Relativistic Hydrodynamics in Heavy-Ion Collisions: part II

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Outline

LECTURE I

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LECTURE II

3. **Comparisons with kinetic theory / Gradient expansion**
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   - 3.2 Massless case
   - 3.3 Massive case
   - 3.4 Gradient expansion
4. **Hydrodynamic equations with spin**
5. **Summary**

3. Comparisons with kinetic theory

Exact solutions of RTA kinetic equation

- Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p^\mu u_\mu \, \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

Bhatnagar, Gross, Krook, Phys. Rev. 94 (1954) 511

- background distribution (Boltzmann statistics)

$$f^{\rm eq} = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{p^\mu u_\mu}{T}\right)$$

- boost-invariant variables

A. Bialas and W. Czyz, Phys. Rev. D30 (1984) 2371

$$w = t p_{||} - zE \qquad\qquad v = tE - z p_{||}$$

- for transversely homogeneous boost-invariant system

$$\frac{\partial f}{\partial \tau} = \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

$$f^{\rm eq}(\tau, w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{\sqrt{(w/\tau)^2 + (m^2 + p_\perp^2)}}{T}\right)$$

Exact solutions of RTA kinetic equation

- formal solution

  G. Baym, Phys. Lett. B138 (1984) 18; Nucl. Phys. A418 (1984) 525c

$$f(\tau, w, p_\perp) = D(\tau, \tau_0) f_0(w, p_\perp) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau, \tau')\, f^{\rm eq}(\tau', w, p_\perp)$$

$$D(\tau_2, \tau_1) = \exp\left[-\int_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\rm eq}(\tau'')}\right]$$

- initial condition (Romatschke-Strickland form)

$$f_0(w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left[-\frac{\sqrt{(1+\xi_0)(w/\tau_0)^2 + (m^2 + p_\perp^2)}}{\Lambda_0}\right]$$

$\xi_0 = \xi(\tau_0)$ - initial value of the anisotropy parameter
$\Lambda_0 = \Lambda(\tau_0)$ - initial transverse-momentum scale

Exact solutions of RTA kinetic equation

- ANISOTROPIC HYDRODYNAMICS in its simplest formulations assumes that the RS form is a good approximation for non-equilibrium distribution function

$$f_{\rm aniso}(\tau, w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left[- \frac{\sqrt{(1+\xi(\tau))(w/\tau_0)^2 + (m^2 + p_\perp^2)}}{\Lambda(\tau)} \right]$$

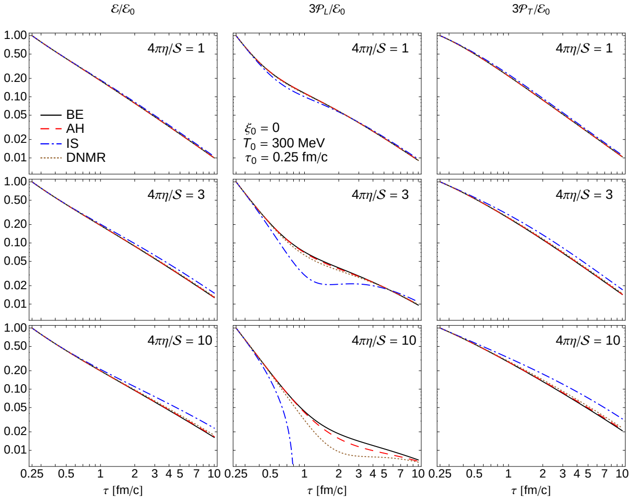
  $\xi(\tau)$ - time dependent anisotropy parameter
  $\Lambda(\tau)$ - time dependent transverse-momentum scale

- several generalisations possible (not discussed here)

Massless case

WF, R. Ryblewski and M. Strickland, Nucl. Phys. A916 (2013) 249; Phys.Rev. C88 (2013) 024903

$m = 0$, boost-invariant, transversally homogeneous system, (0+1) case AH0



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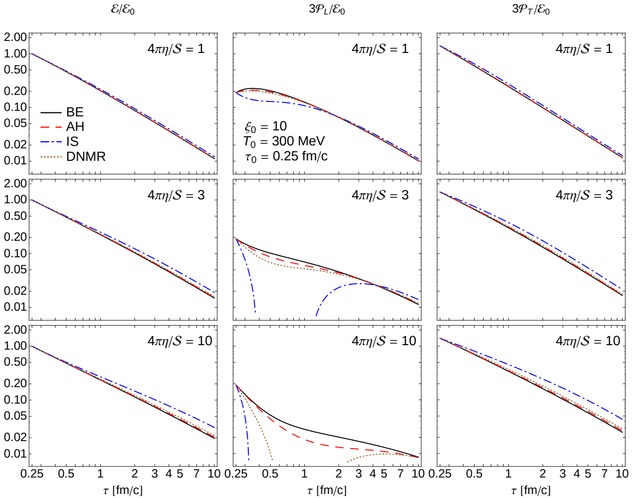

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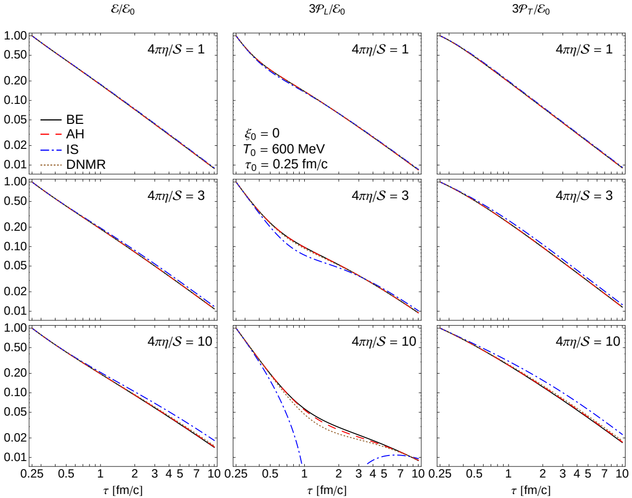

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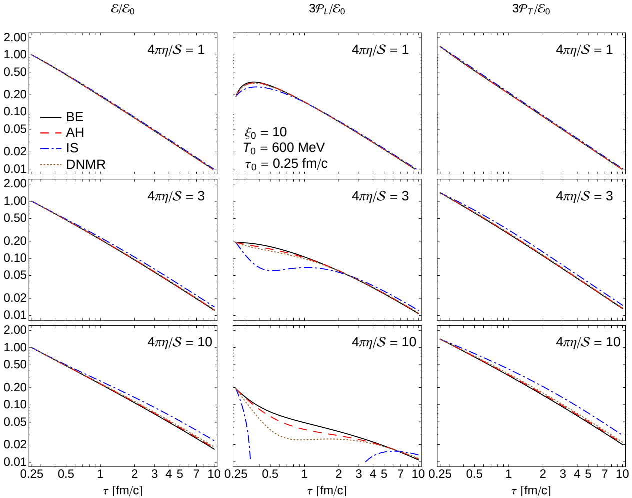
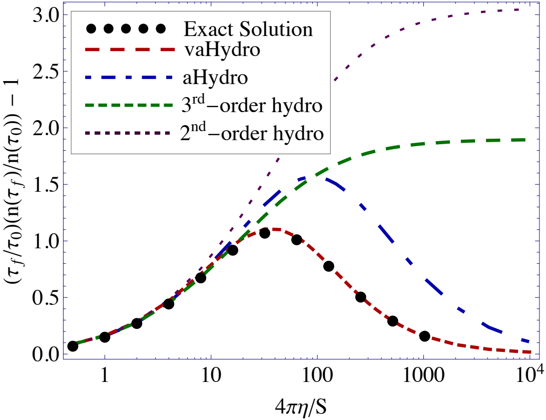


Massless case

D. Bazow, U. W. Heinz, and M. Strickland, Phys.Rev. C90, 044908 (2014)



anisotropic hydro AH0 reproduces two limts:
perfect fluid ($\bar{\eta} \to 0$) and free streaming ($\bar{\eta} \to \infty$)

Massive case

Various, second order hydro equations for the bulk pressure

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{1}{2}\tau_\Pi \Pi \left[\frac{1}{\tau} - \left(\frac{\dot{\zeta}}{\zeta} + \frac{\dot{T}}{T}\right)\right] \qquad (A)$$

MIS A. Muronga, Phys. Rev. C69 (2004) 034903; U. Heinz, H. Song, A.K. Chaudhuri, Phys. Rev. C73 (2006) 034904

$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} - \frac{4}{3}\tau_\Pi \Pi \frac{1}{\tau} \qquad (B)$$

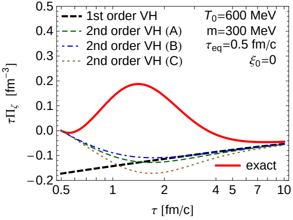
truncated MIS A. Jaiswal, R. Bhalerao, S. Pal, Phys. Rev. C87 (2013) 021901

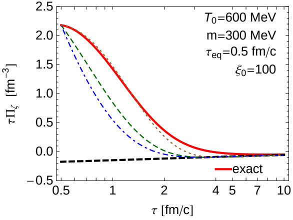
$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{\zeta}{\tau} \qquad (C)$$

IS U. Heinz, H. Song, A.K. Chaudhuri, Phys. Rev. C73 (2006) 034904

Massive case

WF, E. Maksymiuk, R. Ryblewski, M.Strickland, Phys.Rev. C89 (2014) 054908





exact solution and all 2nd order viscous hydrodynamics variations tend toward the 1st order solution at late times

none of the 2nd order viscous hydrodynamics variations seems to qualitatively describe the early-time evolution of the bulk viscous pressure in all cases

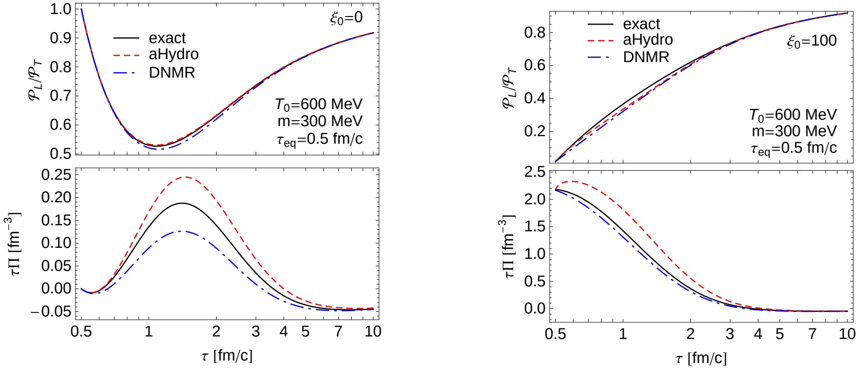
there is something incomplete in the manner in which 2nd order viscous hydrodynamics treats the bulk pressure (neglected shear–bulk coupling)

Massive case

Bulk viscous pressure evolution within DNMR hydrodynamics with SHEAR-BULK COUPLING

G. Denicol, S. Jeon, C. Gale, Phys.Rev. C90 (2014) 024912

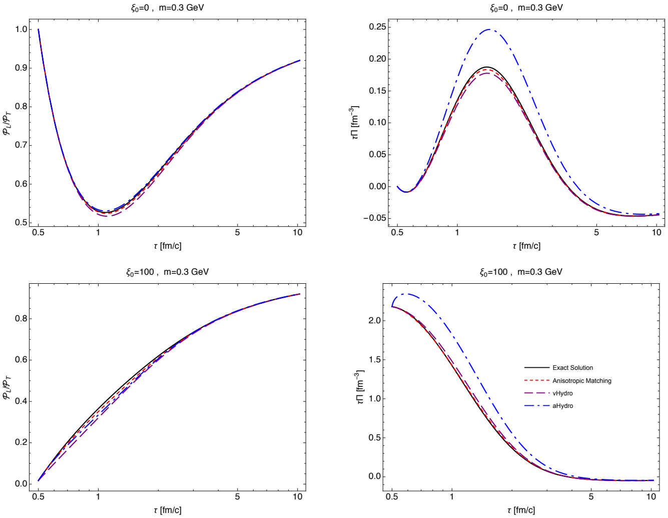
G. Denicol, R. Ryblewski, WF, M.Strickland, Phys.Rev. C90 (2014) 044905



the shear-bulk couplings are extremly important for correct description of the bulk viscous correction

Massive case

Leonardo Tinti, Anisotropic matching principle for the hydrodynamic expansion, Phys.Rev. C94 (2016) 044902



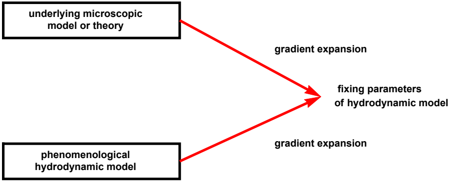
Gradient expansion

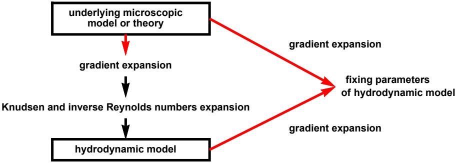
M. P. Heller, R. A. Janik, M. Spalinski, P. Witaszczyk

Formal expansion of $T^{\mu\nu}$ in gradients of hydrodynamic variables T and u^μ

$$T^{\mu\nu} \quad = \quad T^{\mu\nu}_{\rm eq} + \text{powers of gradients of } T \text{ and } u^\mu$$

Formal tool to make comparisons between different theories and check their close to equilibrium behaviour, no useful for finding approximate solutions of the theory, unless completed as a transseries





Gradient expansion

Simple structures for boost-invariant flow with the relaxation time $\tau_\pi = \frac{c}{T}$, for example, T is expanded around the Bjorken flow

$$
\begin{aligned}
T &= T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3} \left(1 + \sum_{n=1}^{\infty} \left(\frac{c}{T_0 \tau_0}\right)^n t_n \left(\frac{\tau_0}{\tau}\right)^{2n/3}\right) \\
\xi(\tau) &= \sum_{n=1}^{\infty} \left(\frac{2c}{\tau_0 T_0}\right)^n \xi_n \left(\frac{\tau_0}{\tau}\right)^{2n/3},
\end{aligned}
$$

first lecture: it is better to use $f(w)$

$$
f = \frac{1}{T}\frac{dw}{d\tau}, \qquad w = \tau T, \quad \Delta = \frac{\Delta P}{P} = 3\frac{P_\parallel - P_\perp}{\varepsilon} = 12\left(f - \frac{2}{3}\right)
$$

The gradient expansion for boost-invariant flow takes the form of an expansion

$$
f(w) = \sum_{n=0}^{\infty} f_n w^{-n}, \quad f_0 = \frac{2}{3}
$$

Gradient expansion

RTA - gradient expansion for the RTA kinetic-theory model

M. P. Heller, Kurkela, Spalinski, arXiv:1609.04803

WF, R. Ryblewski, M. Spalinski, Phys.Rev. D94 (2016) 114025

values of f_n

n	RTA	BRSSS	DNMR	MIS
0	$2/3$	$2/3$	$2/3$	$2/3$
1	$4/45$	$4/45$	$4/45$	$4/45$
2	$16/945$	$16/945$	$16/945$	$8/135$
3	$-208/4725$	$-1712/99225$	-304/33075	$112/2025$
3	-0.044	-0.017	-0.009	0.055

MIS $$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{4\tau_\pi \phi}{3\tau} - \phi$$ ϕ – shear stress component

BRSS $$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{\lambda_1 \phi^2}{2\eta^2} - \frac{4\tau_\pi \phi}{3\tau} - \phi$$

DNMR $$\tau_\pi \dot{\phi} = \frac{4\eta}{3\tau} - \frac{38}{21}\frac{\tau_\pi \phi}{\tau} - \phi$$

Gradient expansion

RTA with $\tau_\pi = c/T$, the n=1 term controlled by viscosity, $\eta/s = (9/4)f_1$

1) BRSS and DNMR equivalent up to n=2, $\eta/s = c/5$, agrees with the kinetic-theory result

2) BRSS has two free parameters that are fitted to RTA

3) DNMR reproduces RTA, since the kinetic coefficients correspond to the RTA kinetic equation

4) MIS good only for n=1, opposite sign for n=3

5) DNMR and BRSS differ for larger values of n and far away from equilibrium
– physics properties should be defined within a given framework

Gradient expansion

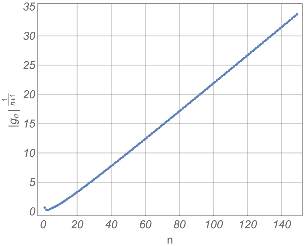
n	RTA	BRSSS	AH I	AH II
0	$2/3$	$2/3$	$2/3$	$2/3$
1	$4/45$	$4/45$	$4/45$	$4/45$
2	$16/945$	$16/945$	$8/945$	$16/945$
3	$-208/4725$	$-1712/99225$	$-184/4725$	$-176/6615$
3	-0.044	-0.017	-0.039	-0.027

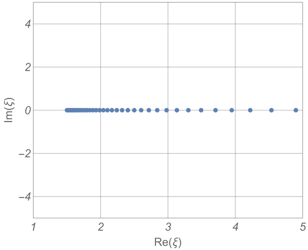
1) AH1, for n=2 too small (by a factor of two) but for n=3 quite close to RTA
2) AH2 reproduces exactly the first three terms of RTA, not too bad for n=3

Gradient expansion

the series f_n, has vanishing radius of convergence, the Borel transform of f is introduced, analytic continuation using diagonal Padé approximants of order 70 is done

$$g_B(\xi) = \sum_{n=0}^{\infty} \frac{f_n}{n!} \xi^n, \tag{1}$$





the cut along the real axis indicates the presence of a single nonhydrodynamic mode, which is is purely decaying, as in MIS theory (Romatschke, Heller).

Gradient expansion

1. Suppose that we treat RTA results on the kinetic coefficients as experimentally measured data, the constructed hydro series does not converge and is bad approximation for real solutions (lack of non hydrodynamic modes) – this case shows the limits of using hydro without any information from the underlying microscopic theory

2. Hydrodynamics tailored to a given microscopic theory has certain advantages – includes non hydrodynamic modes specific for a given theory and can be treated as a good approximation for the original theory. Anisotropic hydro is an example of a hydrodynamic approach that is adjusted to reproduce the RTA kinetic theory.

4. Relativistic fluid dynamics with spin

WF, Bengt Friman, Amaresh Jaiswal, Enrico Speranza, arXiv:1705.00587

Motivation

- **Non-central heavy-ion collisions create fireballs with large global angular momenta** which may generate a spin polarization of the hot and dense matter (Einstein-de Haas and Barnett effects)
- **Much effort has recently been invested in studies of polarization and spin dynamics of particles produced in high-energy nuclear collisions**, both from the experimental and theoretical point of view

L. Adamczyk et al. (**STAR**), (2017), arXiv:1701.06657 [nucl-ex], to appear in **Nature**
Global Λ hyperon polarization in nuclear collisions:
evidence for the most vortical fluid

www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

Local distribution functions

Our starting point: **phase-space distribution functions for spin-1/2 particles** and antiparticles in local equilibrium. In order to incorporate the spin degrees of freedom, they have been **generalized from scalar functions to two by two spin density matrices** for each value of the space-time position x and momentum p, **F. Becattini et al., Annals Phys. 338 (2013) 32**

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^+u_s(p), \quad f^-_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^-v_r(p)$$

Following the notation used by F. Becattini et al., we introduce the matrices

$$X^\pm = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right] M^\pm$$

where

$$M^\pm = \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\hat{\Sigma}^{\mu\nu}\right]$$

Here we use the notation $\beta^\mu = u^\mu/T$ and $\xi = \mu/T$, with the temperature T, chemical potential μ and four velocity u^μ. The latter is normalized to $u^2 = 1$. Moreover, $\omega_{\mu\nu}$ is the spin tensor, while $\hat{\Sigma}^{\mu\nu}$ is the spin operator expressed in terms of the Dirac gamma matrices, $\hat{\Sigma}^{\mu\nu} = (i/4)[\gamma^\mu, \gamma^\nu]$.

Spin/polarization tensor

$$\omega_{\mu\nu} \equiv k_\mu u_\nu - k_\nu u_\mu + \epsilon_{\mu\nu\beta\gamma} u^\beta \omega^\gamma .$$

We can assume that both k_μ and ω_μ are orthogonal to u^μ, i.e., $k \cdot u = \omega \cdot u = 0$,

$$k_\mu = \omega_{\mu\nu} u^\nu , \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \, \omega^{\nu\alpha} u^\beta .$$

It is convenient to introduce the dual spin tensor $\tilde{\omega}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta}$.
One finds $\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} = k \cdot k - \omega \cdot \omega$ and $\frac{1}{2}\tilde{\omega}_{\mu\nu}\omega^{\mu\nu} = 2k \cdot \omega$. Using the constraint

$$k \cdot \omega = 0$$

we find the compact form

$$M^\pm = \cosh(\zeta) \pm \frac{\sinh(\zeta)}{2\zeta} \, \omega_{\mu\nu} \hat{\Sigma}^{\mu\nu} , \tag{2}$$

where

$$\zeta \equiv \frac{1}{2} \sqrt{k \cdot k - \omega \cdot \omega} . \tag{3}$$

We now assume also that $k \cdot k - \omega \cdot \omega \geq 0$, which implies that ζ is real.

Charge current

The **charge current** [S. de Groot, W. van Leeuwen, and C. van Weert]

$$N^\mu = \int \frac{d^3p}{2(2\pi)^3 E_p} p^\mu \left[\mathrm{tr}(\mathrm{X}^+) - \mathrm{tr}(\mathrm{X}^-)\right] = n u^\mu$$

where 'tr' denotes the trace over spinor indices and n is the charge density

$$n = 4\,\cosh(\zeta)\,\sinh(\xi)\,n_{(0)}(T) = 2\,\cosh(\zeta)\left(e^{\xi} - e^{-\xi}\right)\,n_{(0)}(T)$$

Here $n_{(0)}(T) = \langle (u \cdot p)\rangle_0$ is the number density of spin 0, neutral Boltzmann particles, obtained using the thermal average

$$\langle \cdots \rangle_0 \equiv \int \frac{d^3p}{(2\pi)^3 E_p} (\cdots)\, e^{-\beta \cdot p},$$

where $E_p = \sqrt{m^2 + \mathbf{p}^2}$.

Energy-momentum tensor

The **energy-momentum tensor** for a perfect fluid then has the form

$$T^{\mu\nu} = \int \frac{d^3p}{2(2\pi)^3 E_p} p^\mu p^\nu \left[\mathrm{tr}(\mathrm{X}^+) + \mathrm{tr}(\mathrm{X}^-)\right] = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu},$$

where the energy density and pressure are given by

$$\varepsilon = 4\,\cosh(\zeta)\,\cosh(\xi)\,\varepsilon_{(0)}(T)$$

and

$$P = 4\,\cosh(\zeta)\,\cosh(\xi)\,P_{(0)}(T),$$

respectively. In analogy to the density $n_{(0)}(T)$, we define the auxiliary quantities $\varepsilon_{(0)}(T) = \langle (u\cdot p)^2\rangle_0$ and $P_{(0)}(T) = -(1/3)\langle \left[p\cdot p - (u\cdot p)^2\right]\rangle_0$.

Entropy current

The **entropy current** is given by an obvious generalization of the Boltzmann expression

$$S^\mu = -\int \frac{d^3p}{2(2\pi)^3 E_p}\, p^\mu \left(\mathrm{tr}\left[\mathrm{X}^+(\ln \mathrm{X}^+ - 1)\right] + \mathrm{tr}\left[\mathrm{X}^-(\ln \mathrm{X}^- - 1)\right] \right)$$

This leads to the following entropy density

$$s = u_\mu S^\mu = \frac{\mathcal{E} + P - \mu\, n - \Omega w}{T},$$

where Ω is defined through the relation $\zeta = \Omega/T$ and

$$w = 4\,\sinh(\zeta)\cosh(\xi)\, n_{(0)}.$$

This suggests that Ω should be used as a thermodynamic variable of the grand canonical potential, in addition to T and μ. Taking the pressure P to be a function of T, μ and Ω, we find

$$s = \left.\frac{\partial P}{\partial T}\right|_{\mu,\Omega}, \quad n = \left.\frac{\partial P}{\partial \mu}\right|_{T,\Omega}, \quad w = \left.\frac{\partial P}{\partial \Omega}\right|_{T,\mu}.$$

Basic conservation laws

The conservation of energy and momentum requires that

$$\partial_\mu T^{\mu\nu} = 0.$$

This equation can be split into two parts, one longitudinal and the other transverse with respect to u^μ:

$$\begin{aligned} \partial_\mu[(\mathcal{E}+P)u^\mu] &= u^\mu \partial_\mu P \equiv \frac{dP}{d\tau}, \\ (\mathcal{E}+P)\frac{du^\mu}{d\tau} &= (g^{\mu\alpha} - u^\mu u^\alpha)\partial_\alpha P. \end{aligned} \tag{4}$$

Evaluating the derivative on the left-hand side of the first equation we find

$$T\,\partial_\mu(su^\mu) + \mu\,\partial_\mu(nu^\mu) + \Omega\,\partial_\mu(wu^\mu) = 0. \tag{5}$$

The middle term vanishes due to charge conservation,

$$\partial_\mu(nu^\mu) = 0. \tag{6}$$

Thus, in order to have entropy conserved in our system (for the perfect-fluid description we are aiming at), we demand that

$$\partial_\mu(wu^\mu) = 0. \tag{7}$$

Consequently, we self-consistently arrive at the equation for conservation of entropy, $\partial_\mu(su^\mu) = 0$.

Spin dynamics

Since we use a symmetric form of the energy-momentum tensor $T^{\mu\nu}$, the spin tensor $S^{\lambda,\mu\nu}$ satisfies the conservation law,

$$\partial_\lambda S^{\lambda,\mu\nu} = 0.$$

For $S^{\lambda,\mu\nu}$ we use the form taken from the textbook by deGroot,

$$S^{\lambda,\mu\nu} = \int \frac{d^3p}{2(2\pi)^3 E_p} p^\lambda \, \mathrm{tr} \left[(X^+ - X^-) \hat{\Sigma}^{\mu\nu} \right] = \frac{w u^\lambda}{4\zeta} \omega^{\mu\nu}$$

Using the conservation law for the spin density and introducing the rescaled spin tensor $\bar{\omega}^{\mu\nu} = \omega^{\mu\nu}/(2\zeta)$, we obtain

$$u^\lambda \partial_\lambda \bar{\omega}^{\mu\nu} = \frac{d\bar{\omega}^{\mu\nu}}{d\tau} = 0,$$

with the normalization condition $\bar{\omega}_{\mu\nu} \bar{\omega}^{\mu\nu} = 2$.

Vortex solution 1

The hydrodynamic flow is defined by the four-vector u^μ with the components

$$u^0 = \gamma, \quad u^1 = -\gamma\,\tilde{\Omega}\,y, \quad u^2 = \gamma\,\tilde{\Omega}\,x, \quad u^3 = 0,$$

where $\tilde{\Omega}$ is a constant, $\gamma = 1/\sqrt{1-\tilde{\Omega}^2 r^2}$, and r denotes the distance from the center of the vortex in the transverse plane, $r^2 = x^2 + y^2$. Due to limiting light speed, the assumed flow profile may be realised only within a cylinder with the radius $R < 1/\tilde{\Omega}$. The total time (convective) derivative takes the form

$$\frac{d}{d\tau} = u^\mu \partial_\mu = -\gamma\tilde{\Omega}\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right).$$

Equation (8) can be used to find the fluid acceleration

$$a^\mu = \frac{du^\mu}{d\tau} = -\gamma^2\tilde{\Omega}^2(0, x, y, 0).$$

As expected the spatial part of the four-acceleration points towards the centre of the vortex, as it describes the centripetal acceleration.

Vortex solution 1

It is easy to see that the equations of the hydrodynamic background are satisfied if T, μ and Ω are proportional to the Lorentz-γ factor

$$T = T_0\gamma, \quad \mu = \mu_0\gamma, \quad \Omega = \Omega_0\gamma,$$

with T_0, μ_0 and Ω_0 being constants. **One possibility is that the vortex represents an unpolarized fluid with $\omega_{\mu\nu} = 0$ and thus, with $\Omega_0 = 0$.**

Another possibility is that the particles in the fluid are polarized and $\Omega_0 \neq 0$. In the latter case we expect that the spin tensor has the structure

$$\omega_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\tilde{\Omega}/T_0 & 0 \\ 0 & \tilde{\Omega}/T_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where the parameter T_0 has been introduced to keep $\omega_{\mu\nu}$ dimensionless. This form yields $k^\mu = \tilde{\Omega}^2(\gamma/T_0)\,(0, x, y, 0)$ and $\omega^\mu = \tilde{\Omega}(\gamma/T_0)\,(0, 0, 0, 1)$. As a consequence, we find $\zeta = \tilde{\Omega}/(2T_0)$, which, for consistency with the hydrodynamic background equations, implies

$$\tilde{\Omega} = 2\,\Omega_0.$$

Historical remarks

works on fluids with spin started in Krakow in 1930s

Jan Weyssenhoff (1889-1972), professor of physics in Wilno, Lwow, and Krakow, supervisor of Andrzej Bialas

Myron Mathisson (1897-1940), brought by Weyssenhoff from Kazan to Krakow in 1935, left for France and England in 1939, Dirac edited his last papers

Józef Lubański (1914-1946) worked with Mathisson and Weyssenhoff in Krakow in about 1937, later worked in Holland with Kramers, Belinfante and Rosenfeld, known from Pauli-Lubanski four-vector

Antoni Raabe (1915-1941) brought by Weyssenhoff from Lwow to Krakow in 1940, perished at Auschwitz

5. Summary

1. Enormous progress in the field of relativistic hydrodynamics
2. No fast thermalisation required to describe (most of) the data