Jet fragmentation within and without a medium

Boris Kopeliovich
Valparaiso

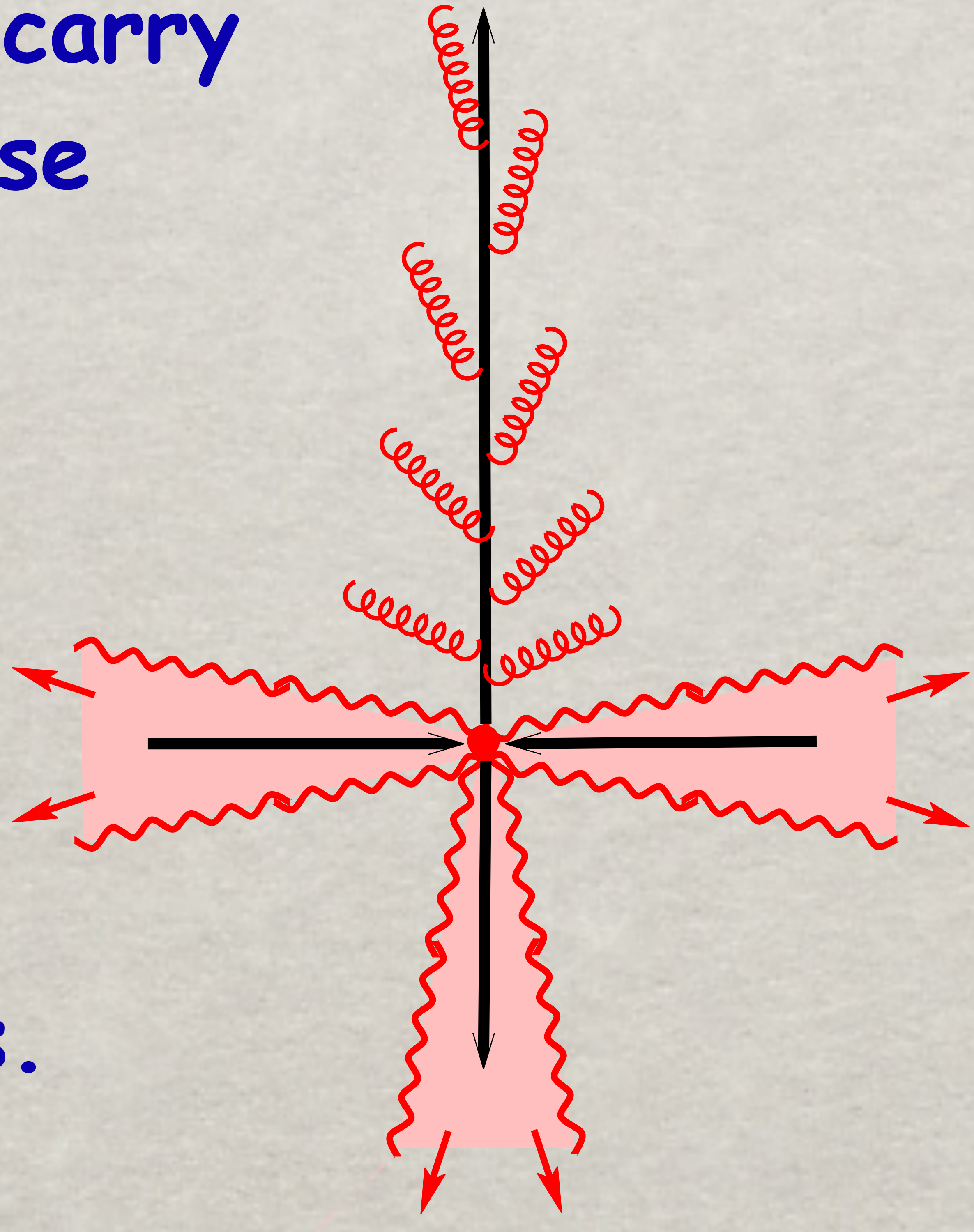
Hard parton collision

High-pt parton scattering leads to formation of 4 cones of gluon radiation:

(i) the color field of the colliding partons is shaken off in forward-backward directions.

(ii) the scattered partons carry no field up to transverse momenta kt<pt.

The final state partons are regenerating the lost color field by radiating gluons and forming the up-down jets.



In terms of the Fock state representation all radiated gluons pre-exist in the initial bare parton, and are liberated on mass shell later on in accordance with the coherence length/time of gluon radiation

$$l_c = \frac{2E\,x(1-x)}{k_T^2 + x^2 m_q^2} \approx \frac{2\,\omega}{k_T^2}$$

First are radiated gluons with small longitudinal and large transverse momenta.

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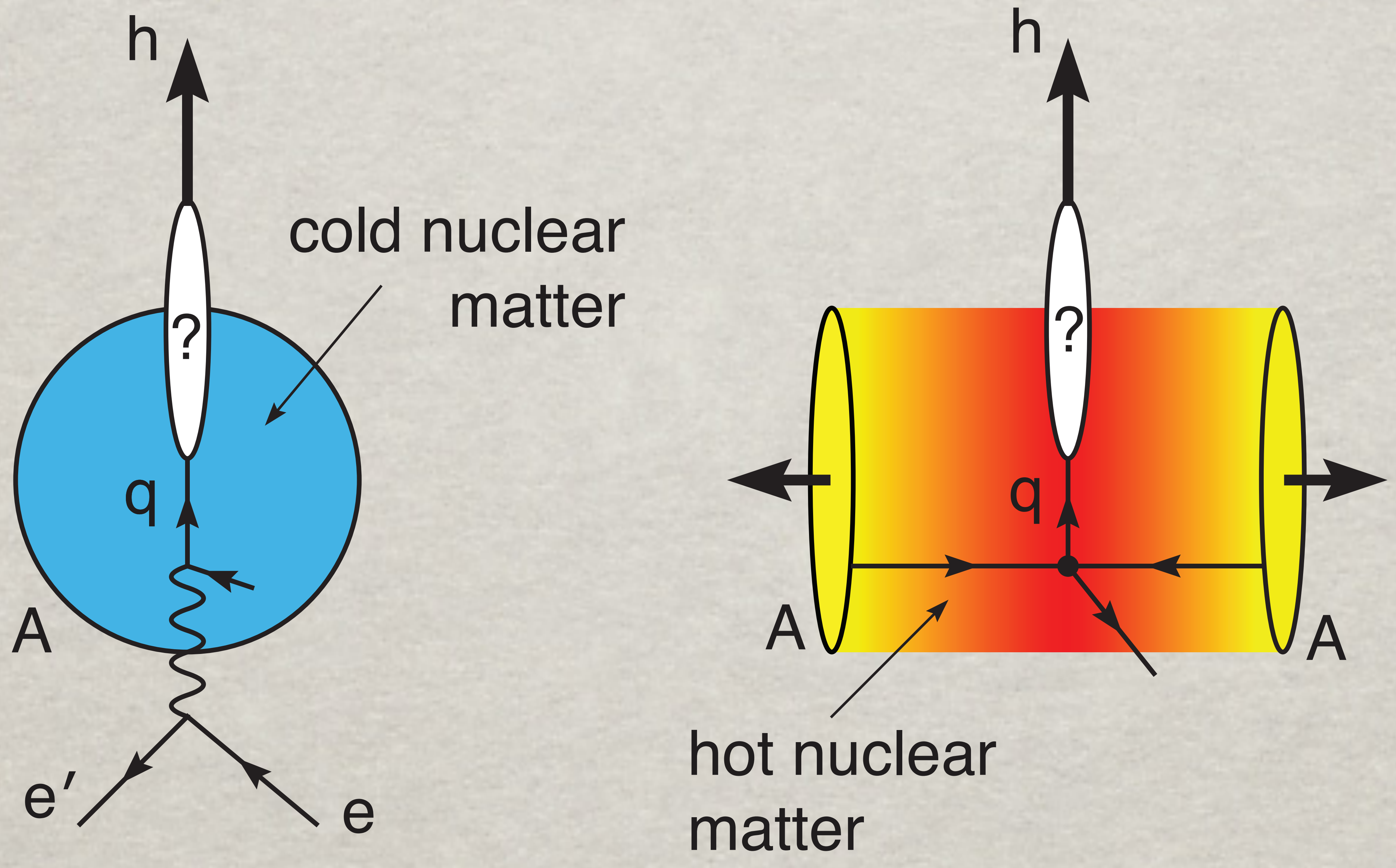
SIDIS: testing hadronization models

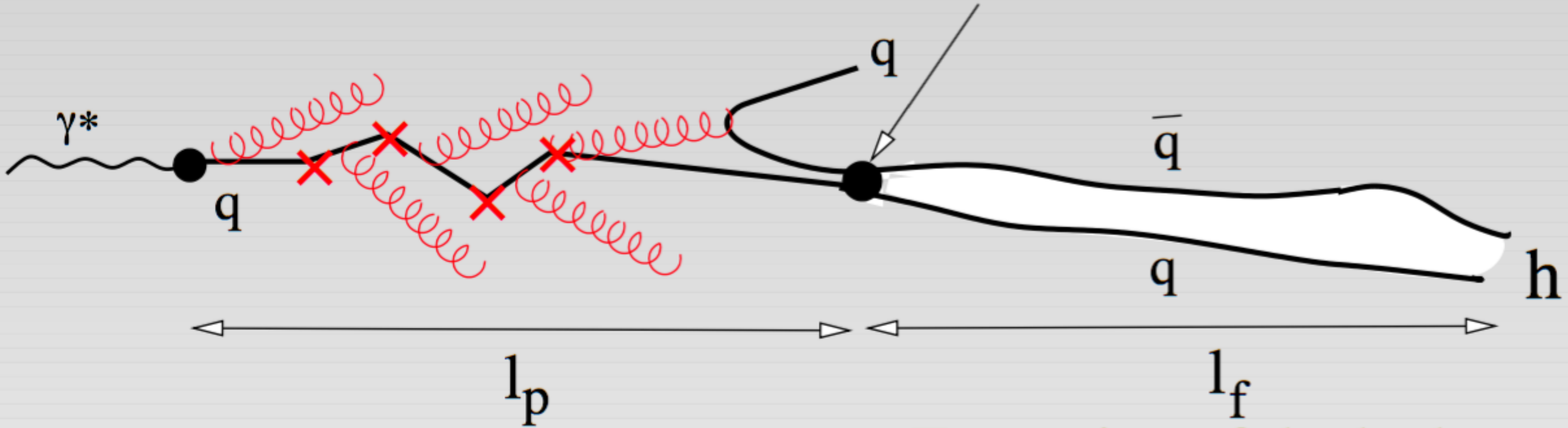
Semi-inclusive deep-inelastic processes (SIDIS) can be used as a testing ground for the models describing final-state attenuation of high-pT hadrons produced of nuclear collisions



Similar kinematics

Jet quenching in DIS

Two–step picture



Perturbative color neutralization

Production length:
energy loss, broadening

Formation of the hadron
no energy loss, no broadening

Two sources of jet quenching:

(i) energy loss of the parton prior production of a pre-hadron (no absorption);

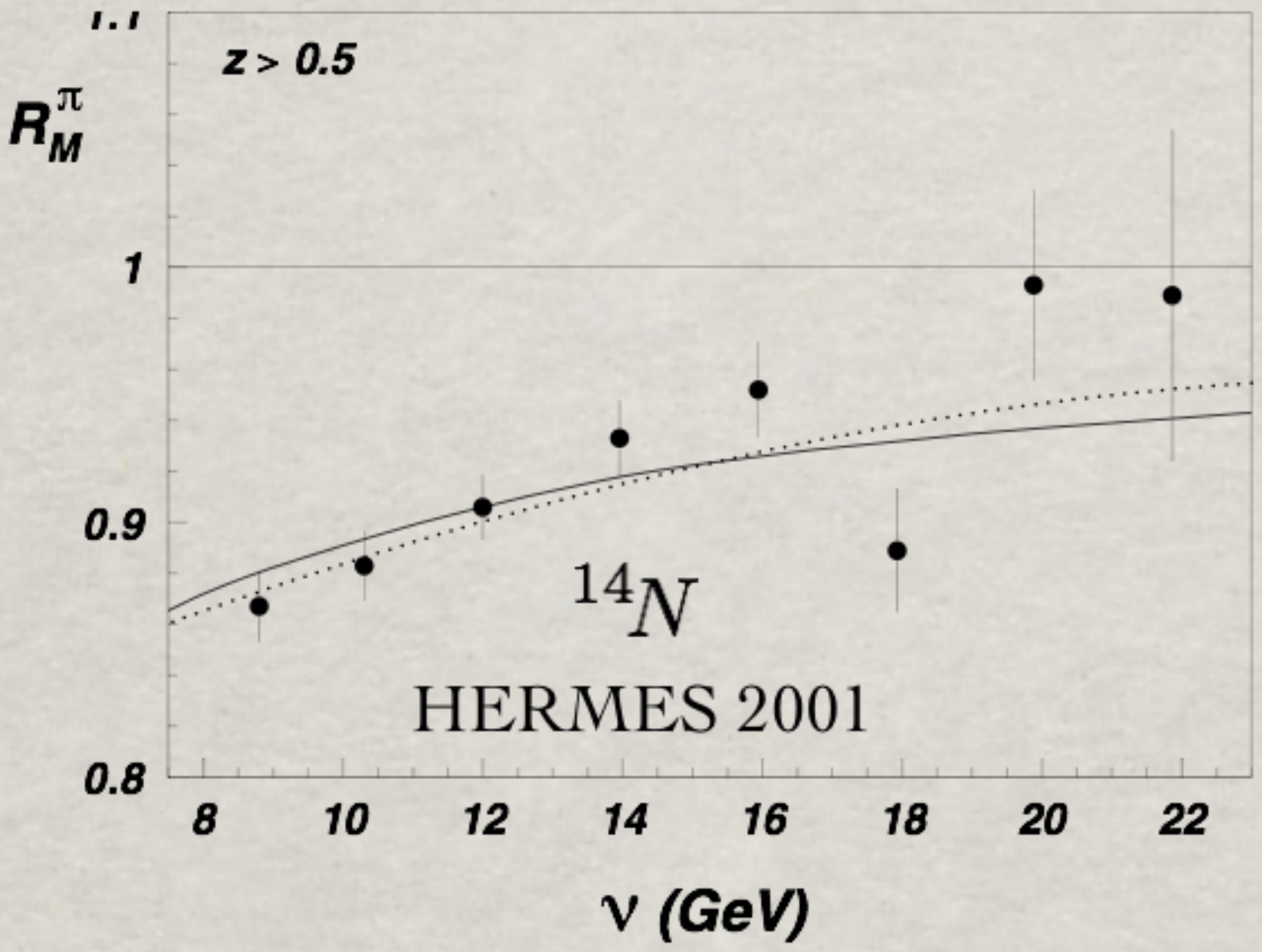
(ii) attenuation of the pre-hadron in the medium (absorption)

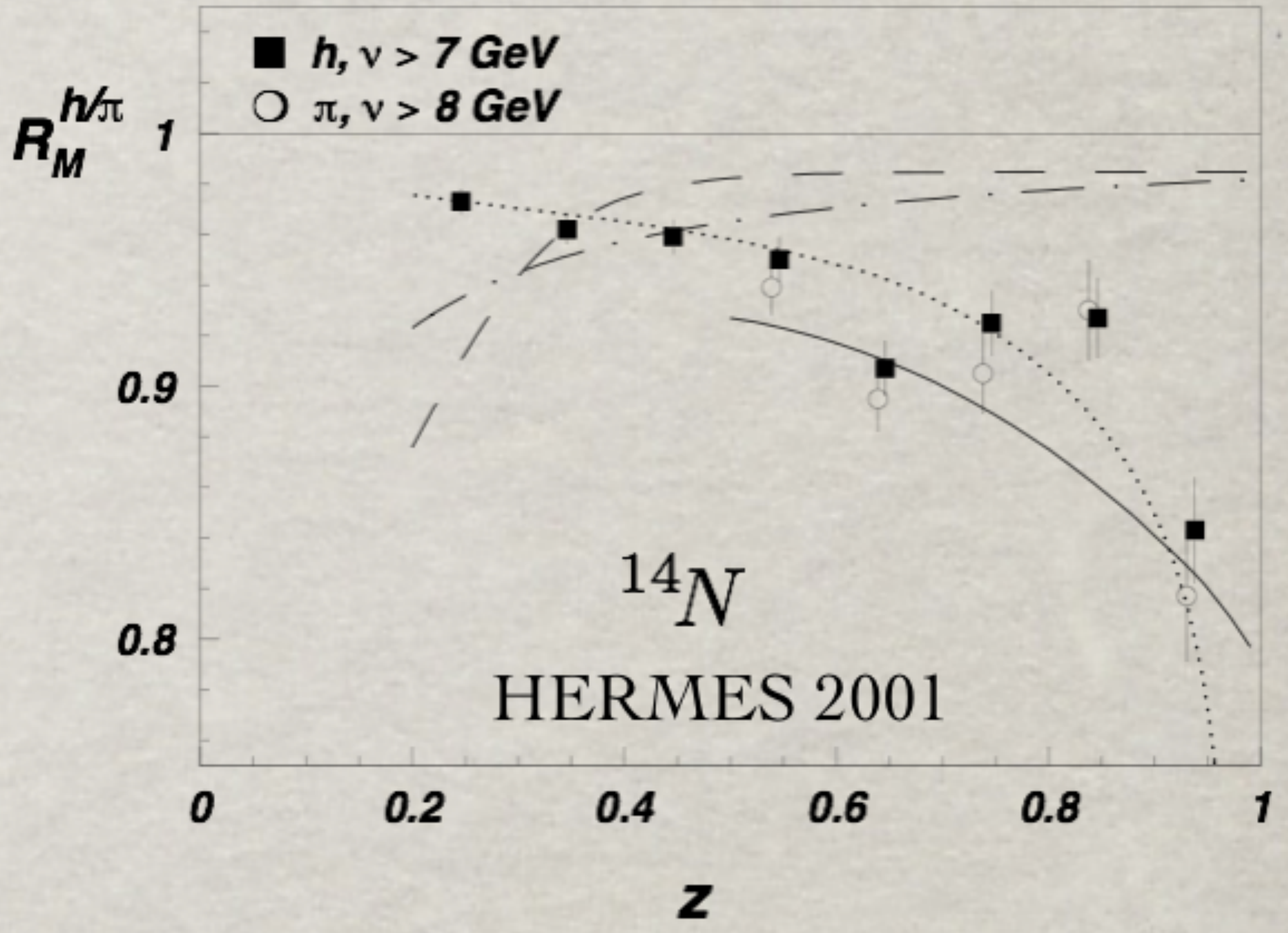
Testing the model in SIDIS

Finite l_p:
E-loss + absorption

Predictions

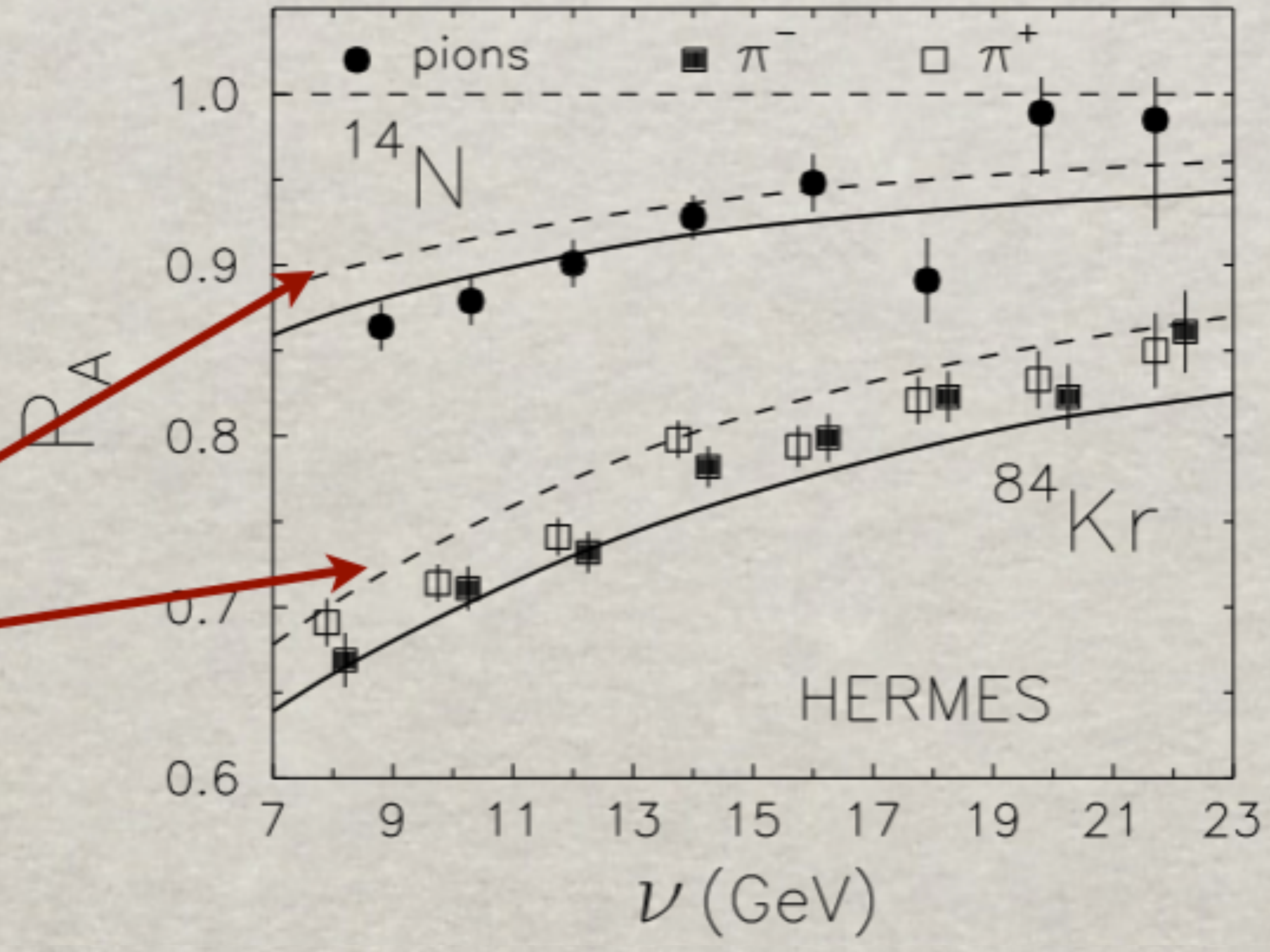
B.K., J. Nemchik, E. Predazzi
1996

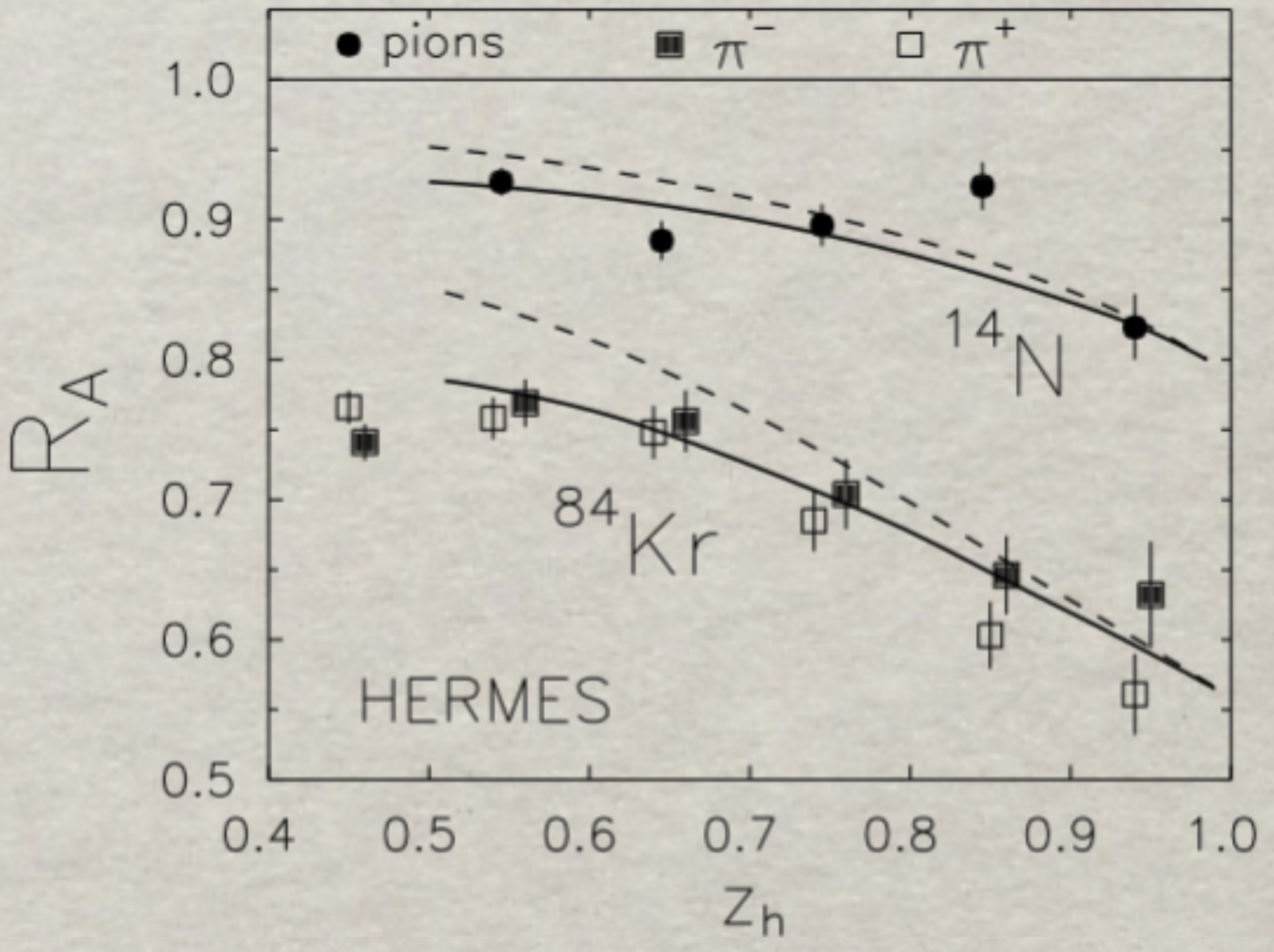




B.K., J. Nemchik, E. Predazzi
&A. Hayashigaki
Nucl. Phys. A 740, 211 (2004)

Only absorption





Vacuum energy loss

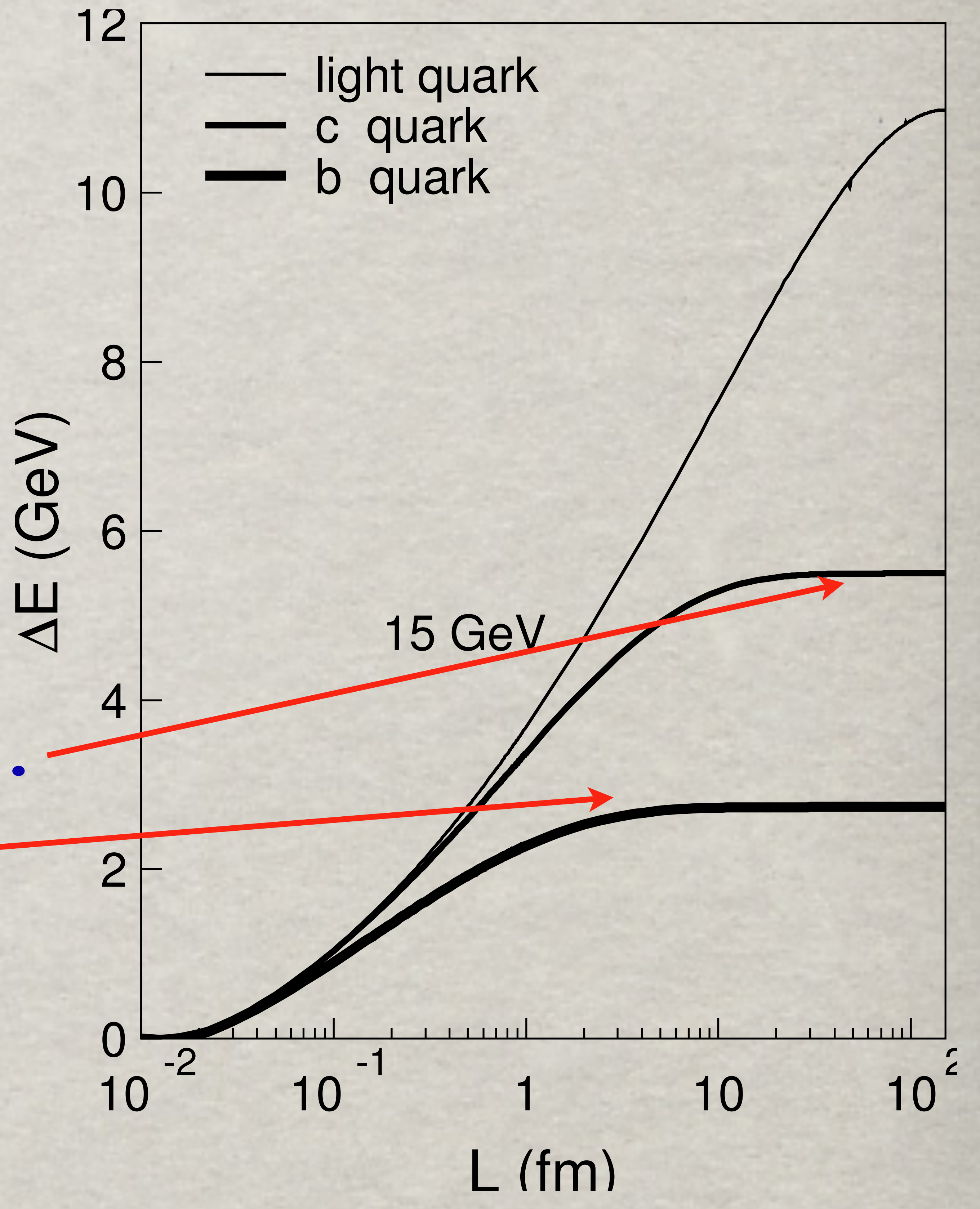
How much energy is radiated over the path length L?

$$\Delta E(L) = E \int_{\Lambda^2}^{Q^2} dk^2 \int_0^1 dx\, x \,\frac{dn_g}{dx\, dk^2}\Theta(L - l_c)$$

$$\frac{dn_g}{dx\, dk^2} = \frac{2\alpha_s(k^2)}{3\pi\, x}\, \frac{k^2[1+(1-x)^2]}{[k^2 + x^2 m_q^2]^2}$$

Dead-cone effect: gluons with $k^2 < x^2 m_q^2$ are suppressed.

Heavy quarks radiate less energy than the light ones.



B.K., I.Potashnikova,I.Schmidt,
PRC 82(2010)037901

Another dead cone: soft gluons cannot be radiated at short path length

$$k^2 > \frac{2Ex(1-x)}{L} - x^2 m_q^2$$

This is why heavy and light quarks radiate with similar rates at short time scales $L \lesssim \frac{Ex(1-x)}{x^2 m_q^2}$

Peculiar features of high-p_T jets

Energy conservation: $l_p \lesssim \frac{E}{dE/dl}(1 - z_h)$ (in vacuum)

Energy and scale dependences of the production length in SIDIS:

(i) Energy dependence at fixed Q^2

$\langle dE/dl \rangle$ is fixed, so $l_p \propto E$

(ii) Scale dependence at fixed energy

$\langle dE/dl \rangle$ rises with Q^2, so $l_p(Q^2)$ is falling

---

Specifics of high-pT jets: the energy and scale strongly correlate:

★ $E = p_T$

$Q^2 \sim p_T^2$

Is the pT-dependence of $l_p(p_T)$ rising or falling?

- the answer is not obvious...

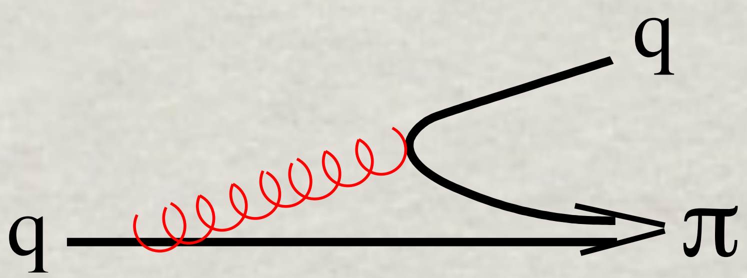
Hadronization in vacuum

E. Berger, PLB 89(1980)241

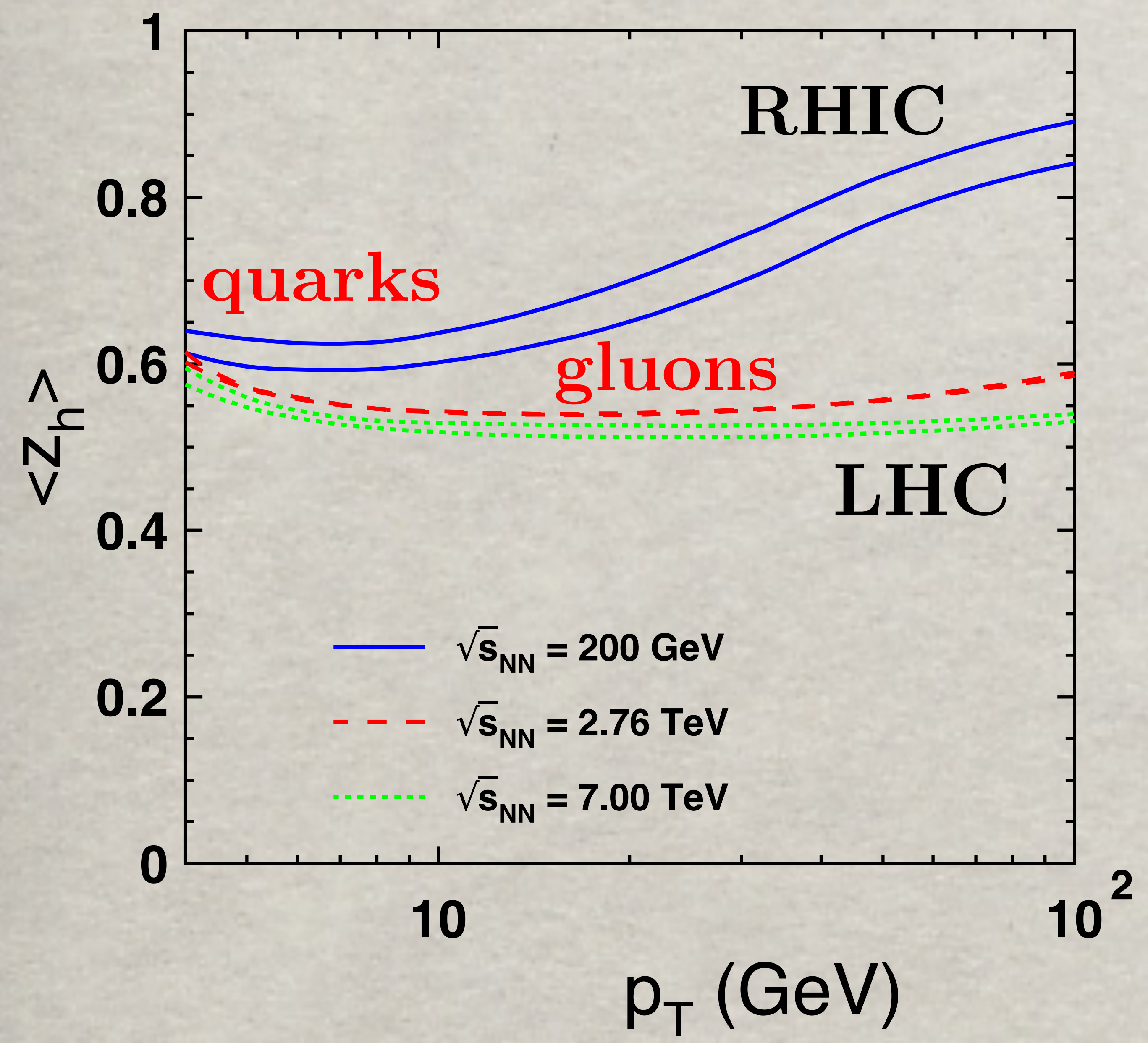
B.K., H.J.Pirner,I.Schmidt,A.Tarasov
PRD 77(2008)054004

B.K., H.J.Pirner,I.Potashnikova,I.Schmidt,
PLB 662(2008)117

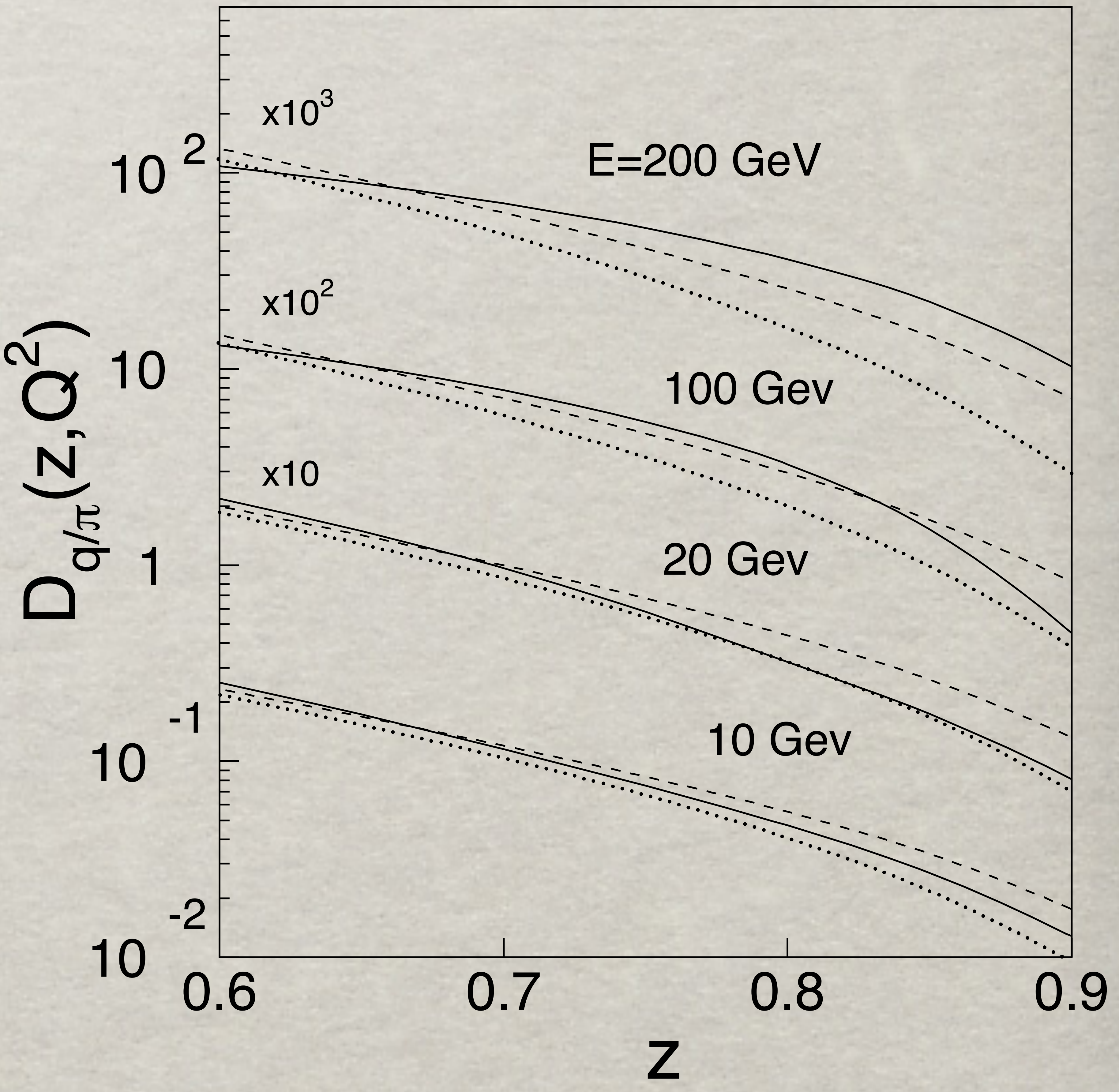
Perturbative hadronization at large z



The mean value $\langle z_h \rangle$



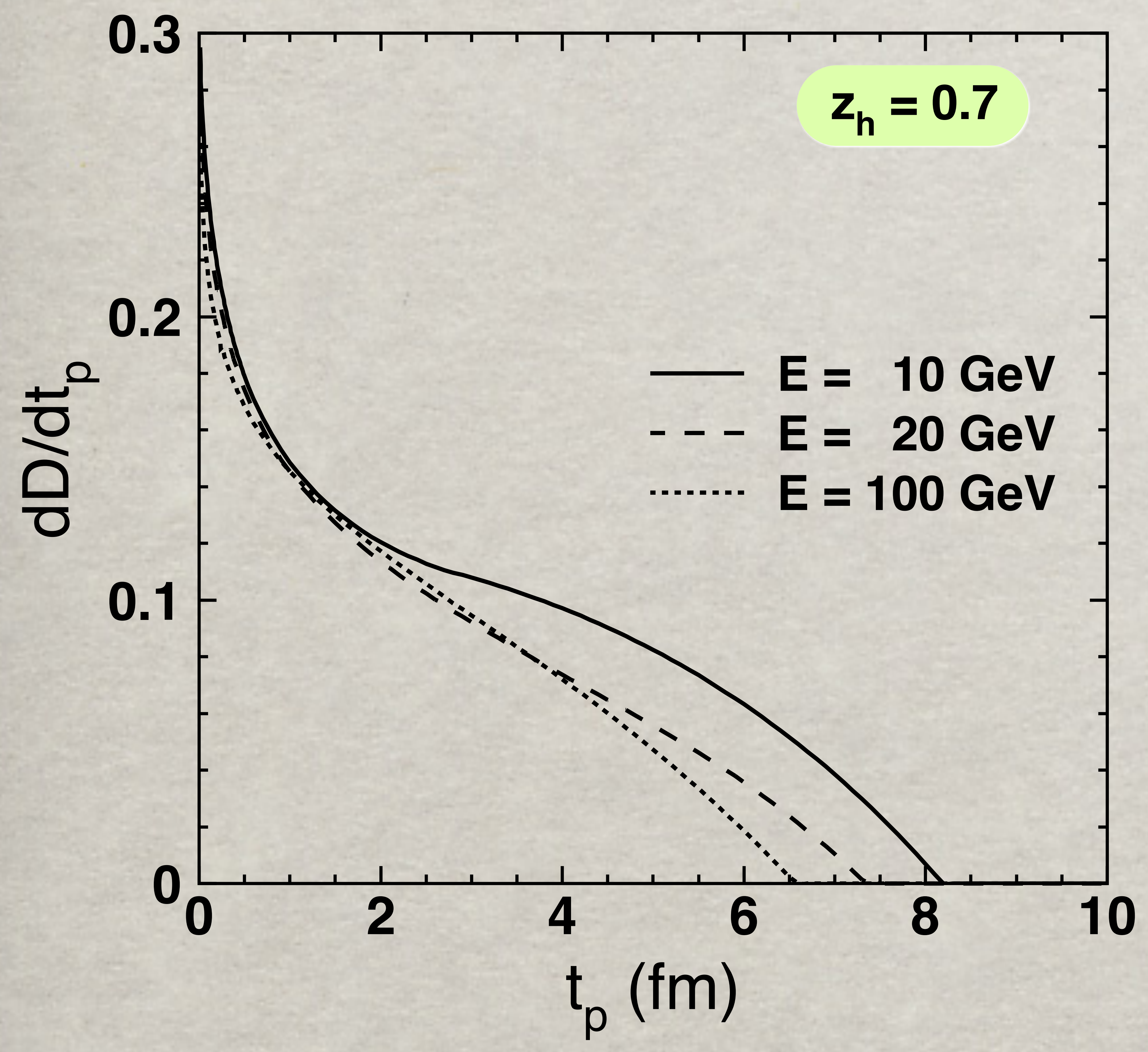
Test with phenomenological FF, KKP and BKK



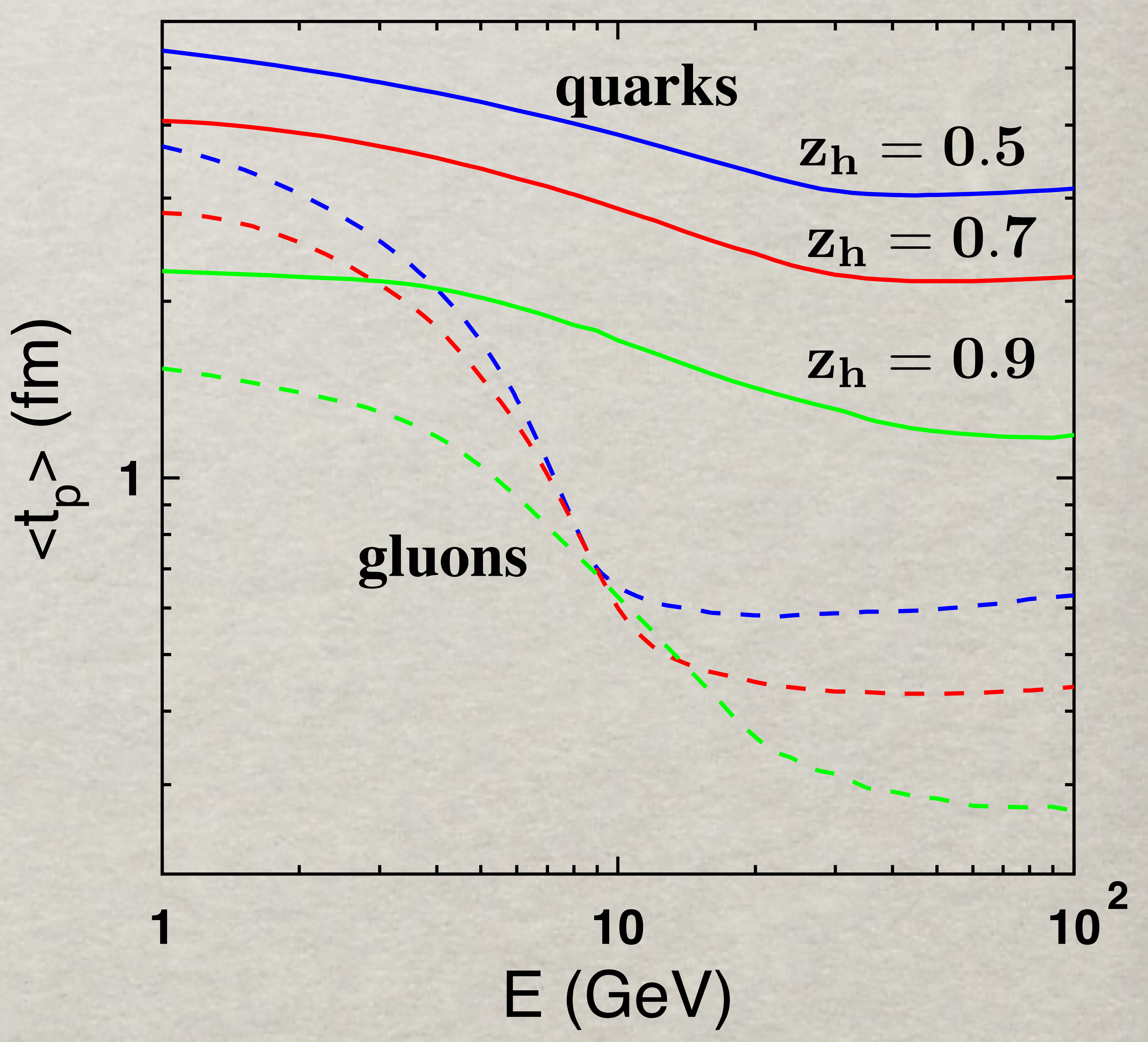
Production time/length

t_p-dependent fragmentation function

$$\frac{\partial D_{\pi/q}(z_h, E)}{\partial t_p}$$



$$\langle t_p(z_h, E)\rangle = \frac{1}{D_{\pi/q}} \int dt_p\, t_p \frac{\partial D_{\pi/q}(z_h, E^2)}{\partial t_p}$$



Production time/length

Why the Lorentz factor does not make l_p longer at large p_T ?

Jet features depend on two parameters, the hard scale Q^2 and jet energy E.

For the leading hadron energy conservation constraint: $l_p \lesssim \frac{E}{dE/dl}(1 - z_h)$

Energy and scale dependences of l_p in **SIDIS:**

(i) Energy dependence at fixed Q^2

$\langle dE/dl \rangle$ is fixed, so $l_p \propto E$

(ii) Scale dependence at fixed energy

$\langle dE/dl \rangle$ rises with Q^2, so $l_p(Q^2)$ is falling

★ For high-pT jets: $E = p_T$ $Q^2 = p_T^2$

Quenching of high-p_T hadrons

BK,,I.Potashnikova, I.Schmidt
Phys.Rev.C83(2011)021901

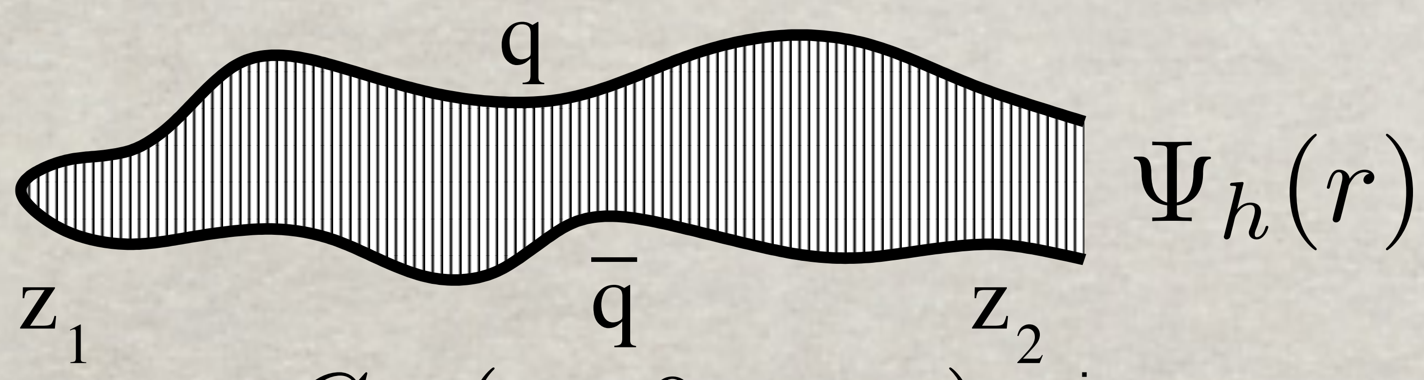
BK, J.Nemchik, I.Potashnikova, I.Schmidt
Phys.Rev. C86(2012)054904

As far as l_p is short, the in-medium attenuation of the produced dipoles becomes the main source of the observed suppression of high-p_T hadrons.

Exact solution: path integrals

BK, B.Zakharov, Phys.Rev. D44(1991)3466

One has to sum up all quark trajectories.

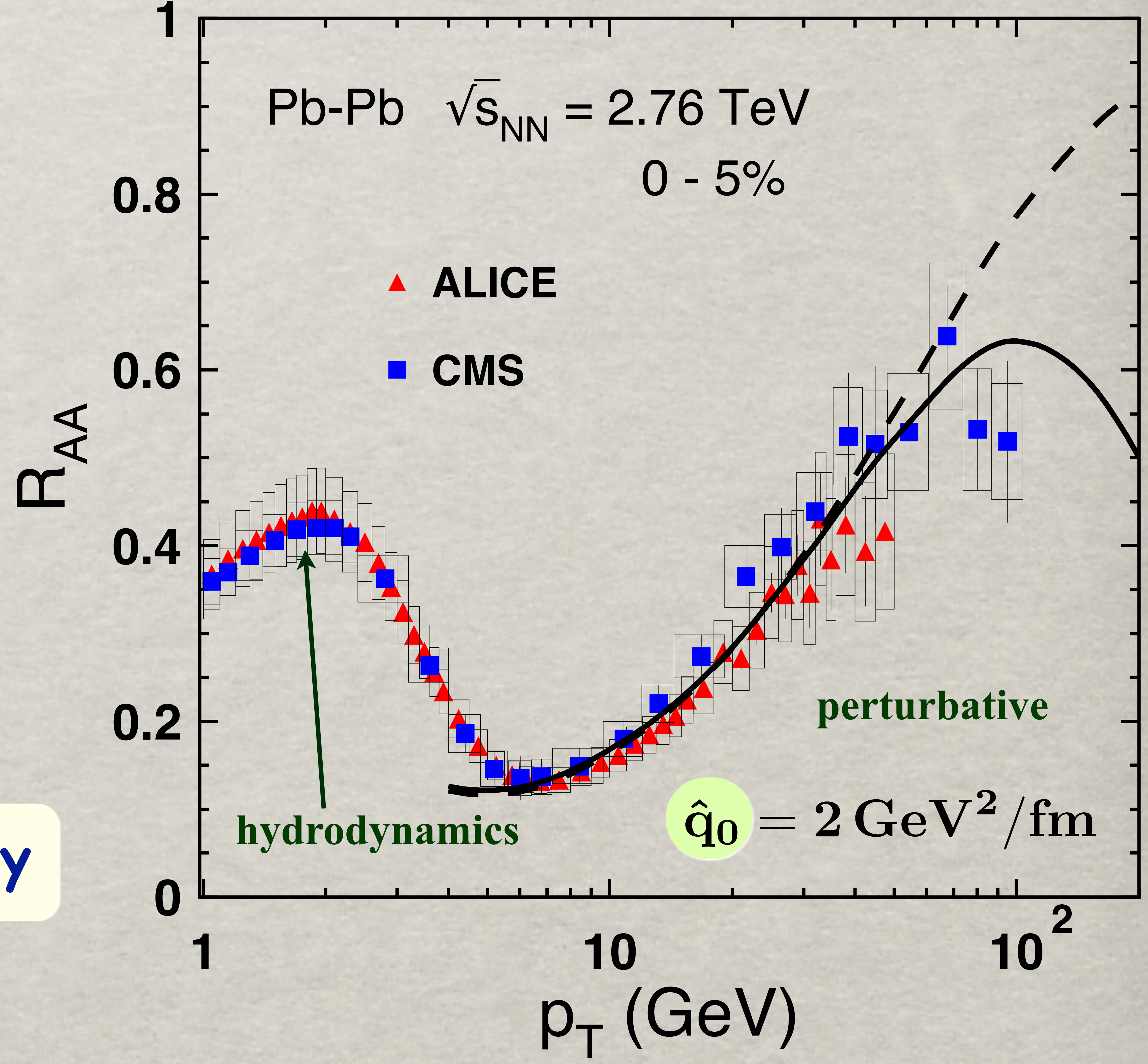


$$G_{\bar{q}q}(z_1,0;z_2,r)$$

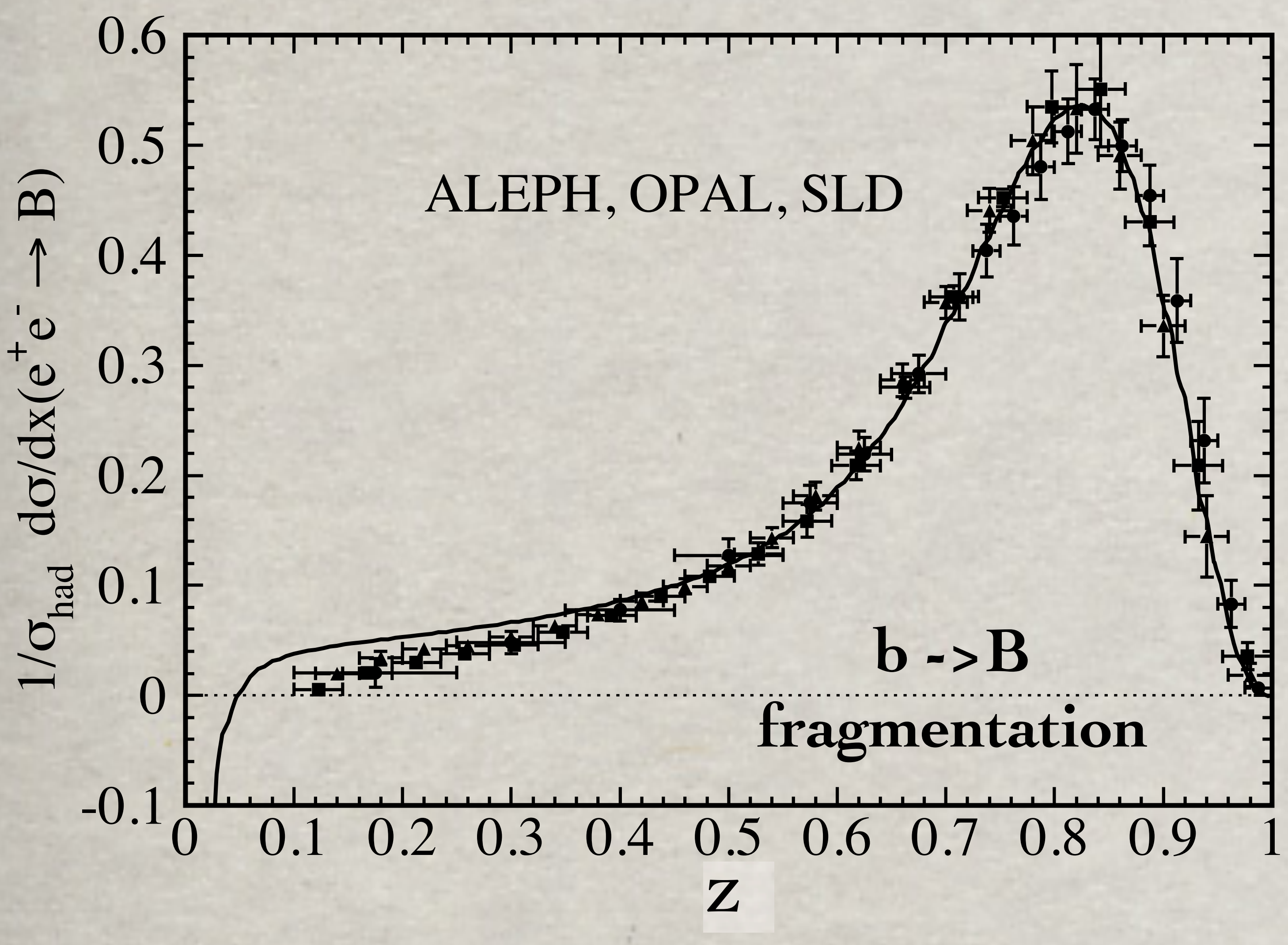
$$\mathrm{Im}\, V_{\bar{q}q}(l,\vec{r}) = -\frac{1}{4}\,\hat{q}(l)\, r^2$$

R_{AA} rises with p_T due to color transparency

$$\hat{q}(l,\vec{b},\vec{\tau}) = \frac{\hat{q}_0\, l_0}{l}\,\frac{n_{part}(\vec{b},\vec{\tau})}{n_{part}(0,0)}$$



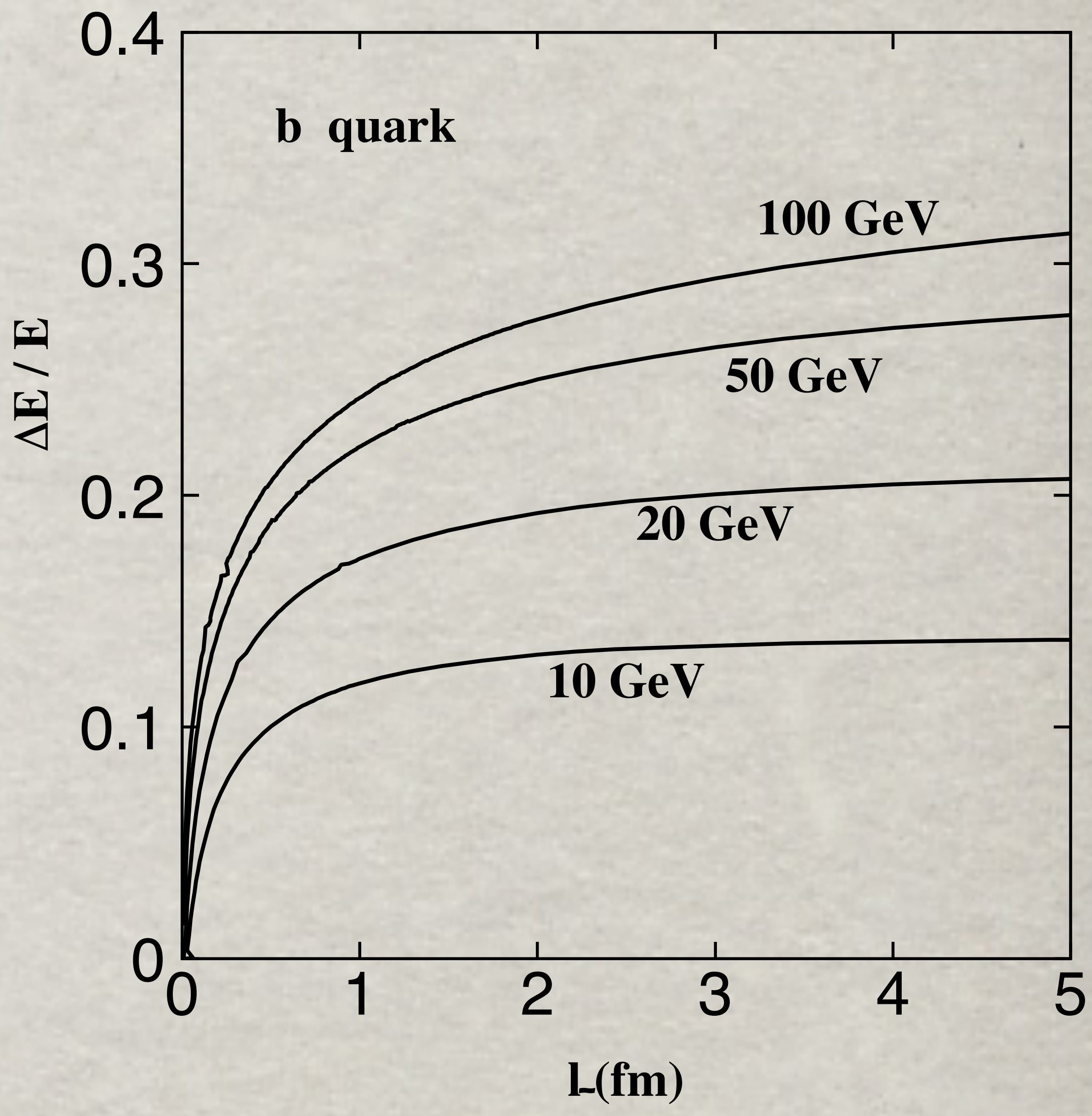
Quenching of high-p_T heavy flavored hadrons

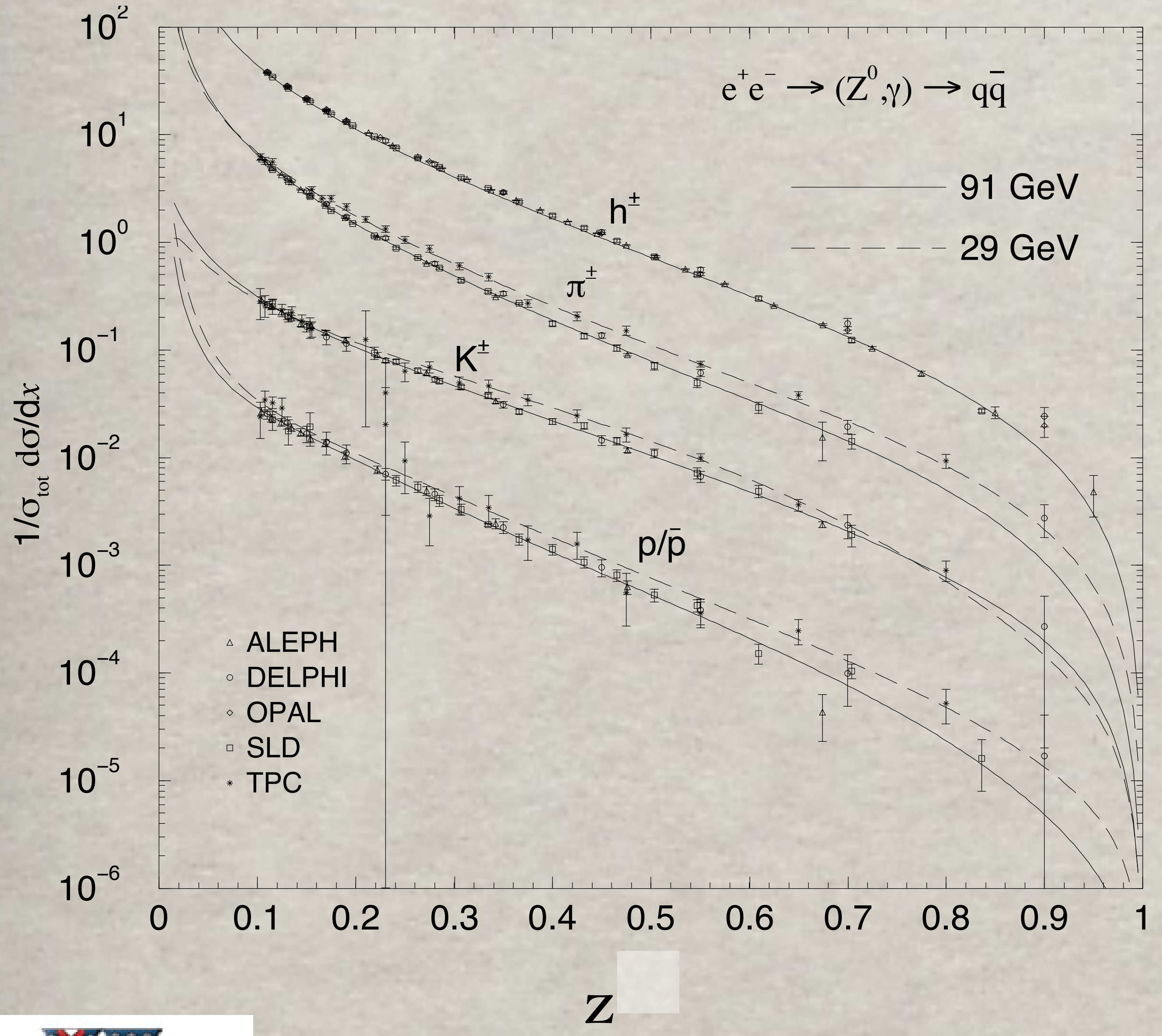


BK, J.Nemchik, I.Potashnikova, I.Schmidt
arXiv:1701.07121

Large z are enhanced in the fragmentation function of heavy flavors, b->B, c->D.

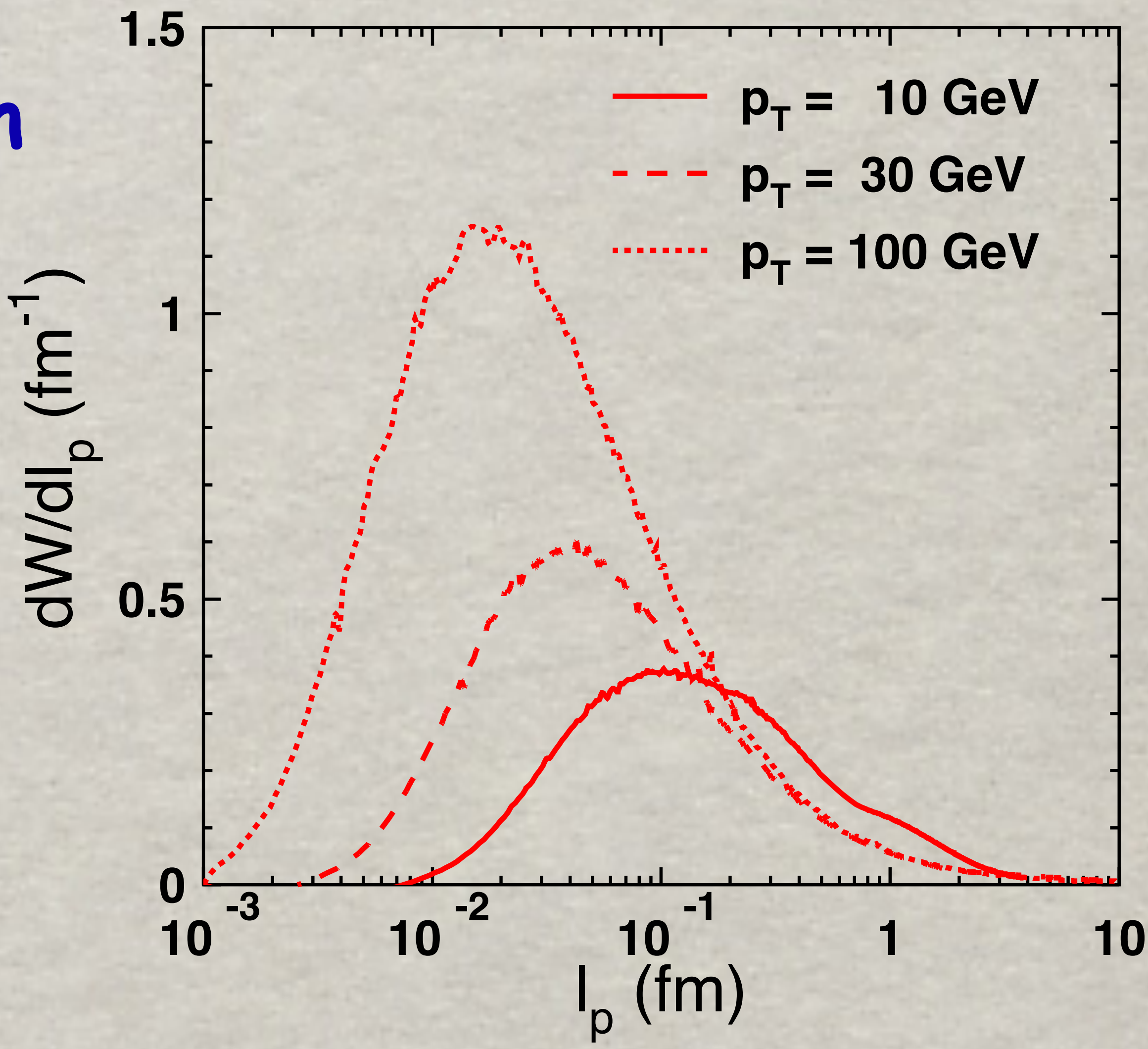
On the contrary, large z fragmentation of light quarks is strongly suppressed.





As far as we are able to calculate **Δz(L)**, we can extract the production length l_p of B-mesons directly from data for $D_{b/B}(z)$

Remarkably, the mean value of l_p shrinks with rising p_T



Attenuation in a hot medium

The light quarks in the B-meson carries a tiny fraction of the momentum,
$\mathbf{x \sim m_q/m_b \approx 5\%}$

Therefore, even if the produced b-q dipole has a small transverse separation, its size expands with a high speed, enhanced by 1/x. The formation time of the B-meson wave function (in the medium rest frame) is very short,

$$\mathbf{t_f^B = \frac{\sqrt{p_T^2 + m_B^2}}{2m_B\omega}}$$

(ω=300MeV)

The mean free path of such a meson in a hot medium is very short

$$\mathbf{\lambda_B \sim \frac{1}{\hat{q}\,\langle r_T^2\rangle}}\,,$$

where $\mathbf{\langle r_T^2\rangle = \frac{8}{3}\,\langle r_{ch}^2\rangle}$

B meson is nearly as big as a pion, $\mathbf{\langle r_{ch}^2\rangle_B = 0.378\,fm^2}$ [Ch.-W. Hwang (2001)]

E.g. at $\mathbf{\hat{q} = 1\,GeV^2/fm}$ $\mathbf{\lambda_B = 0.04\,fm}$, i.e. the b-quark propagates through the hot medium, picking up and losing light quarks. Meanwhile the b-quark keeps losing energy with a rate, enhanced by medium-induced effects. Eventually the detected B-meson is formed and survives in the dilute medium at the surface.

Where the nuclear suppression comes from?

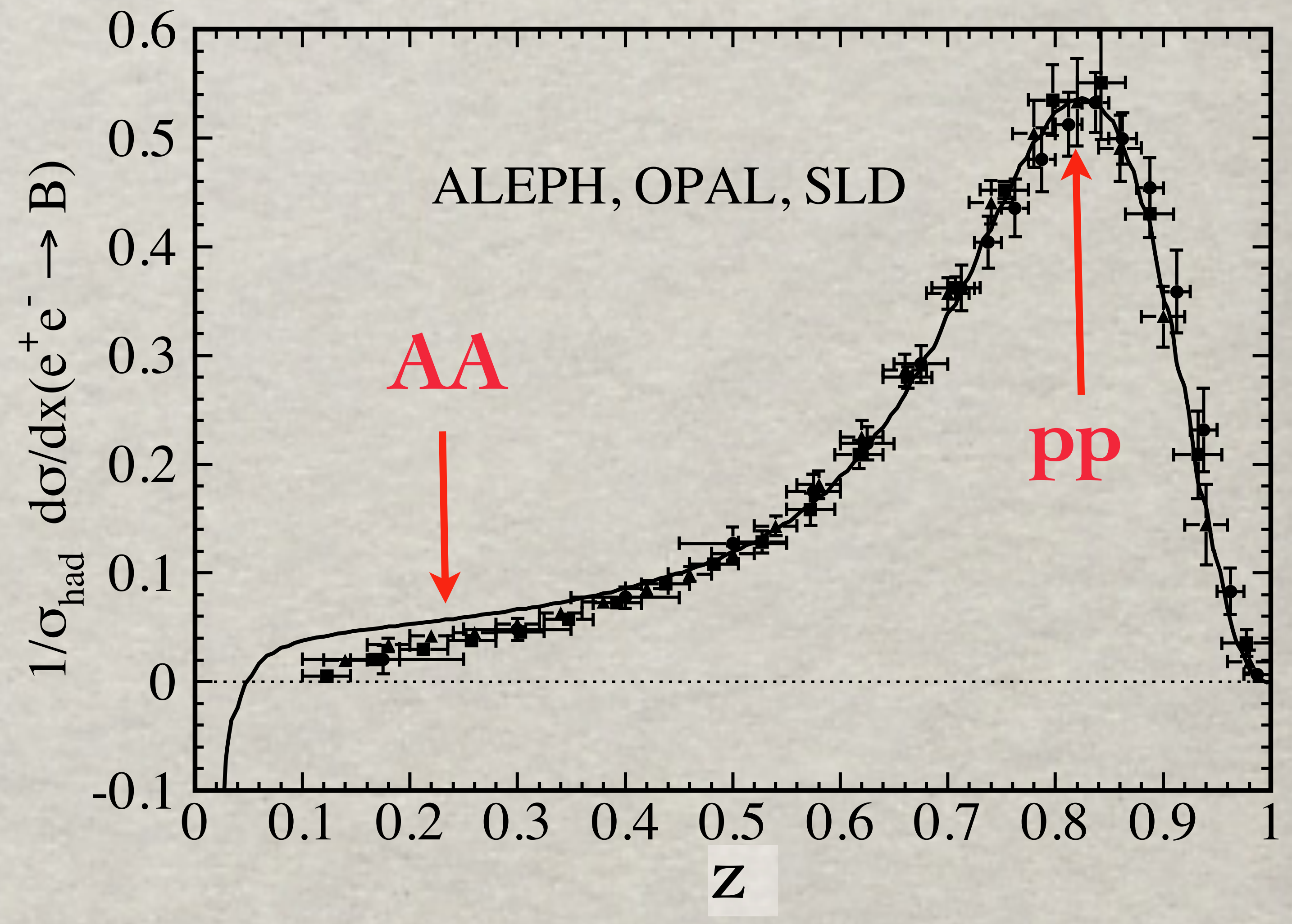
A high-pT b-quark, produced in pp collisions, starts radiating so intensely, that loses 20-30% of its initial energy on a very short distance, then picks-up a light antiquark. The produced colorless B-meson stops radiating and retains its fractional momentum z.

If, however, the b-quark is produced in a dense environment, it has to propagate a long distance up to the medium surface, where the final B-meson can survive.
All this long path the quark keeps losing energy and eventually produces a B-meson with reduced fractional momentum z, which is suppressed by the fragmentation function.

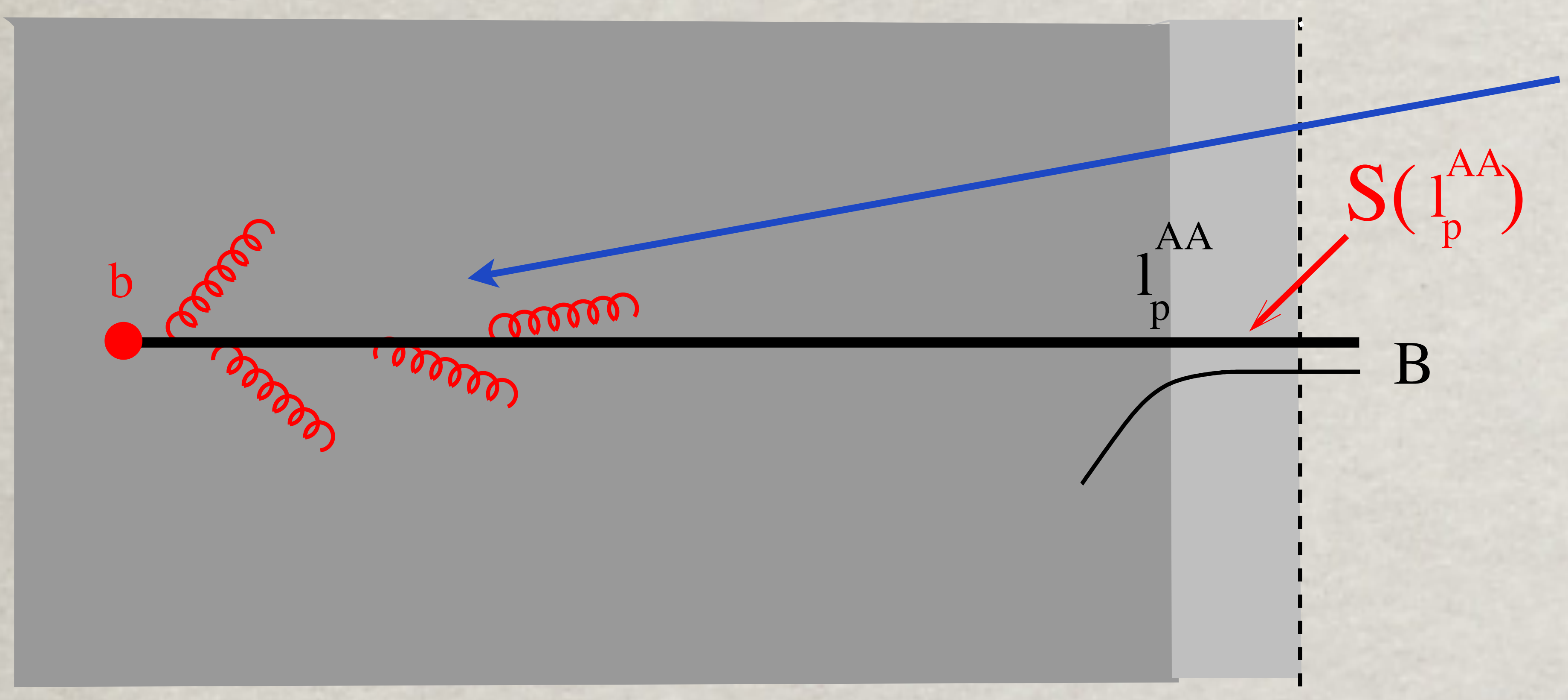
$$\frac{d\sigma(pp\to BX)}{d^2p_T} = \int d^2p_T^b\, \frac{d\sigma(pp\to bX)}{d^2p_+^b}\, \frac{1}{z} D_{b/B}(z),$$

$$\frac{d\sigma(AA\to BX)}{d^2p_T} = \int d^2p_T^b\, \frac{d\sigma(pp\to bX)}{d^2p_T^b}\, \frac{1}{z_{AA}} D_{b/B}(z_{AA})\, S(l_p^{AA})$$

$$S(l_p^{AA}) = \exp\left[-\int\limits_{l_p^{AA}}^{\infty} \frac{dl}{\lambda_B(l)}\right]$$



Interplay between energy loss & absorption



Energy loss in the medium: radiational vacuum and induced, collisional,string.

In vacuum: gluon radiation plus string

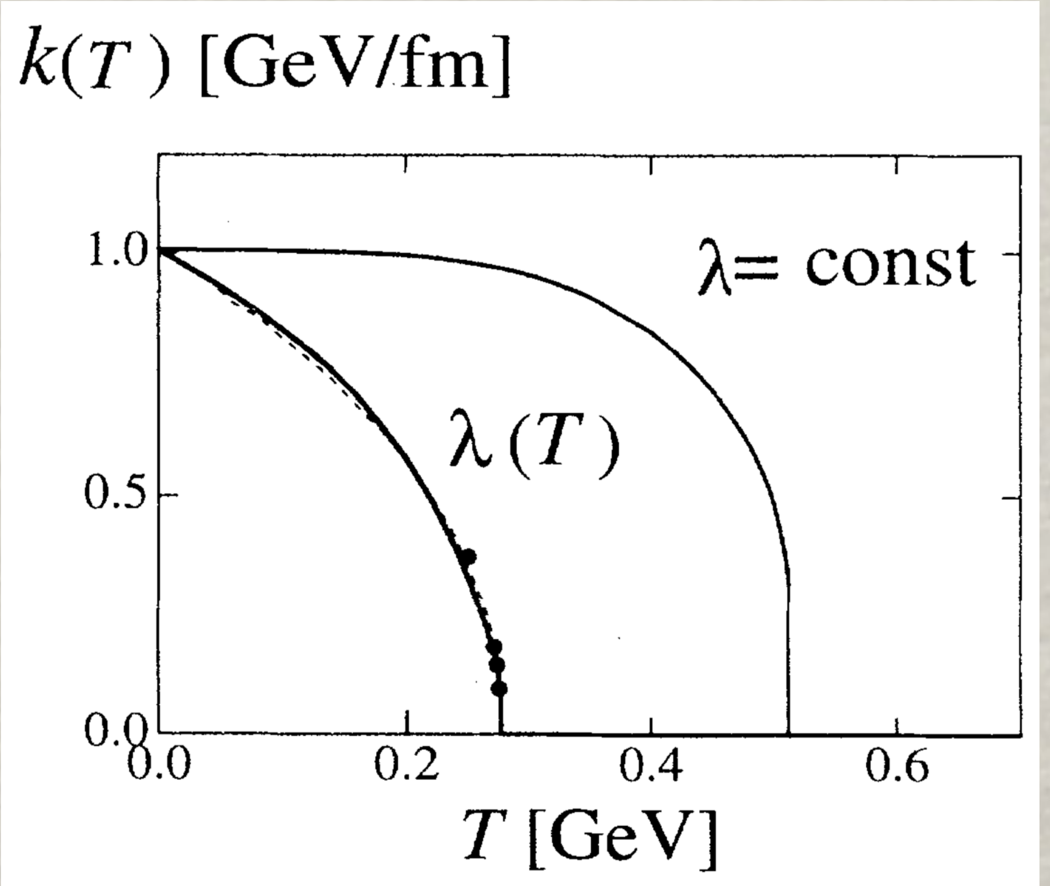
$$\mathbf{dE_{string}/dl = -\kappa \approx -1\,GeV/fm}$$

String tension is falling with temperature:

$$\kappa(\mathbf{T}) = \kappa\,(\mathbf{1 - T/T_c})^{\mathbf{1/3}}$$

While in vacuum a B-meson is produced on a very short length $l_p \ll 1\,\mathrm{fm}$, in a hot medium strong absorption pushes the production point to the dilute medium surface. However, energy loss on a longer $\mathbf{l_p^{AA}} \gg \mathbf{l_p}$ causes a large shift down to small z, suppressed by D(z).

Thus, the two sources of suppression act in opposite directions



H.Ichie, H.Suganuma & H.Toki(1996)

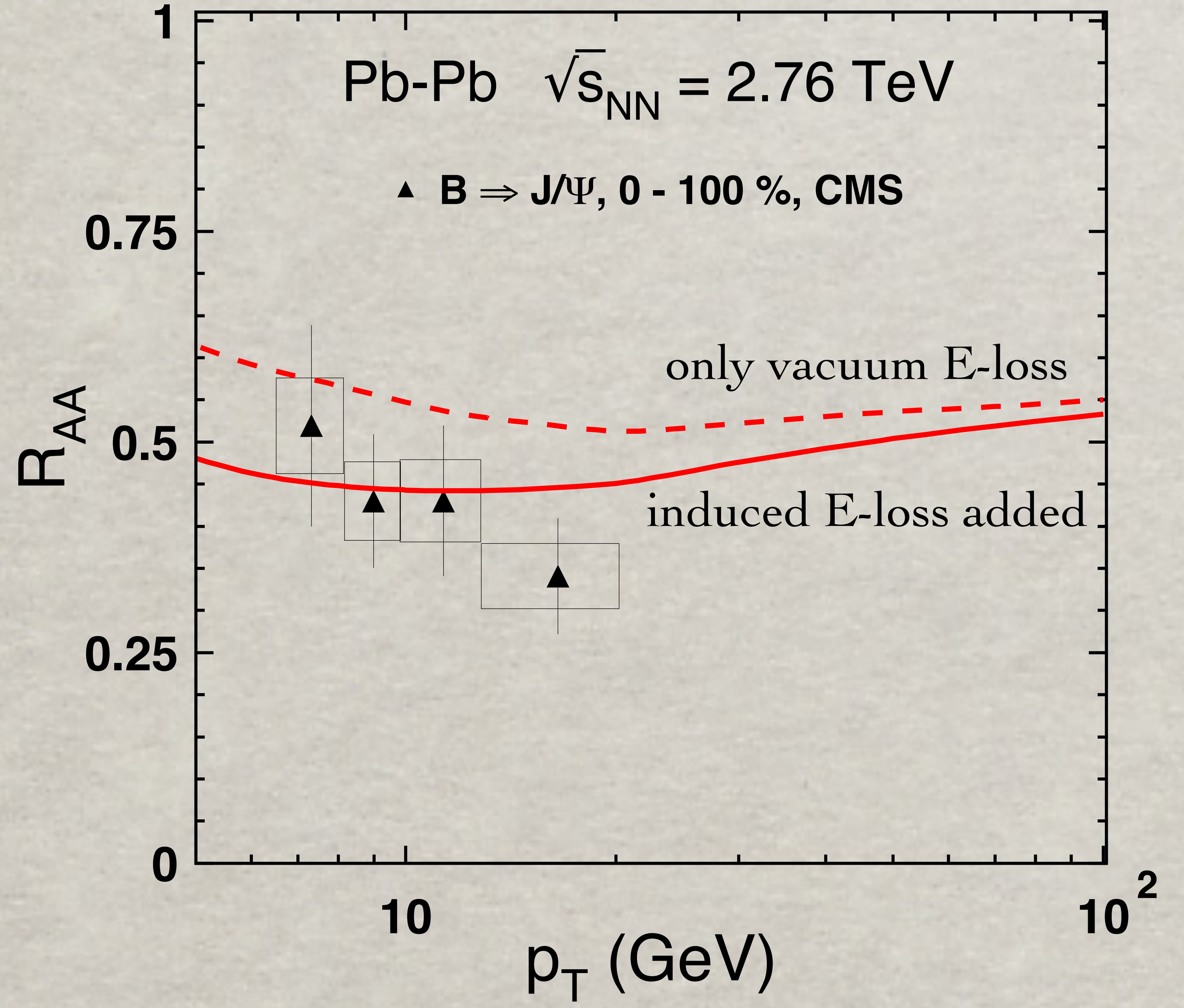
Results

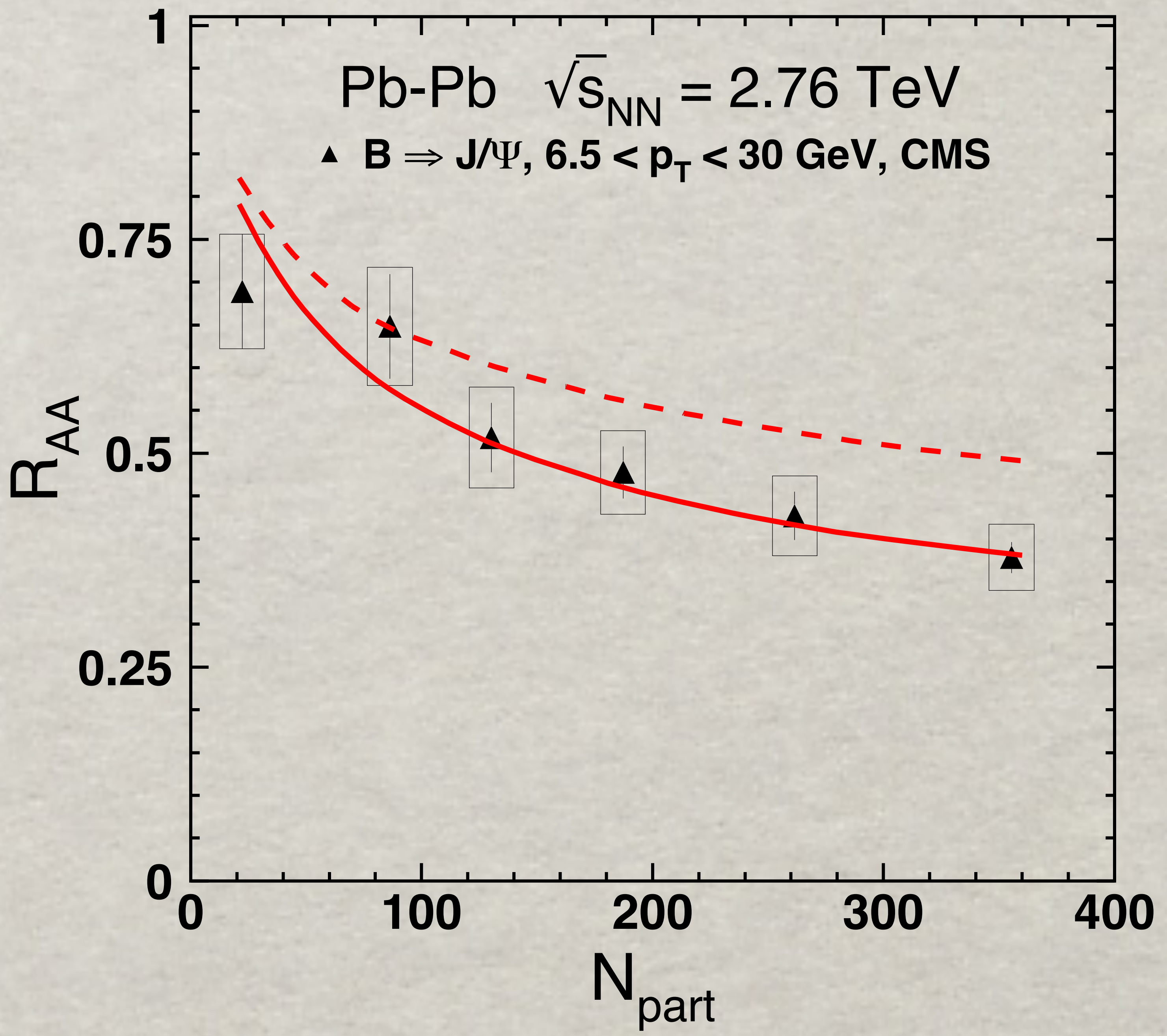
$q_0 = 2\,\mathrm{GeV^2/fm}$
$(1.6\,\mathrm{GeV^2/fm})$

fixed by quenching of pions
at LHC (RHIC)

J.Nemchik,I.Potashnikova,I.Schmidt & B.K.
PRC 86(2012)054904

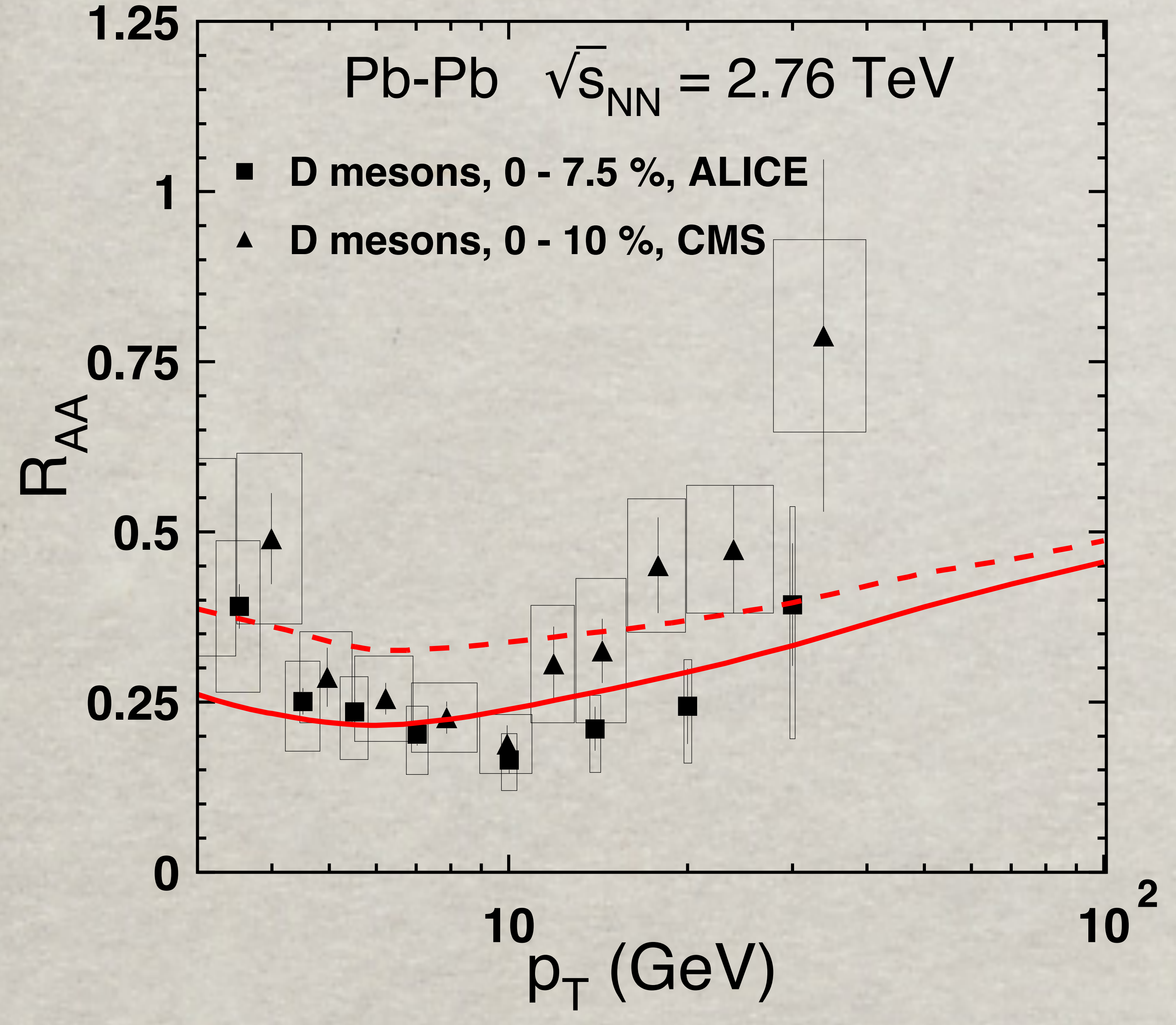
Different sources of time-dependent energy loss should be added up. Medium-induced energy loss is much smaller than the vacuum one, and should not produce a dramatic effect. They are particularly small for heavy flavors (Yu.Dokshitzer & D.Kharzeev (2001)

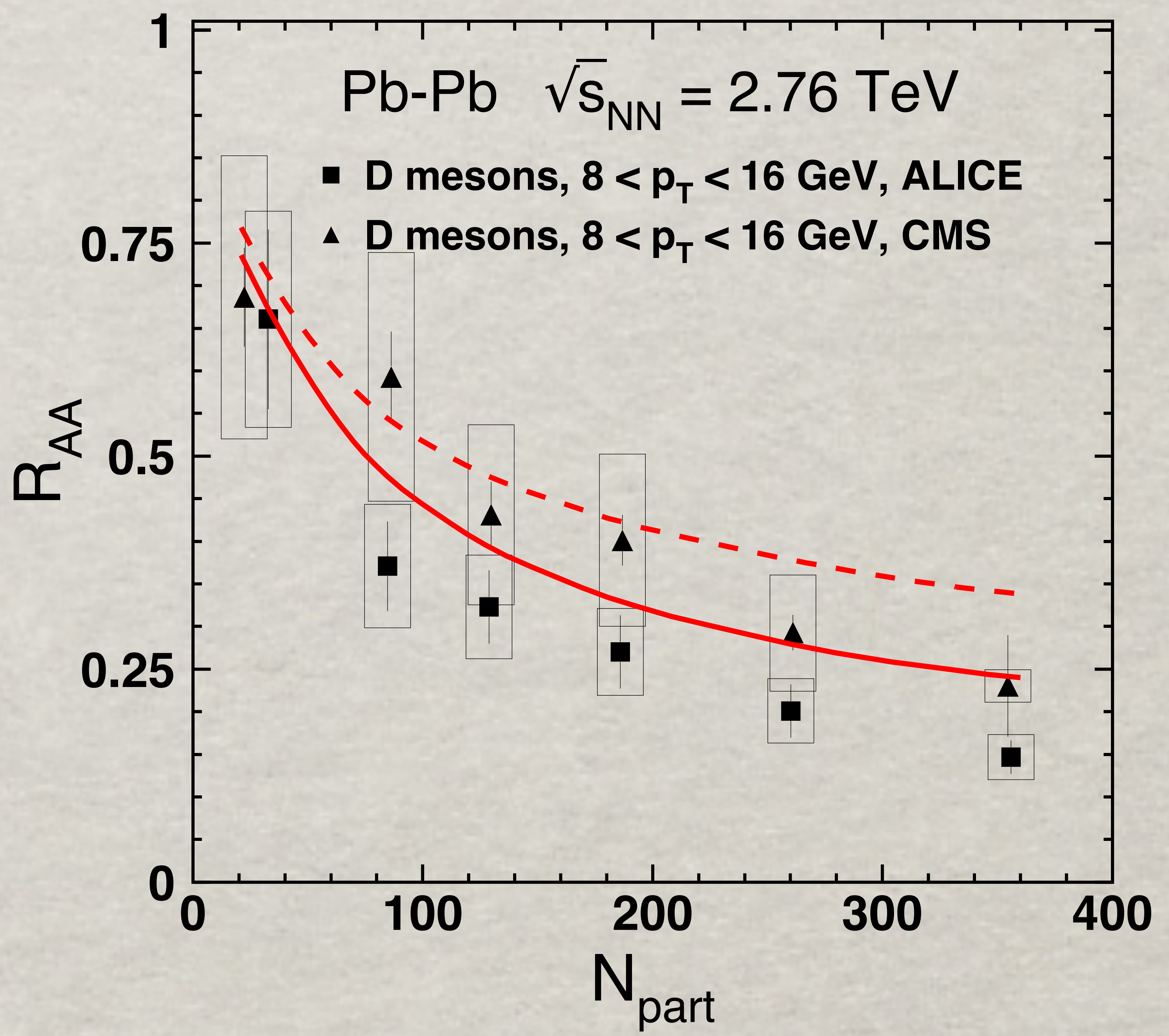




Results

c-quarks radiate in vacuum much more energy than b-quarks, while the effects of absorption of c-q and b-q dipoles in the medium are similar. Therefore D-mesons are suppressed in AA collisions more than B-mesons.





J/Ψ in a hot medium: melting or absorption?

No signal of J/Psi melting has been observed so far

The main flaws of the melting scenario

- Once a bound level disappears,
  the charmonium dissociates and is terminated.
- Screening of the potential is the only reason for charmonium disintegration in a dense medium.

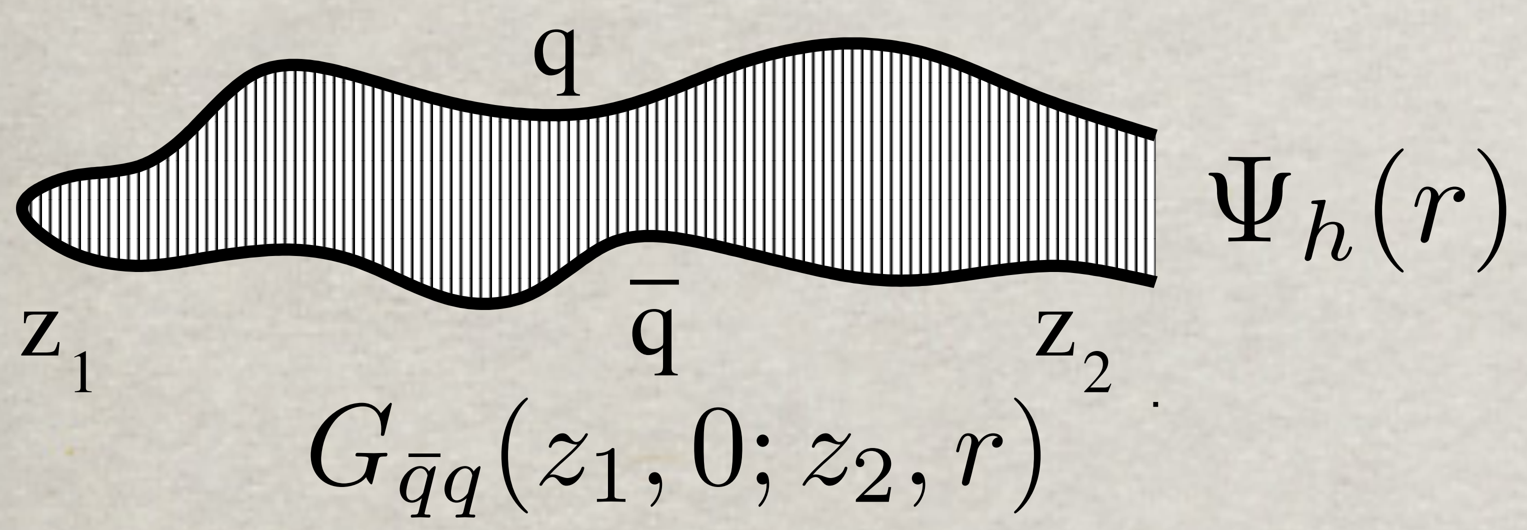
Most of charmonia at RHIC-LHC have large $\langle p_T^2\rangle \approx 4-16\,\mathrm{GeV}^2$, so they move with relativistic velocities and the Schrödinger equation and lattice results cannot be applied.

Charmonium propagation through a medium

Path integral technique

B. Zakharov & B.K. PRD44(1991)3466

$$\left[i\frac{d}{dz}-\frac{m_c^2-\Delta_{r_\perp}}{E_\Psi/2}-V_{\bar{q}q}(z,r_\perp)\right]G_{\bar{q}q}(z_1,r_{\perp 1};z,r_\perp)=0$$



$G_{\bar{q}q}(z_1,0;z_2,r)$

The Green function $G_{\bar{q}q}(z_1,r_1;l_2,r_2)$ describes propagation of the dipole.

$\mathrm{Re}V_{\bar{q}q}(z,r)$ corresponds to the binding potential, which is known only in the rest frame of the dipole.

The imaginary part of the light-cone potential describes color-exchange interaction of the dipole with the surrounding medium, missed in previous consideractions.

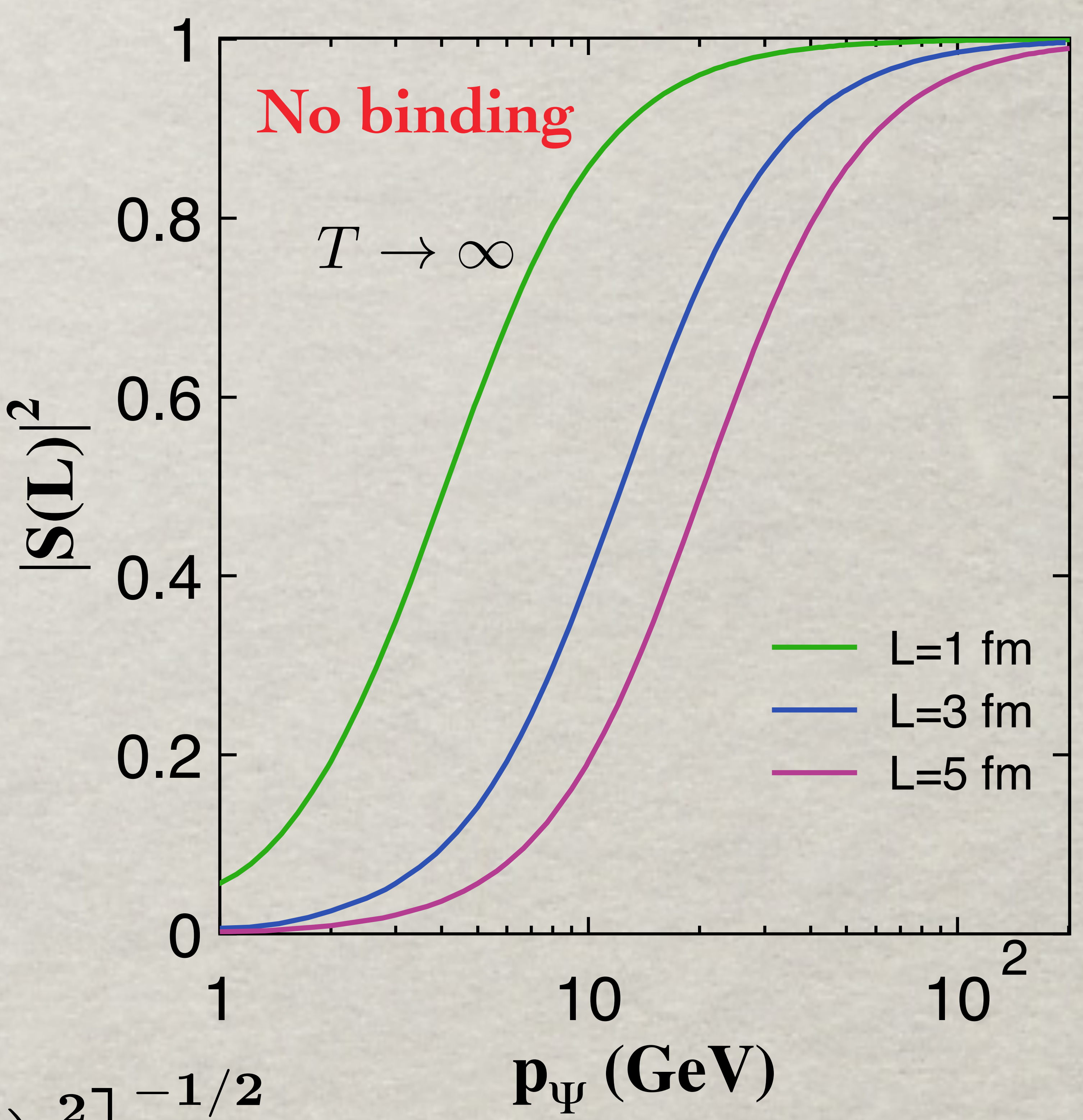
$$\mathrm{Im}V_{\bar{q}q}(z,r_\perp)=-\frac{1}{4}\,\hat{q}(z)\,r_\perp^2$$

Transport coefficient $\hat{q}\approx 3.6\,T^3$ is to be adjusted to data.

Survival of an unbound $c\bar{c}$

Even in the extreme case of lacking any potential between c and $\bar{c}$ ($T \to \infty$), still the J/Ψ can survive.

I.Potashnikova, I.Schmidt, M.Siddikov & B.K.
PRC91 (2015) 2, 024911



Path-integral description of J/Ψ attenuation

$$|S(L)|^2 = \frac{m_c^2\, p_\psi}{16\pi^2 L}\left(1+\frac{\omega}{2m_c}\right)\frac{8\pi^2}{m_c^2}\left[\omega^2 m_c^2 + \frac{p_\psi^2}{4L^2}\left(1+\frac{\omega}{2m_c}\right)^2\right]^{-1/2}$$

$$\omega = (M_{\psi'} - M_\psi)/2$$

Lorentz boosted Schrödinger equation

E.Levin, I.Schmidt, M.Siddikov & B.K. arXiv:1501.01607, PRD(2015)

The light cone fractional momentum distribution of quarks in a charmonium sharply peaks around x=1/2. With a realistic potential

$$\langle\lambda^2\rangle \equiv \left\langle\left(\mathbf{x}-\frac{1}{2}\right)^2\right\rangle = \frac{\langle \mathbf{p}_L^2\rangle}{4\mathbf{m}_c^2} = \frac{1}{4}\langle {v_L}^2\rangle \approx 0.017$$

Introducing a variable ζ Fourier conjugate to λ,

$$\tilde{\Psi}_{\bar{c}c}(\zeta,\mathbf{r}_\perp) = \int\limits_0^1 \frac{\mathbf{dx}}{2\pi}\,\Psi_{\bar{c}c}(\mathbf{x},\mathbf{r}_\perp)\,\mathbf{e}^{2\mathbf{i}\mathbf{m}_c\zeta(\mathbf{x}-1/2)}$$

and making use of smallness of λ and of the binding energy, we arrive at the boost-invariant Schrödinger equation for the Green function

$$\left[\frac{\partial}{\partial \mathbf{z}^+} + \frac{\Delta_\perp + (\partial/\partial\zeta)^2 - \mathbf{m}_c^2}{\mathbf{p}_\psi^+/2} - \mathbf{U}\left(\mathbf{r}_\perp,\zeta\right)\right]\mathbf{G}(\mathbf{z}^+,\zeta,\mathbf{r}_\perp;\mathbf{z}_1^+,\zeta_1,\mathbf{r}_{1\perp}) = 0$$

Lorentz boosted binding potential

Debye screening of the potential for J/Ψ at rest relative to the medium can be modeled,

$$\mathbf{V_{\bar{c}c}}\left(\mathbf{r}=\sqrt{\mathbf{r}_\perp^2+\zeta^2}\right)=\frac{\sigma}{\mu(\mathbf{T})}\left(\mathbf{1}-\mathbf{e}^{-\mu(\mathbf{T})\mathbf{r}}\right)-\frac{\alpha}{\mathbf{r}}\mathbf{e}^{-\mu(\mathbf{T})\mathbf{r}}$$

$$\mu(\mathbf{T})=\mathbf{g}(\mathbf{T})\mathbf{T}\sqrt{\mathbf{1}+\frac{\mathbf{N_f}}{\mathbf{6}}},\qquad \mathbf{g}^2(\mathbf{T})=\frac{\mathbf{24}\pi^2}{\mathbf{33}\ln\left(\mathbf{19T}/\Lambda_{\overline{\mathrm{MS}}}\right)}$$

F. Karsch, M. Mehr and H. Satz, Z.Phys.C37(1988)617

However, most of J/Ψs are fast moving, at the LHC $\langle \mathbf{p}_\psi^2\rangle=\langle \mathbf{p}_\mathbf{T}^2\rangle\approx 10\,\mathrm{GeV}^2$

$\mathbf{V}(\mathbf{r})$ is not Lorentz invariant **r** is 3-dimensional

The procedure of Lorentz boosting of the Schrödinger equation was developed recently in E.Levin, I.Schmidt, M.Siddikov & B.K. arXiv:1501.01607, PRD2015

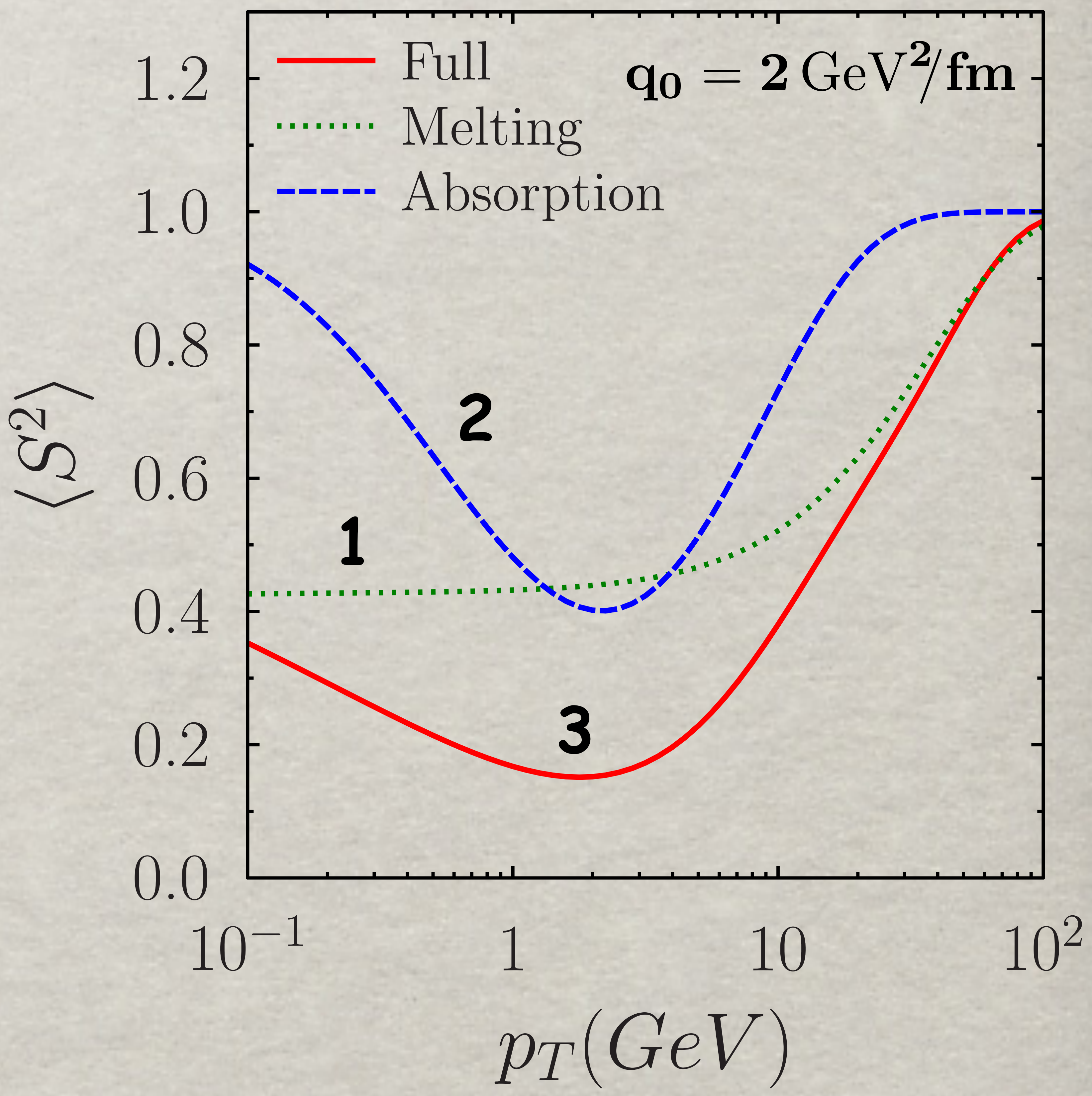
Results for J/Ψ

Survival probability

$$\mathbf{S^2_{J/\Psi}(b) = \int\limits_0^{2\pi} \frac{d\phi}{2\pi} \int \frac{d^2s\, T_A(\tilde{s})T_B(\tilde{b}-\tilde{s})}{T_{AB}(b)}}$$

$$\mathbf{\times \left| \frac{\int d^2r_1 d^2r_2 d\zeta_1 d\zeta_2 \Psi_f^\dagger(\zeta_2,\tilde{r}_2) G(\infty,\zeta_2,\tilde{r}_2; l_0,\zeta_1,\tilde{r}_1)\Psi_{in}(\zeta_1,\tilde{r}_1)}{\int d^2r d\zeta\, \Psi_f^\dagger(\zeta,\tilde{r})\, \Psi_{in}(\zeta,\tilde{r})} \right|^2}$$

1. Net melting: $\mathbf{ReU \neq 0; ImU = 0.}$
2. Net absorption: $\mathbf{ReU = 0; ImU \neq 0.}$
3. Total suppression: $\mathbf{ReU \neq 0; ImU \neq 0.}$

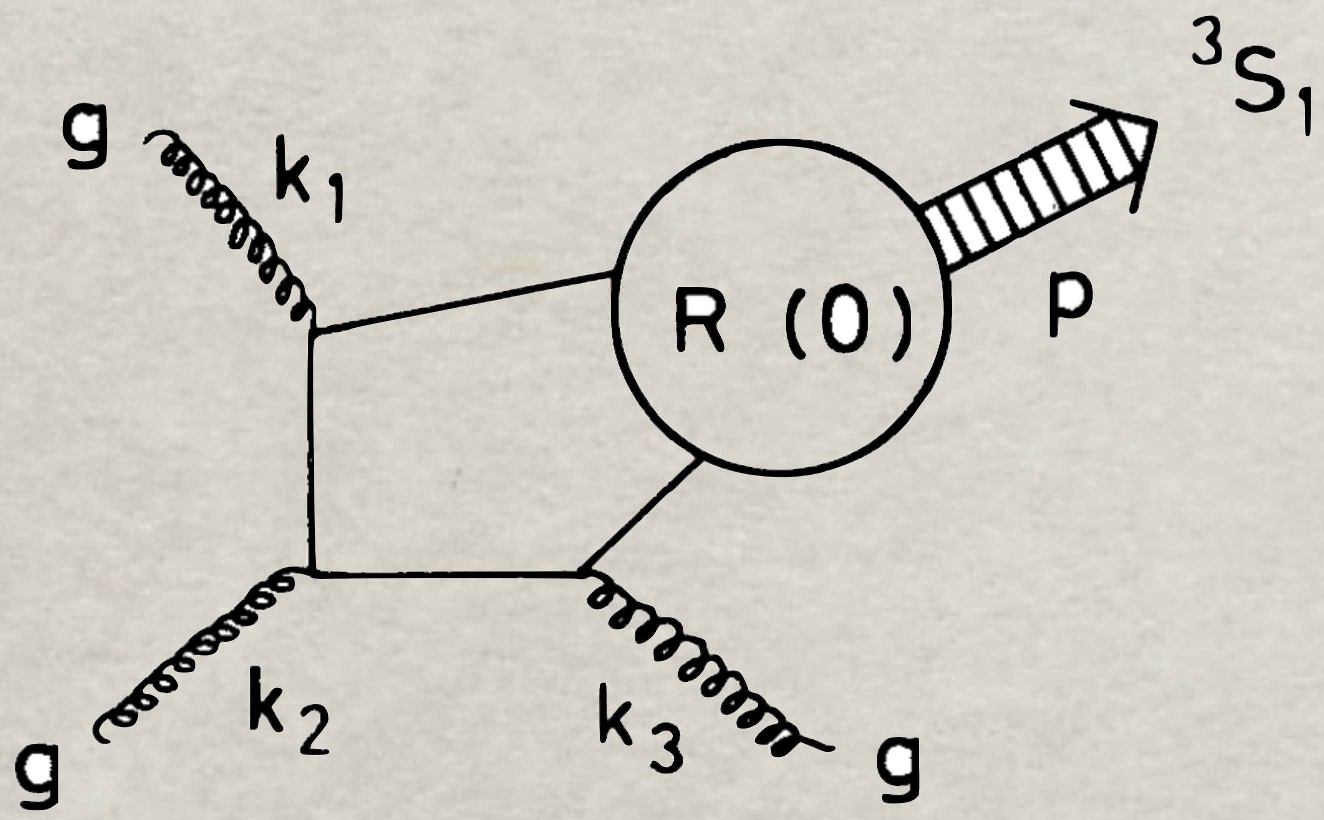


Calculations are done for central Pb-Pb collisions with realistic nuclear density. No ISI effects are added.

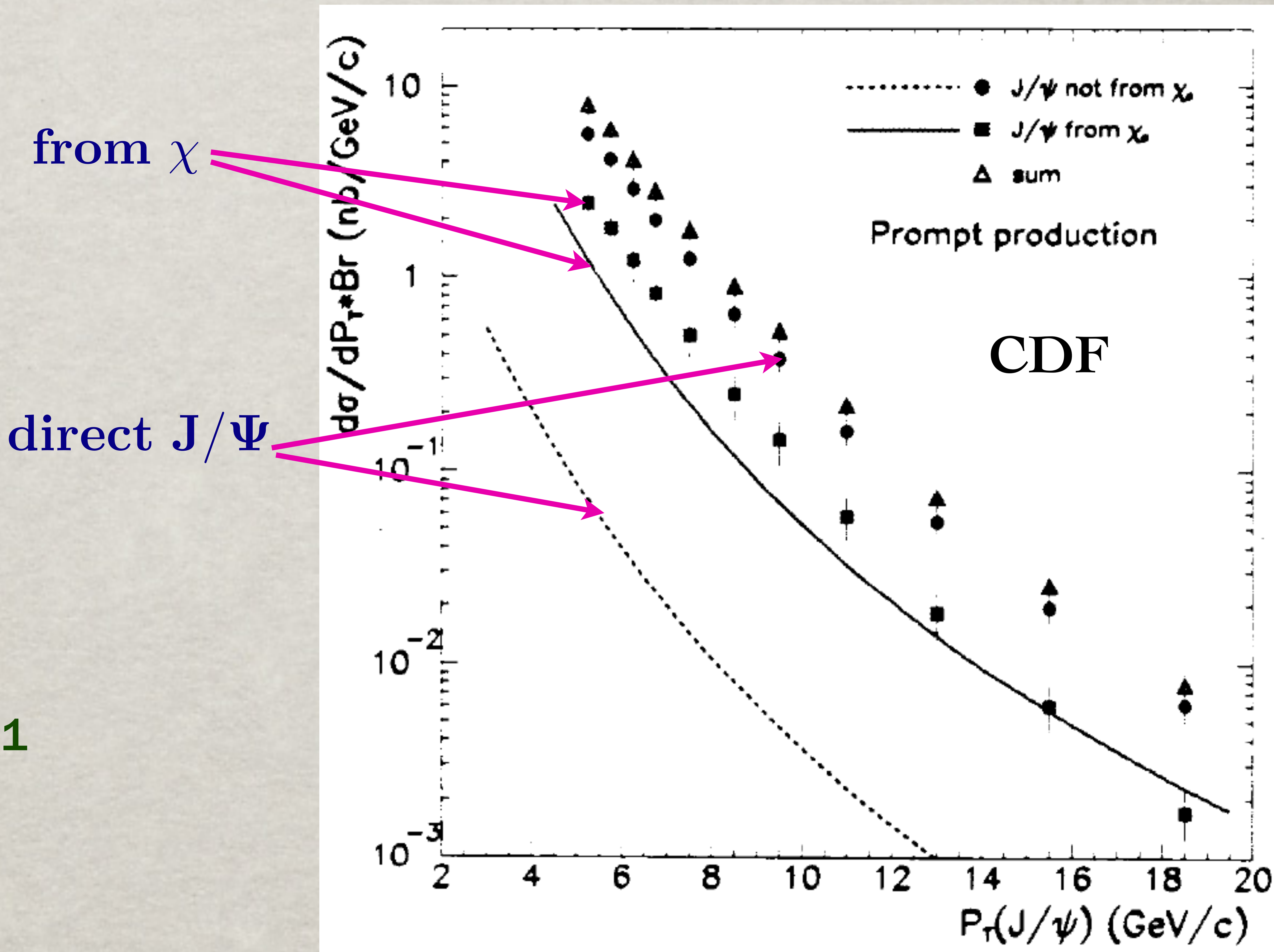
I.Potashnikova, I.Schmidt, M.Siddikov & B.K. PRC91 (2015) 2, 024911

Charmonium with high p_T

Color singlet mechanism



E.Berger & D.Jones PRD 23(1981)1521
R.Baier & R.Ruckl PLB102(1981)364



F. Abe et al., PRL 79(1997)572

Charmonium with high p_T

Color-singlet model fails, because the strong kick from the target breaks-up the c-cbar pair.

Color-octet model: the projectile gluon can easily accept a strong kick, and then fragment to J/ψ via production of a color-octet c-cbar. Fragmentation is assumed to happen on a long time scale, by a soft mechanism, which cannot be calculated, but fitted.

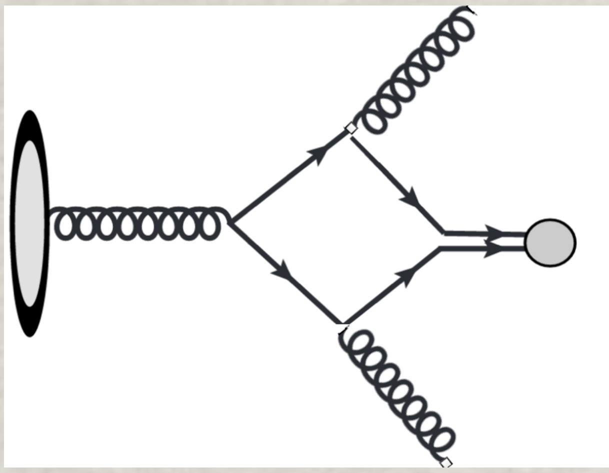
However, we demonstrated that energy conservation restricts the time of color neutralization and the colorless c-cbar dipole is produced promptly, in the perturbative regime. Therefore this contribution can be evaluated in pQCD.

Gluon fragmentation

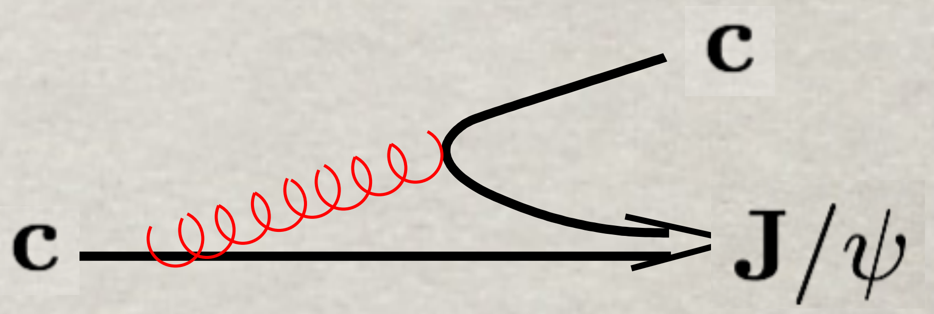
S. Baranov & B.K. 2017

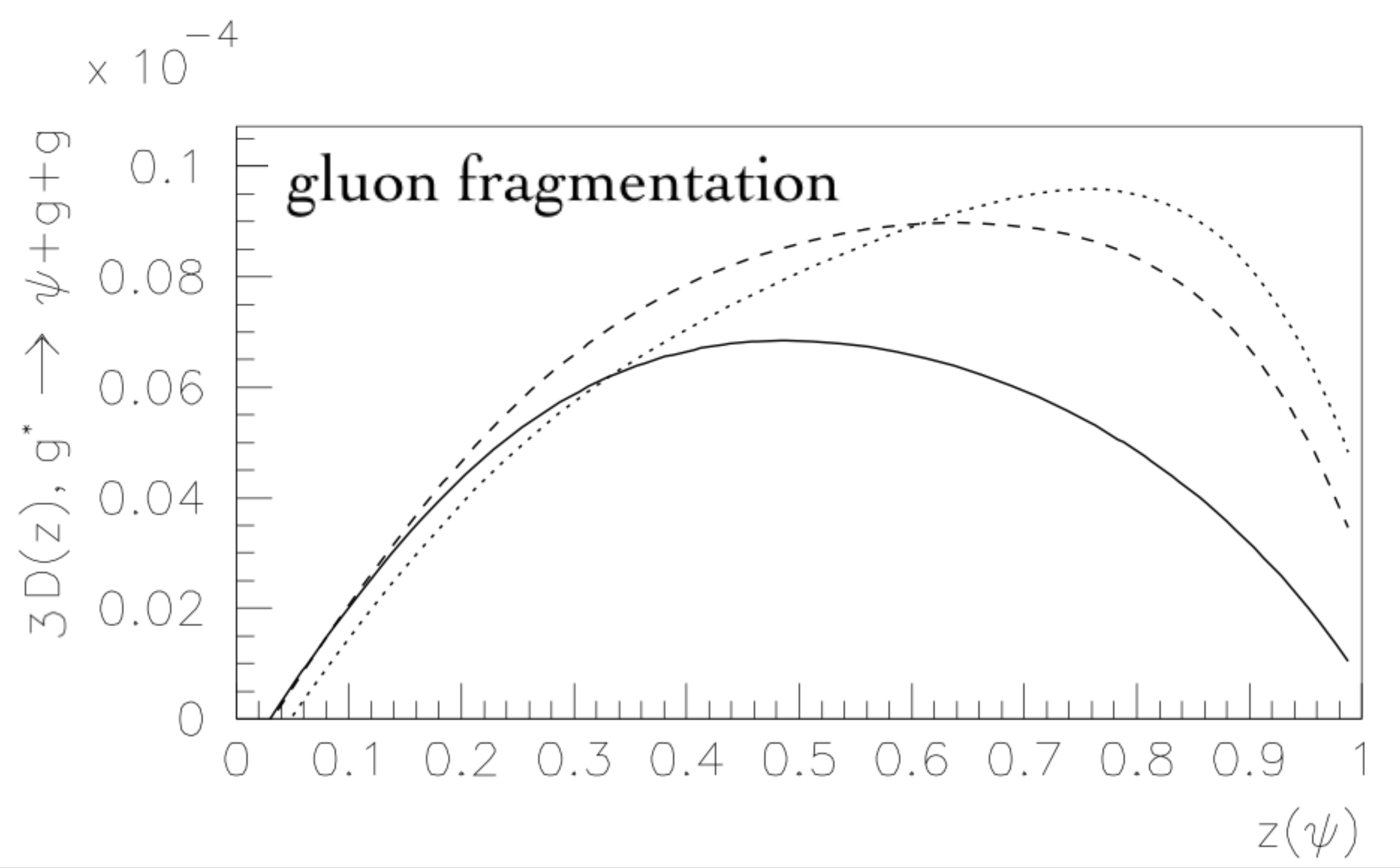
Perturbative fragmentation

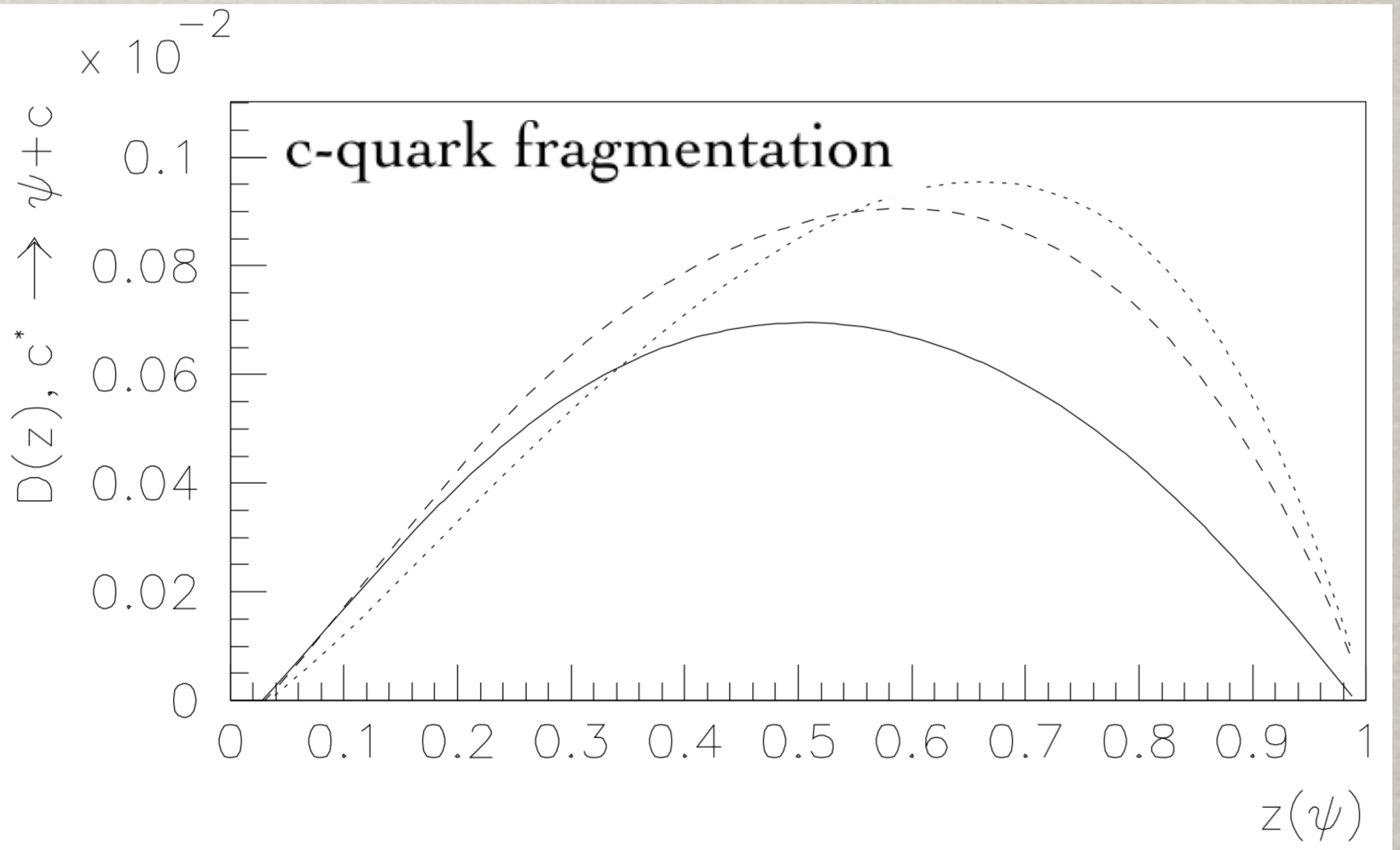
$$\mathbf{g} \rightarrow \mathbf{J}/\psi + \mathbf{2g}$$



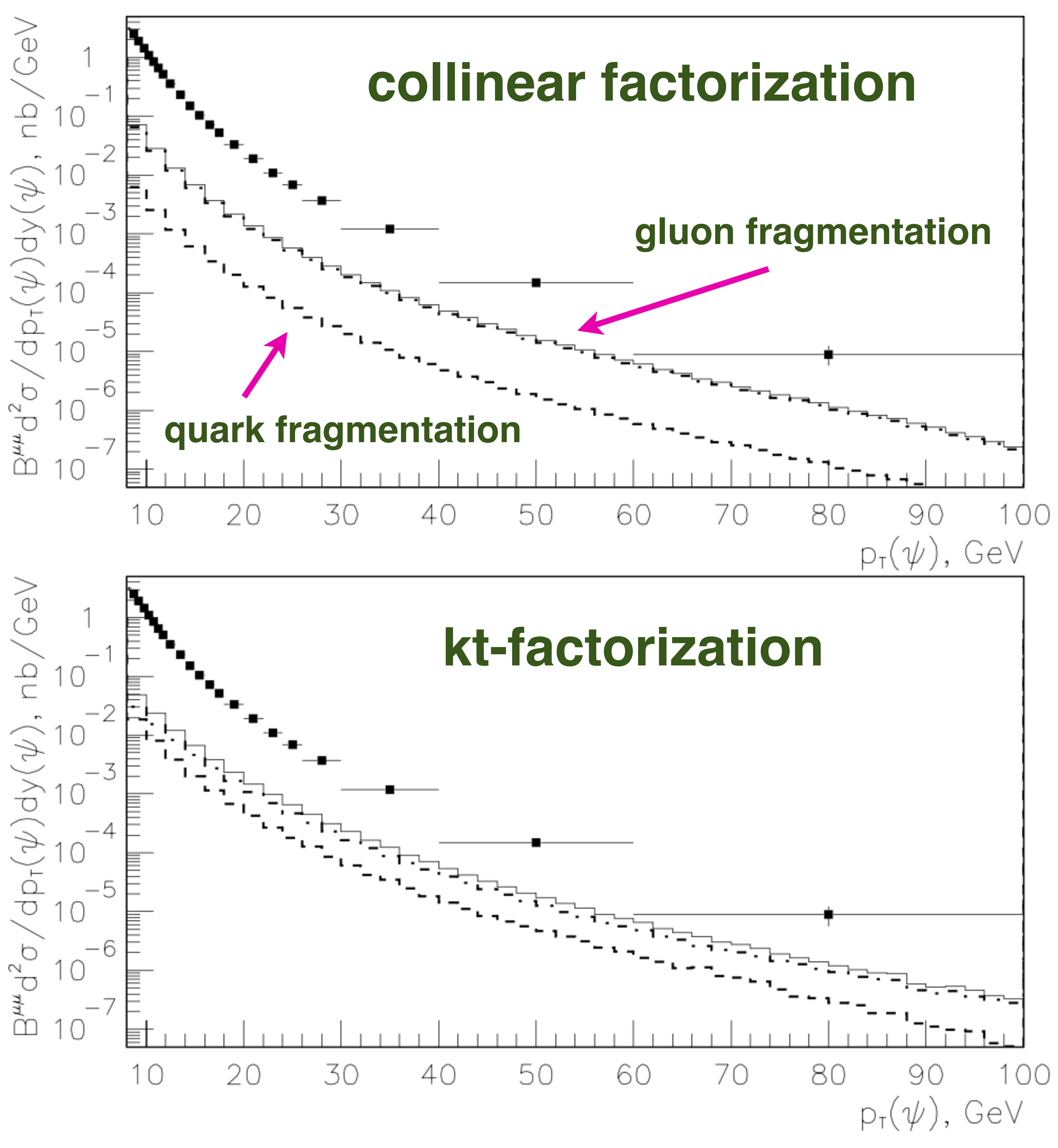
A single high-pT c-quark can fragment into J/ψ similar to q->πq transition







Gluon vs quark fragmentations

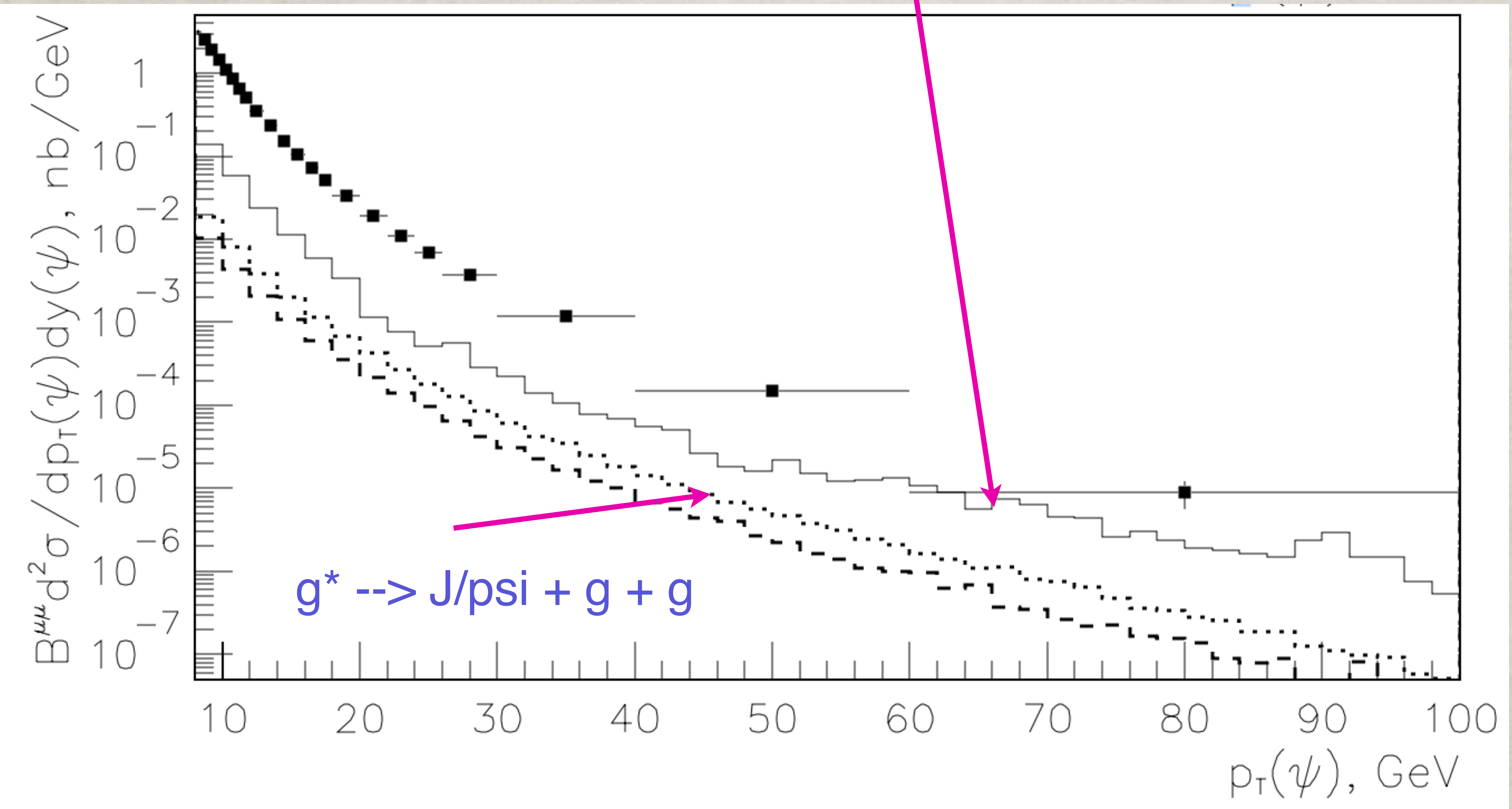


Although the quark to J/ψ FF is much larger than gluon FF, gluons win because the cross section of gluon production is much higher.

Nevertheless altogether they essentially underestimate data.

Challenging fragmentation

Full calculation of 36 LO graphs for g+g-> J/ψ+g+g



g* --> J/psi + g + g

No severe disagreement remains

Summary

Fragmentation of high-pT quarks expose nontrivial features.

- The FF of pion production at large z in the Berger's mechanism, dressed by gluon re-summation, complies with data.
- A high-p jet with virtuality equal to its energy dissipates energy so intensively, that has to produce a leading hadron (colorless dipole) with large z promptly, on a very short time scale, which does not rise with p .
- Heavy and light quarks produced in high-pT partonic collisions radiate differently. Heavy quarks regenerate their stripped-off color field much faster than light ones and radiate a significantly smaller fraction of the initial energy.
- This peculiar feature of heavy-quark jets leads to a specific shape of the fragmentation functions. Differently from light flavors, the heavy quark fragmentation function strongly peaks at large fractional momentum z, i.e. the produced heavy-light meson, B or D, carry the main fraction of the jet momentum. This is a clear evidence of a short production time of a heavy-light mesons.

Summary

- Contrary to the propagation of a small q-$\bar{q}$ dipole, which survives in the medium due to color transparency, a $\bar{q}$-Q dipole promptly expands to a large size. Such a big dipole has no chance to survive intact in a hot medium. On the other hand, a breakup of such a dipole does not suppress the production rate of $\bar{q}$-Q mesons, differently from light qq mesons.

- Melting of a charmonium in QGP does not lead to its disappearance. The survival probability is still high and rises with pT.

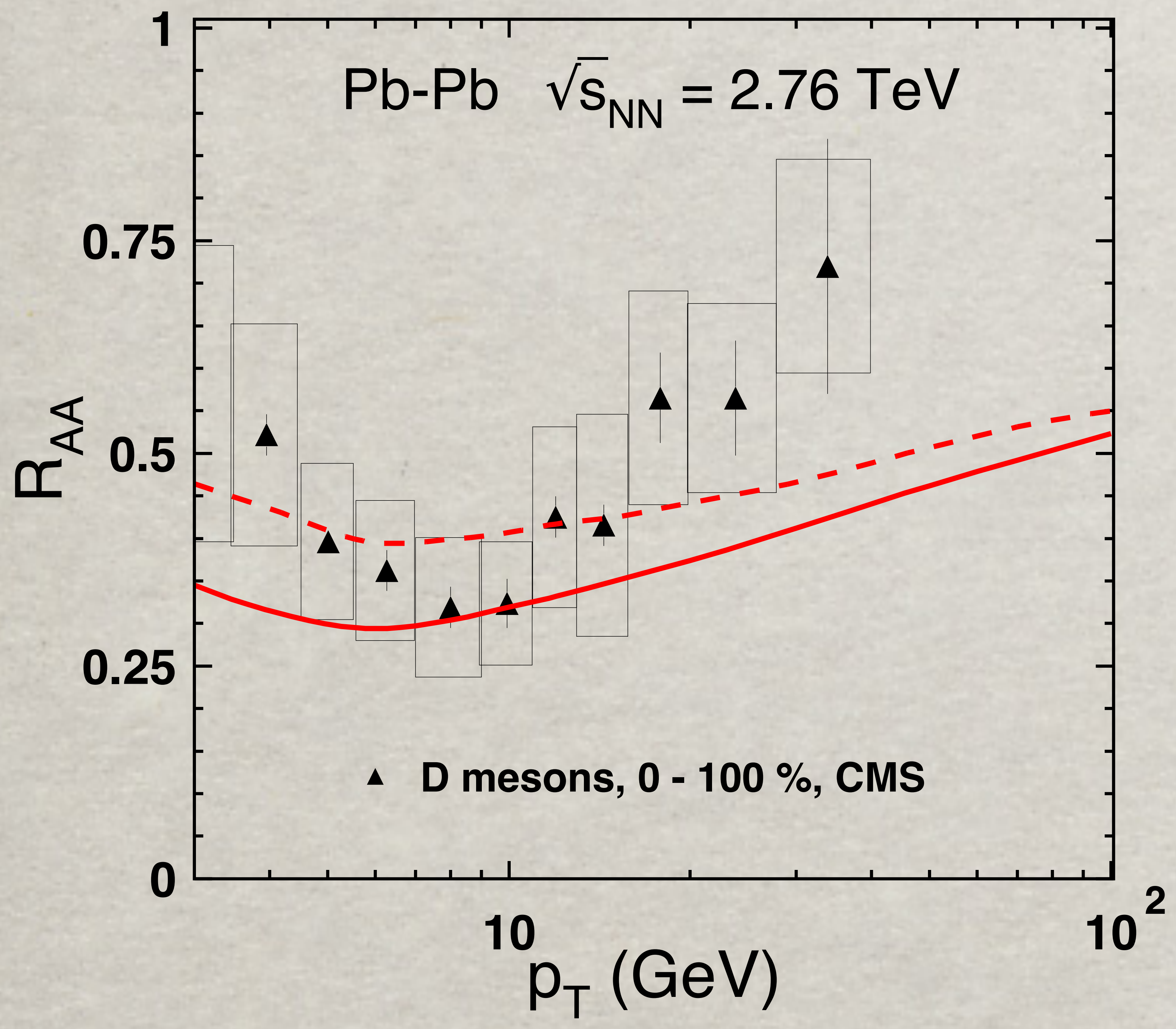
- Another source of charmonium suppression is color-exchange interaction with the medium, which breaks-up the colorless dipole.

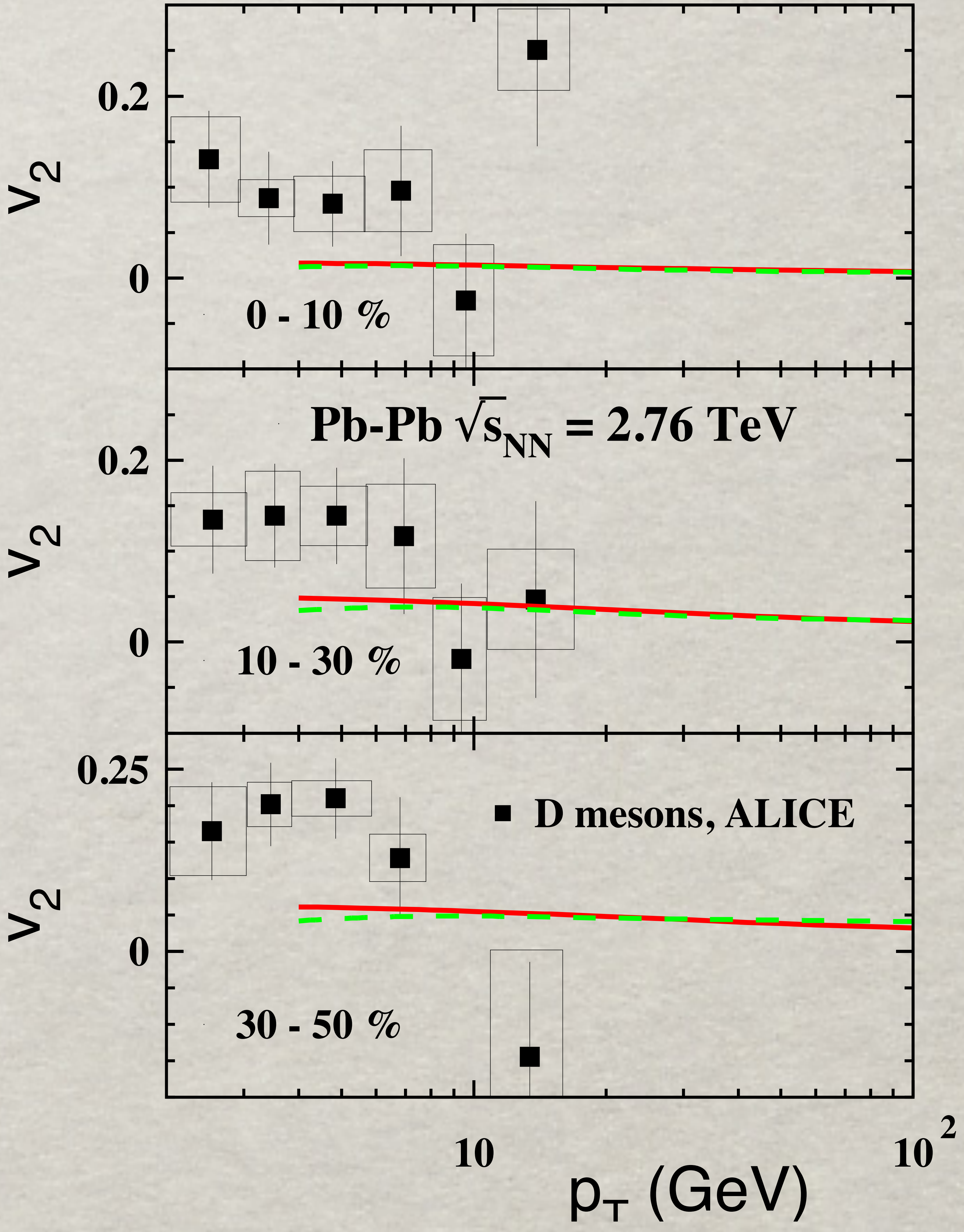
- A novel procedure for boosting the Schrödinger equation to a moving reference frame is proposed.

- A high-p J/ψ appears to result from perturbative fragmentation of either a gluon, or a quark

BACKUPS

Results





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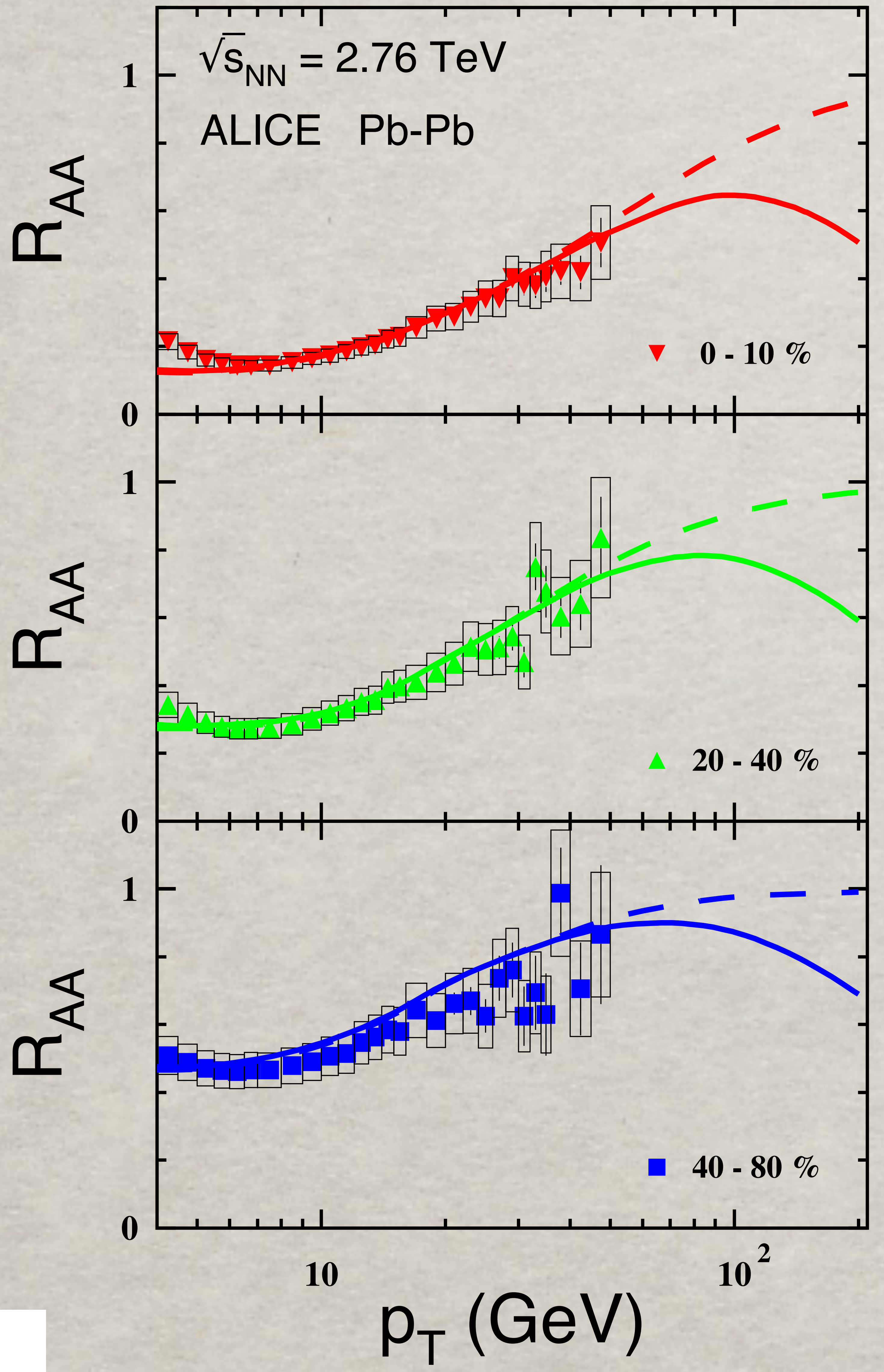
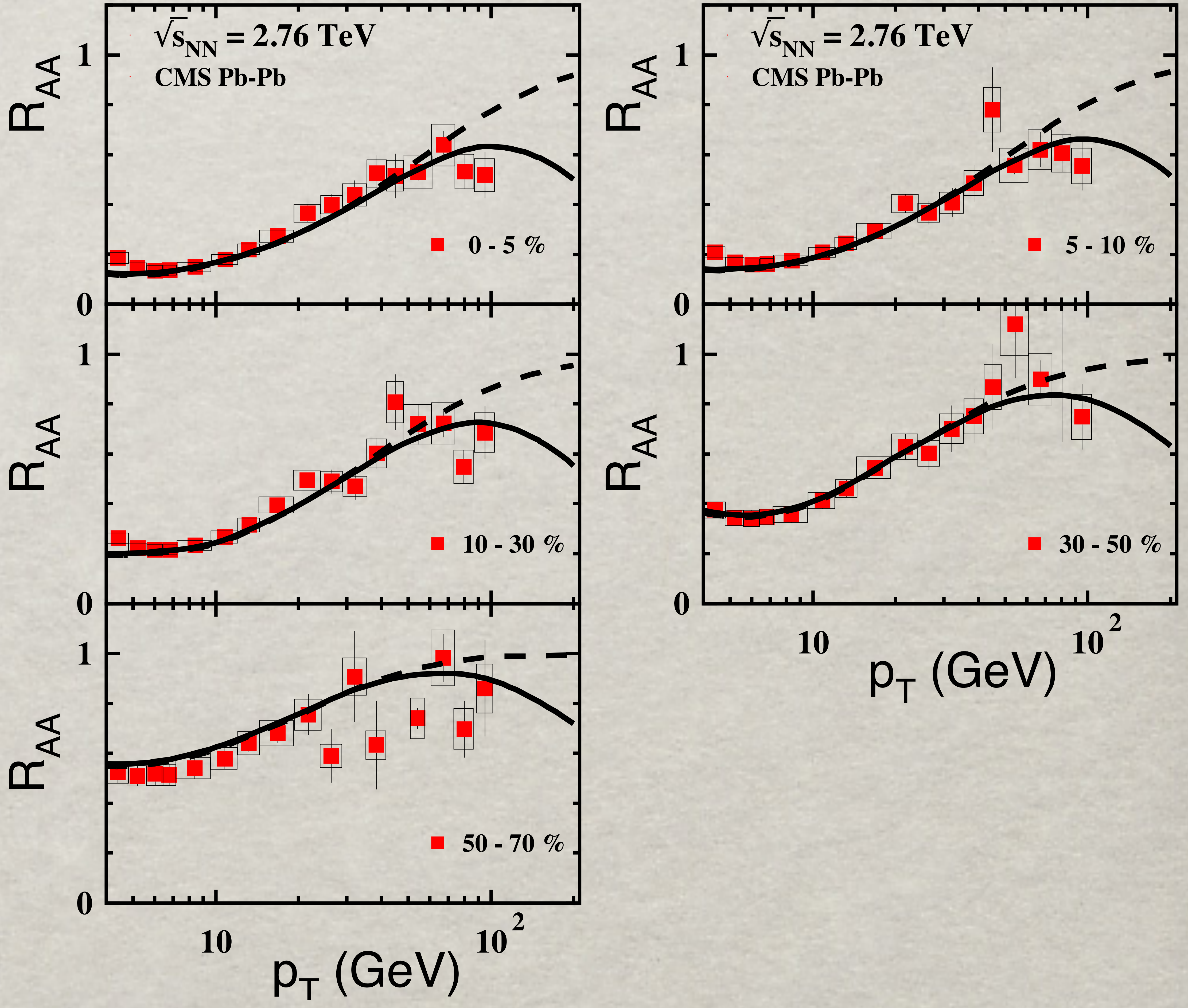
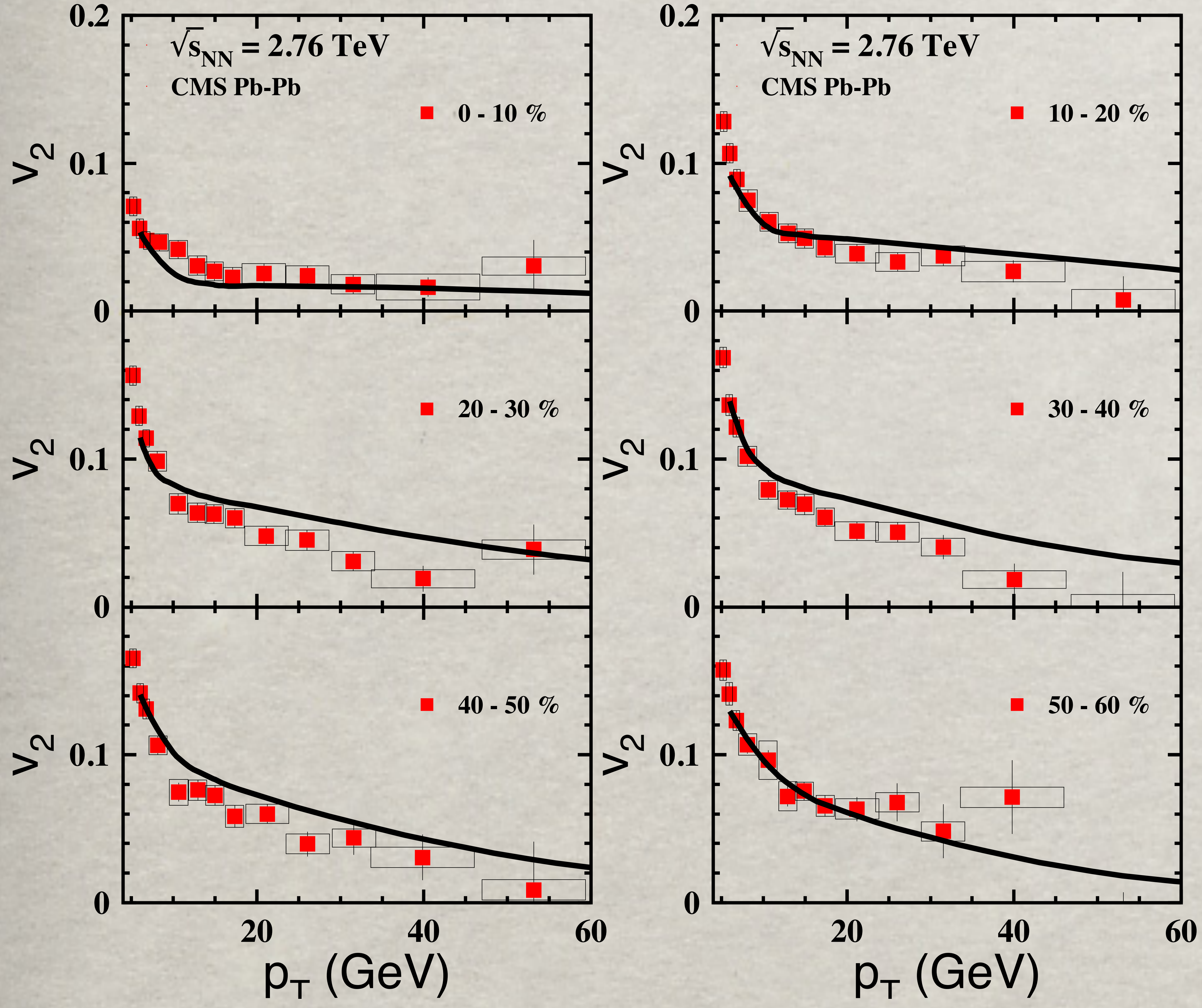
Quenching of high-p_T hadrons

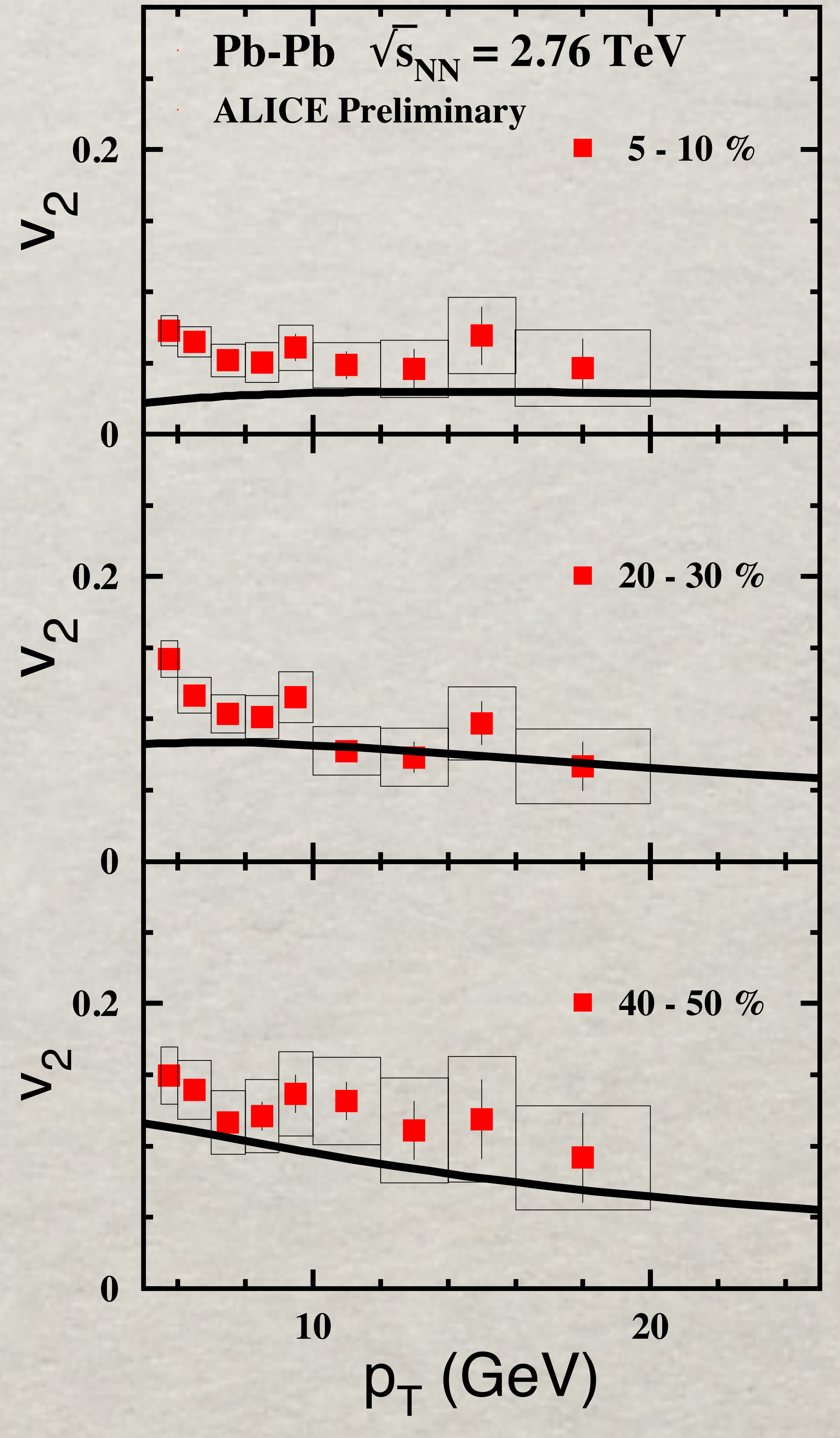


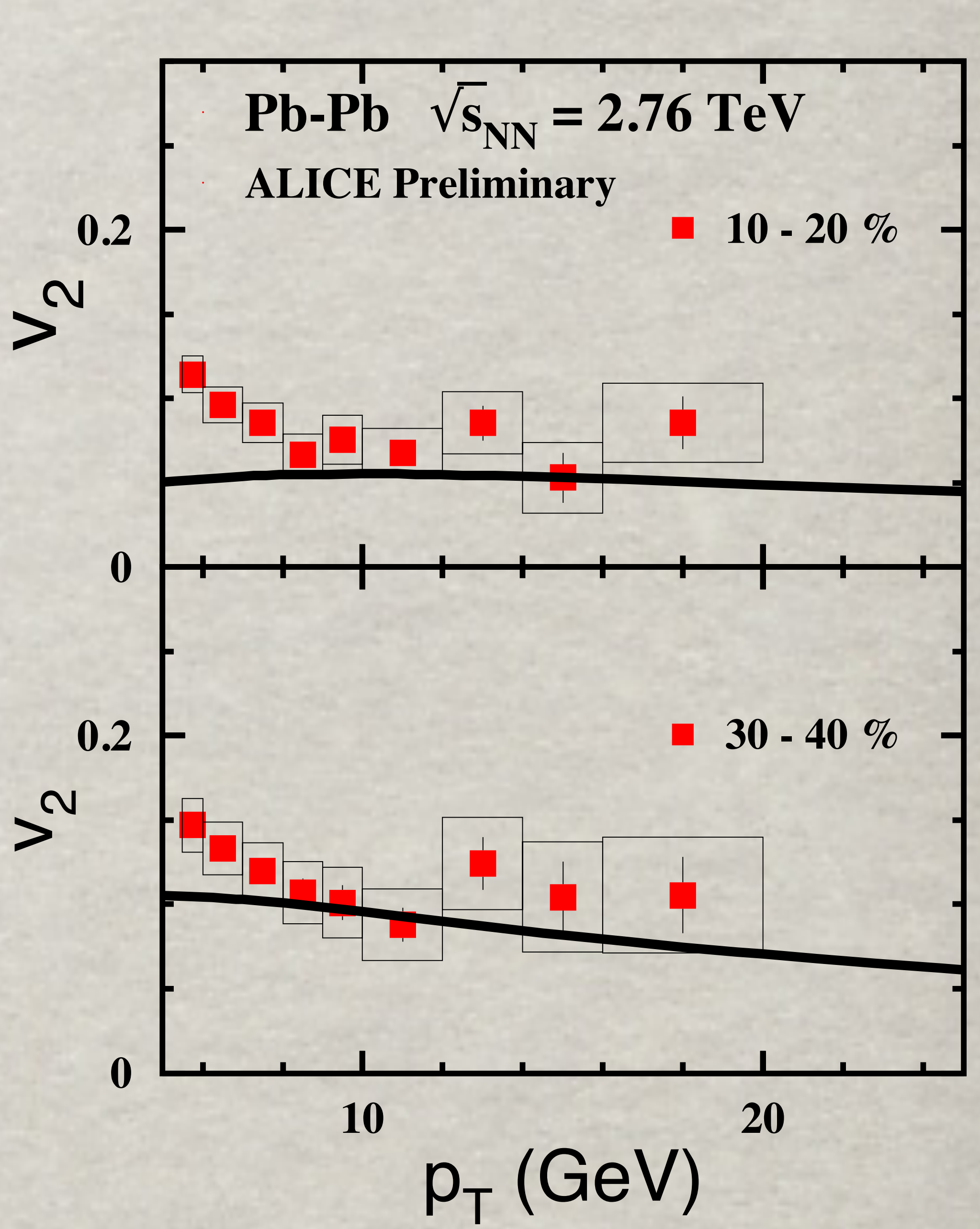


BACKUPS

Azimuthal asymmetry



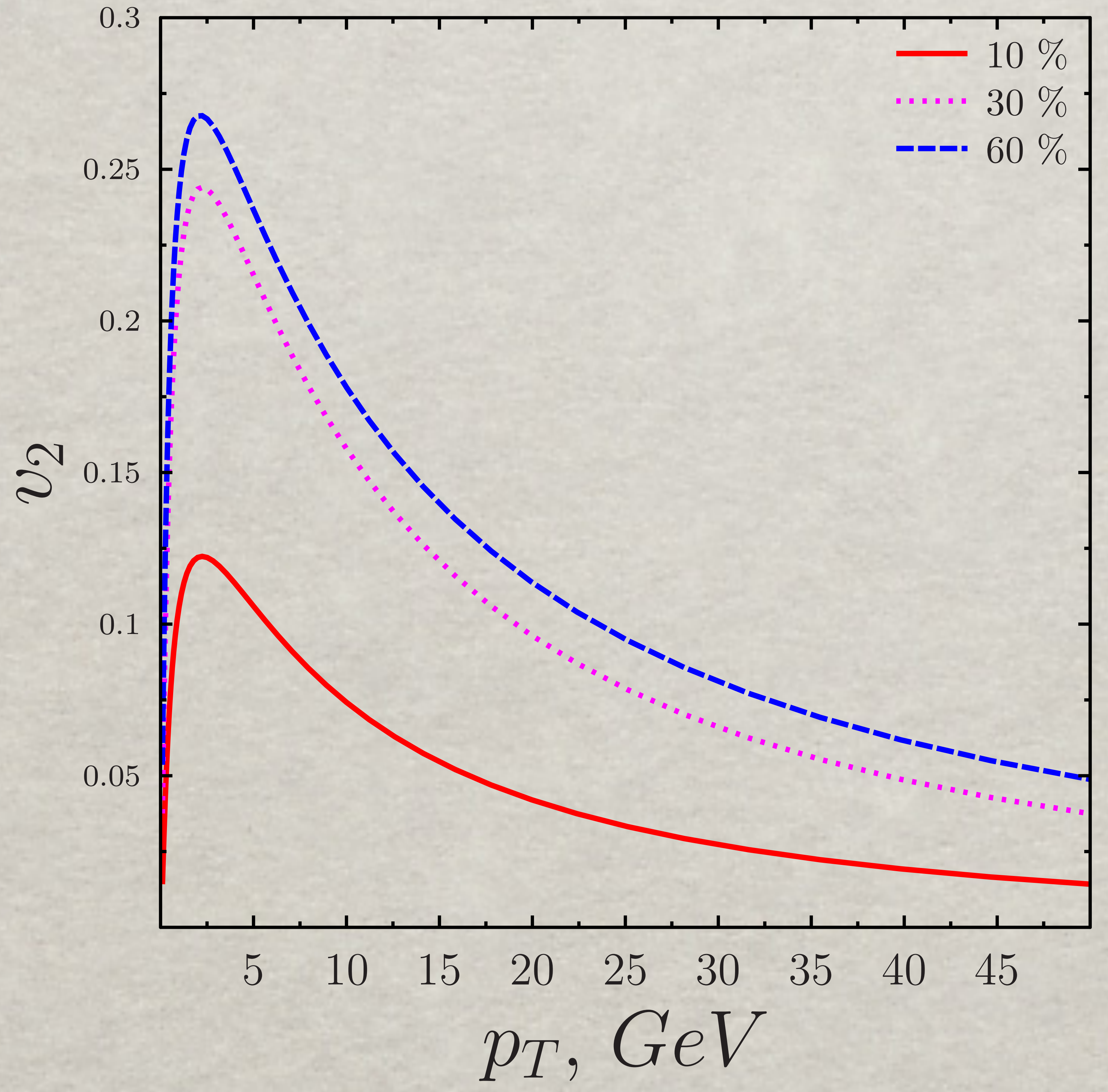




Azimuthal asymmetry

$$\mathbf{v_2(b)} = \frac{\mathbf{1}}{\mathbf{S^2_{J/\Psi}(b)}} \int\limits_{\mathbf{0}}^{\mathbf{2\pi}} \frac{\mathbf{d\phi}}{\mathbf{2\pi}} \cos(\mathbf{2}\phi) \int \frac{\mathbf{d^2s\, T_A(s) T_B(b-s)}}{\mathbf{T_{AB}(b)}}$$

$$\times \left| \frac{\int \mathbf{d^2r_1 d^2r_2 d\zeta_1 d\zeta_2 \Psi^\dagger_f(\zeta_2, \tilde{r}_2) G(\infty, \zeta_2, \tilde{r}_2; l_0, \zeta_1, \tilde{r}_1) \Psi_{in}(\zeta_1, \tilde{r}_1)}}{\int \mathbf{d^2r d\zeta\, \Psi^\dagger_f(\zeta, \tilde{r})\, \Psi_{in}(\zeta, \tilde{r})}} \right|^{\mathbf{2}}$$



Results for Ψ’

Projecting to the wave function of Ψ(2S) one gets a stronger suppression