EMMI Nuclear and Quark Matter Seminar

GSI Darmstadt, 07. 12. 2016

Progress on the QCD phase diagram

(is slow…but steady)

HIC for FAIR
Helmholtz International Center

Owe Philipsen

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Introduction: Lattice QCD and the sign problem
- Overview: the finite T transition from direct lattice simulations
- Towards cold and dense QCD: effective lattice theories

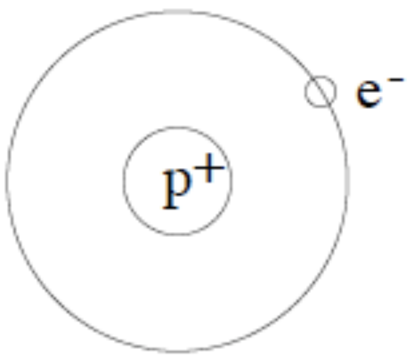
Quantum Chromodynamics, theory of strong interactions

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2}\,\text{Tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{3}\bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i$$

$$m_u \sim 3\text{MeV}, \quad m_d \sim 6\text{MeV}, \quad m_s \sim 120\text{MeV} \quad \Rightarrow \quad N_f \approx 2+1$$

weak vs. strong coupling:

QED



$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{bind} = 13.6$ eV

hydrogen (e.m. force)

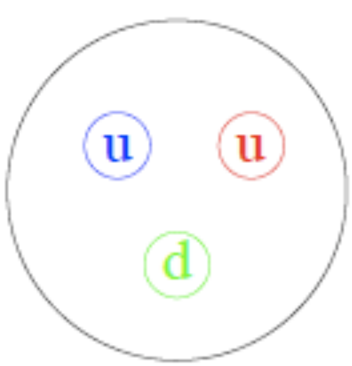
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

photons → gluons

e,p → quarks

gauge group U(1) → gauge group SU(3)

QCD



$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

proton (strong force)

$$\alpha_s = \frac{g^2}{4\pi} \approx 1$$

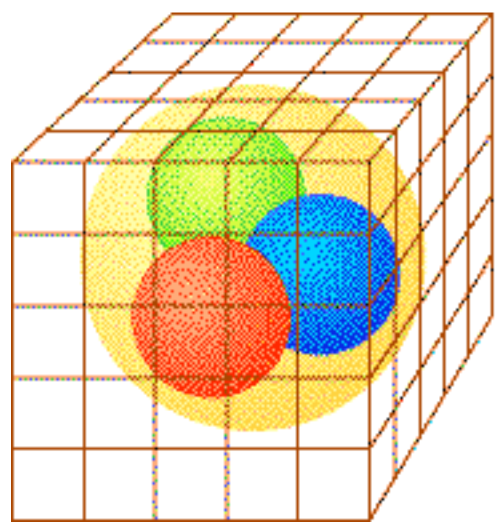
$\Rightarrow$ **Confinement, non-perturbative**

gluon self-interaction!

Lattice gauge theory + Monte Carlo method

QCD partition fcn: $$Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, e^{-S_{gauge}(\beta;U)}$$

links=gauge fields

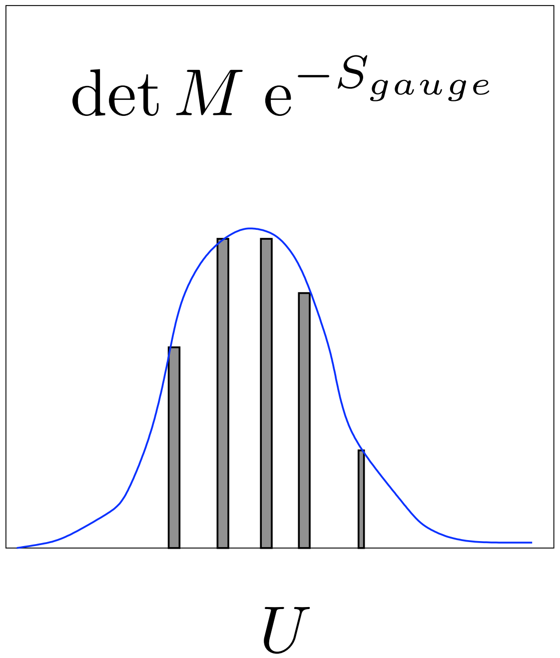


$4d : N_s^3 \times N_t$

lattice spacing a<< hadron << L !

typically $> 10^8 - 10^{10}$ dim. integral

→ Monte Carlo, importance sampling



Euclidean time:

$$T = \frac{1}{aN_t}$$

Continuum limit: $N_t \to \infty, a \to 0$

Phase diagram: $N_t = 4, 6$

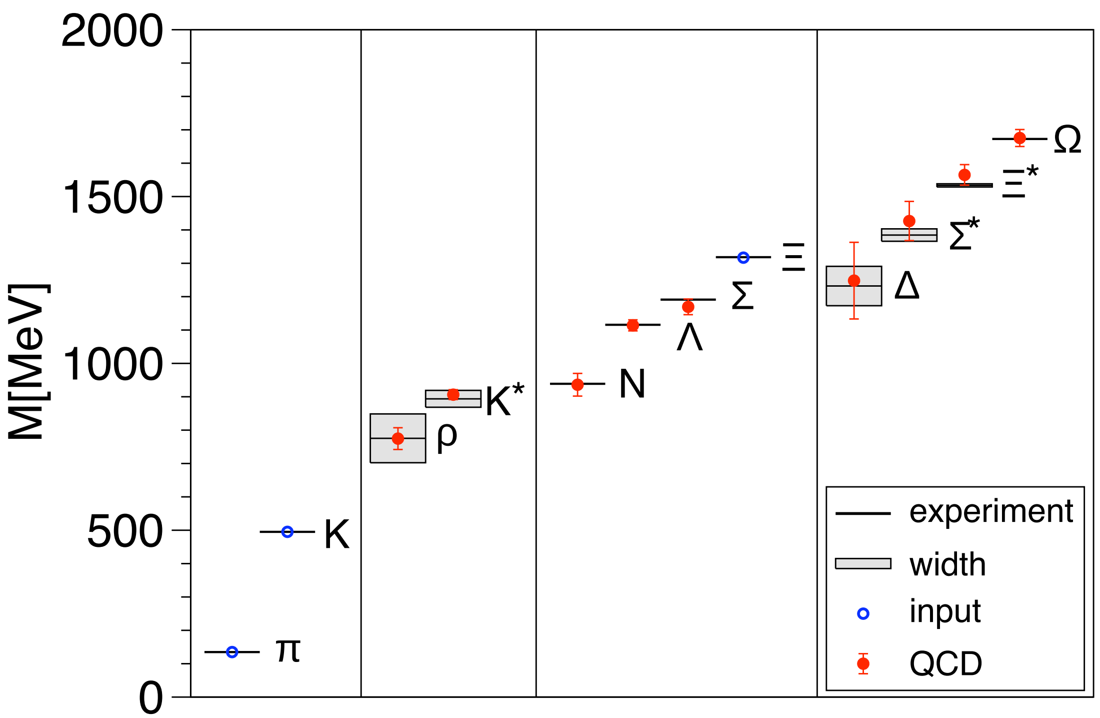
$a \sim 0.3, 0.2$ fm

Light fermions expensive, $\text{cost}(\det M) \sim \frac{1}{m^6}$

Directly calculable: particle masses, decay constants, equilibrium thermodynamics

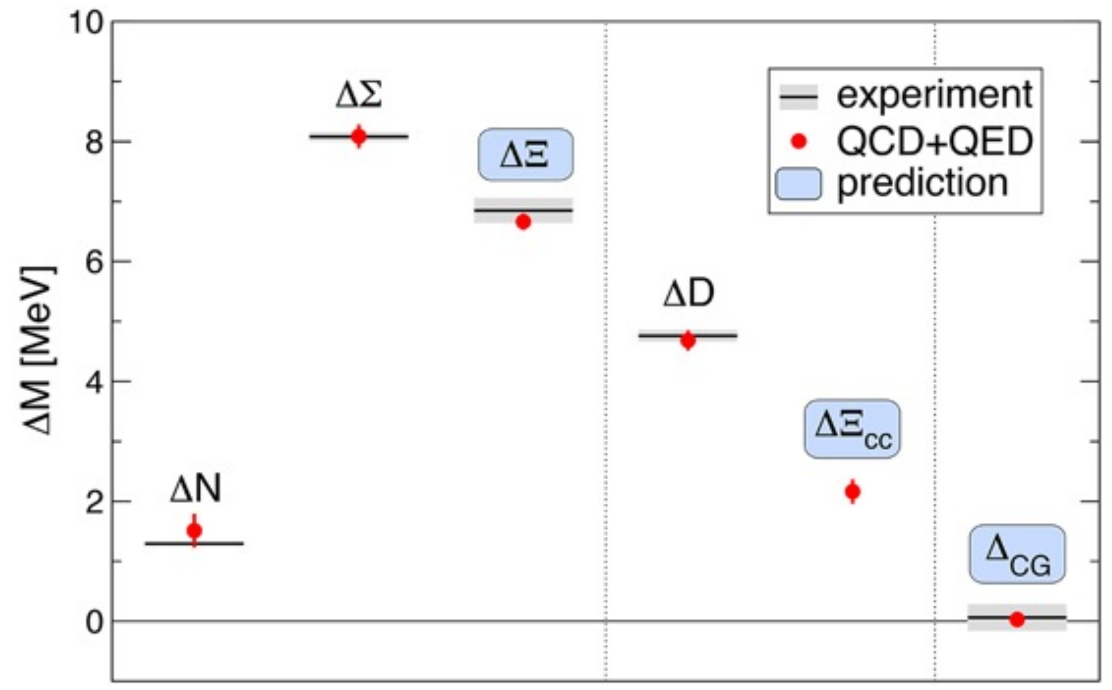
Success of lattice QCD: hadron spectrum

Nf=2+1 QCD



(Budapest, Marseille, Wuppertal) 2010

including isospin splitting + QED



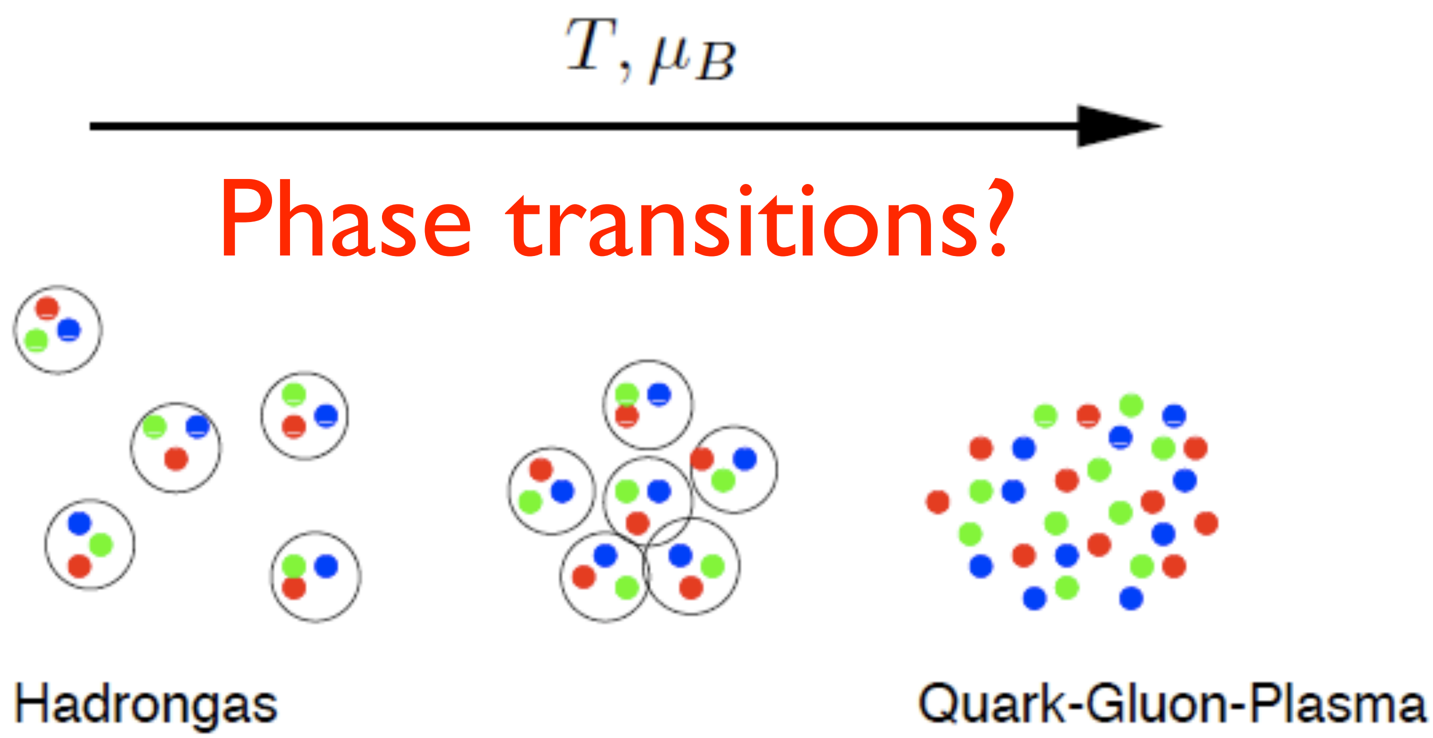
Budapest - Wuppertal 2014

- High precision hadron spectrum
- QED effects included
- LQCD used as discovery tool:

exotic hadron states, hadronic matrix elements for g-2, beyond SM models, axions+relic WIMP abundance….

QCD at high temperature/density: change of dynamics

asymptotic freedom $\alpha_s(p \to \infty) \to 0$

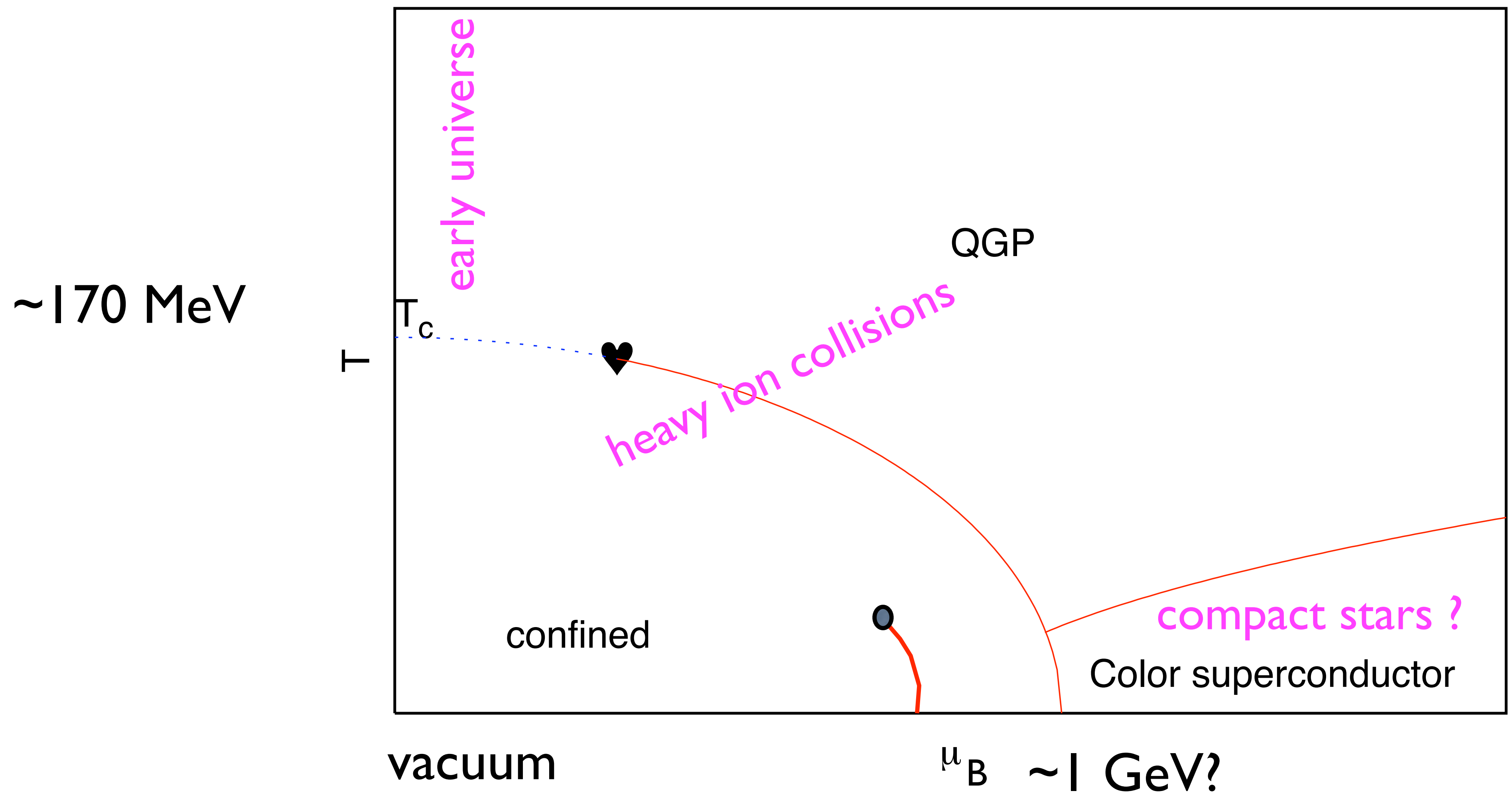


Chiral symmetry: broken (nearly) restored

Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

chiral condensate , Cooper pairs

QCD phase diagram: theorist's view (science fiction)



Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

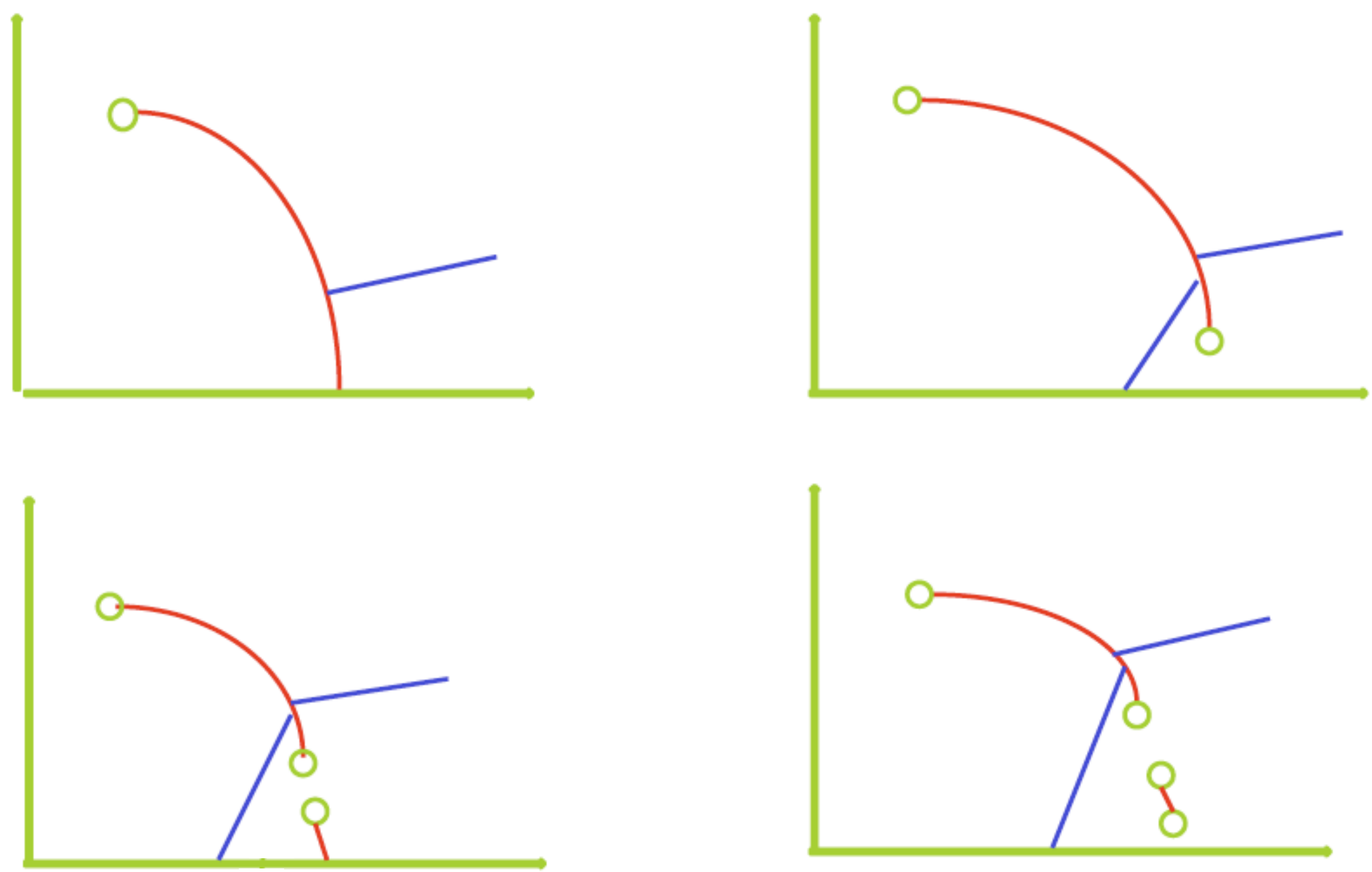
Check this from first principles QCD!

Less conservative views….

NJL with vector interactions,
Ginzburg-Landau approach
for quark condensates,
beyond mean field methods...

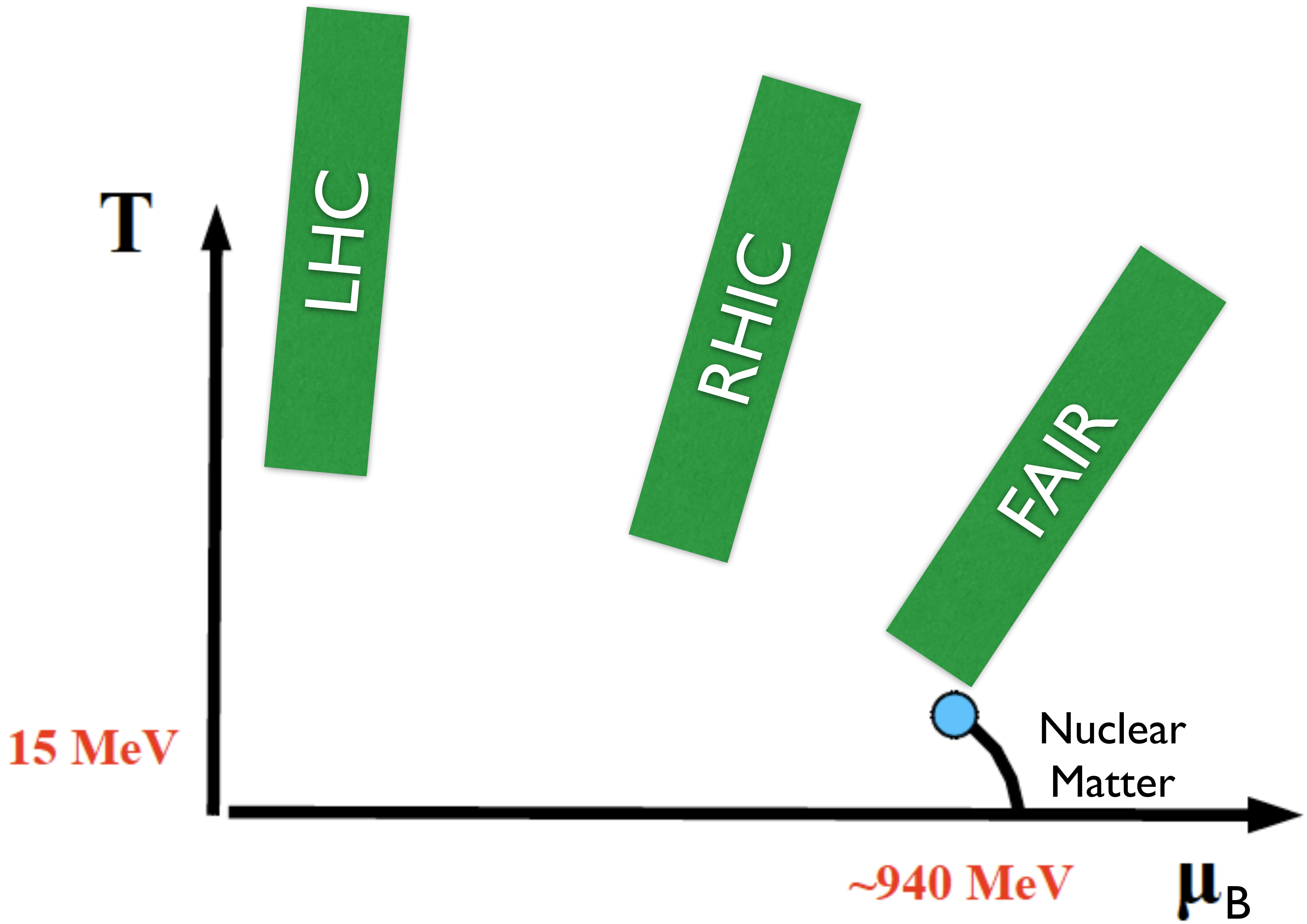
Zhang, Kunihiro, Fukushima 09
Baym et al. 06

Ferroni, Koch, Pinto 10



+ inhomogeneous phases, quarkyonic phases,…. you name it!

The QCD phase diagram established by experiment:

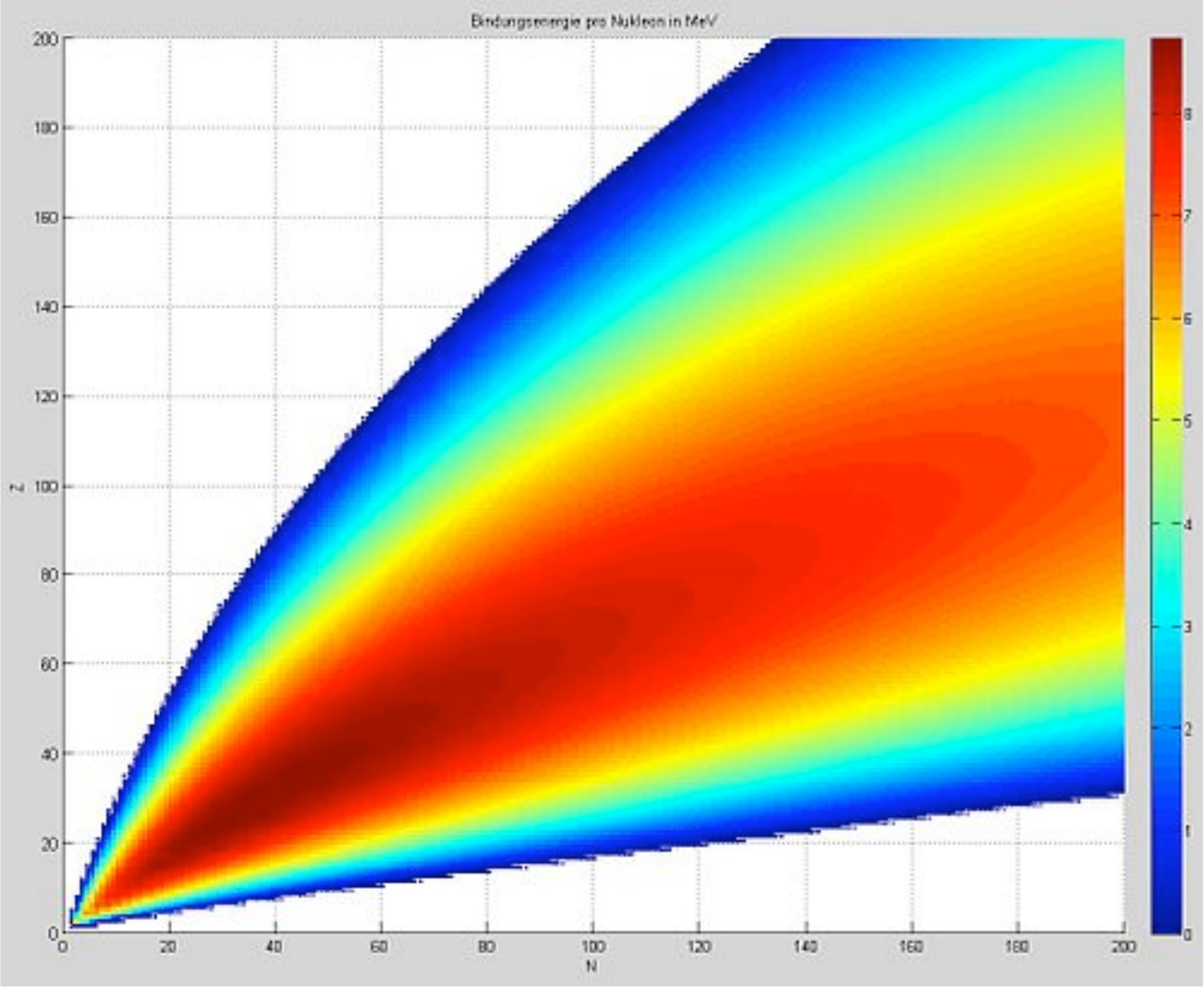


Nuclear liquid gas transition with critical end point

Unsolved from QCD: nuclear matter

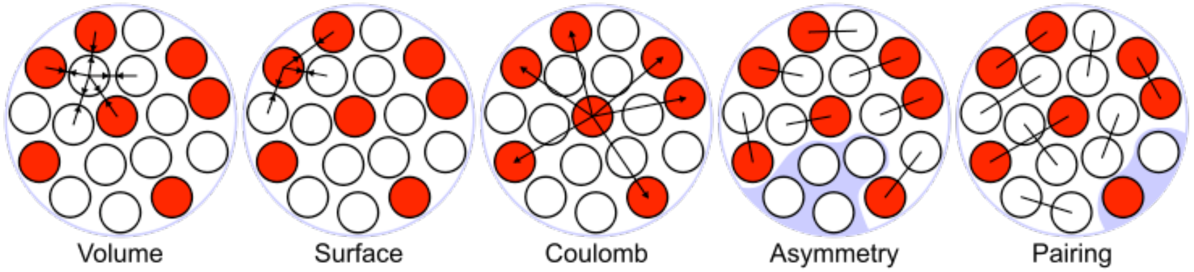
~100 years old, still no fundamental description, Bethe-Weizsäcker droplet model:

Z



N

Binding energy per nucleon



QFT descriptions: Fetter-Walecka model, Skyrme model, ...

Theory: how to calculate p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

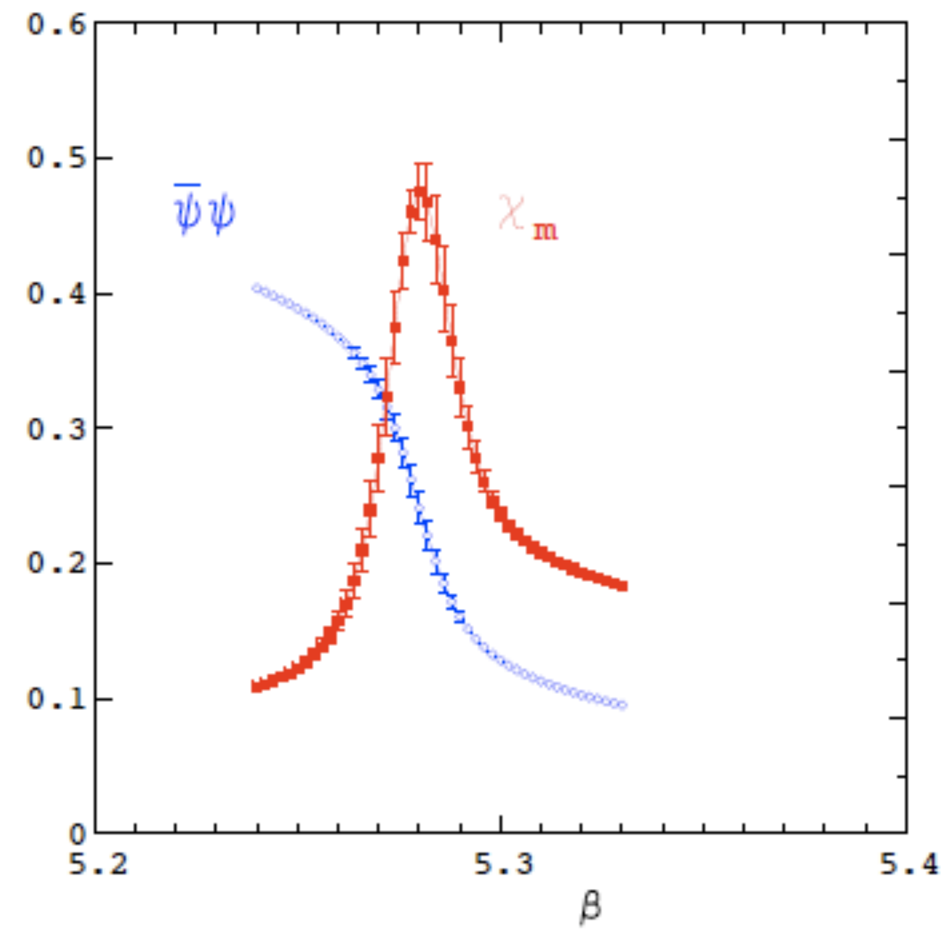
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz. T

only pseudo-critical on finite V!

Order of transition:

finite volume scaling

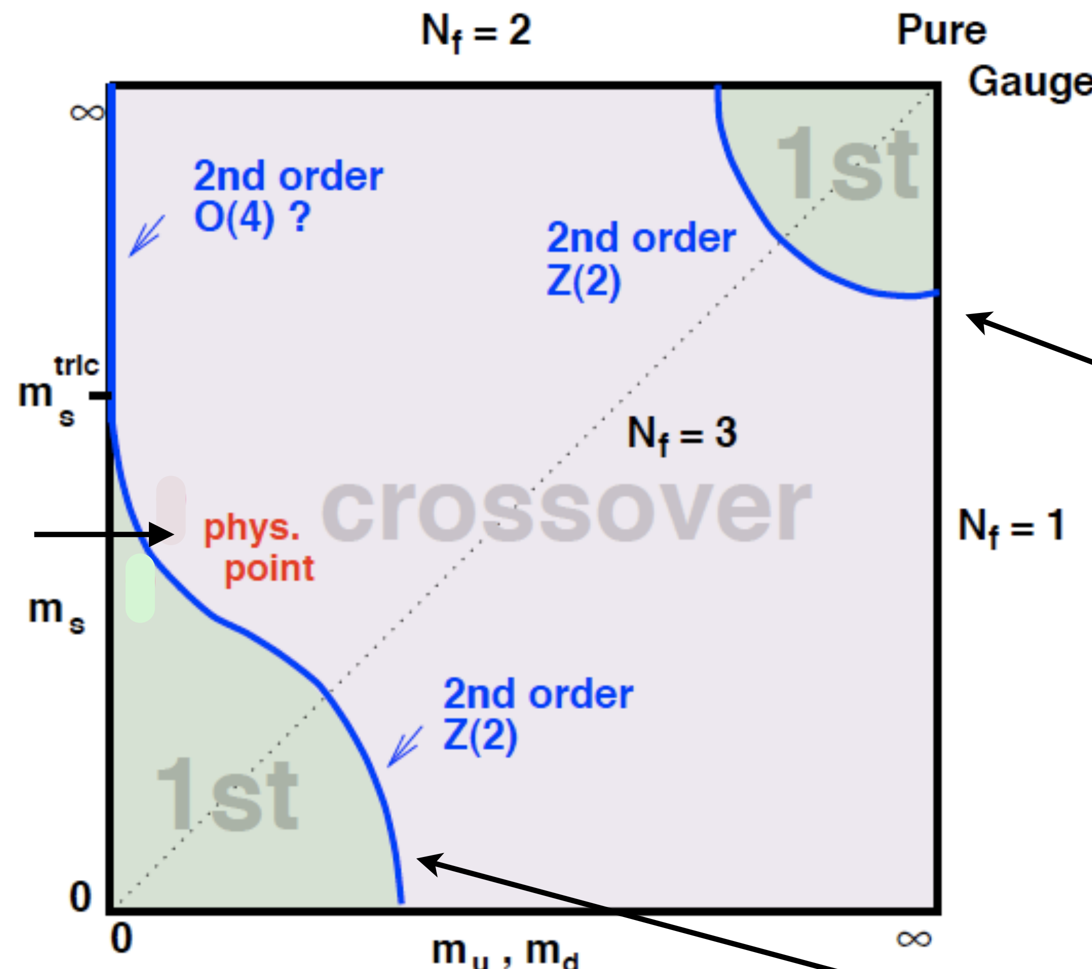
$$\chi_{max} \sim V^{\sigma}$$

$\sigma = 1$	1st order
σ = crit. exponent	2nd order
$\sigma = 0$	crossover

The order of the p.t., arbitrary quark masses $\mu = 0$

deconfinement p.t.:
breaking of global $Z(3)$ symmetry

Aoki et al. 06



deconfinement critical line

chiral critical line

de Forcrand, O.P. 07

$N_t = 4, a \sim 0.3$ fm

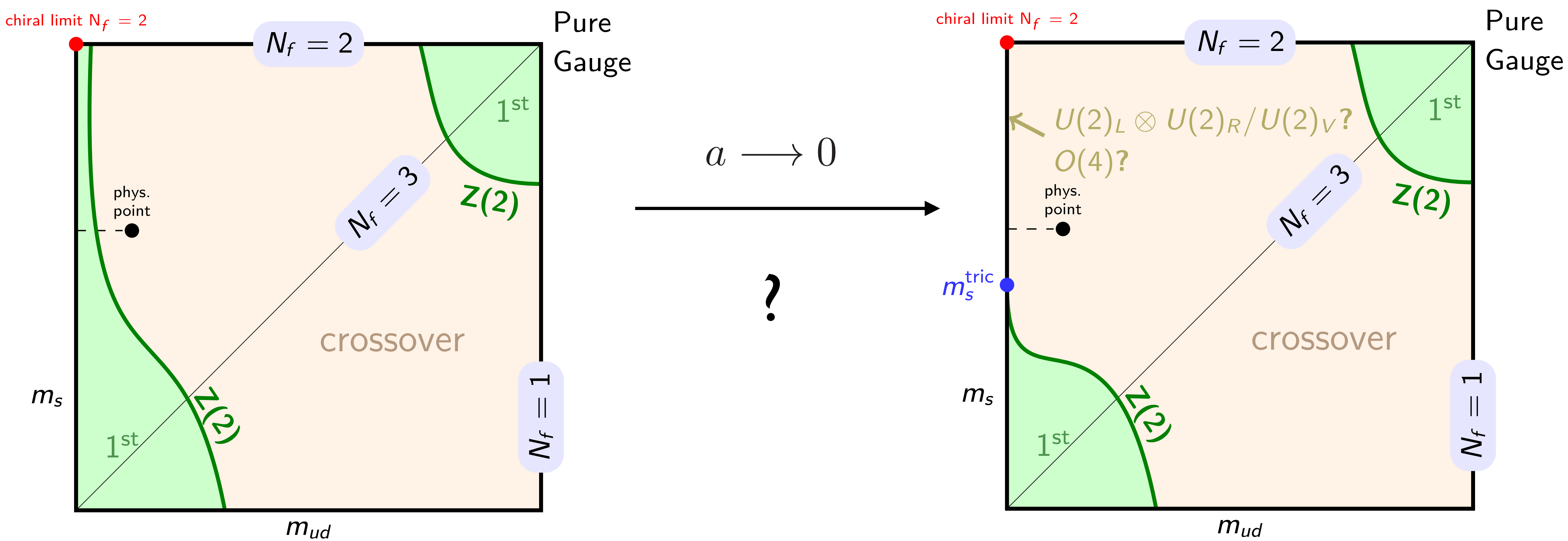
chiral p.t.
restoration of global symmetry in flavour space

$SU(2)_L \times SU(2)_R \times U(1)_A$

anomalous

no continuum extrapolation yet!

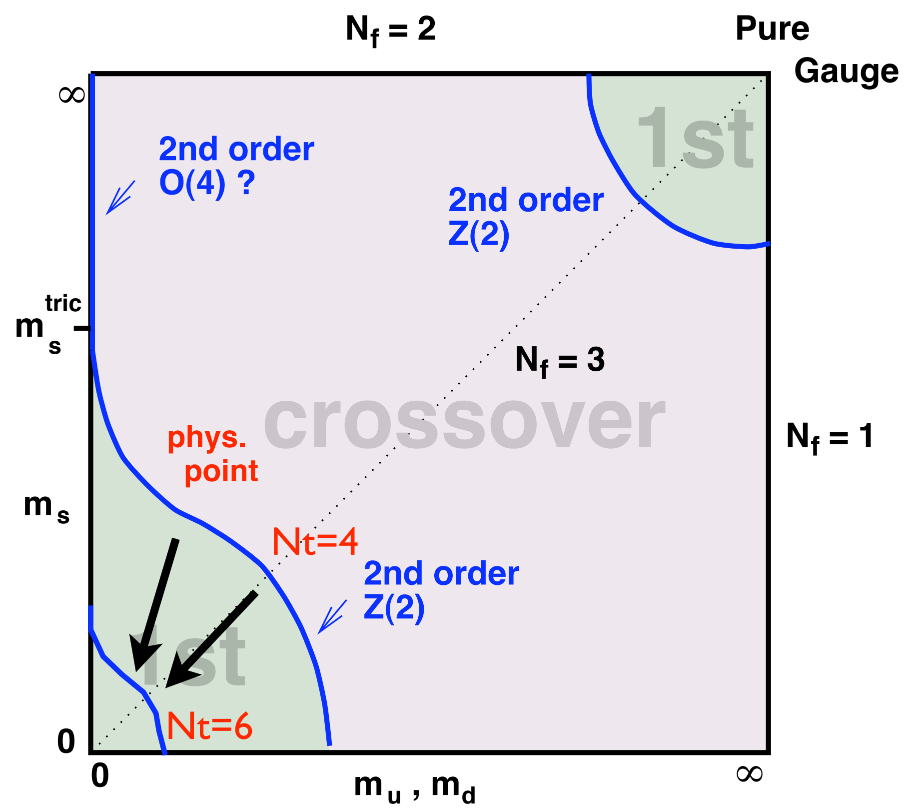
Order of the transition in the chiral limit is not yet settled!



Coarse lattices:
chiral limit is first order!

Unimproved staggered: Bonati et al. 14
Unimproved Wilson: Pinke, O.P. 14

Large cut-off effects on critical lines!



de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \to 0$

critical pion mass shrinks by factor ~1.8 from a=0.3 fm to a=0.2 fm!
no continuum limit yet!

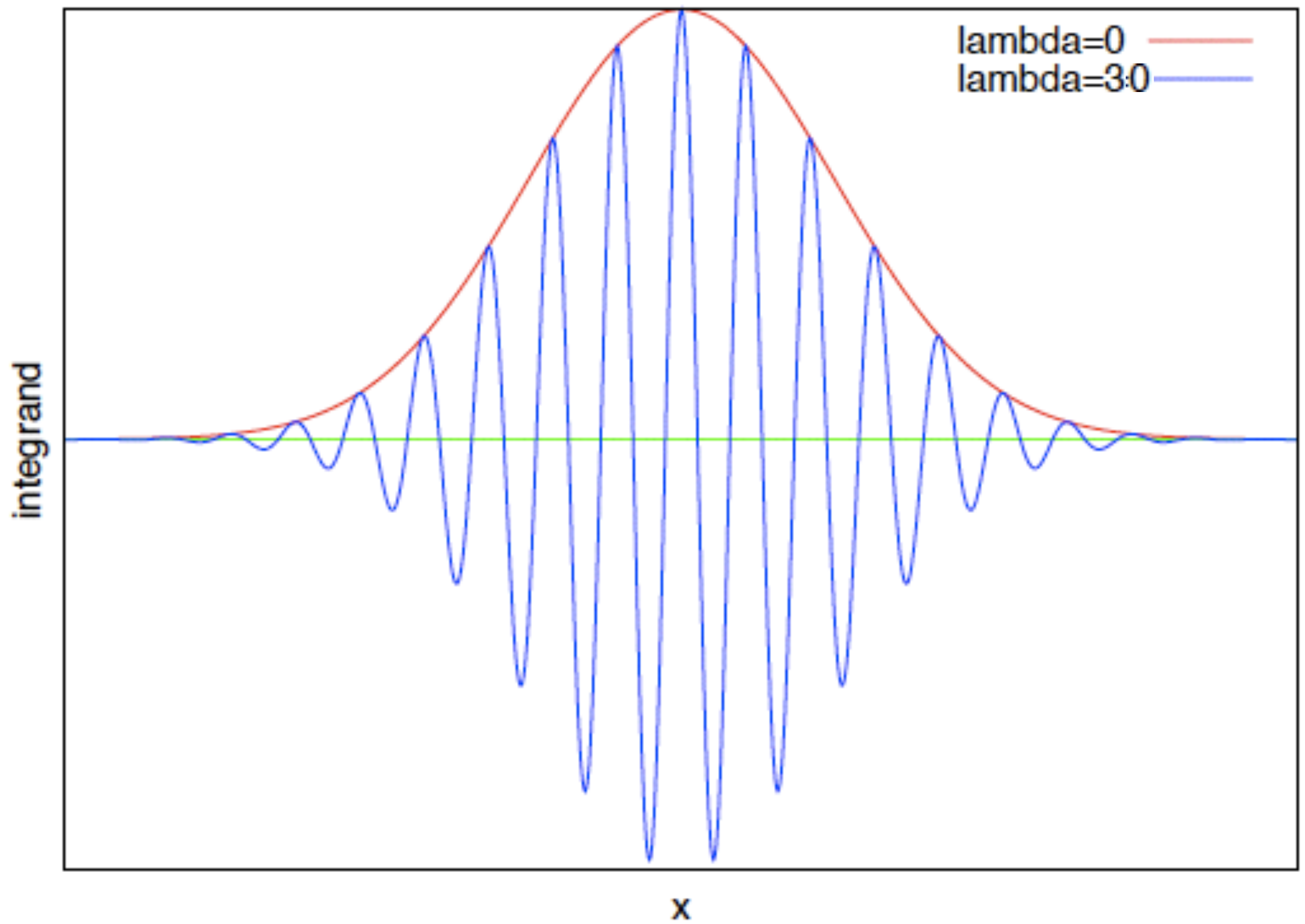
The sign problem for finite density QCD

$$Z = \int DU\, [\det M(\mu)]^f e^{-S_g[U]}$$

importance sampling requires
positive weights

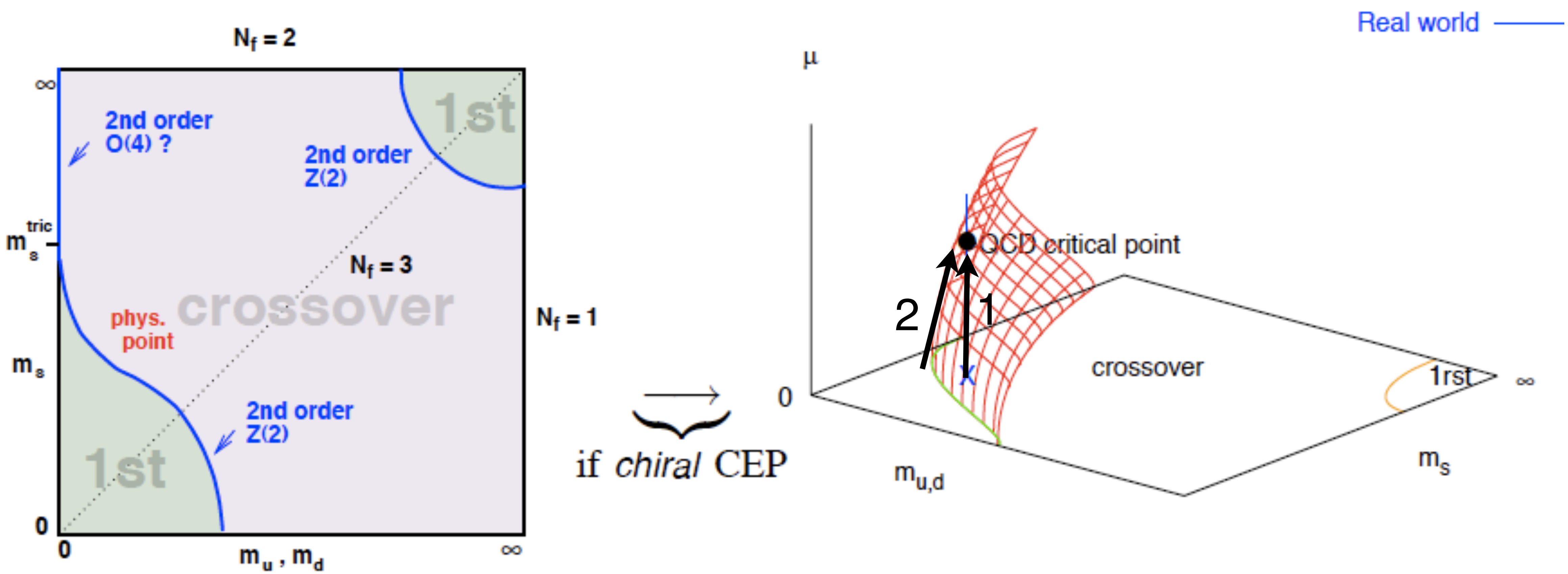
$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$ real positive for $\mu = i\mu_i$

Example: $Z(\lambda) = \int dx \exp(-x^2 + \mathbf{i}\lambda\mathbf{x})$



$Z(\lambda)/Z(0) = \exp(-\lambda^2/4)$: exponential cancellations

Much harder: is there a QCD critical point?



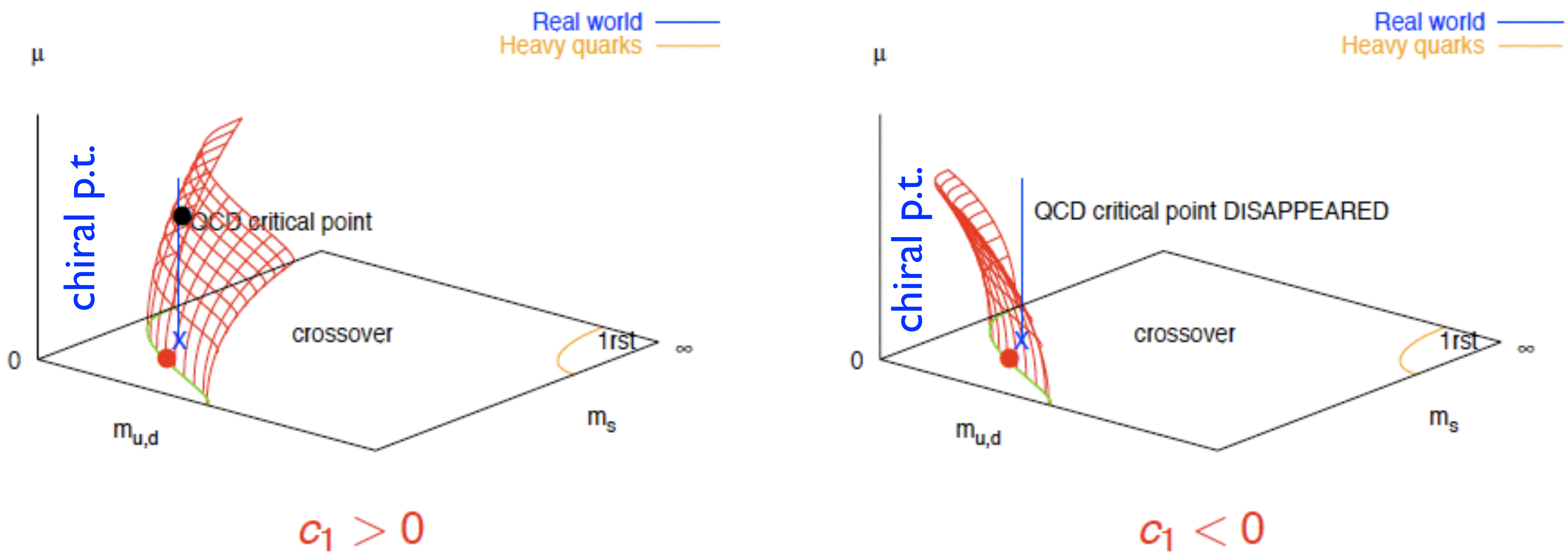
Two strategies:

1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

2 follow critical surface: $m = m_{\text{crit}}(\mu)$

Some methods trying (1) give indications of critical point, but systematics not yet controlled

Approach 2: follow chiral critical line ➜ surface



$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0$, $T = T_c$
   known universality class: 3d Ising

2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

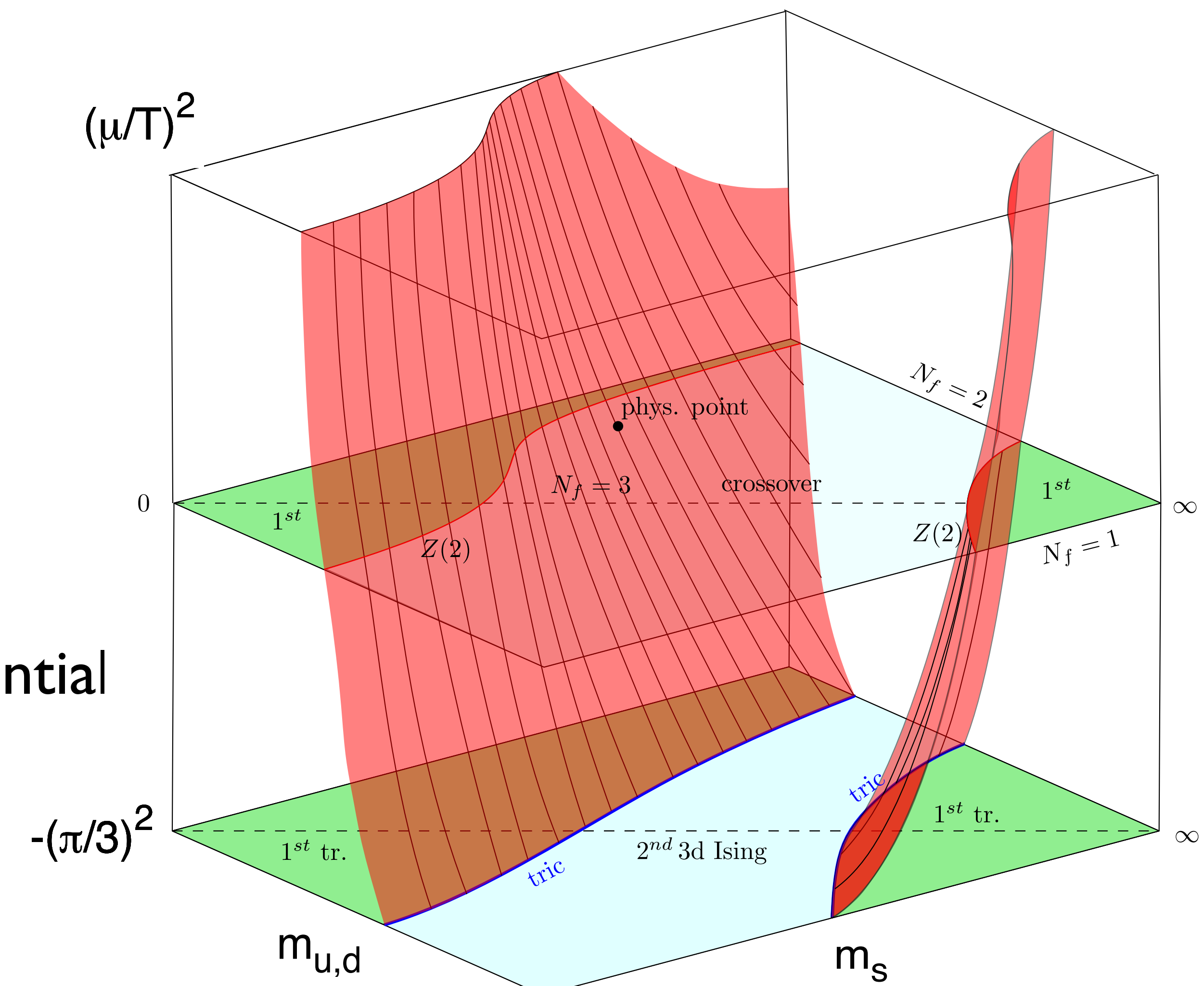
Critical surfaces from imaginary chemical potential

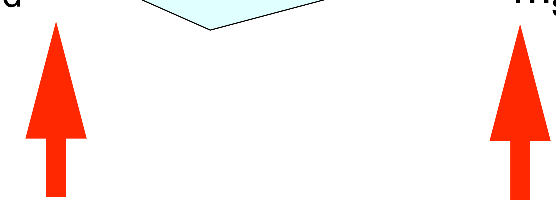
Real and imaginary chemical potential, coarse Nt=4 lattices

Real chemical potential:
sign problem

Imaginary chemical potential
no sign problem

Non-trivial phase structure
Roberge-Weiss Z(3) symmetry!





staggered,Nf=3:

de Forcrand, O.P. 10

staggered,Nf=2:

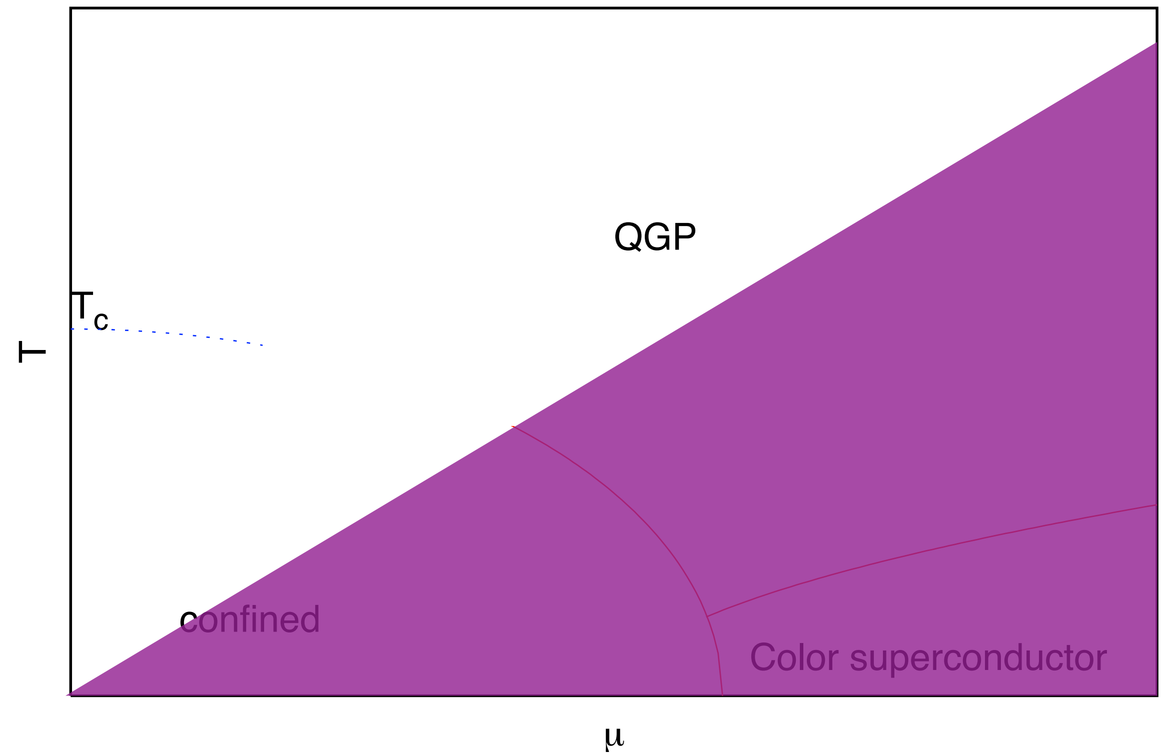
D'Elia, Sanfillippo 11

Wilson,Nf=2:

Pinke, O.P. 14

shape, sign of curvatures determined by tricritical scaling!

Lattice-calculable region of the QCD phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond
- Complex Langevin: lots of progress, but not in all parameter space, no "guarantees"

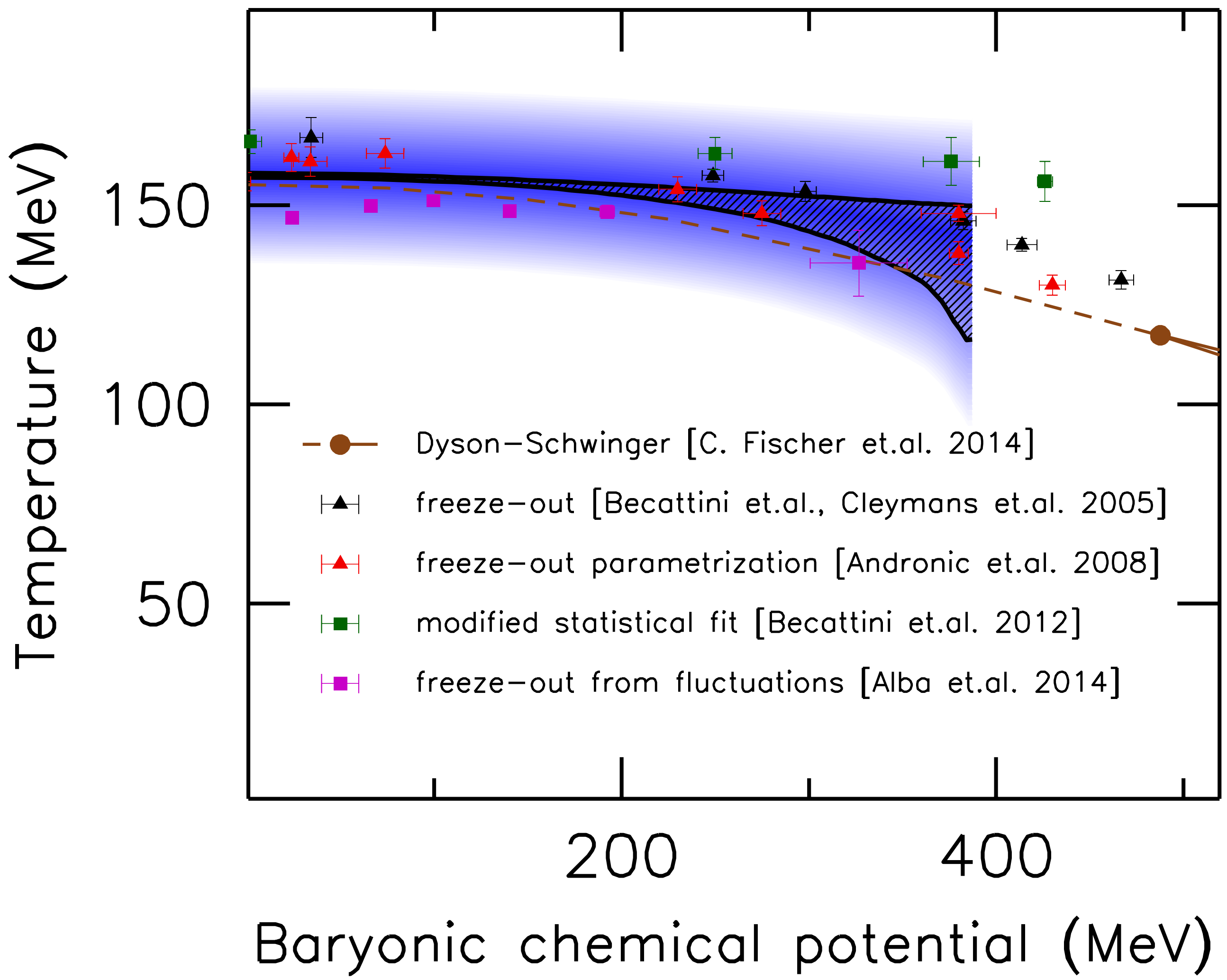
So far only "heavy dense QCD", i.e. static quarks Aarts et al. 16
cf. density of states Langfeld et al. 16

The crossover for physical masses

In the continuum:

$$\frac{T_c(\mu_B)}{T_c(\mu=0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \mathcal{O}(\mu_B^4)$$

Budapest - Wuppertal 15



Consistent with other simulations and different actions

Bonati et al., 15
Cea et al. 15
Bielefeld-Brookhaven 14

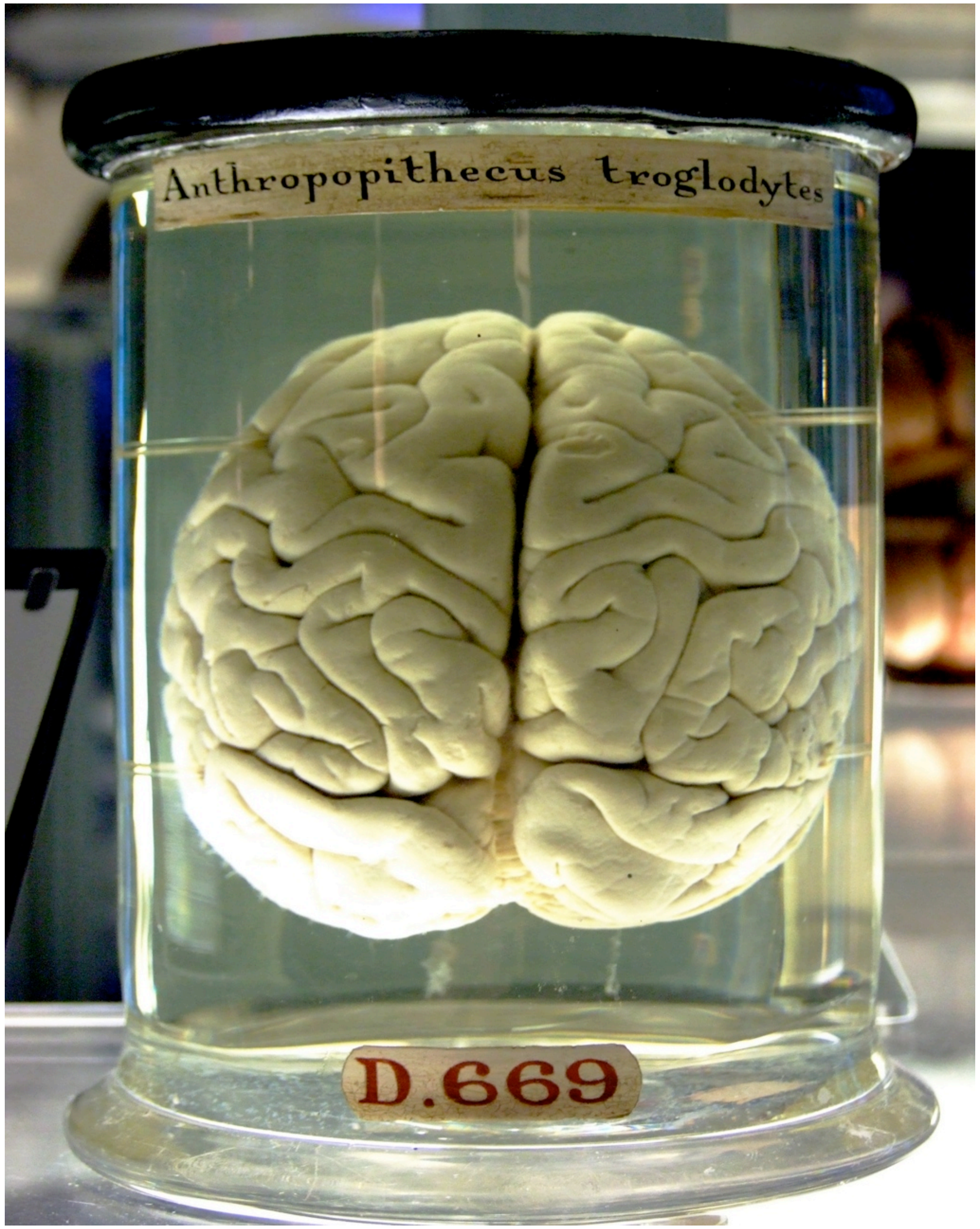
New computational avenues in LQCD:

"(Wall)Time is Money (CPU hrs)"

CPU ⟶ GPU

Here, very old-fashioned approach: BPU!

Biological Processing Unit!



Large densities?   Effective theories!

Effective lattice theory for heavy and dense QCD

with M.Fromm, J.Langelage, S.Lottini, M.Neuman, J.Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \ \det Q \ e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \ e^{-S_{eff}[L]}$$

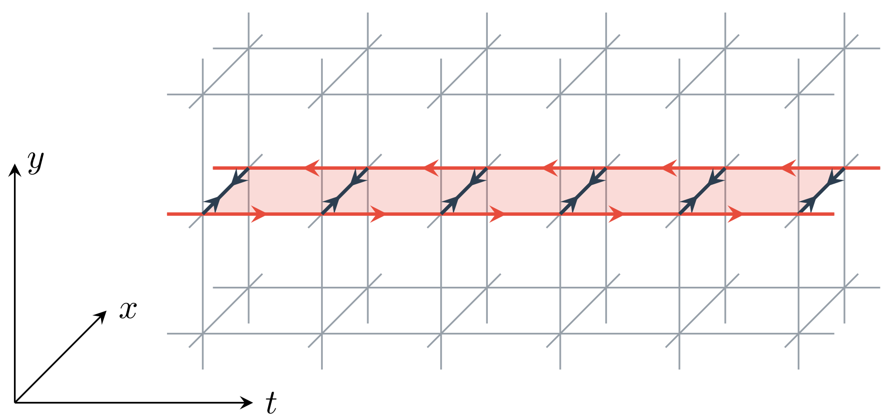
  Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$
  (Numerical versions: Greensite et al. ; Bergner et al.)

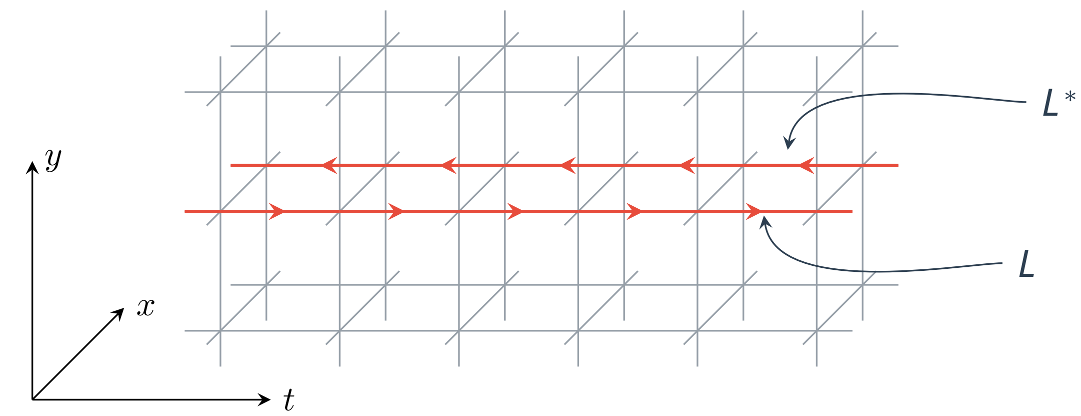
- Truncation valid for heavy quarks on reasonably fine lattices, a~0.1 fm

- Step II.: Mild sign problem, complex Langevin, Monte Carlo
  Check in SU(2): Scior, von Smekal 15

- New Step II.: Analytic solution by cluster expansion!

LO 3d effective theory for lattice YM





Integrate over all spatial gauge links

What remains is an interaction between Polyakov Loops

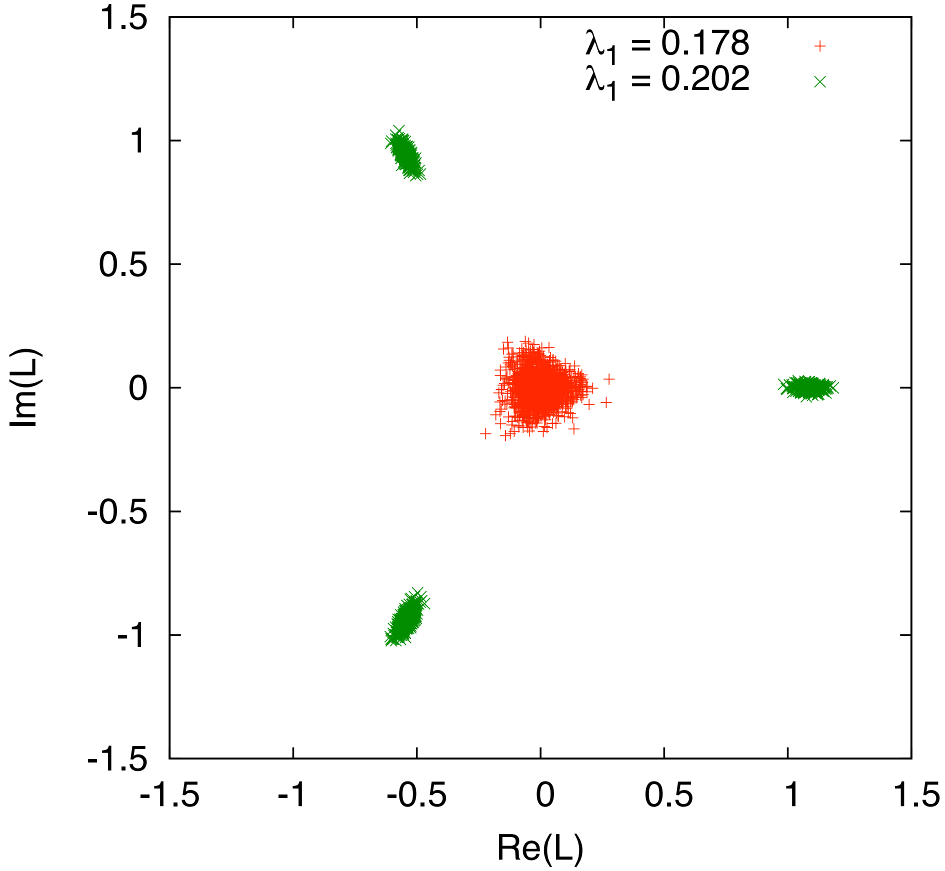
$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \,\mathrm{tr}\, W_j$$

Polonyi, Szachlanyi 82

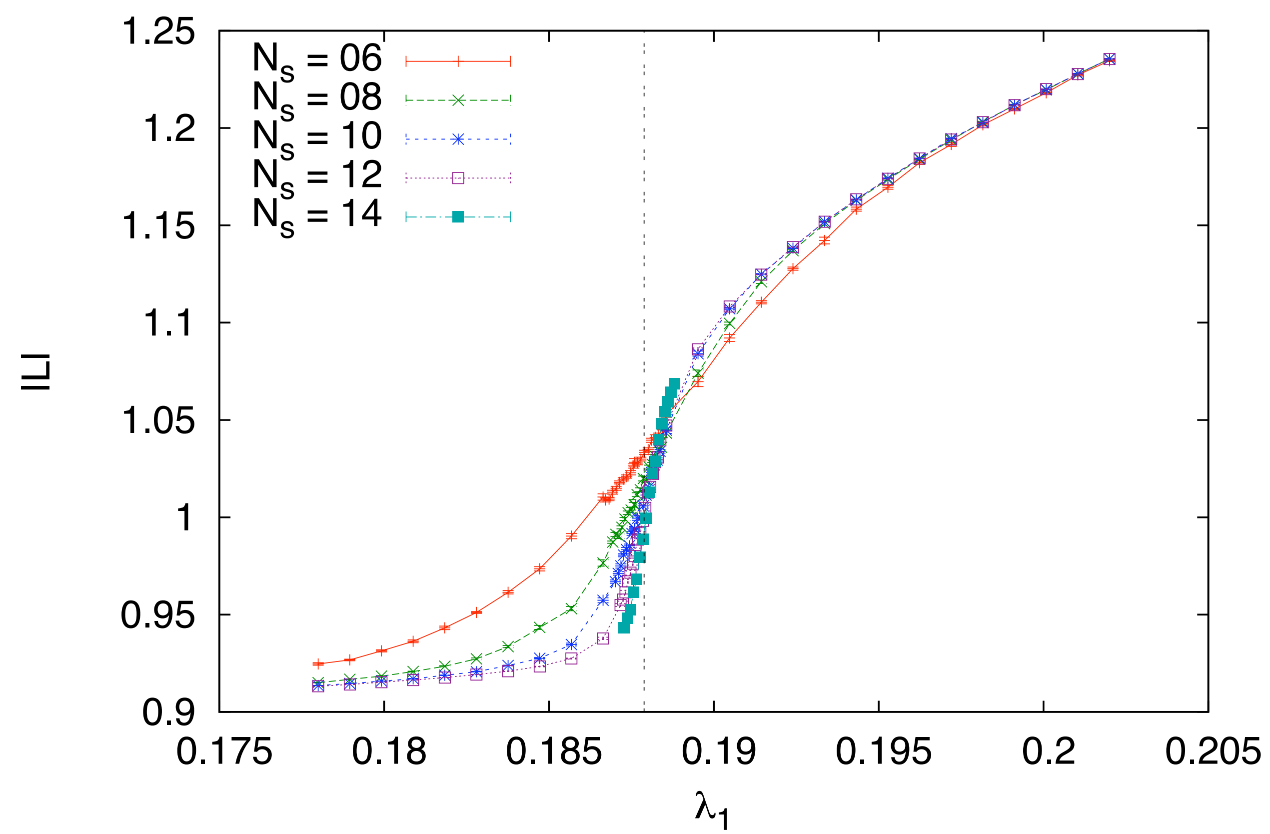
Character expansion: $u = \frac{\beta}{18} + O(\beta^2) < 1$ $\quad \beta = \frac{2N}{g^2}$

- larger distances between loops, higher powers of loops
- higher representations of loops
- decorations of LO graphs by additional plaquettes

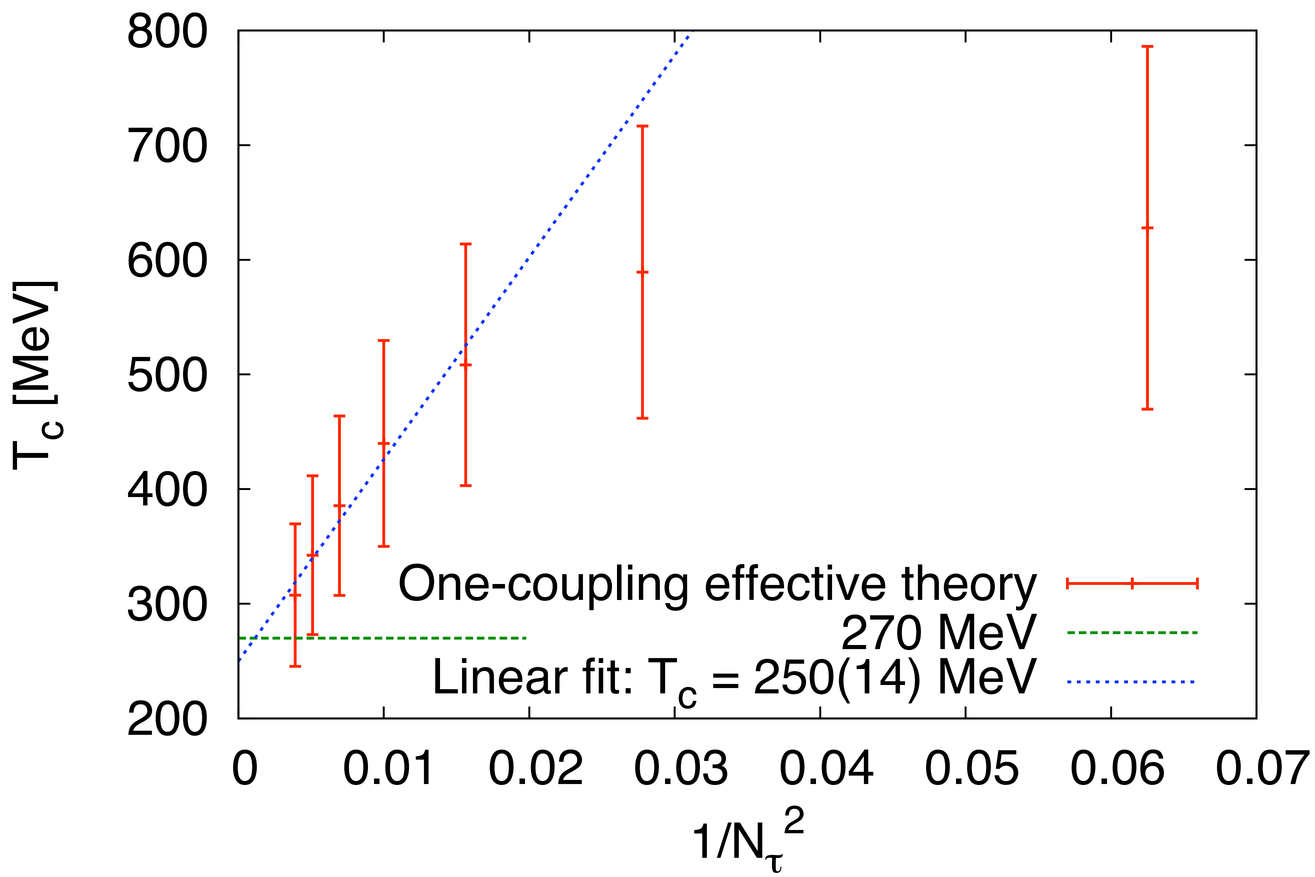
Numerical results for SU(3), one coupling



Order-disorder transition
=Z(3) breaking



Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

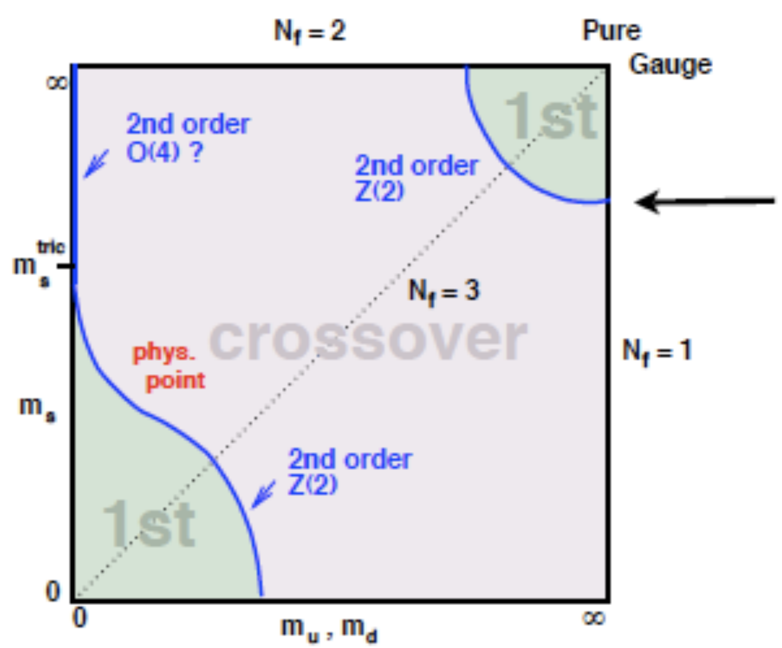
Including heavy, dynamical Wilson fermions

Expand in the *hopping parameter* $\kappa = 1/(2aM+8)$:

$$-S_{\rm eff} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\rm S} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^{\rm A} + \bar{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger \rm A} \right]$$

Deconfinement transition for heavy quarks

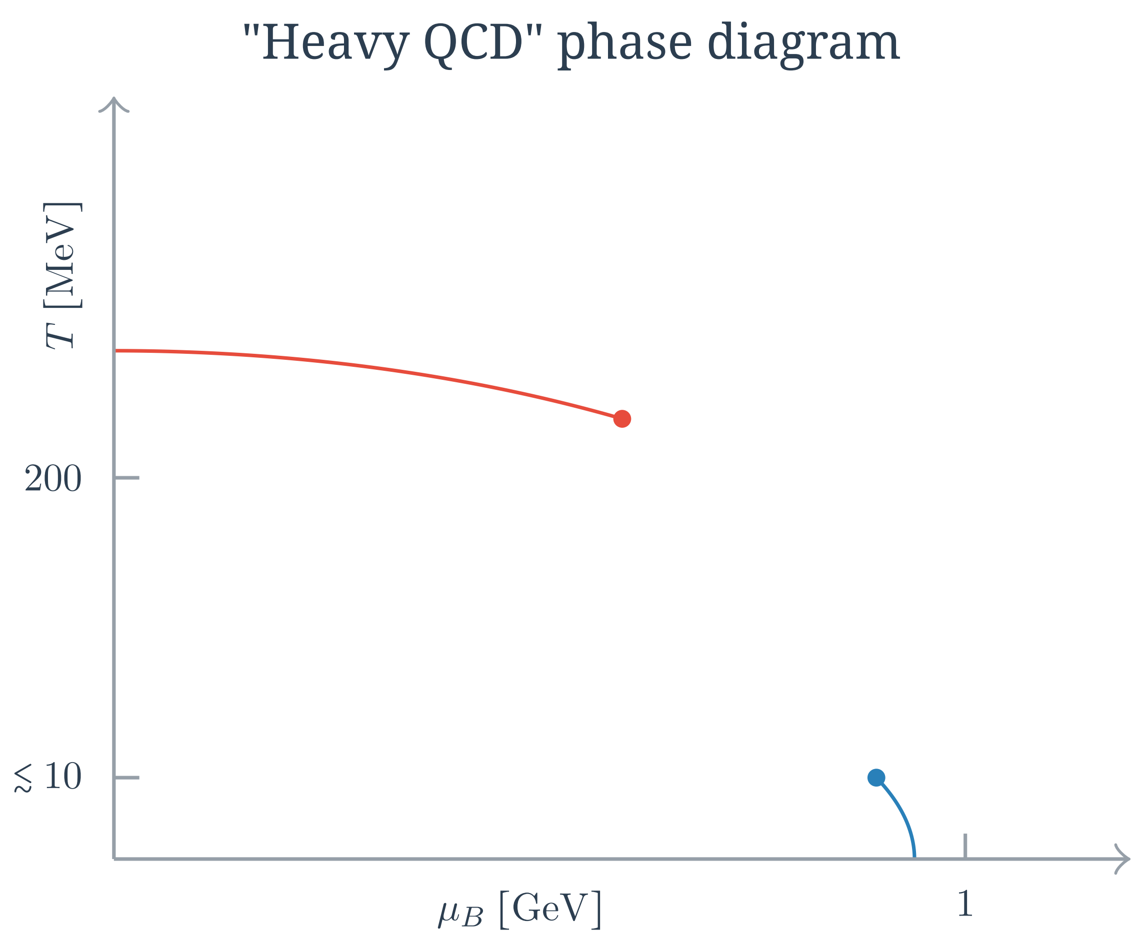
NLO: $\sim \kappa^2$

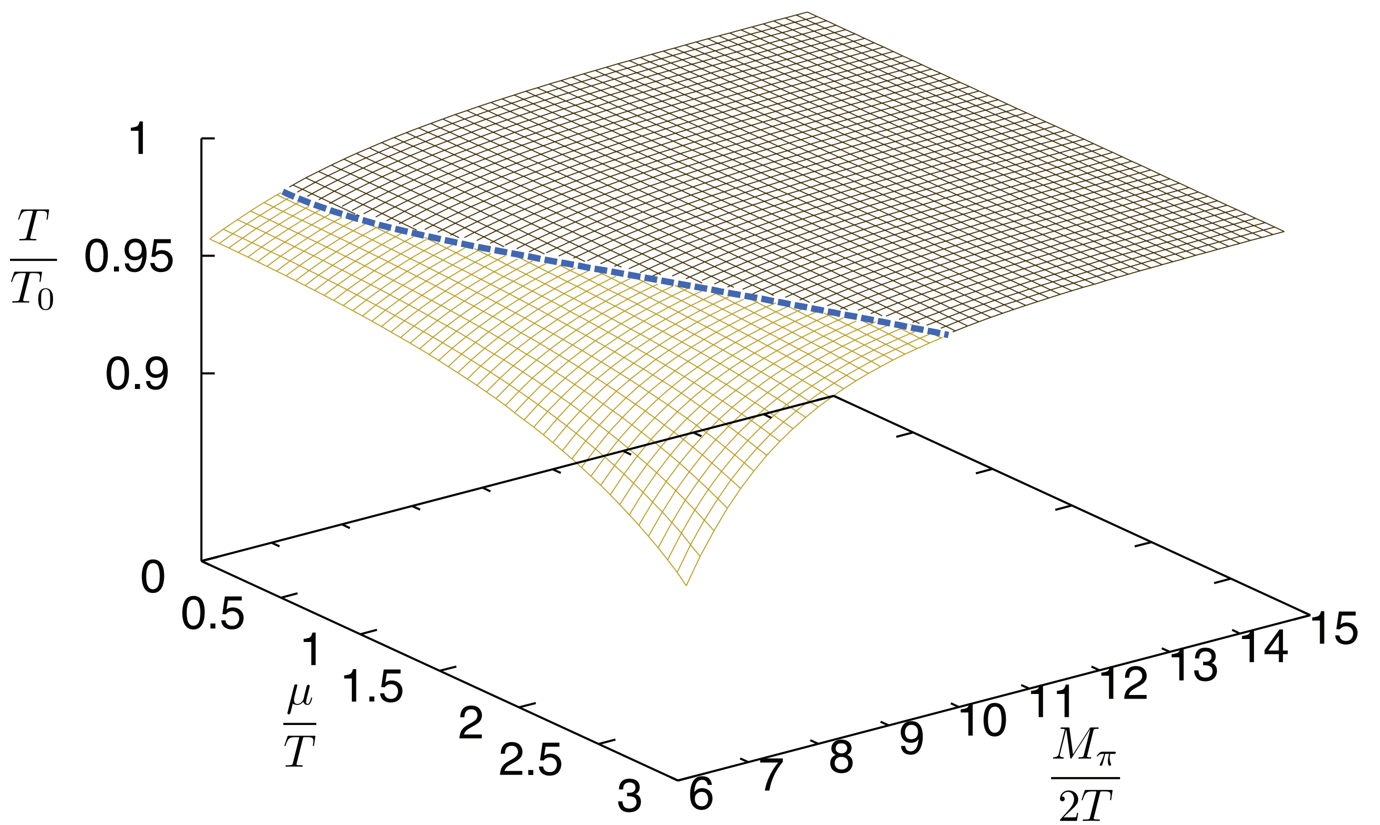


	eff. theory		4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau=4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

The fully calculated deconfinement transition





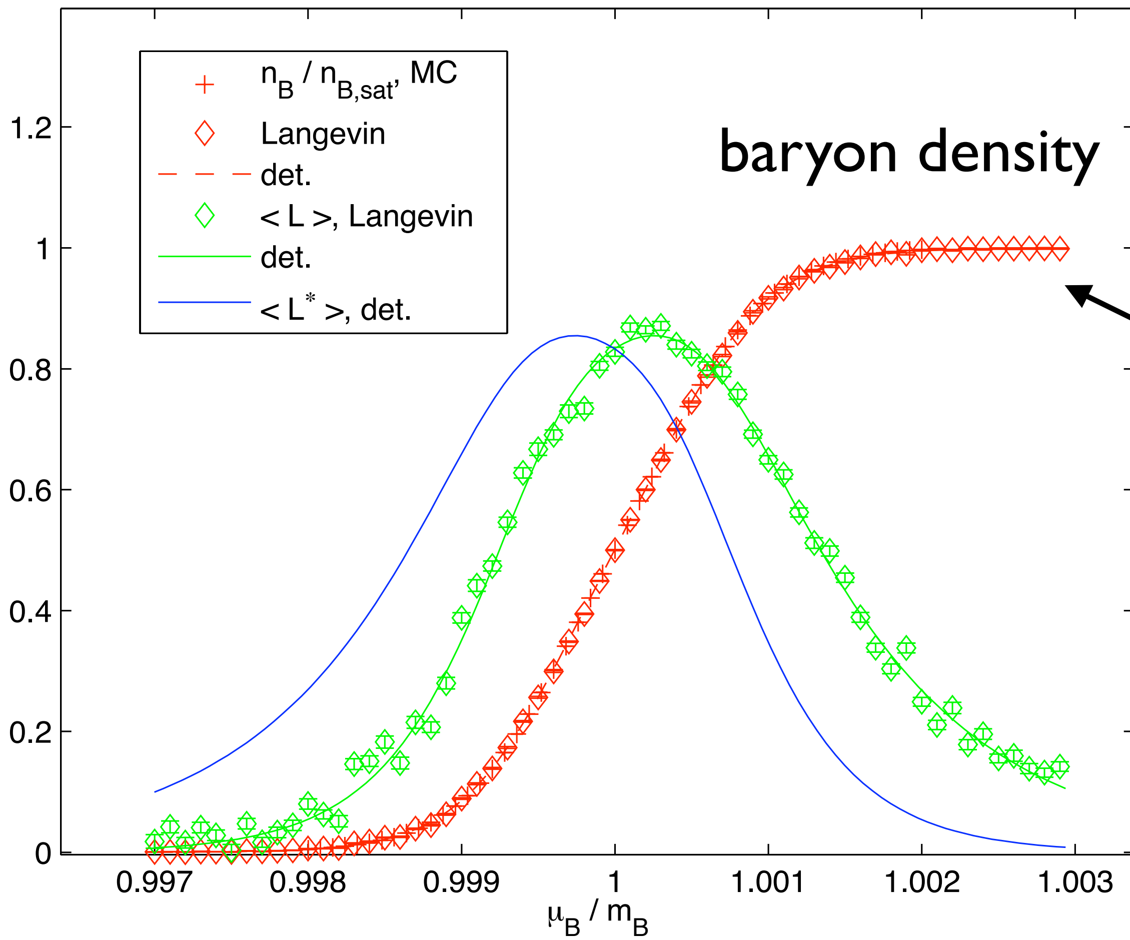
Continuum:

Friman, Lo, Redlich 14
Fischer, Lücker, Pawlowski 15

Fromm, Langelage, Lottini,O.P. 11

Cold and dense: onset to nuclear matter

Fromm, Langelage, Lottini, Neuman, O.P. 13



$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}, a = 0.17 \text{ fm}$$

$$\beta = 5.7, \kappa = 0.0000887, N_\tau = 116$$

- Silver blaze property

  no dependence on chem. pot. until onset

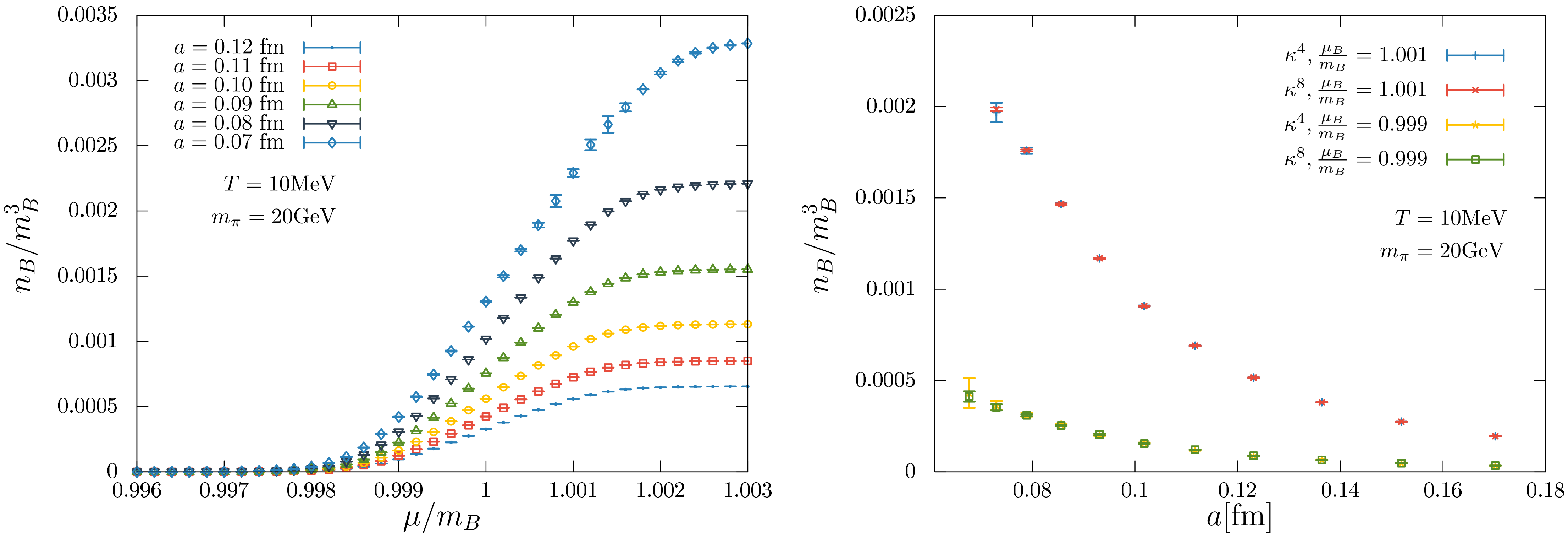
- Lattice saturation

  Pauli principle, cut-off effect!

- Screening of Polyakov loop

  But no deconfinement!

Continuum approach

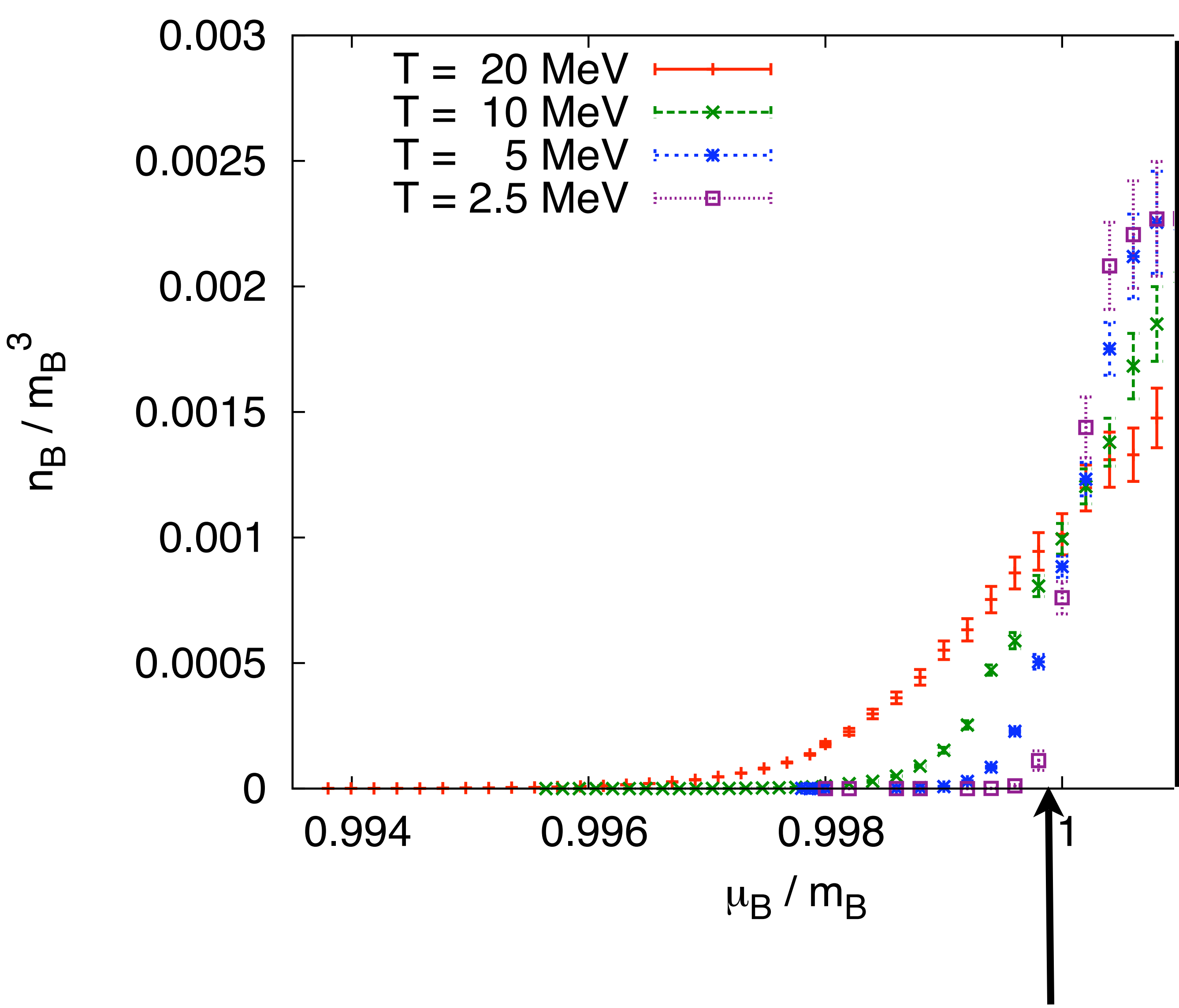


- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition
- Finer lattice necessary for larger density to avoid saturation

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$$m_\pi = 20\ \mathrm{GeV}$$



Effect of binding between baryons: $\mu_c < m_B$

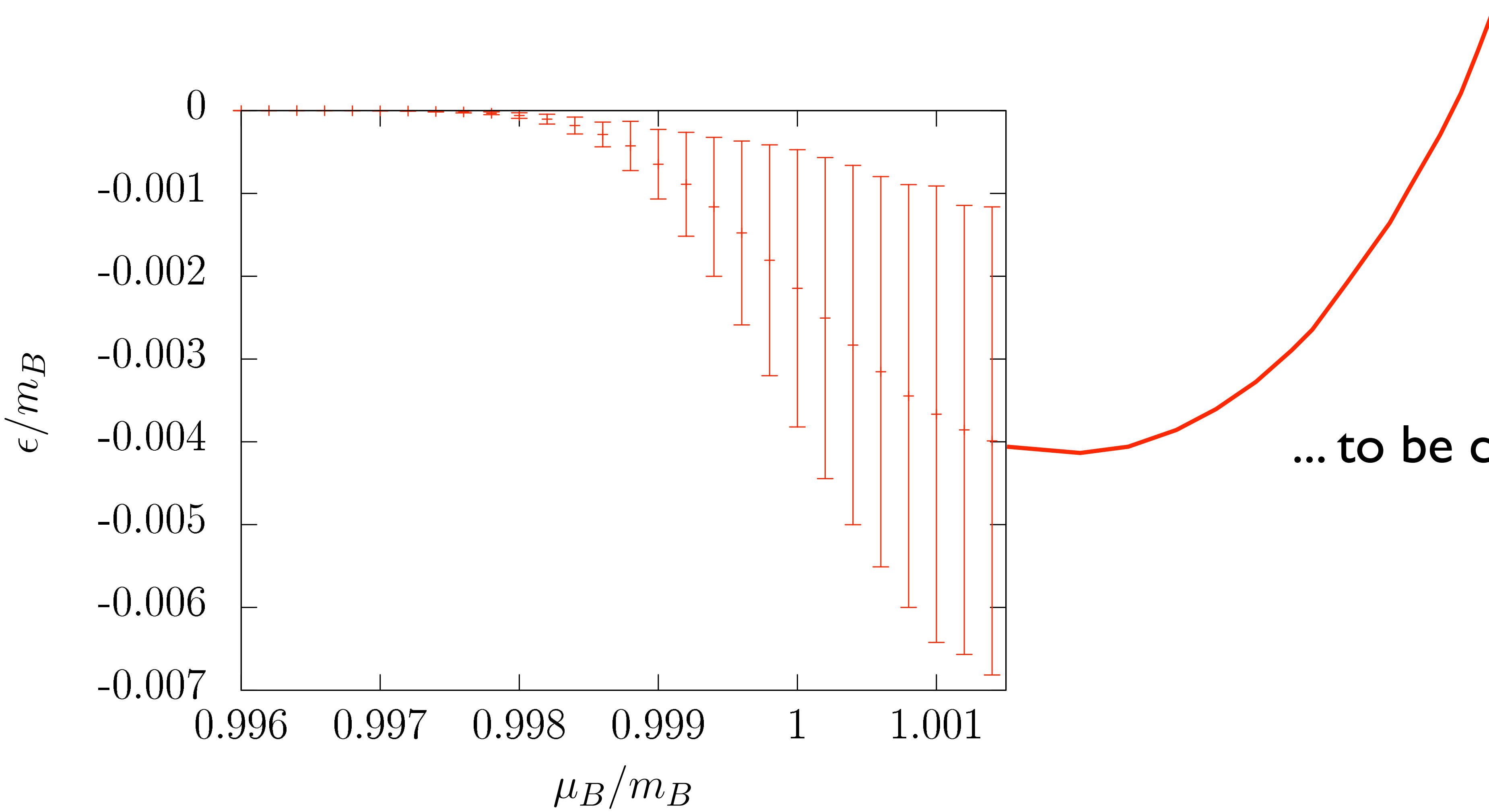
$$\frac{\mu}{T} \sim 4000$$

Binding energy per nucleon: $\epsilon = \frac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$

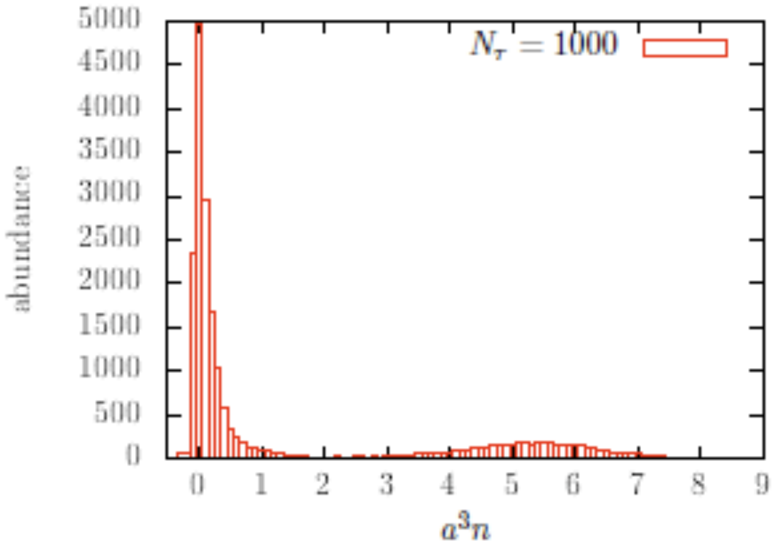


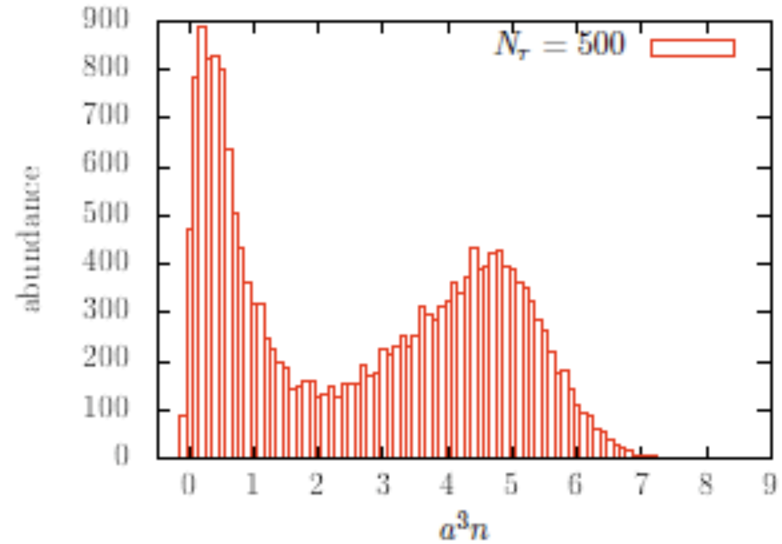
... to be continued...

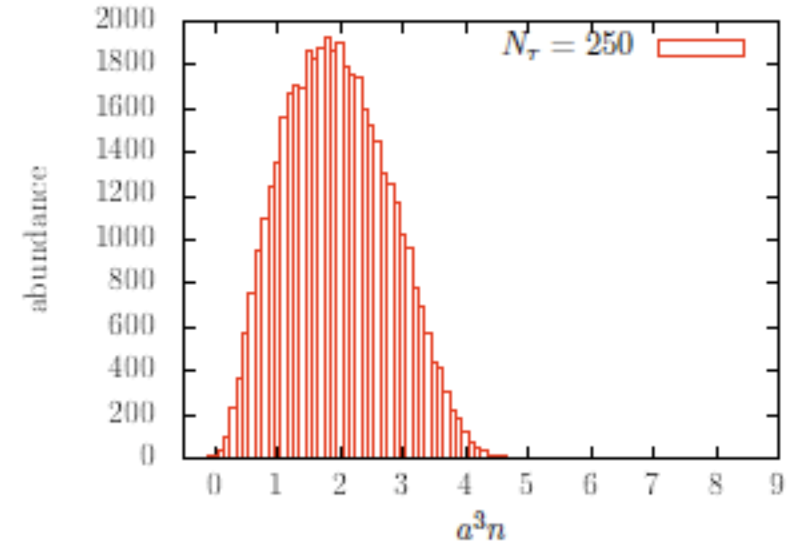
Minimum: access to nucl. binding energy, nucl. saturation density!

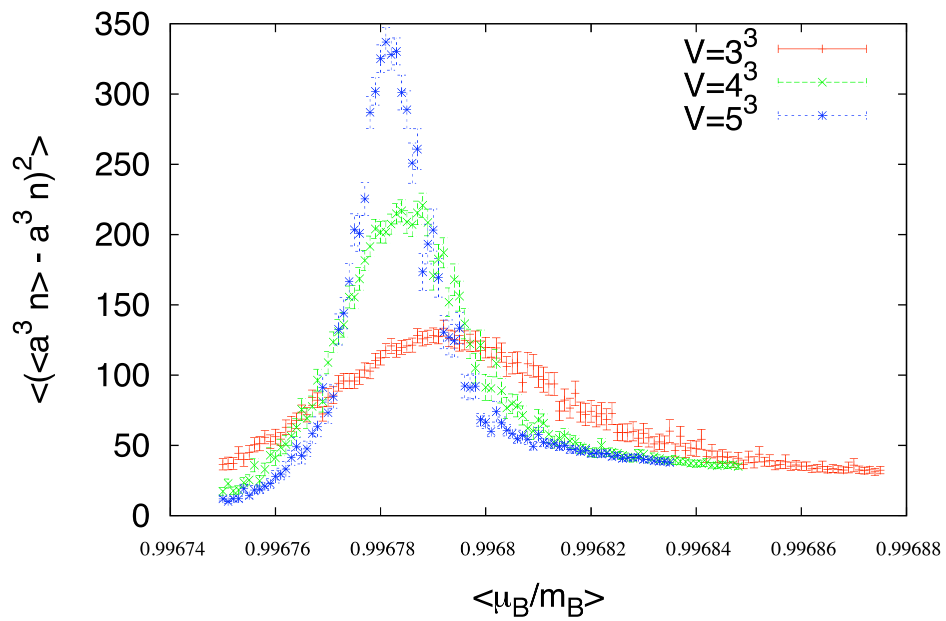
$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

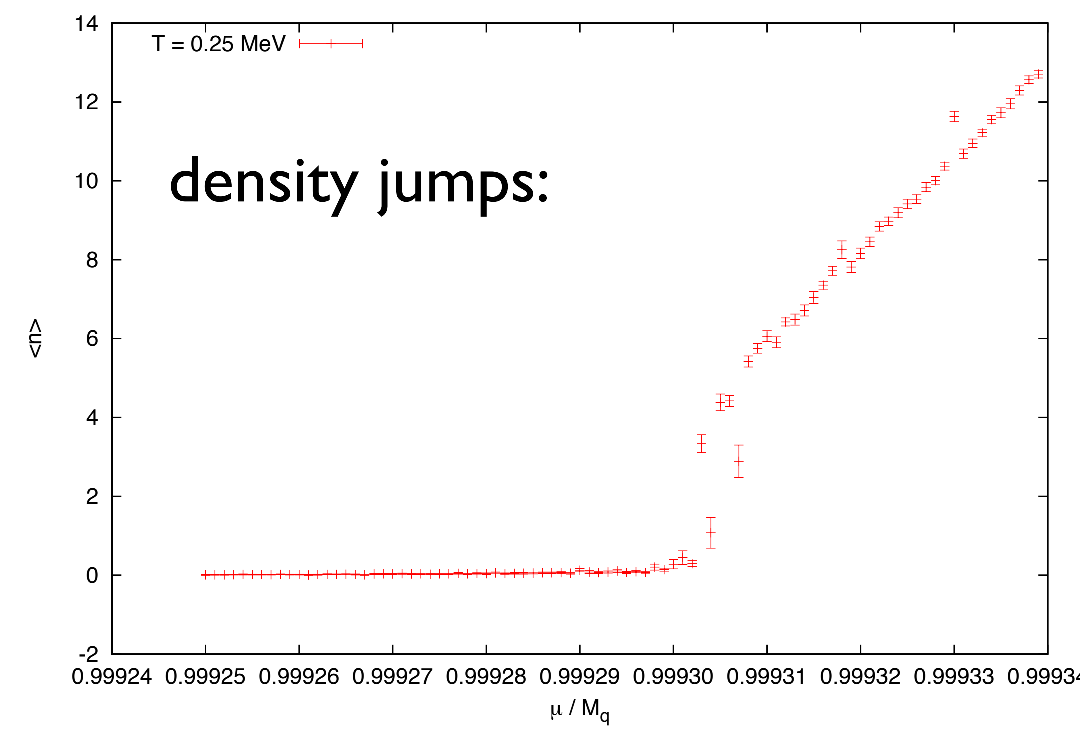
Light quarks: first order transition + endpoint











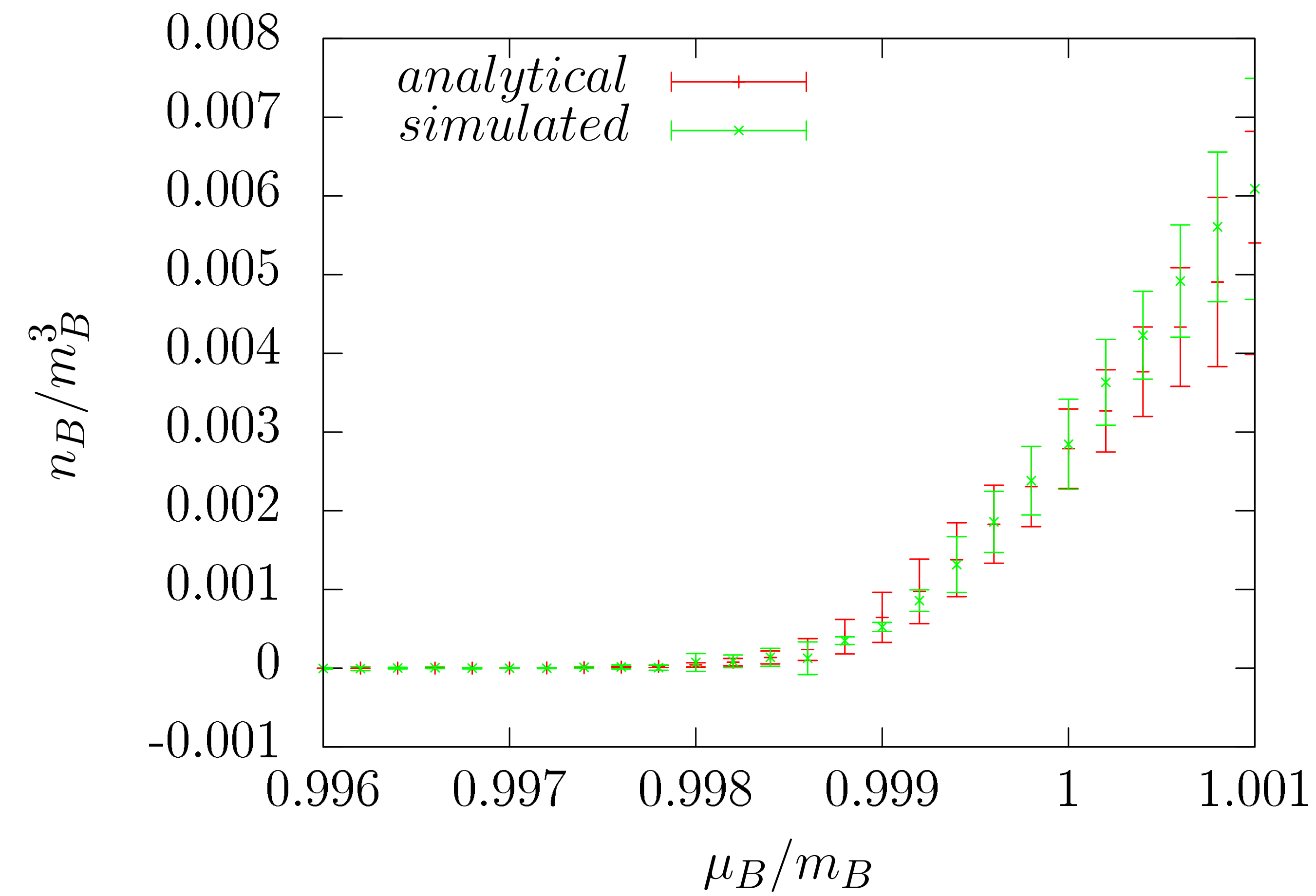
For sufficiently light quarks: $\kappa \sim 0.1$

- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!

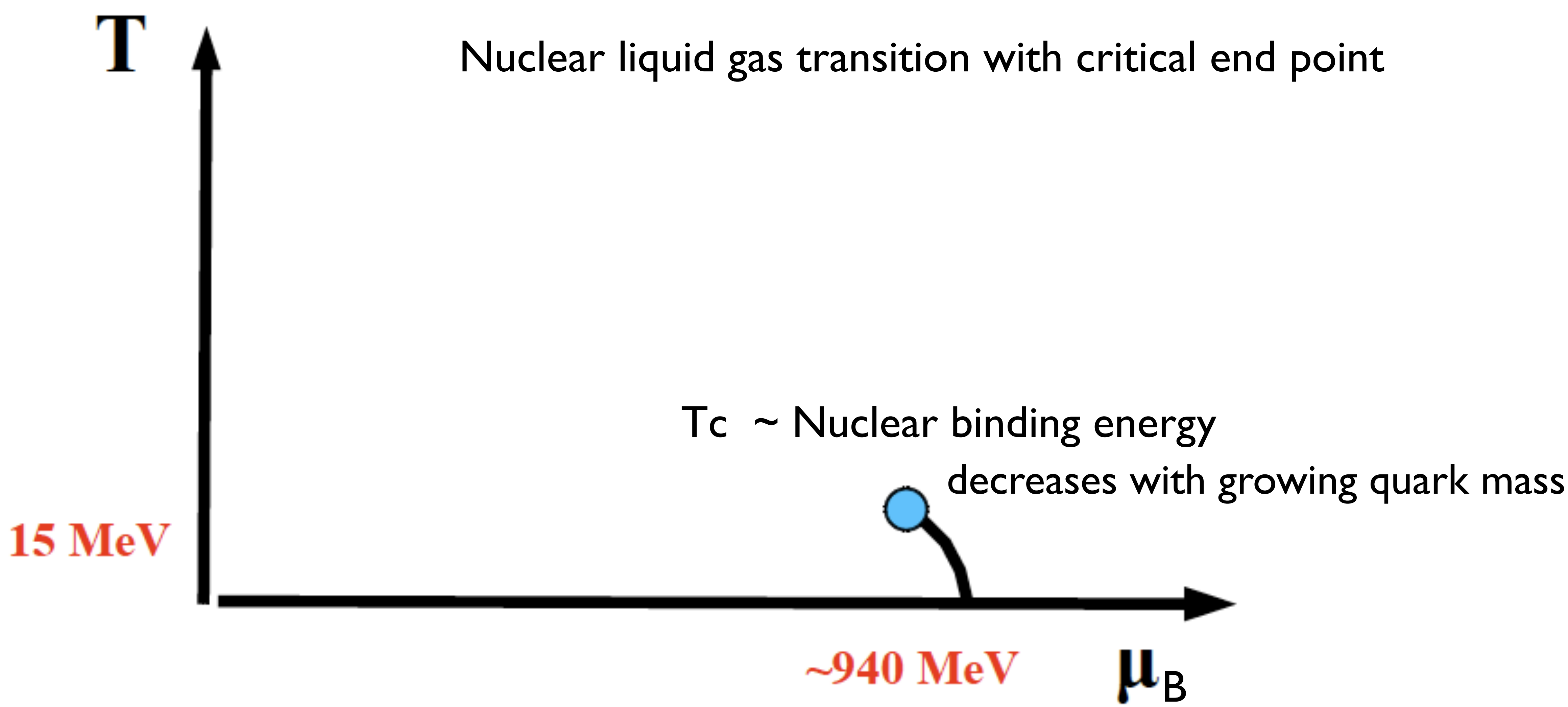
Perturbation theory possible in effective theory!

- Effective couplings small
- Linked cluster expansion in effective couplings
- Error bars systematic: difference between orders in effective action



Binding energy per nucleon: shrinks with growing quark mass

$$\epsilon = -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2 = -\frac{1}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 e^{-am_M} + \ldots$$



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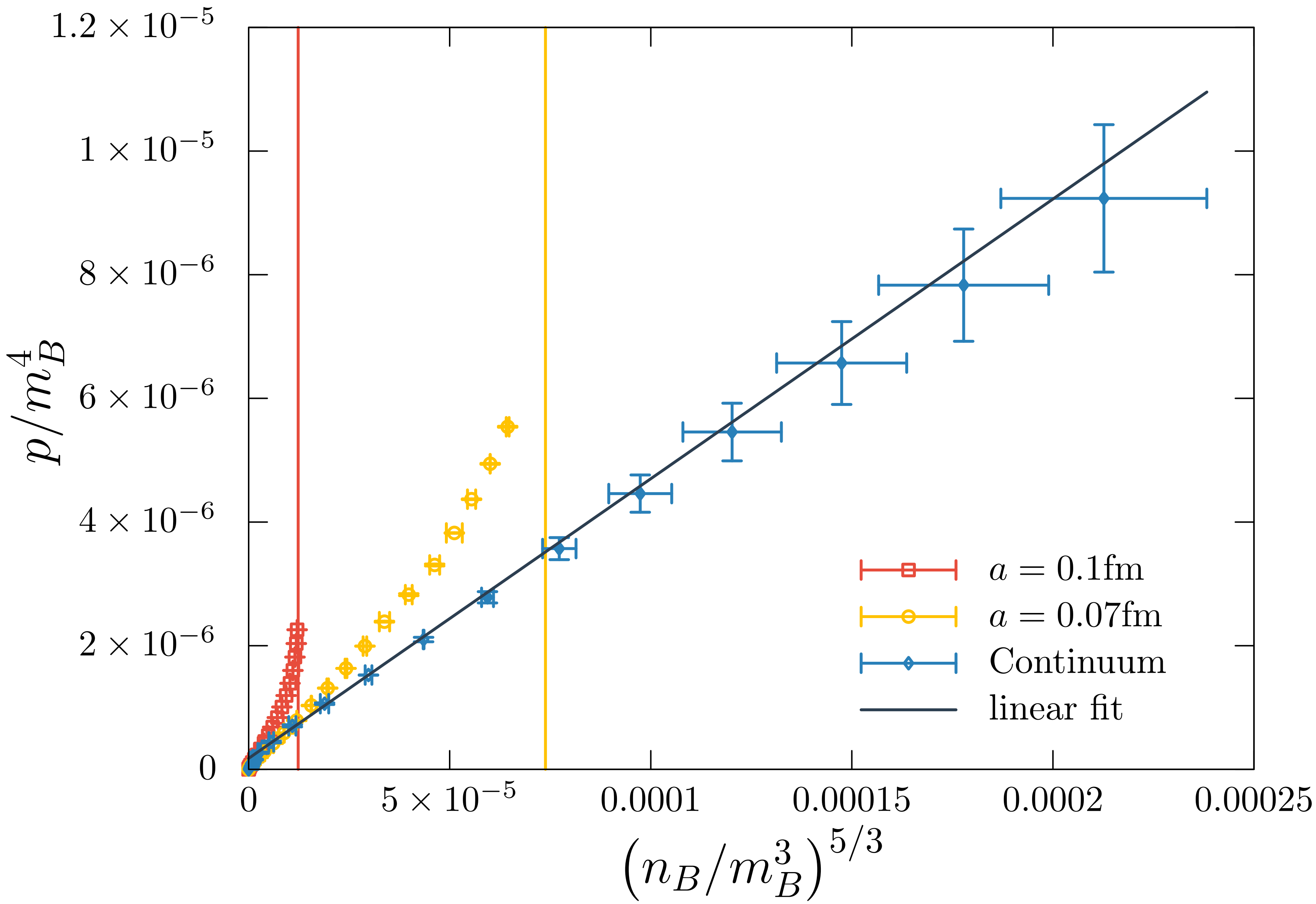
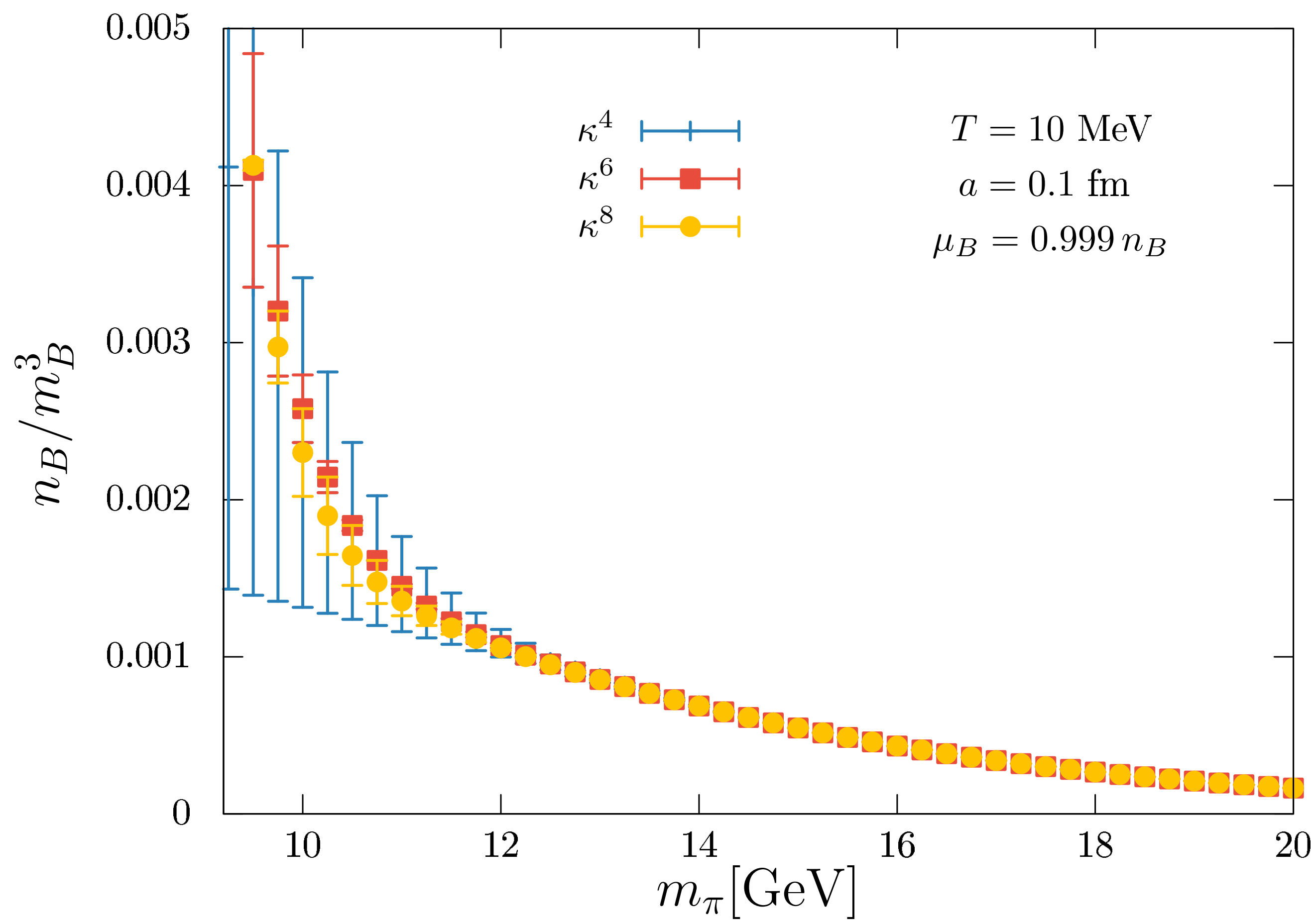
Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

Mass dependence of convergence



The effective lattice theory approach II

- Two-step treatment:

  de Forcrand, Langelage, O.P., Unger
  Phys.Rev.Lett. 113 (2014) 152002

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: integrate over gauge links in strong coupling expansion, leave fermions

  Wolff; Karsch, Mütter

$$Z_{\rm QCD} = \int d\psi d\bar{\psi} dU \, e^{S_F + S_G} = \int d\psi d\bar{\psi} Z_F \left\langle e^{S_G} \right\rangle_{Z_F}$$

$$\left\langle e^{S_G} \right\rangle_{Z_F} \simeq 1 + \langle S_G \rangle_{Z_F} = 1 + \frac{\beta}{2N_c} \sum_P \left\langle {\rm tr}[U_P + U_P^\dagger] \right\rangle_{Z_F} \qquad Z_F(\psi, \bar{\psi}) = \int dU \, e^{S_F}$$

- Result: 4d "polymer" model of QCD (hadronic degrees of freedom!)
  Valid for all quark masses (also m=0!), at strong coupling (very coarse lattices)

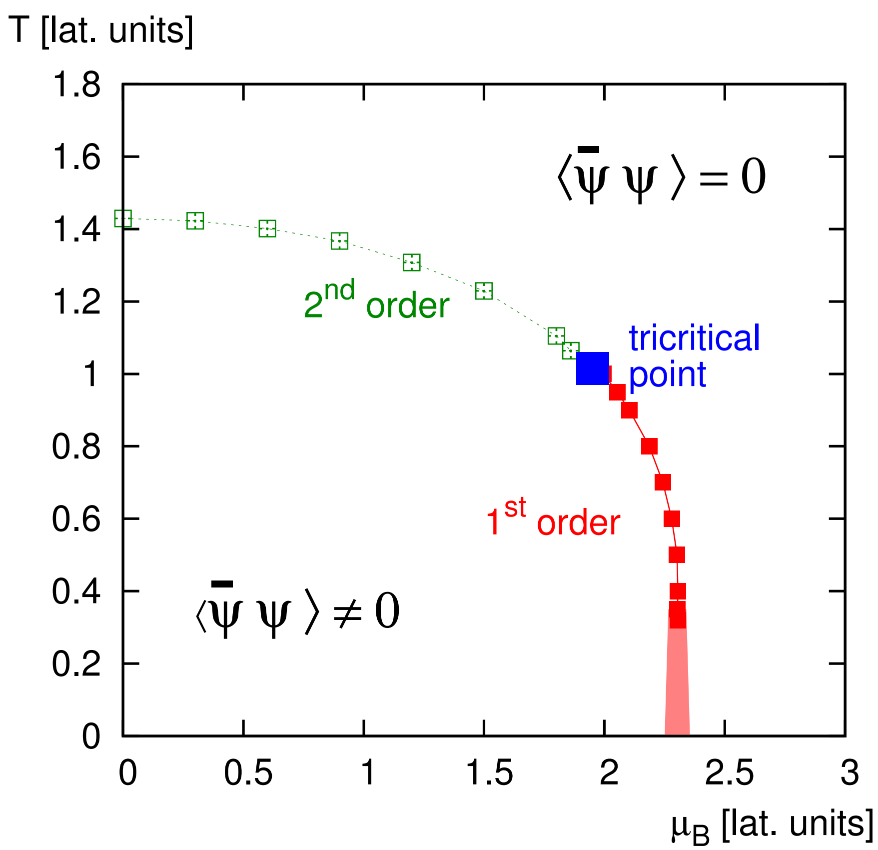
- Step II: sign problem milder: Monte Carlo with worm algorithm

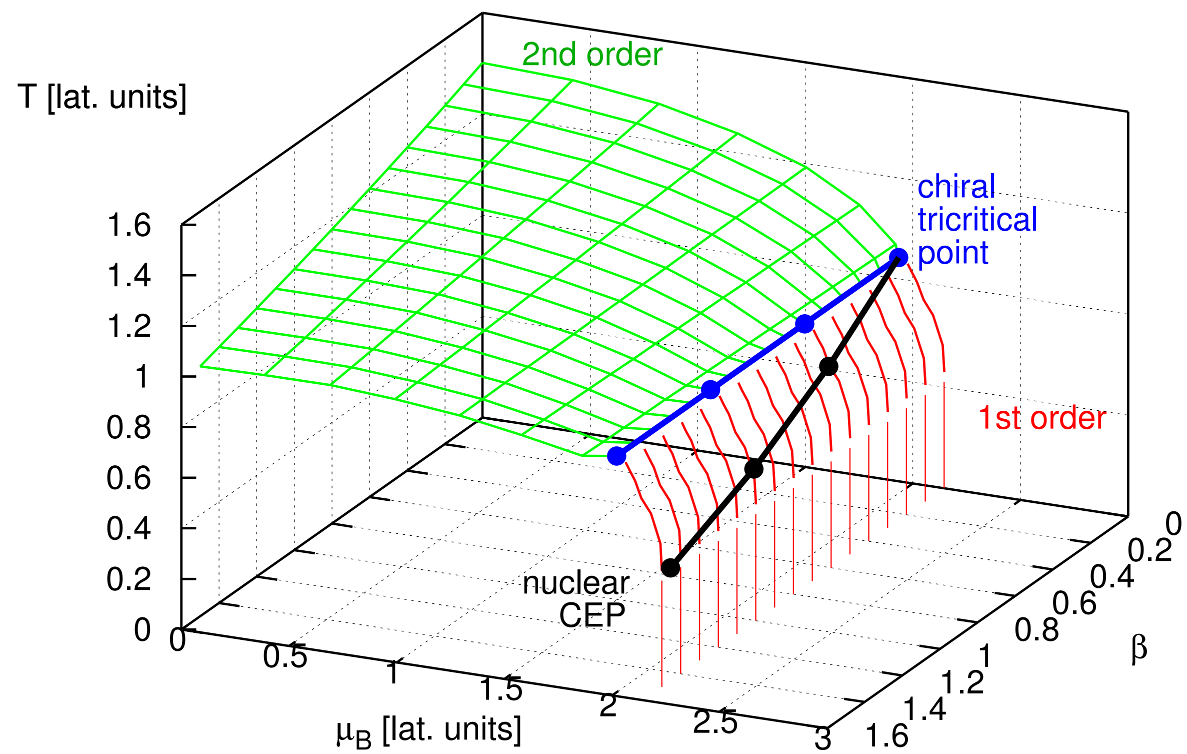
- Numerical simulations without fermion matrix inversion, very cheap!

From strong coupling limit to finite coupling

Unrooted staggered fermions: Nf=4

de Forcrand, Langelage, O.P., Unger 14



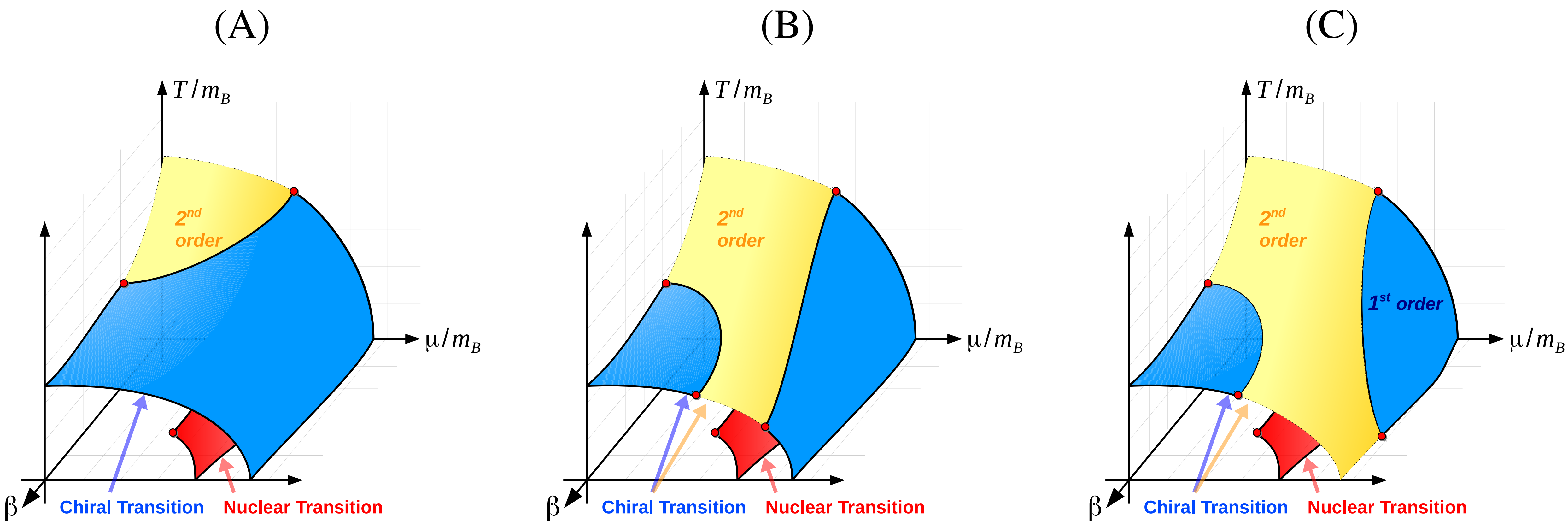


Strong coupling limit: $\beta = 0$

Including leading gauge corrections

Nucl. and chiral transition coincide!

Possibilities for continuum Nf=4 phase diagram:



Nf=4 is known to have first order transition at zero density

Conclusions

- Growing control over phase diagram in generalised parameter space:

  Nf, quark mass, imaginary chemical potential, lattice spacing

- "Physical" QCD: finite T transition is crossover that softens with density

- Complete phase diagram + baryon matter directly from QCD for:

  -Heavy dense QCD near continuum with fully analytic methods

  -Chiral dense QCD on coarse lattices