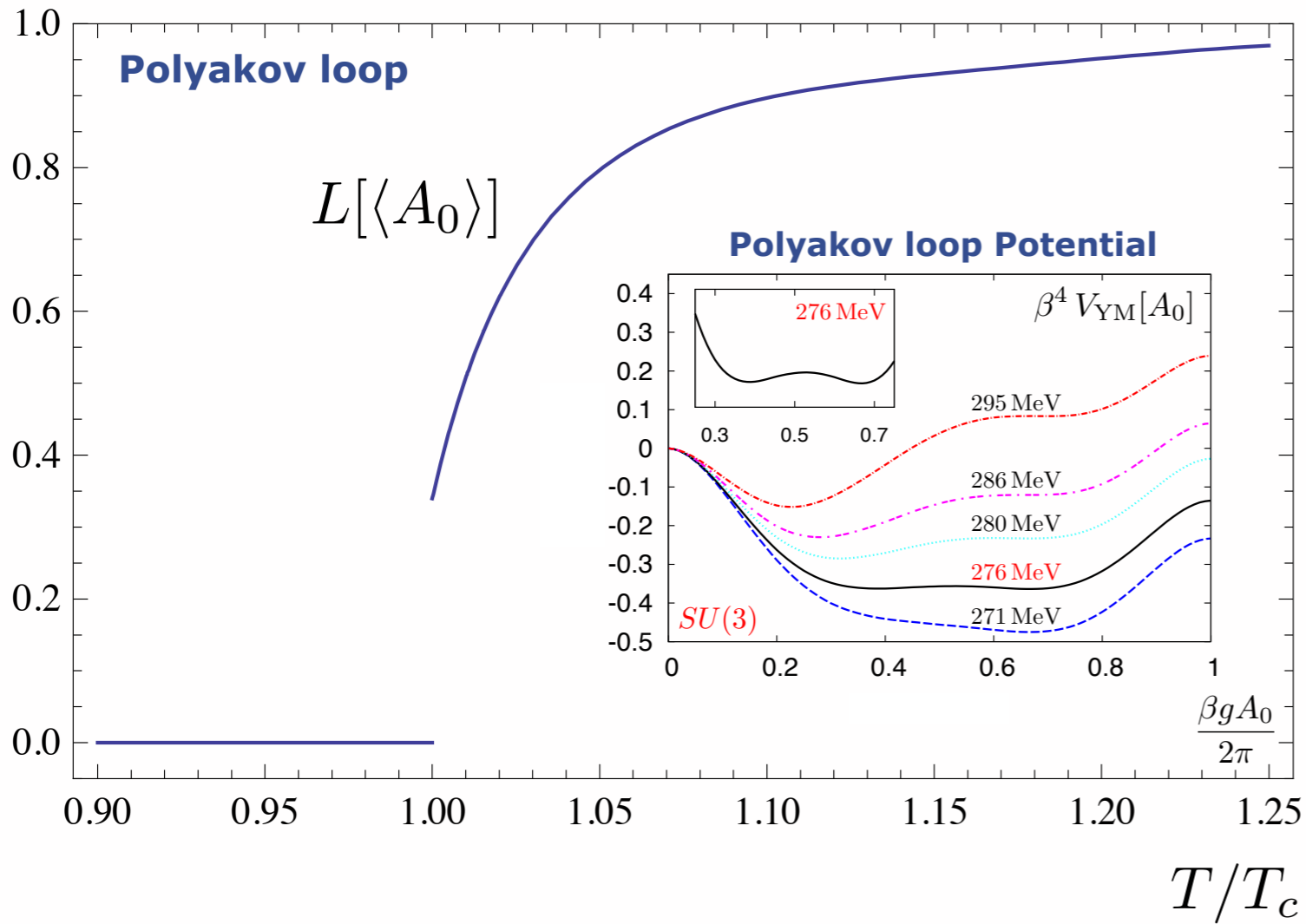
**gluon propagator
not gapped too much**

Fister, Mitter, JMP, Strodthoff '14

Confinement

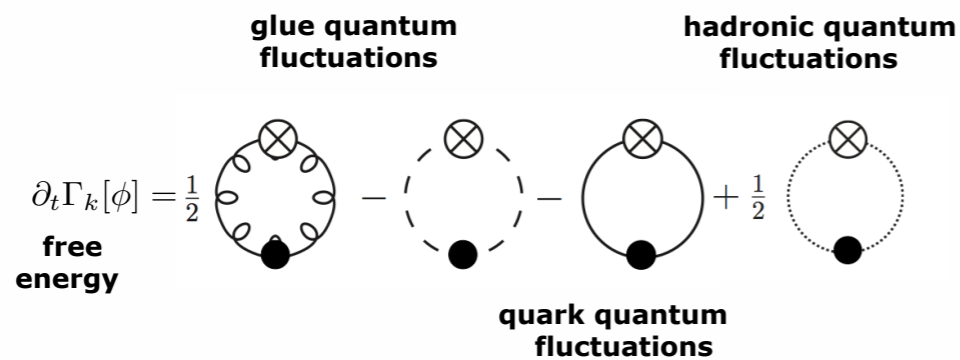
FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

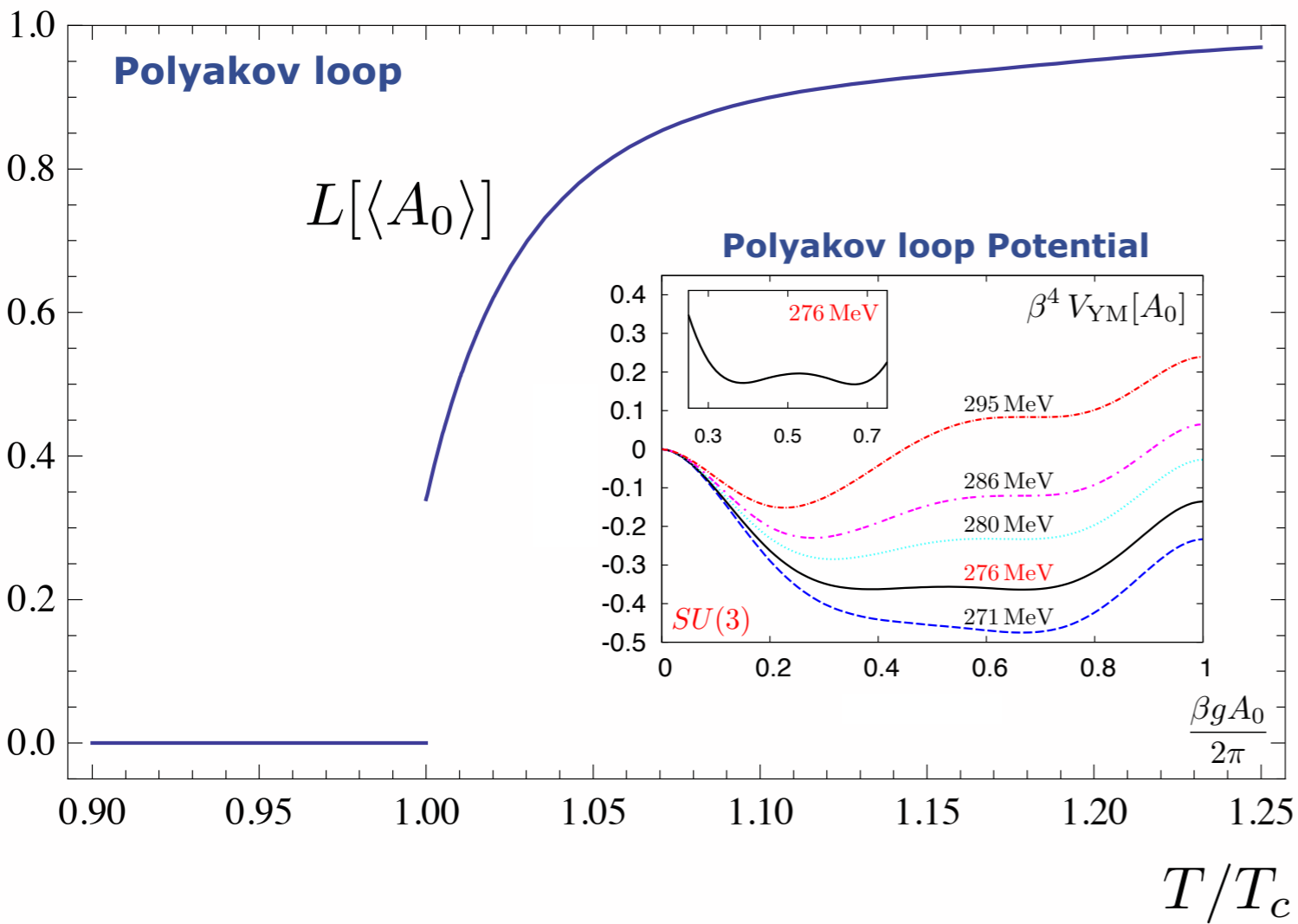
$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$



Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

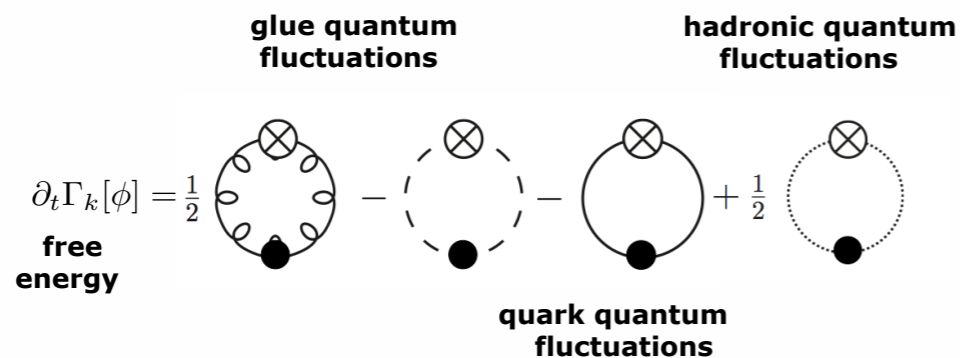


$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$

confinement

gluon propagator
gapped relative to
ghost propagator

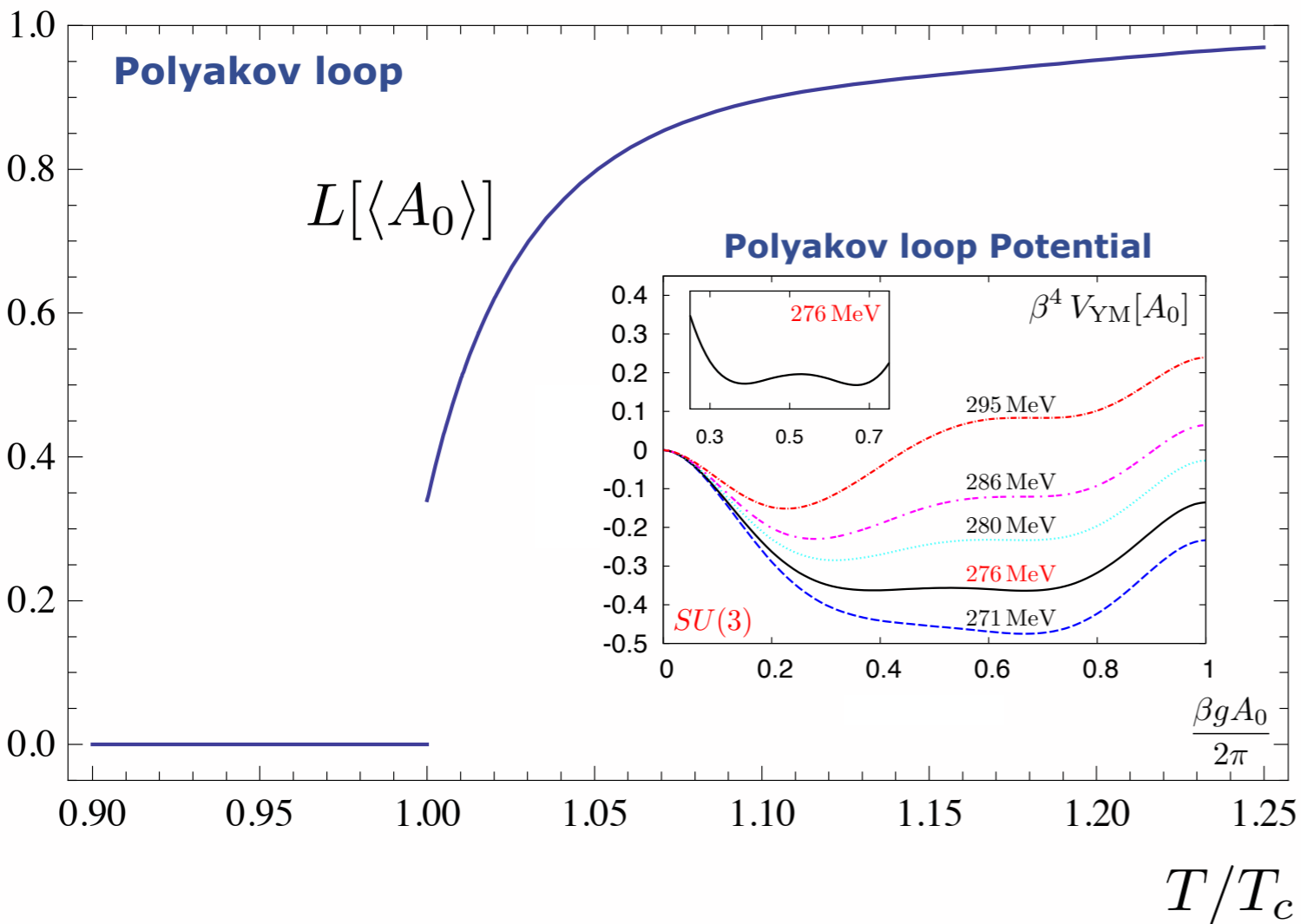


Braun, Gies, JMP '07
 Marhauser, JMP '08
 Fister, JMP '13

Confinement

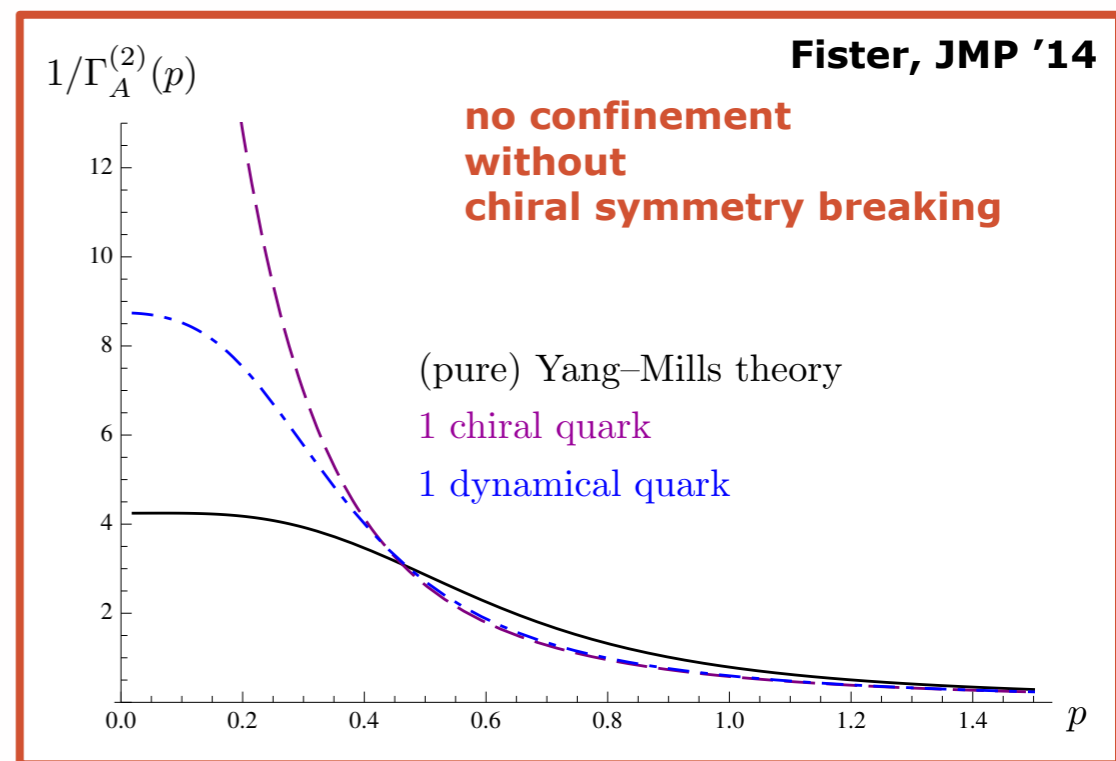
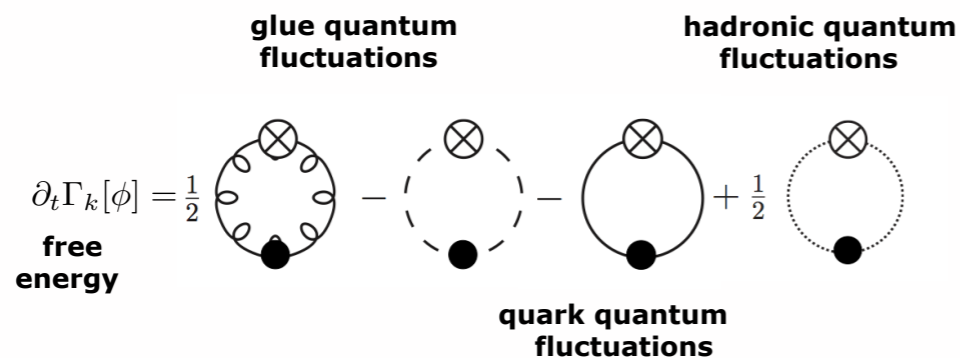
FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$

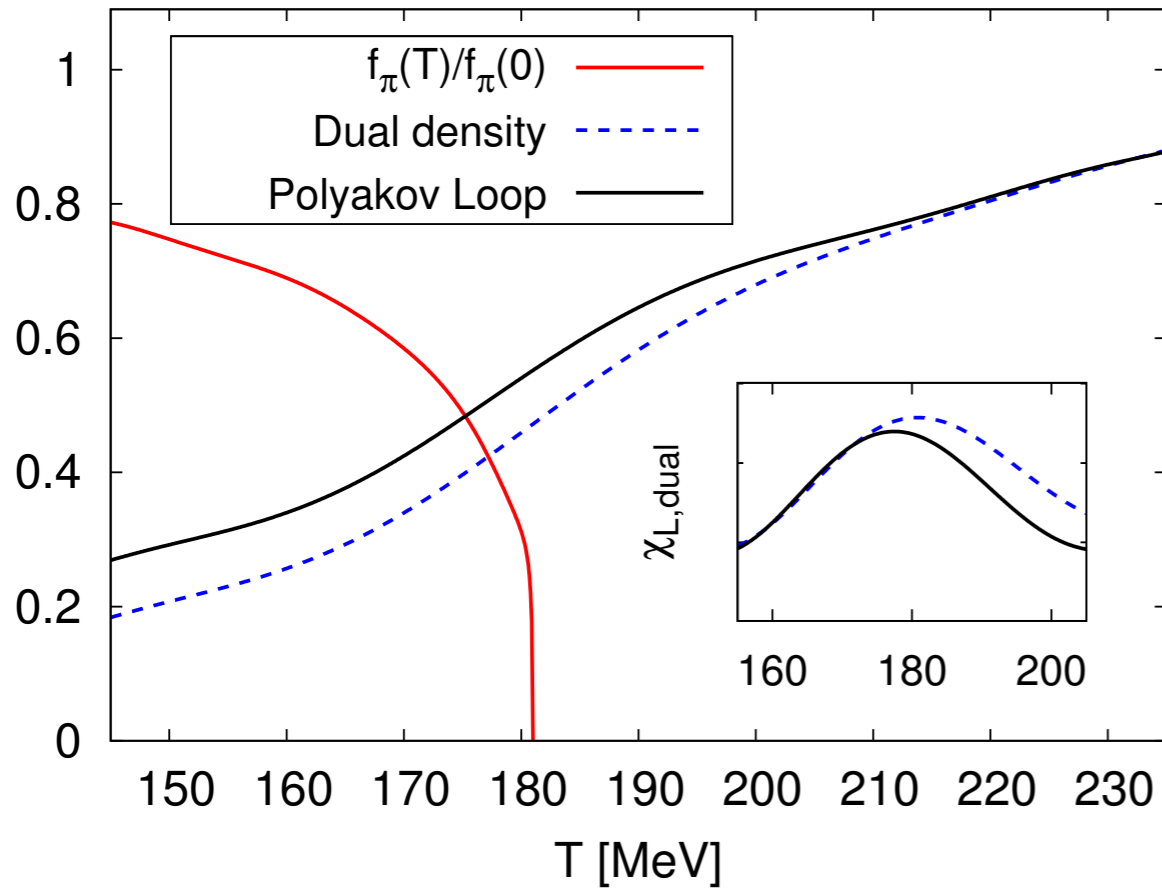


QCD

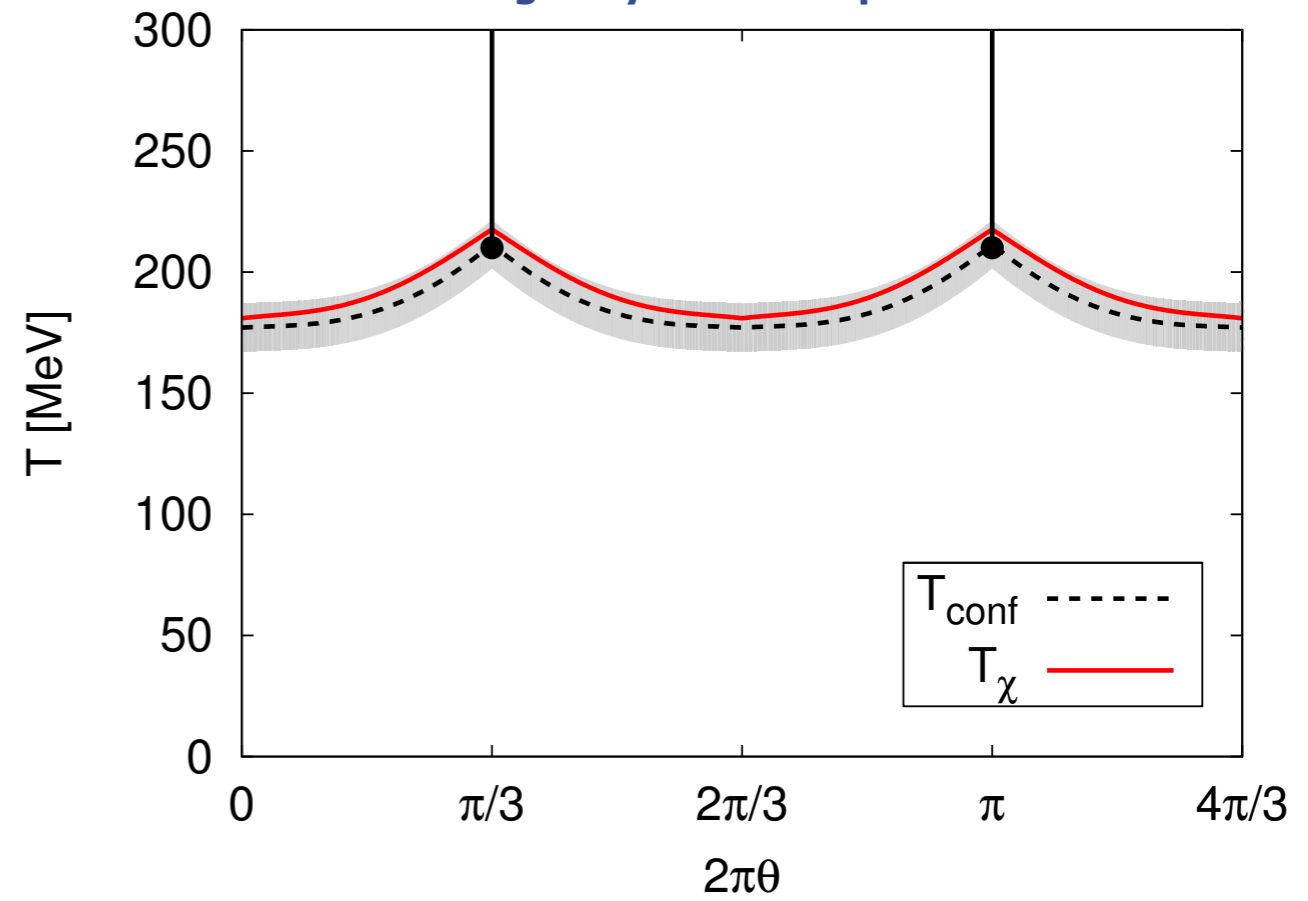
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Loop 1} - \text{Loop 2} - \text{Loop 3} + \frac{1}{2} \text{Loop 4} \right)$$

2 flavors & chiral limit

vanishing density



imaginary chemical potential



Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002