Shallow Bound States and Universality

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Outline

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- Threshold bound states and the unitary limit
- Applications
  - Ultracold atoms
  - Halo nuclei
  - Factorization in break-up and recombination
- Summary and outlook

Review articles: HWH, Ji, Phillips, arXiv:1702.08605

Braaten, HWH, Phys. Rep. **428** (2006) 259

Physics Near the Unitary Limit

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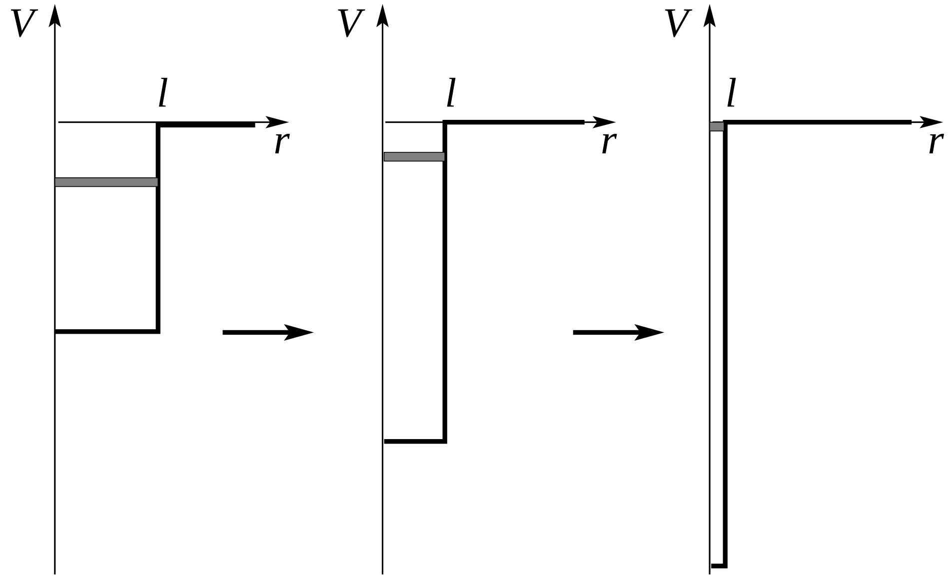
- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \sim r_e \to 0$ (cf. Bertsch problem, 2000)



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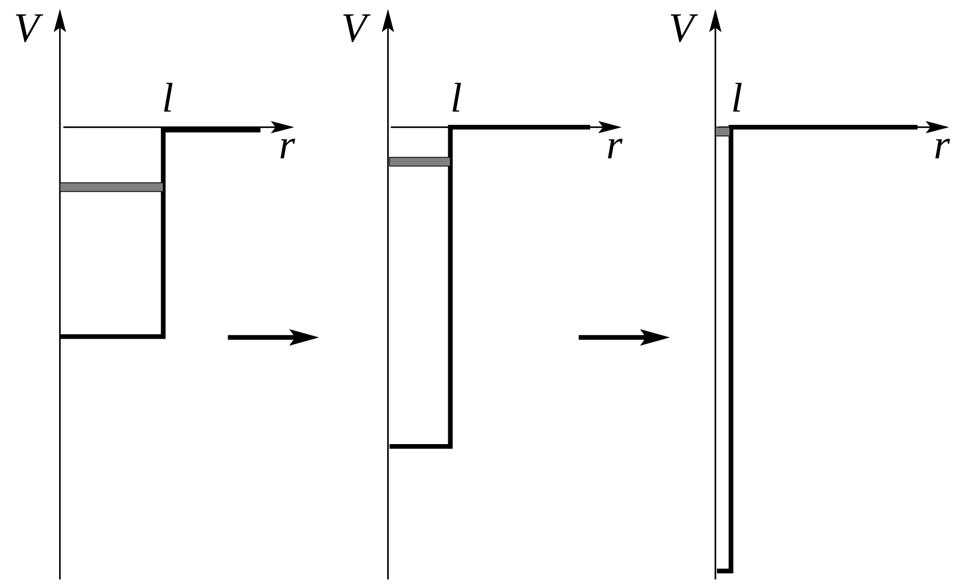

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \Longrightarrow i/k$$

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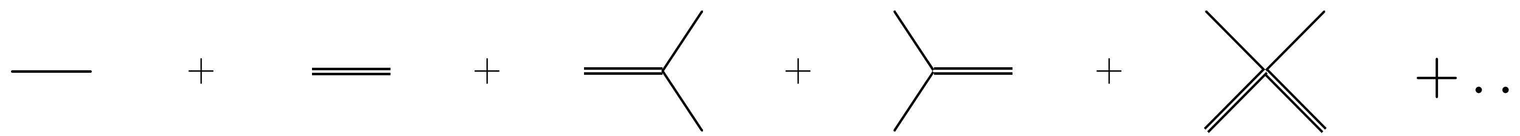
- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $B_2 = -1/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} \sim a$ $\Rightarrow$ halo state

EFT Framework

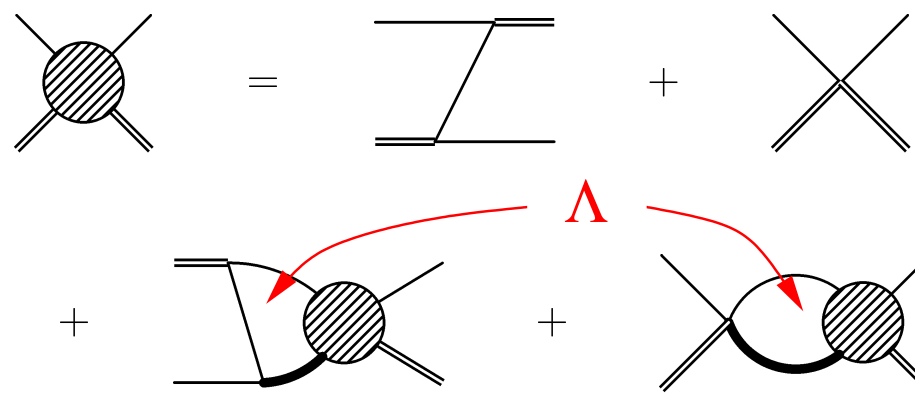
- Exploit scale separation from shallow states: $B \sim 0$
- Here: S-wave case, higher L states can also be treated

- Effective Lagrangian

$\mathcal{L}_{eff} =$ 

- 2-body amplitude: 
- 2-body coupling g_2 near fixed point $(B_2 = 0)$ $\Longleftrightarrow$ unitary limit

- 3-body amplitude:



$g_3(\Lambda)$ $\Rightarrow$ limit cycle

$\Rightarrow$ discrete scale inv.

Limit Cycle

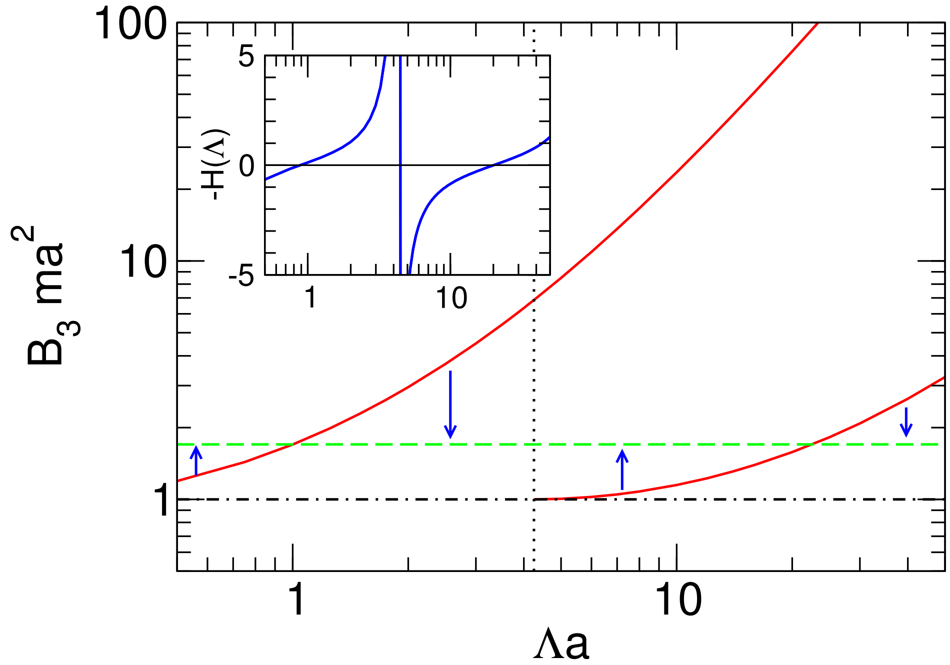
- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



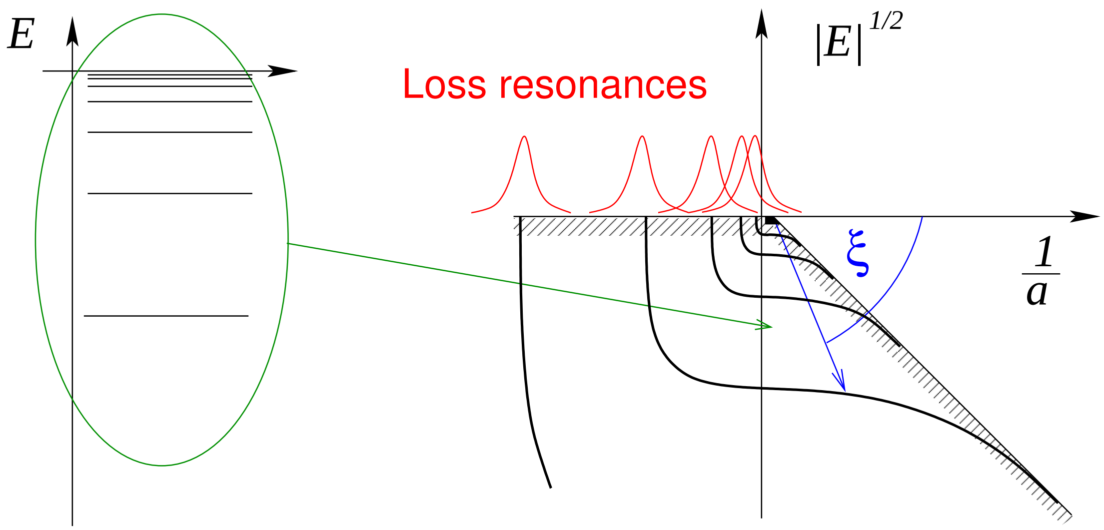
$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states (Efimov, 1970)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \overset{1/a\to 0}{\longrightarrow} \left(e^{\pi/s_0}\right)^2 = 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances
  $\Longrightarrow$ drive RF transitions

Three-Body Recombination

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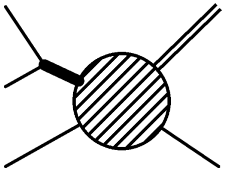

- Three-body recombination:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$

- K_3 has log-periodic dependence on scattering length

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

- Deep dimers: Efimov trimers aquire width $\Rightarrow$ resonances

- Loss term in short distance b.c.: $\Lambda_* \longrightarrow \Lambda_* \exp^{i\eta_*/s_0}$

- Universal line shape of recombination resonance $(a<0,\, T=0)$

$$K_3^{deep} = \frac{64\pi^2(4\pi - 3\sqrt{3})\, \coth(\pi s_0) \sinh(2\eta_*)}{\sin^2\left[s_0 \ln(a/a_-)\right] + \sinh^2 \eta_*} \frac{\hbar\, a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

and other observables . . .

Efimov States in Ultracold Atoms

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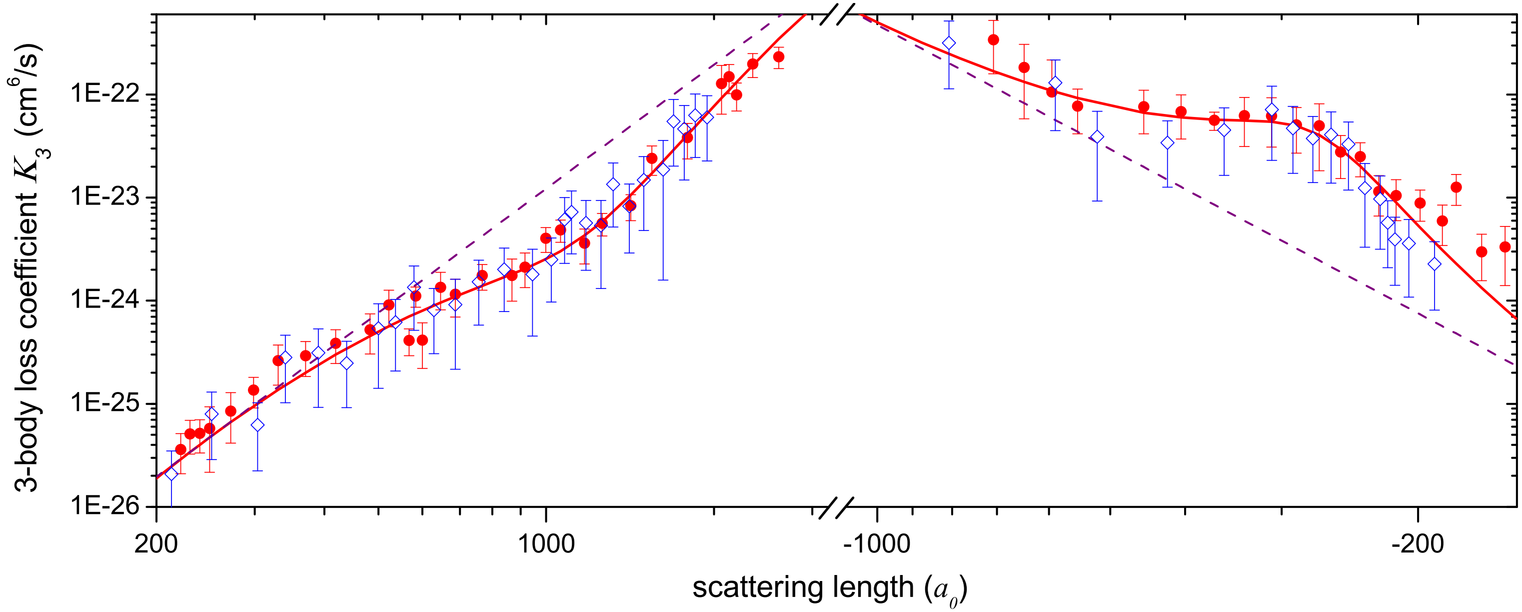
- First experimental evidence in ^{133}Cs (Krämer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb, ^{6}Li/^{133}Cs
- Example: Efimov spectrum in ^{6}Li/^{133}Cs mixture
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)



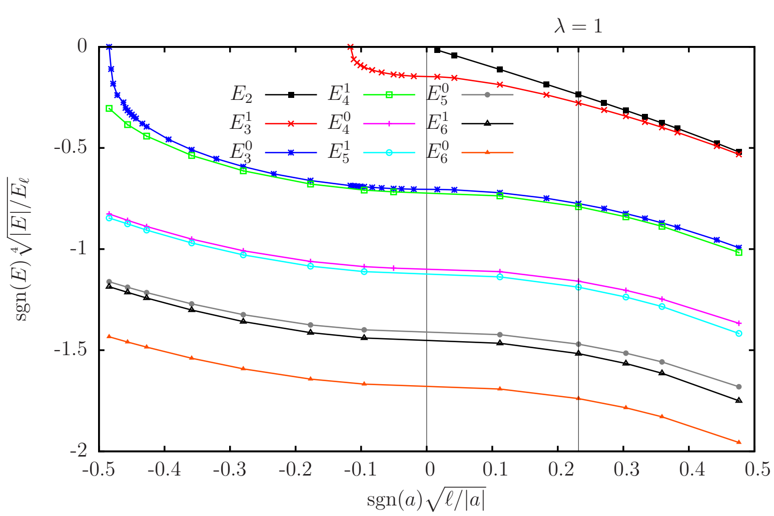
- Van der Waals tail determines $a_-/l_{vdW} \approx -10\ (\pm 15\%)$
  (Wang et al., 2012; Naidon et al. 2012, 2014; ...)
  ... but not η_* ...

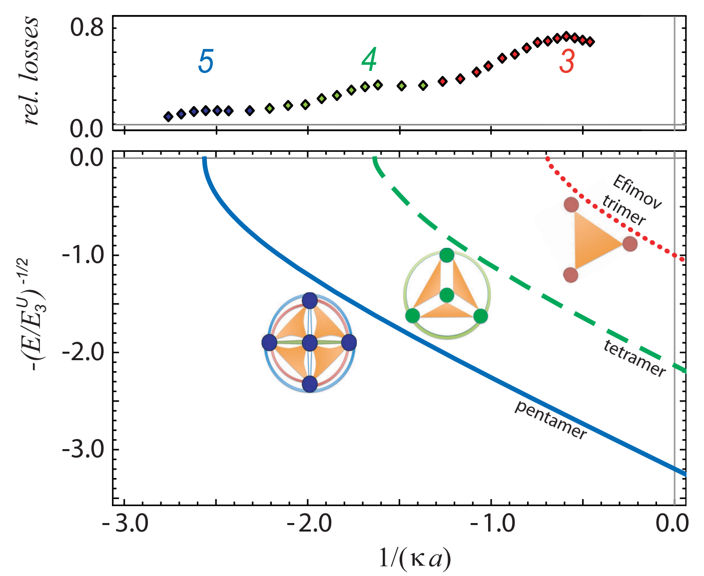
Universal Tetramers and Beyond

- Universal tetramers: $B_4^{(0)} = 4.610(1)\ B_3\,, \quad B_4^{(1)} = 1.00227(1)\ B_3$

  (Platter, HWH, 2004, 2007; von Stecher et al., 2009; Deltuva 2010-2013)
- Two tetramers attached to each trimer
- Universal states up to $N = 16$ calculated

  (von Stecher, 2010, 2011; Gattobigio, Kievsky, Viviani, 2011-2014)
- Observation up to $N = 5$ in Cs losses (Grimm et al. (Innsbruck), 2009, 2013)

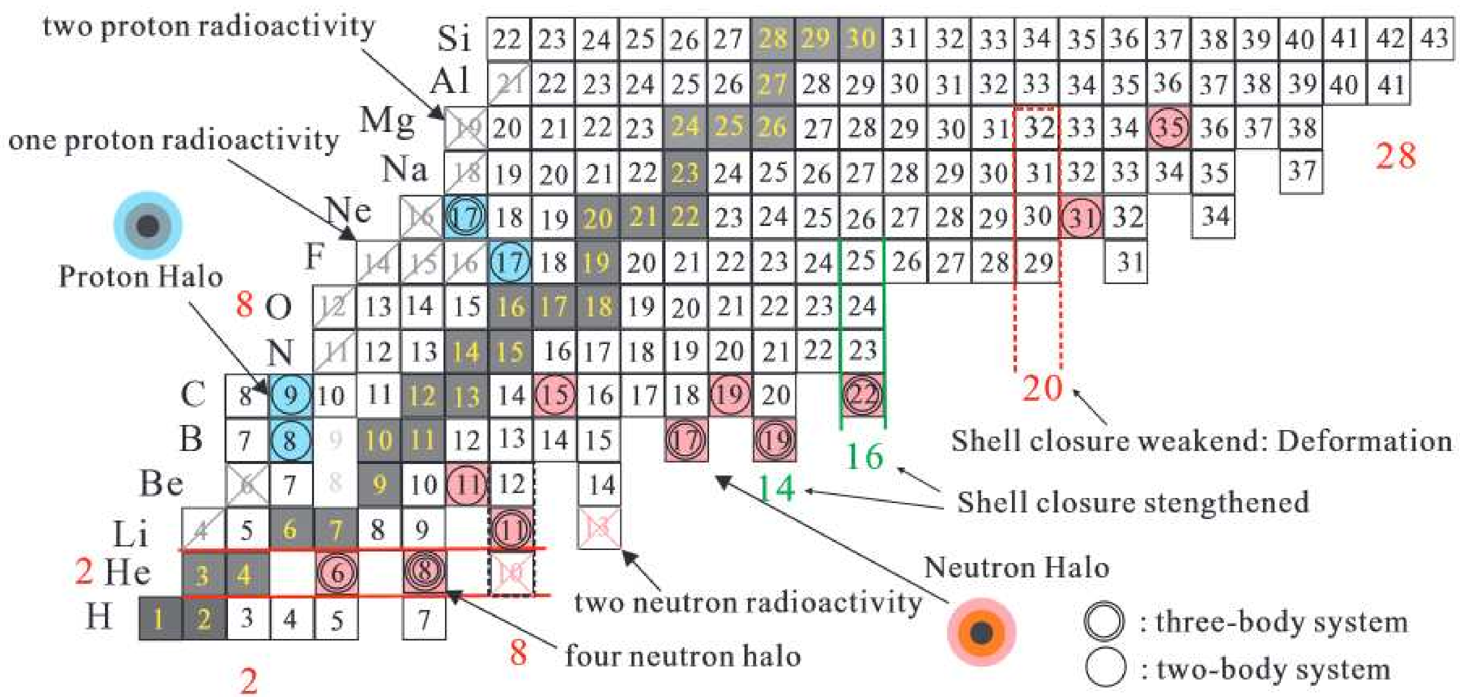




Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to “nucleon drip line” $\longrightarrow$ scale separation $\longrightarrow$ EFT



C.-B. Moon, Wikimedia Commons

- EFT for halo nuclei

  (Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

Halo Effective Field Theory

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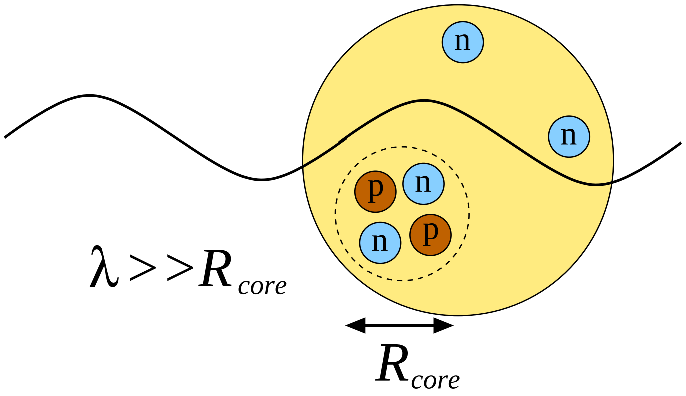
- Scales: $E \sim p^2/(2\mu) \sim 1/(2\mu R^2)$
- Separation of scales:
  $1/k = \lambda \gg R_{core}$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR_{core}



- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\Longrightarrow$ universal properties
- Very low energies: only short-range physics $\Longrightarrow$ pionless EFT
- Exploit cluster substructures $\Longrightarrow$ Halo EFT

Efimov Physics in Halo Nuclei

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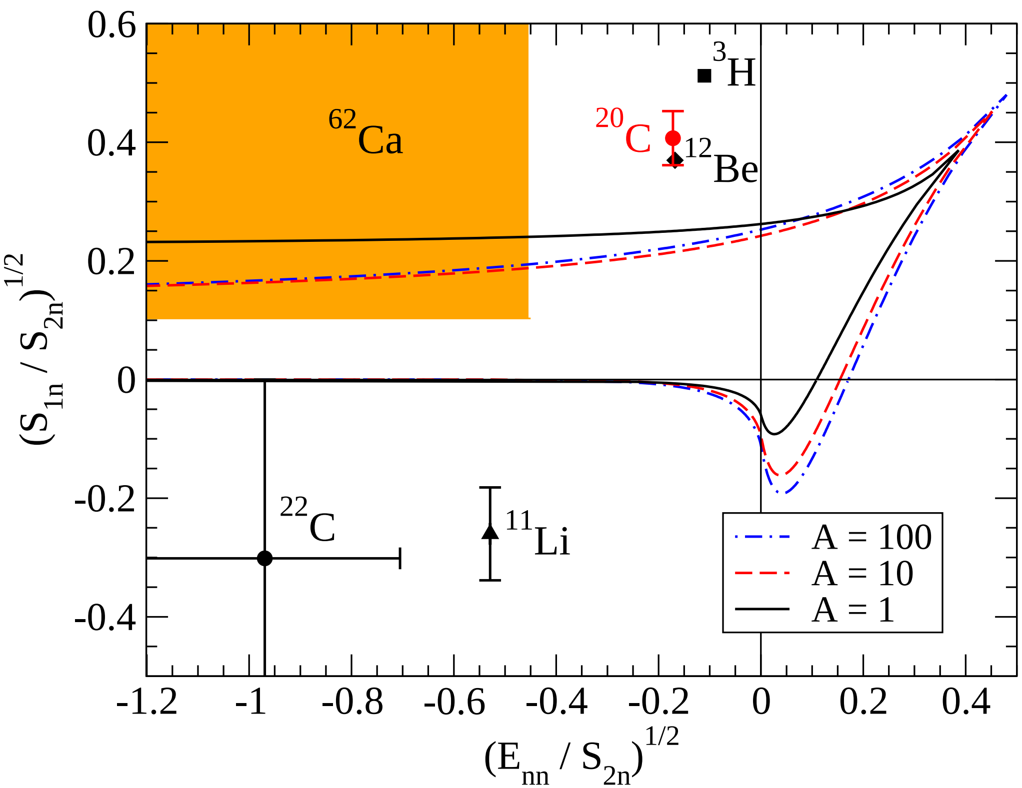
- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)

  $\Longrightarrow$ excited states obeying scaling relations

- Correlation plot: $E_{nn} \leftrightarrow S_{1n}$ (Amorin, Frederico, Tomio, 1997)



adapted from Canham, HWH, Eur. Phys. J. A **37** (2008) 367

Efimov Physics in ^{62}Ca

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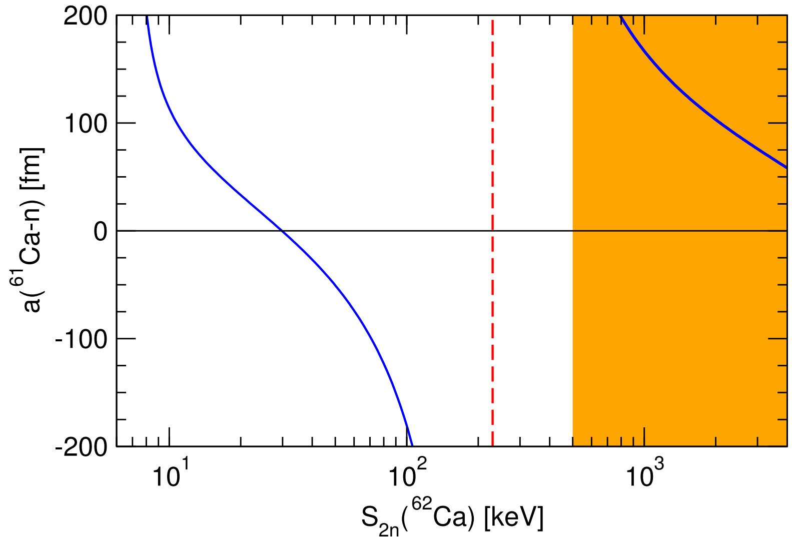
(G. Hagen, P. Hagen, HWH, Platter, Phys. Rev. Lett. **111** (2013) 132501)

- From many to few: emergence of halo degrees of freedom
- Coupled cluster calculations of ^{60}Ca and ^{61}Ca using chiral N2LO two-body force and schematic three-body force:

$\Rightarrow$ ^{61}Ca is a weakly bound S-wave state (or virtual state)



- Prospects for excited Efimov states in ^{62}Ca:

$S_{\rm deep} = 1/(\mu_{cn} r_{cn}^2) \approx 500$ keV

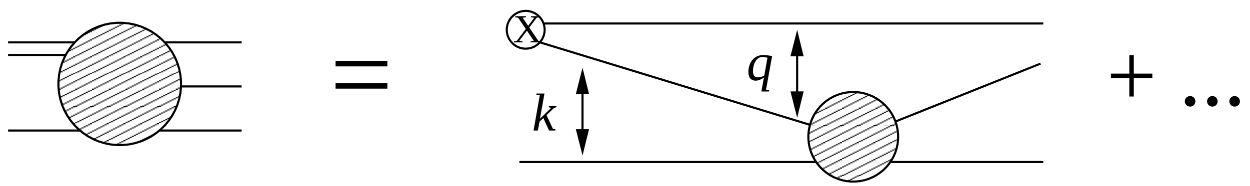
scaling factor $\lambda_0 \approx 16$

$\Longrightarrow$ possible if $S_{2n} \gtrsim 230$ keV

Recombination and Breakup

- Breakup and recombination reactions of shallow atomic bound states at higher energy E
  (Braaten, Zhang, Phys. Rev. A **73**, 042707 (2006))
- Factorization of amplitude if typical momenta between atom and bound state satisfy $k \gg 1/a \sim q,\ E \sim k^2/m$

$$\mathcal{T} \approx \frac{4(\pi/a)^{1/2}}{q^2 + 1/a^2} \times \frac{8\pi/m}{(\frac{1}{2}r_e k^2 + \ldots) - ik}$$



- Final state interaction suppressed
- Works well for breakup reactions of ^{4}He atoms
- Compare with exact calculation for recombination

Recombination and Breakup

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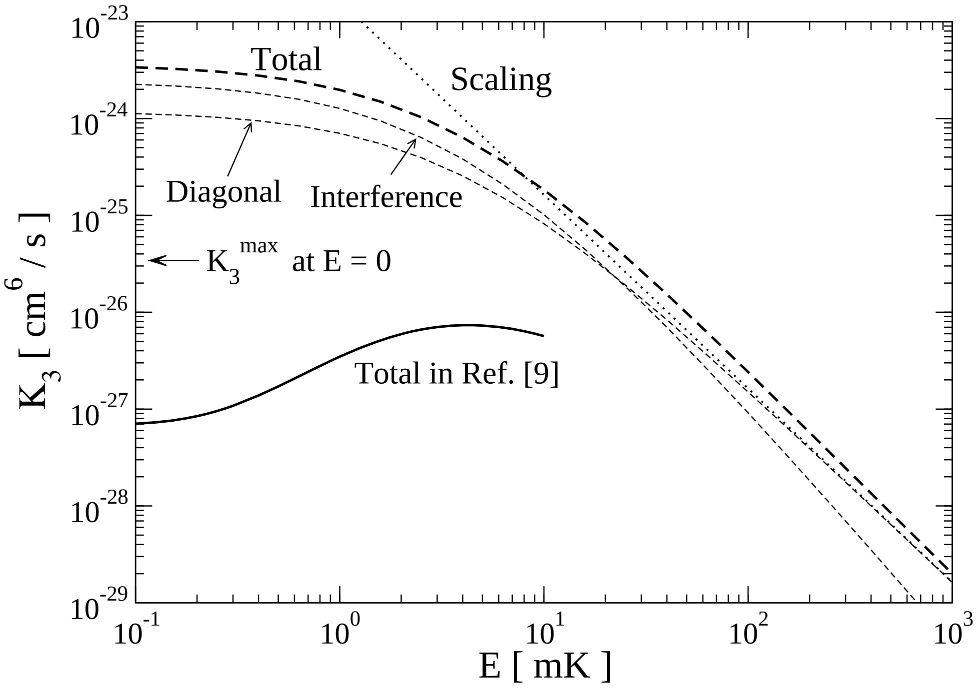
- Three-body recombination rate coefficient K_3

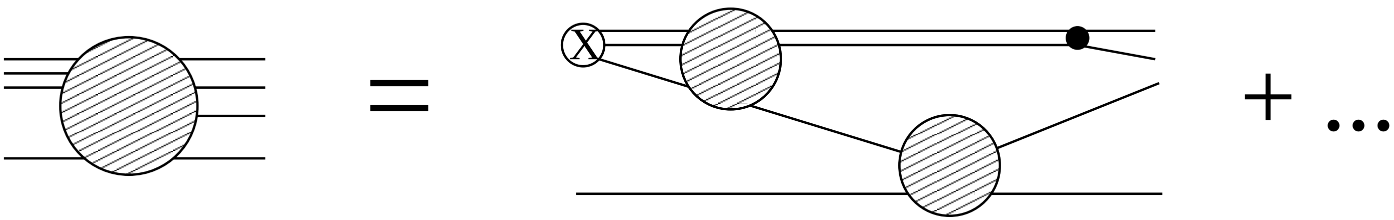


(Braaten, Zhang, Phys. Rev. A **73**, 042707 (2006))

- Interference terms important at low energies ($B_2 = 1.6$ mK)
- Higher orders in atom-atom scattering amplitude

Recombination and Breakup

- Extension to three-body bound states



- Requires momentum space wave function of trimers

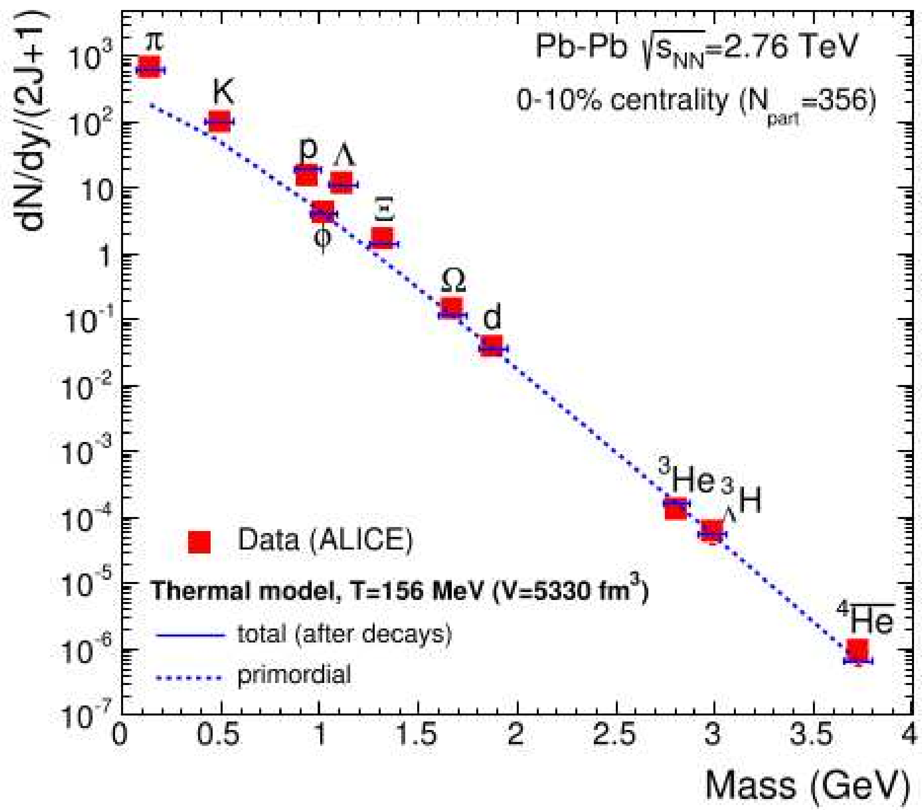
  $\Longrightarrow$ three-body problem requires numerical solution
  $\Longrightarrow$ dependence on a and Λ_*

- Extension to higher-body states possible
- Only bound state calculations with $N-1$ bodies required
- Closure relation for sum over all (initial/final) states

$$\int d^3r \sum_{X_{N-1}} |\langle X_{N-1}|\psi(r)|M\rangle|^2 = \int d^3r \, \langle M|\psi^\dagger\psi(r)|M\rangle = N$$

Recombination and Breakup

- Application to heavy ion collisions?



Andronic et al. (2016)

- Short-distance factor cancels in ratios?

Summary

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- Effective field theory for large scattering length
- Universal aspects of (Discrete) Scale Invariance $\Leftrightarrow$ Efimov physics
  - Effective field theory for threshold states
  - ....
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei: halo nuclei, ...
  - Hadronic molecules: $X(3872)$, ...
- Factorization for breakup and recombination reactions
  - Application to production of weakly-bound objects in heavy ion collisions?