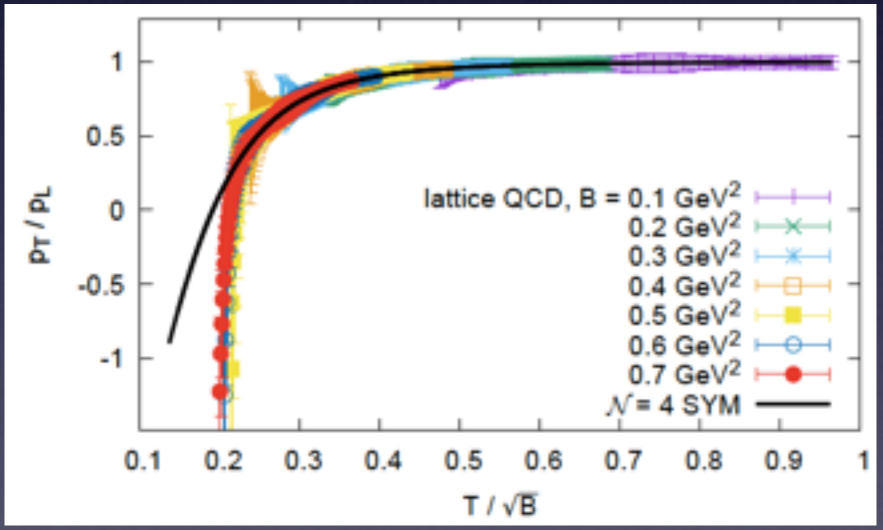
Strengths of the holographic approach at high baryon densities

EMMI Nuclear and Quark Matter Seminar, GSI, Darmstadt

July 25th, 2018

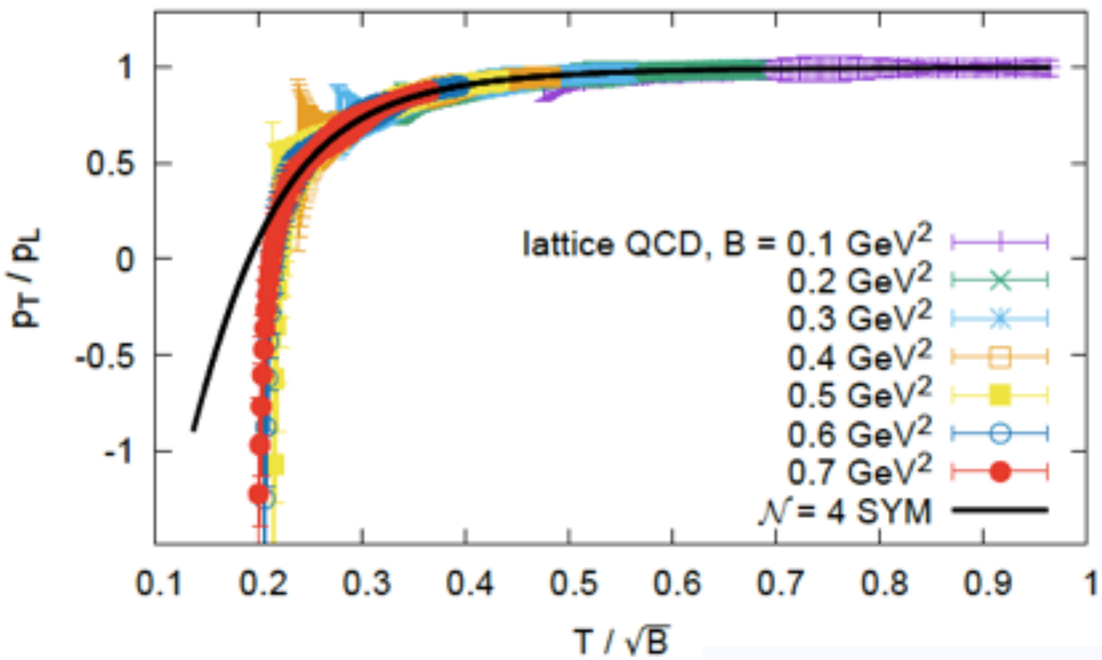


Matthias Kaminski

University of Alabama

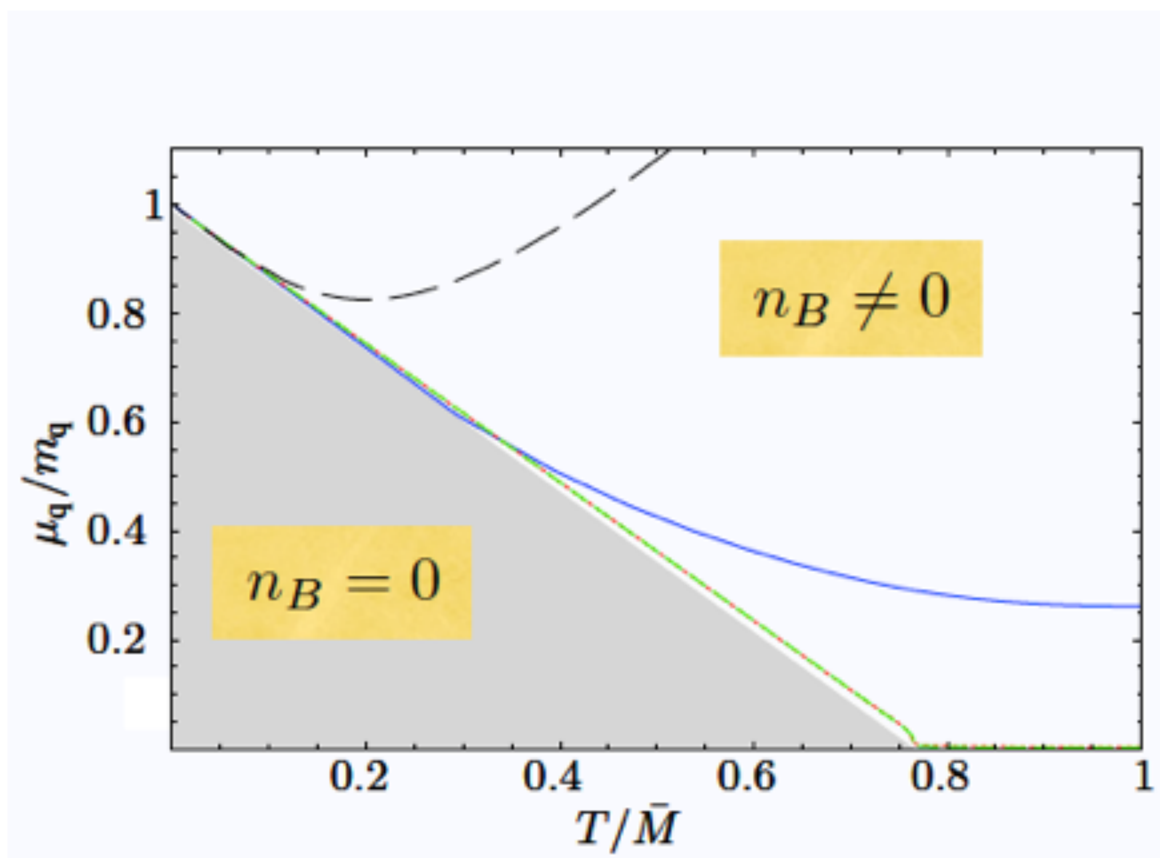
Experiments

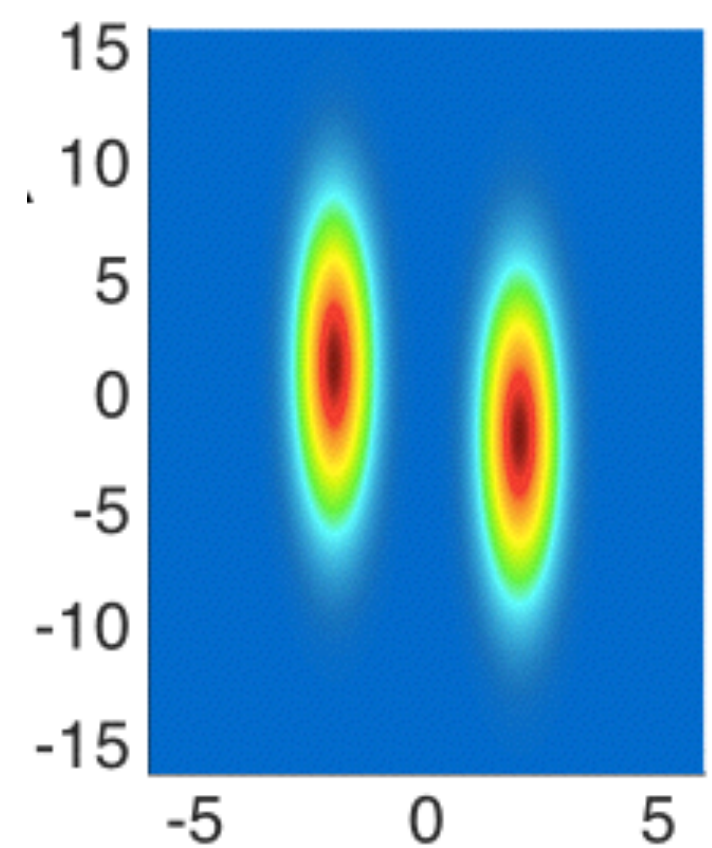
Teaser



Problems at strong coupling

- non-perturbative regime
- lattice sign problem
- far from equilibrium
- near critical point



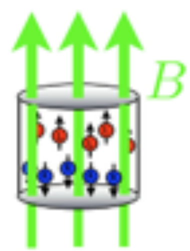


Goals

- compare to experimental data
- understand physical mechanisms

Outline

1. Holography primer 

2. Results 

    2.1 Phase transitions and critical points
    2.2 Mesons at finite densities
    2.3 Magnetic field — anisotropic hydrodynamics
    2.4 Chiral transport effects
    2.5 Holographic heavy ion collision - correlations

3. Discussion

1. Holography primer

Basic idea

Assume we have a hard problem that is difficult to solve in a given theory, for example the **standard model**

model or effective description

(Hard) problem in "similar" theory

holography (gauge/gravity correspondence)

Simple problem in a particular gravitational theory

Basic idea

Assume we have a hard problem that is difficult to solve in a given theory, for example the **standard model**

?

- gravity dual to QCD or standard model?
- not known yet

model or effective description

holography (gauge/gravity correspondence)

(Hard) problem in "similar" theory

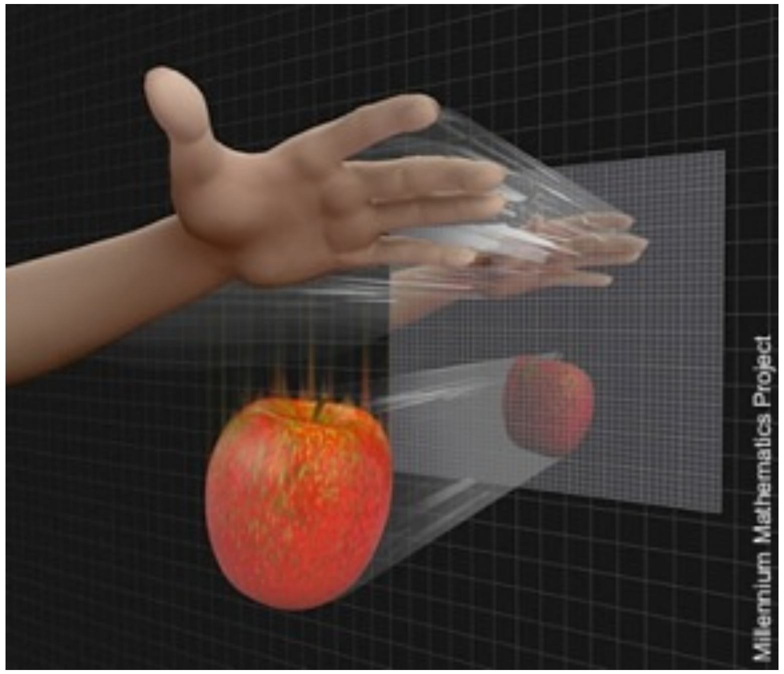
Simple problem in a particular gravitational theory

➡ Holography is good at predictions that are qualitative or universal.

Gauge/Gravity concepts

The Gauge/Gravity correspondence is based on the holographic principle. *['t Hooft (1993)]*

$$S_{max}(\text{volume}) \propto \text{surface area}$$



Gauge/Gravity concepts

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$$S_{max}(\text{volume}) \propto \text{surface area}$$

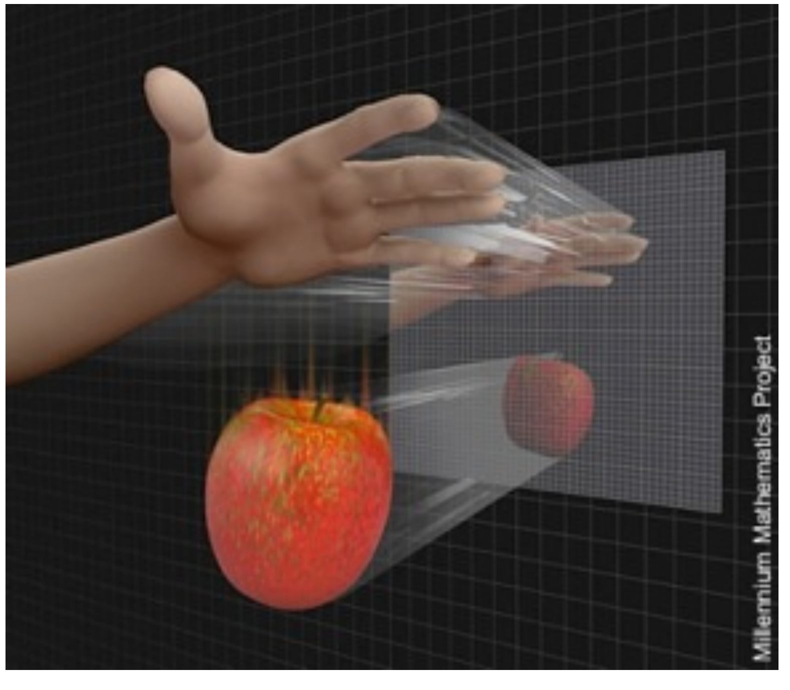
String theory gives one example (AdS/CFT).

N=4 Super-Yang-Mills in 3+1 dimensions (CFT) $\Leftrightarrow$ Typ II B Supergravity in (4+1)-dimensional Anti de Sitter space (AdS)

[Susskind (1995)]
[Maldacena (1997)]

't Hooft limit: number of colors $N \to \infty$
large coupling constant λ

limit and symmetries only needed for this exact conjecture



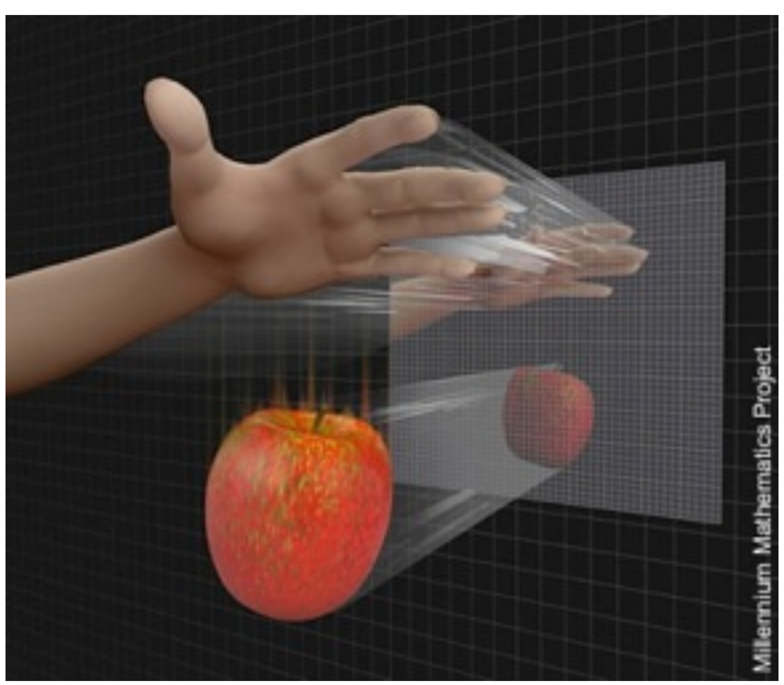
Gauge/Gravity Correspondence

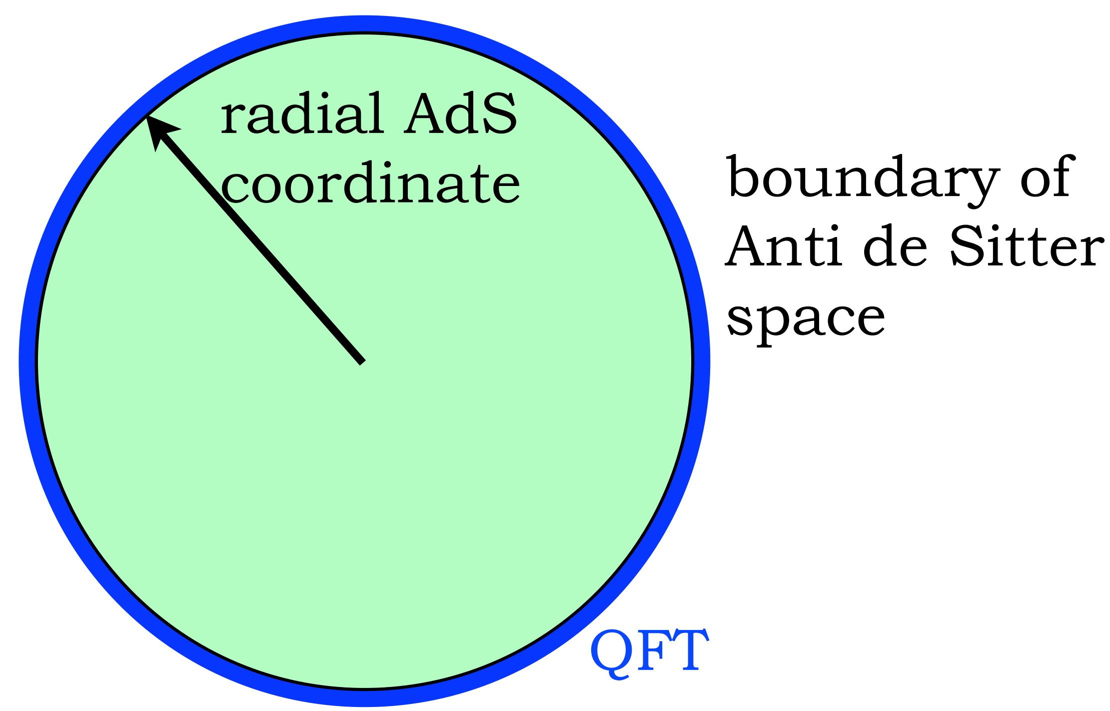
strongly coupled quantum field theory (QFT) —*"CFT"*

correspondence

weakly curved gravity theory —*"AdS"*

renormalization scale ⟷ radial AdS coordinate





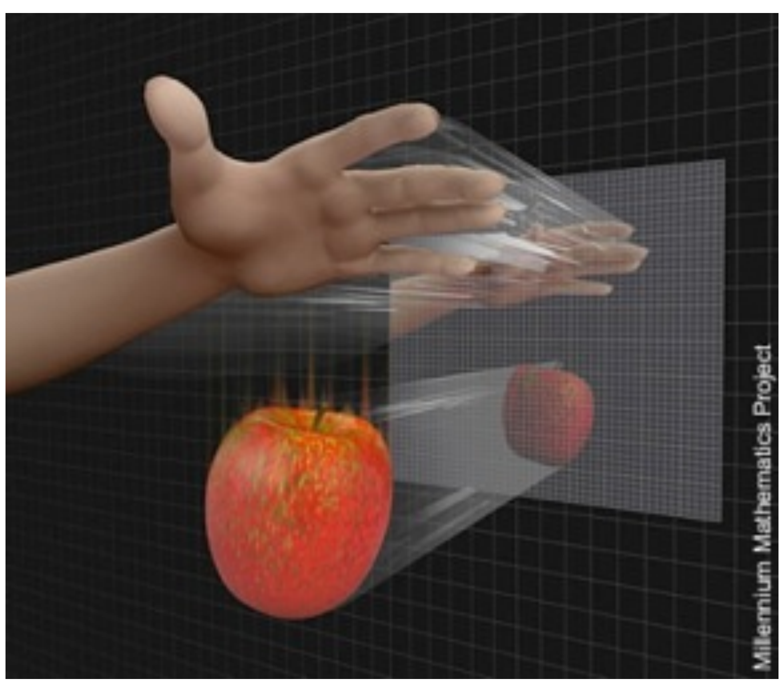
Gauge/Gravity Correspondence

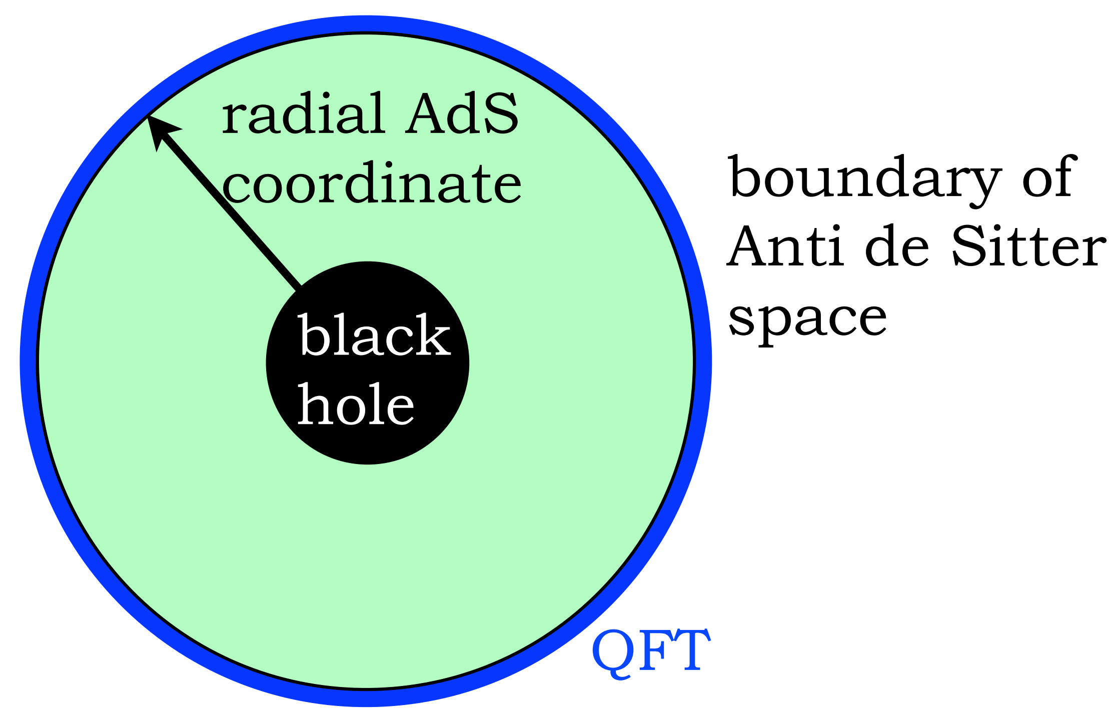
strongly coupled quantum field theory (QFT) —*"CFT"* ⟷ *correspondence* ⟷ weakly curved gravity theory —*"AdS"*

renormalization scale ⟷ radial AdS coordinate

QFT temperature ⟷ Hawking temperature

[Witten (1998)]





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correspondence ⟷

weakly curved gravity theory —*“AdS”*

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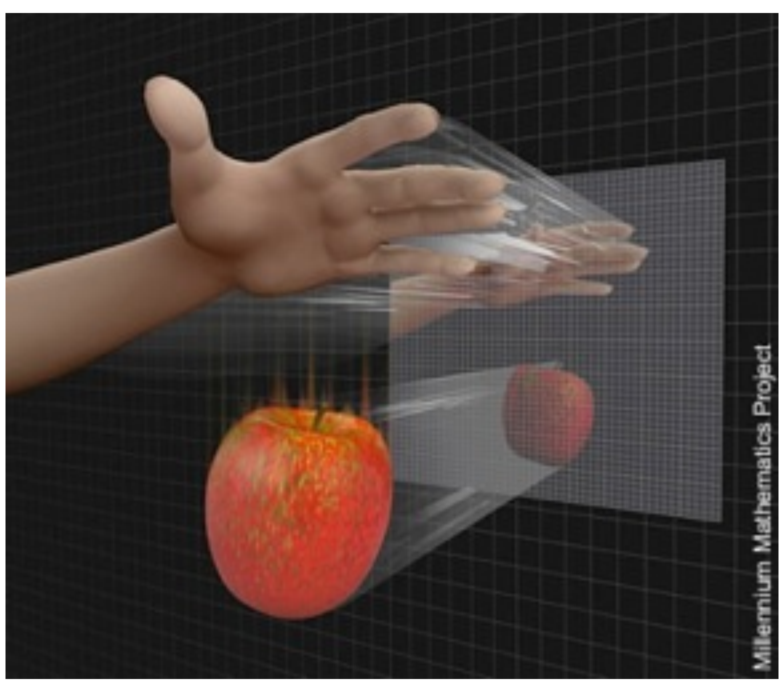
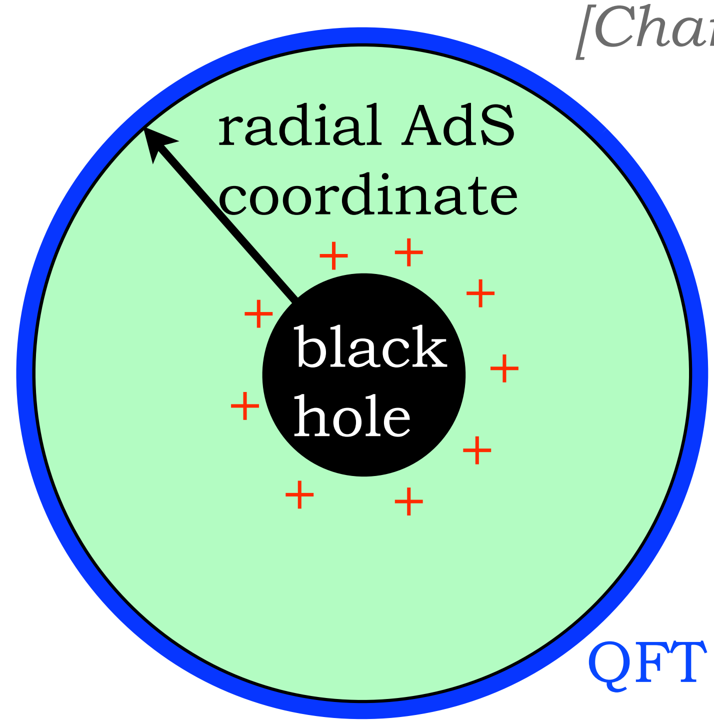
[Witten (1998)]

conserved charge ⟷ charged black hole/brane

[Chamblin et al (1999)]





boundary of Anti de Sitter space

Example: Reissner-Nordström black hole

N=4 Super-Yang-Mills theory at nonzero temperature & charge

correspondence ⟷

metric & gauge field defining a RN black brane (solve Einstein-Maxwell eq's)

metric: $$ds^2 = \frac{r^2}{L^2}\left(-f\,dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f}dr^2$$

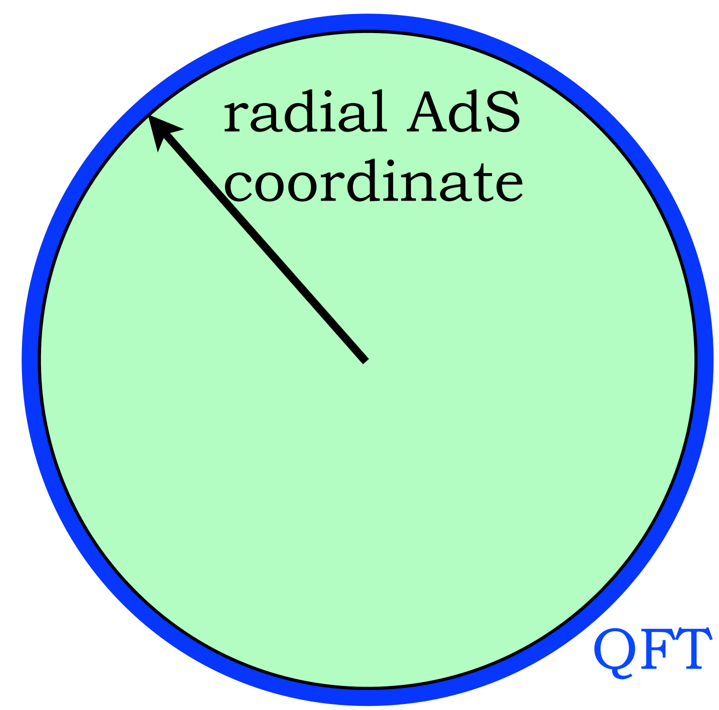
$$T = r_H^2 \frac{|f'(r_H)|}{4\pi}$$

$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2L^2}{r^6}$$

gauge field:

$$A_t = \mu - \frac{Q}{Lr^2}$$

$$\mu = \frac{\sqrt{3}q}{2r_H^2}$$



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QFT temperature:

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metric: $ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2$

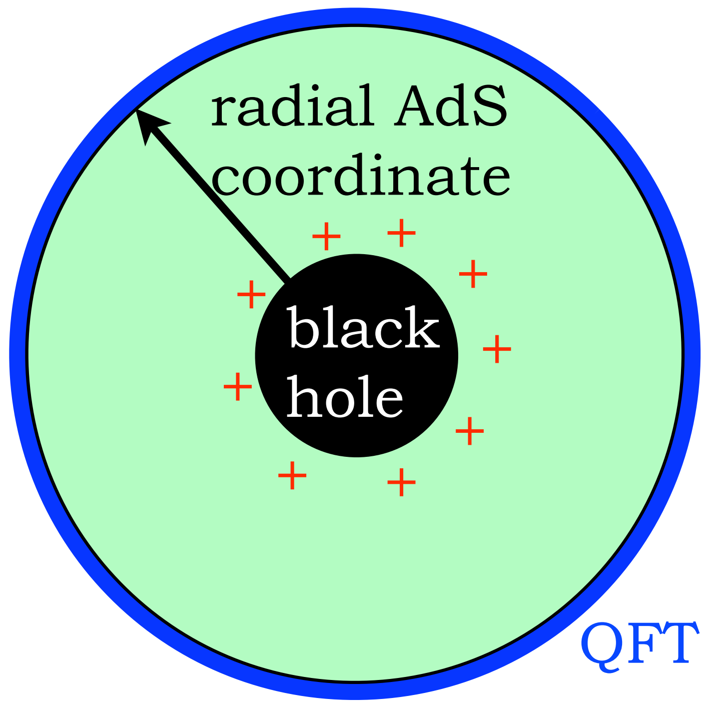
$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

conserved charge Q, thermodynamically dual to chemical potential:

$$\mu = \frac{\sqrt{3}q}{2r_H^2}$$

gauge field:

$$A_t = \mu - \frac{Q}{Lr^2}$$



2. Results

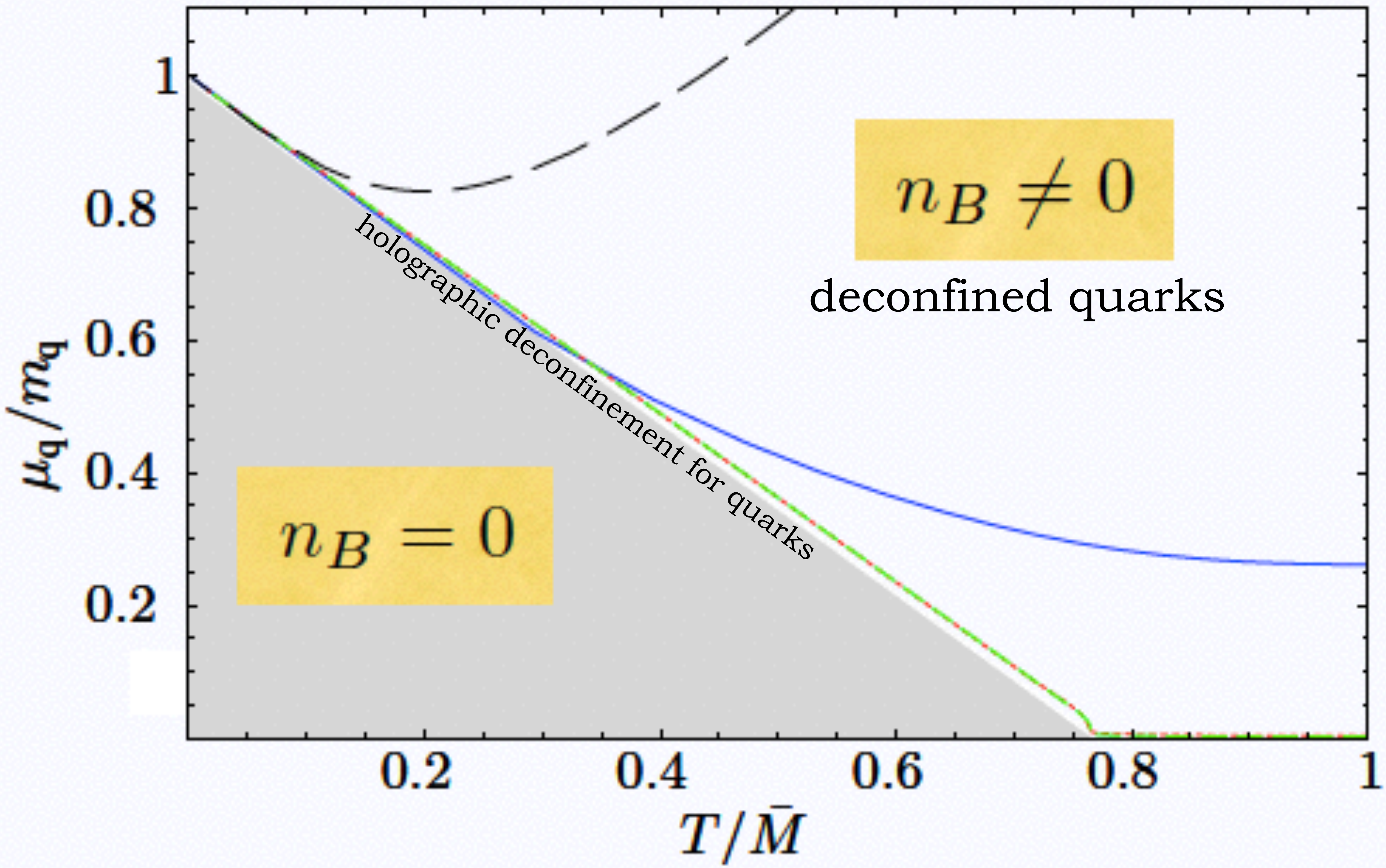
2.1 Phase transitions and critical points
2.2 Mesons at finite densities
2.3 Magnetic field — anisotropic hydrodynamics
2.4 Chiral transport effects
2.5 Holographic heavy ion collision - correlations

2.1 Results - Phase transitions and critical points

Phase diagram for holographic quarks at nonzero baryon potential

[O'Bannon, Karch; (2007)]
[Kobayashi et al; JHEP (2007)]
[Mateos et al; JHEP (2007)]

[Erdmenger, Kaminski, Rust; PRD (2007)]

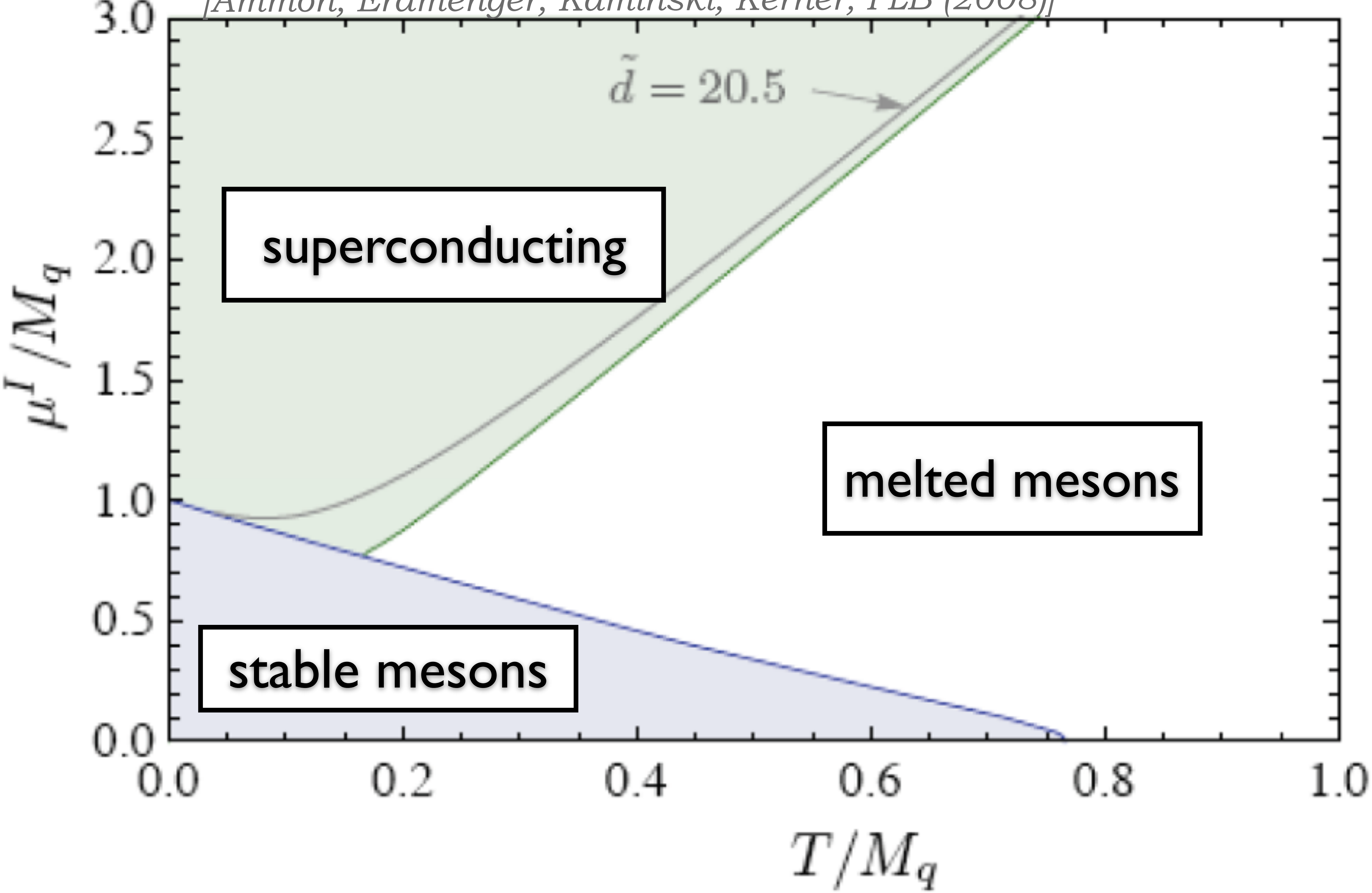


2.1 Results - Phase transitions and critical points

Phase diagram for holographic quarks at nonzero isospin potential

[Erdmenger, Kaminski, Kerner, Rust; JHEP (2008)]

[Ammon, Erdmenger, Kaminski, Kerner; PLB (2008)]



2.1 Results - Phase transitions and critical points

Phase diagram for holographic quarks at nonzero isospin potential

[Erdmenger, Kaminski, Kerner, Rust; JHEP (2008)]

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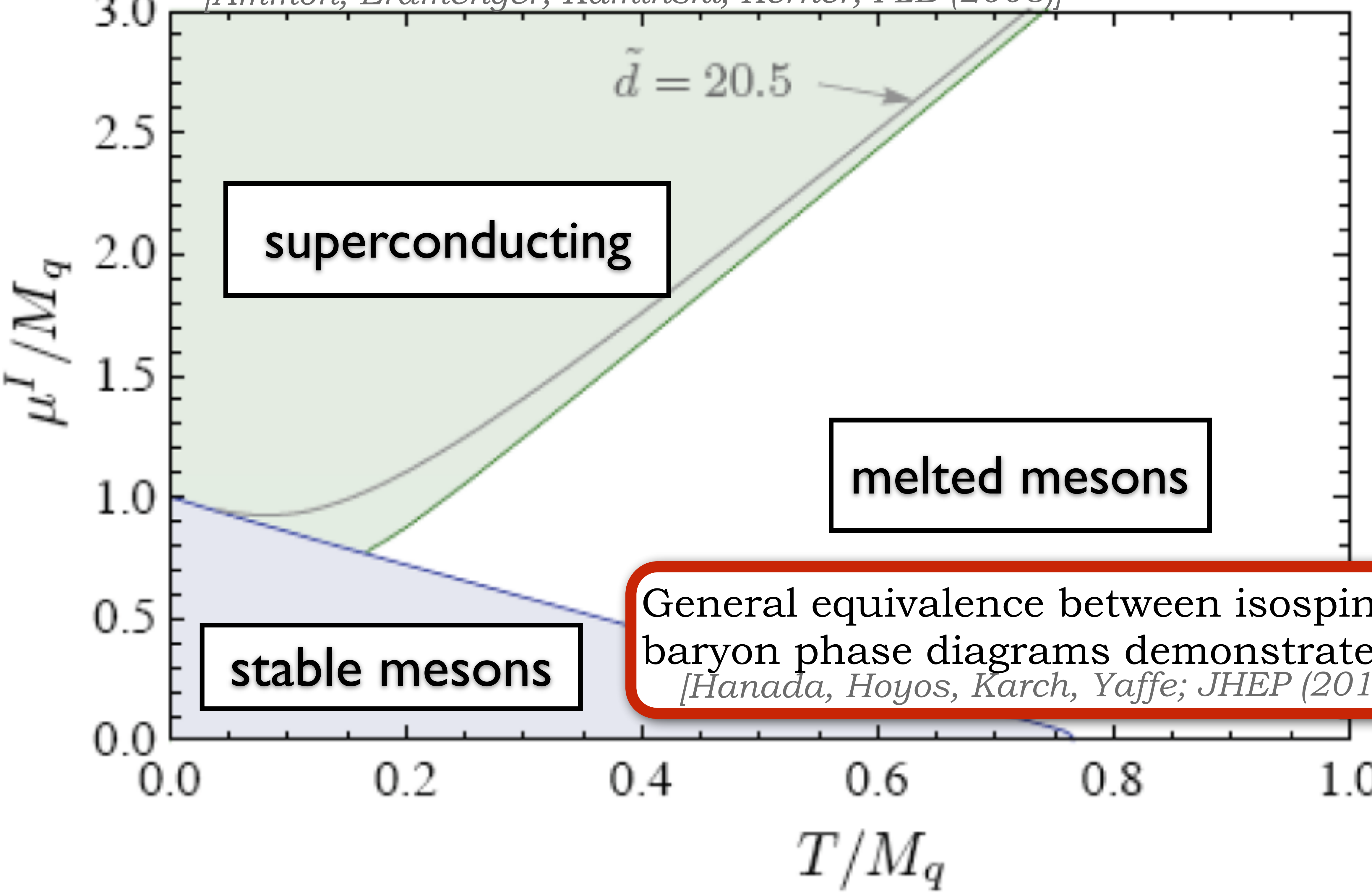
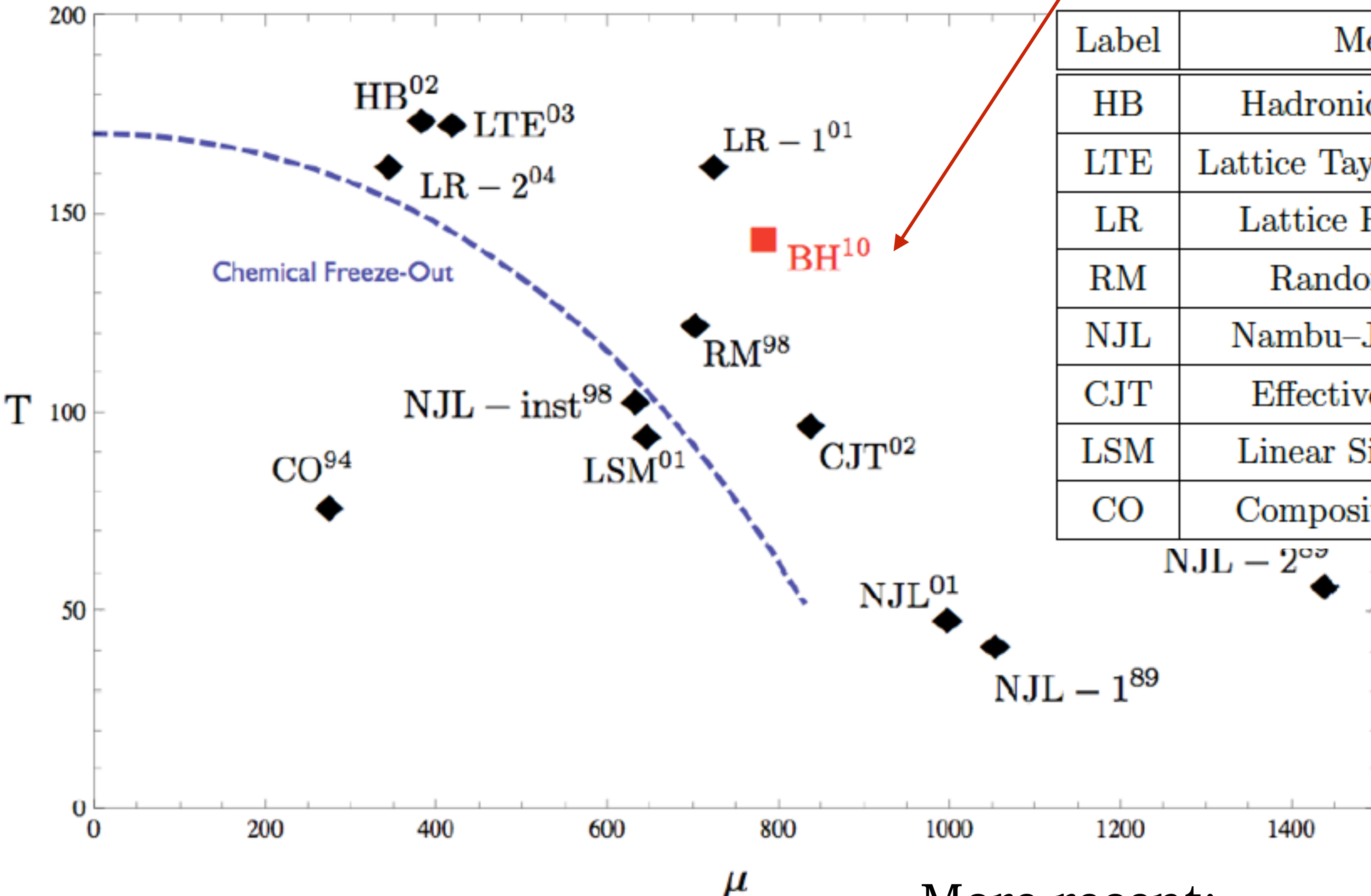


General equivalence between isospin and baryon phase diagrams demonstrated:
[Hanada, Hoyos, Karch, Yaffe; JHEP (2012)]

2.1 Results - Phase transitions and critical points

A holographic critical point

[Gubser, Rosen; PRD (2016)]



Label	Method
HB	Hadronic Bootstrap
LTE	Lattice Taylor Expansion
LR	Lattice Reweighting
RM	Random Matrix
NJL	Nambu–Jona-Lasinio
CJT	Effective Potential
LSM	Linear Sigma Model
CO	Composite Operator

More recent:

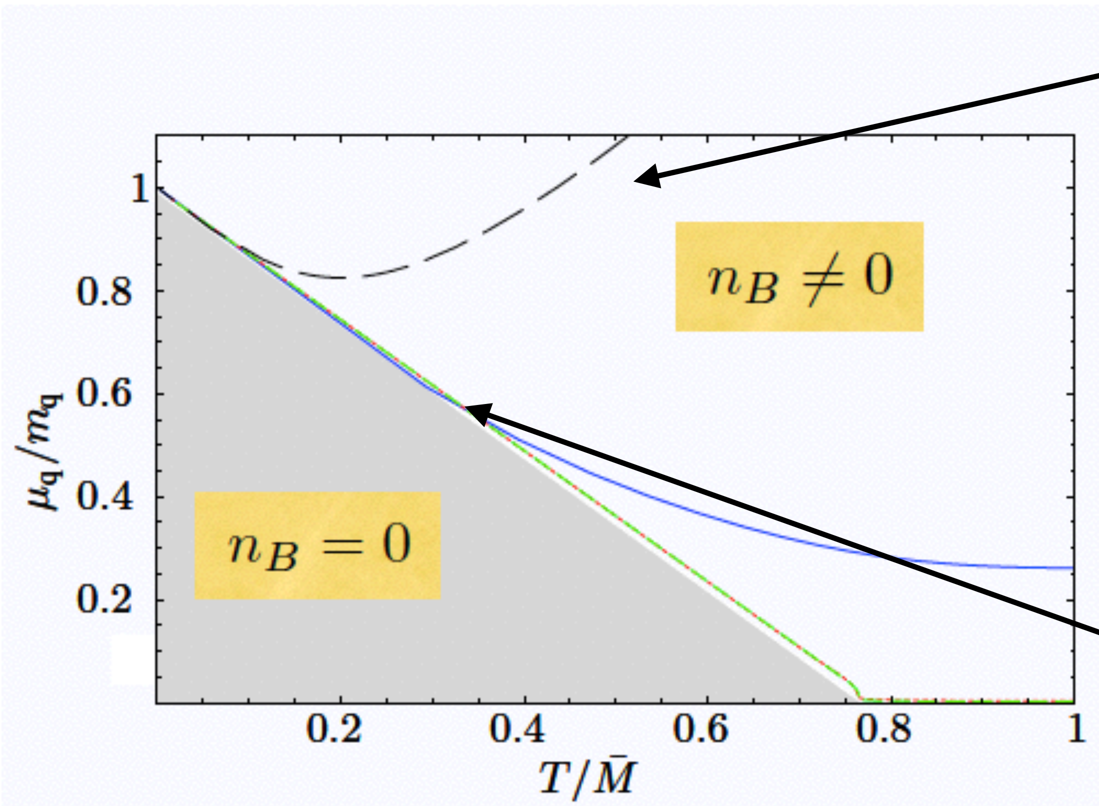
[Knaute, Yaresko, Kaempfer; PLB (2018)]

[Critelli et al; PRD (2017)]

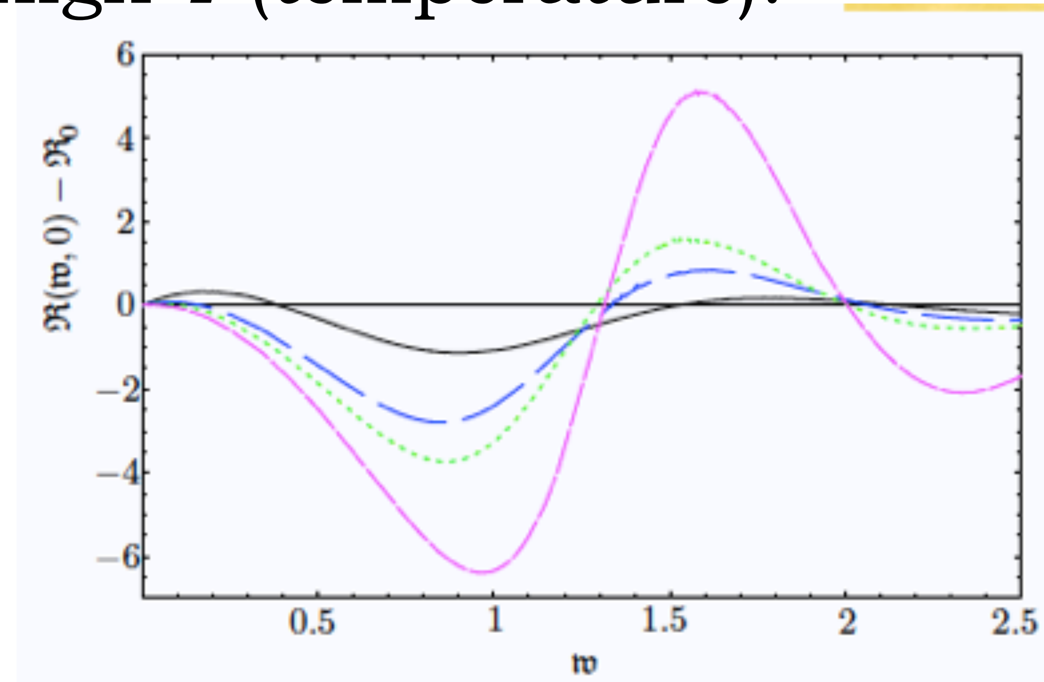
2. Results

2.2 Results - Holographic mesons at finite densities

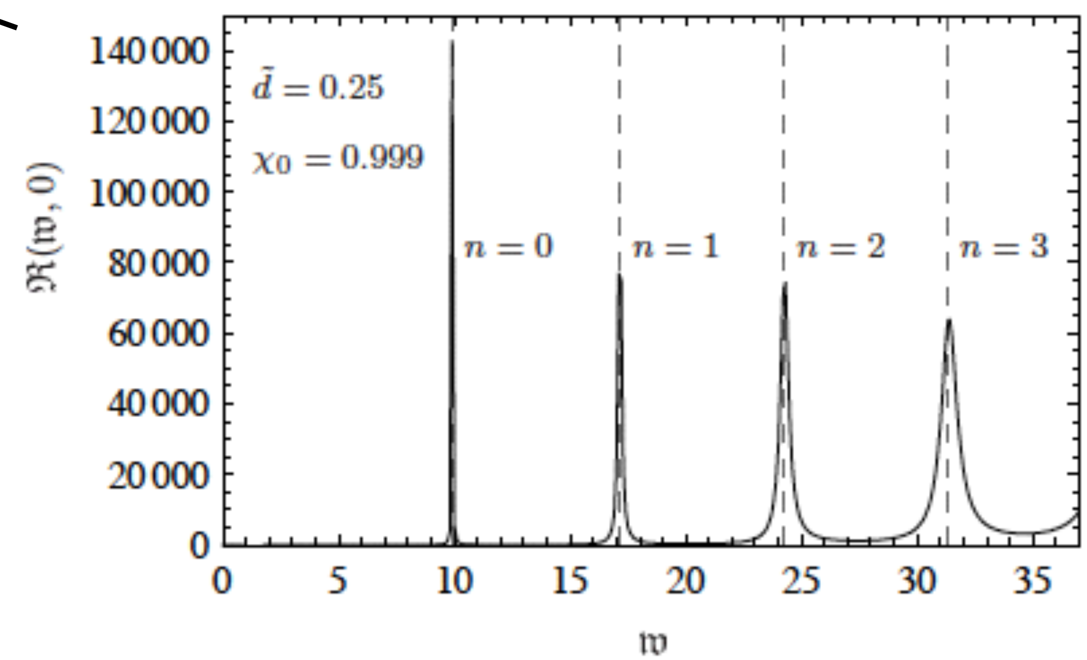
[Erdmenger, Kaminski, Rust; PRD (2007)]



Deconfined mesons at high T (temperature):

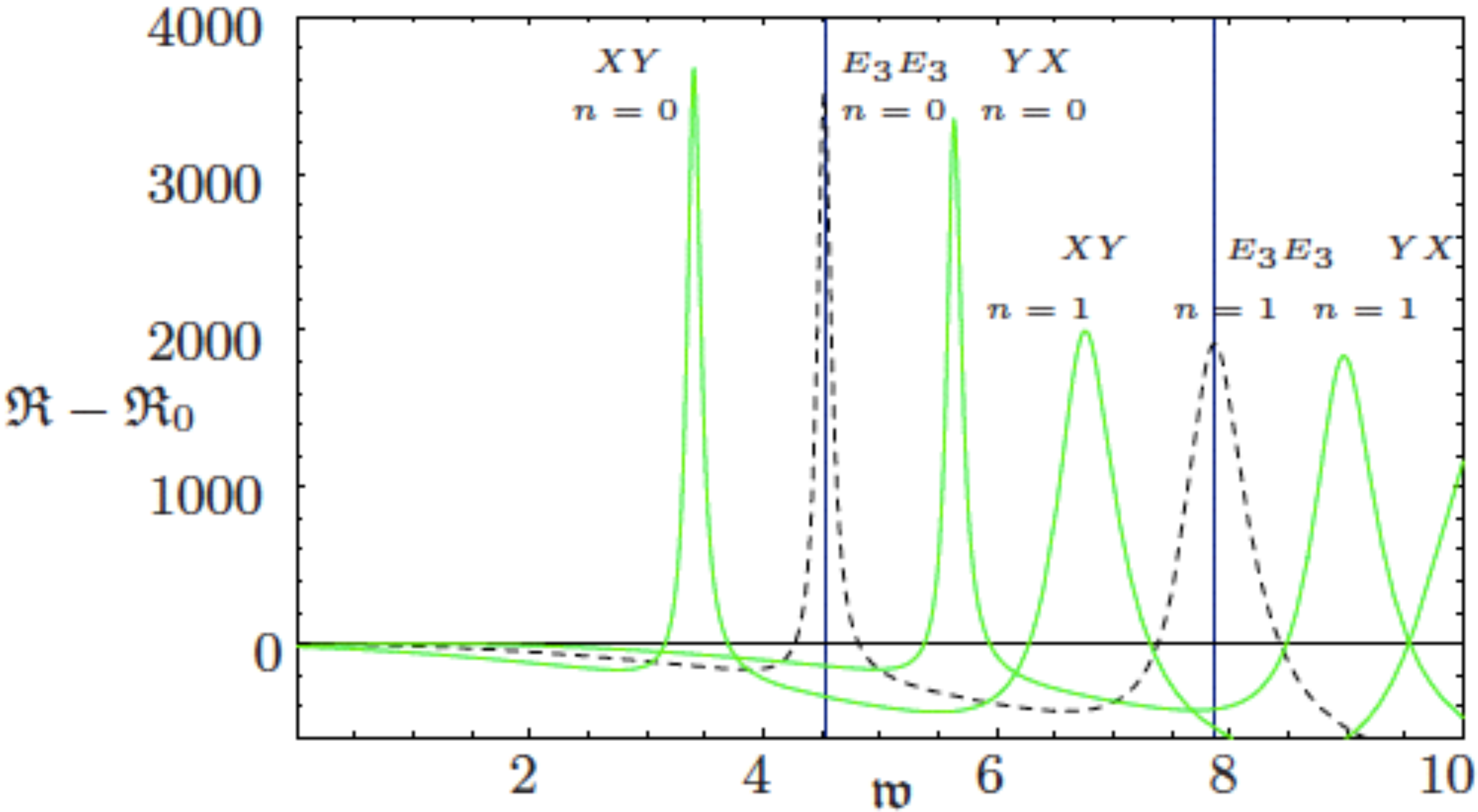


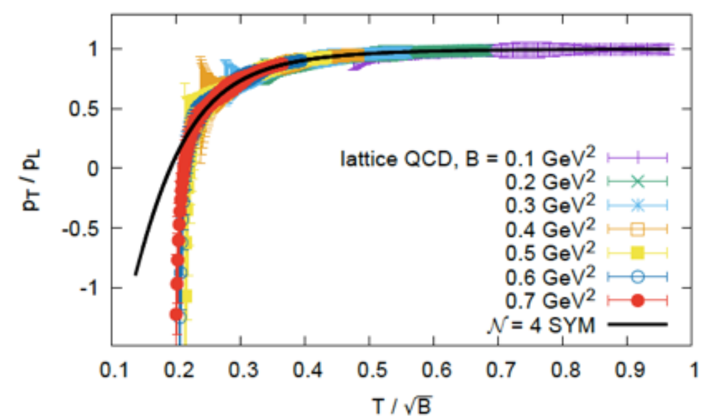
Deconfined mesons near the deconfinement transition, low T:

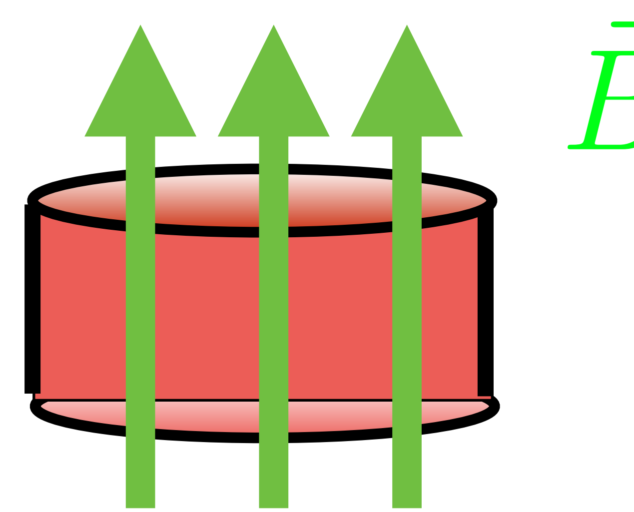


2.2 Results - Holographic mesons at finite densities

Vector meson triplet splitting:







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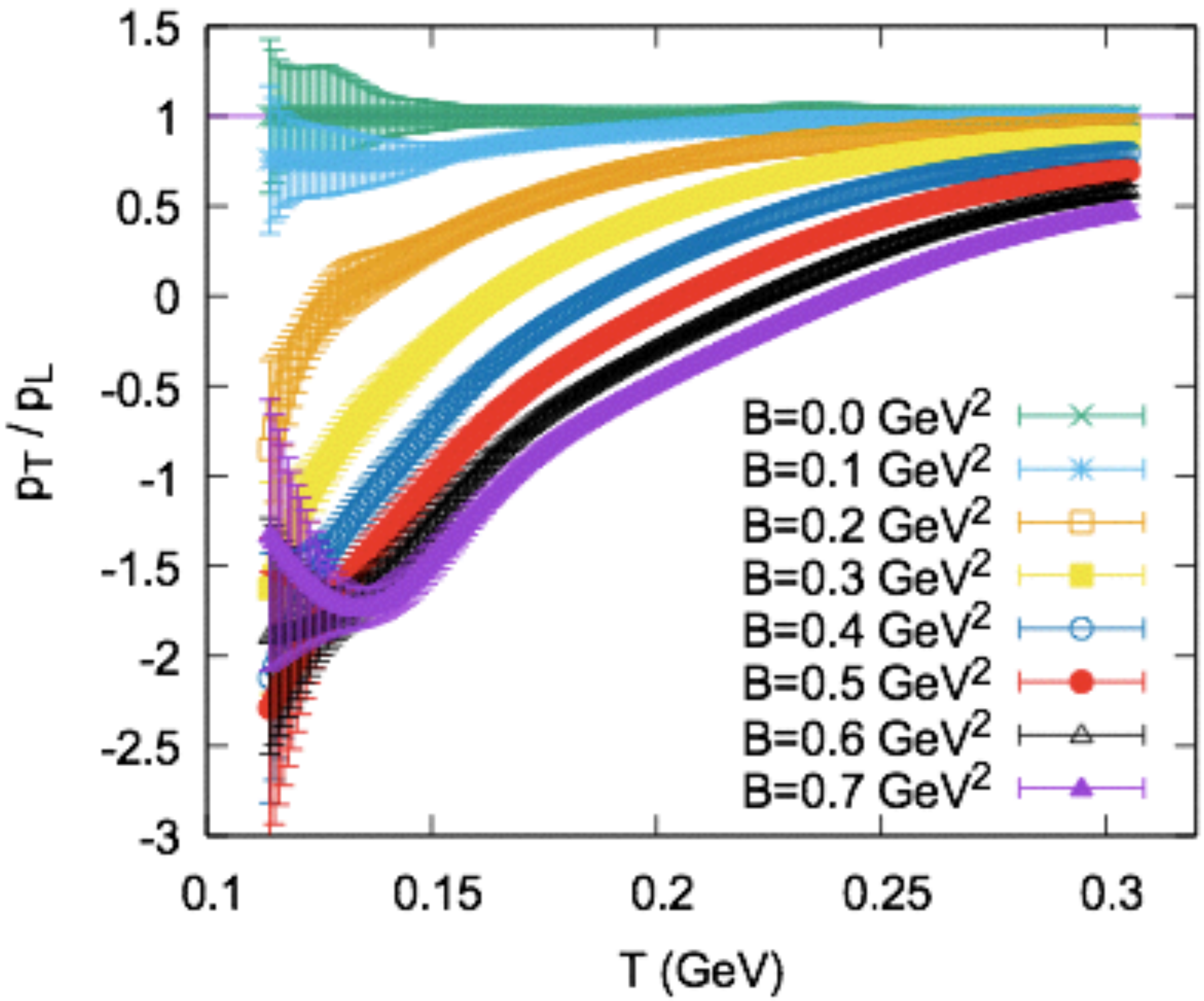
Universal magnetoresponse in QCD ...

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\rm T} = -\frac{L_{\rm T}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm T}}$

longitudinal pressure: $p_{\rm L} = -\frac{L_{\rm L}}{V}\frac{\partial F_{\rm QCD}}{\partial L_{\rm L}}$

$F_{\rm QCD}$ ... *free energy*

$L_{\rm T}$ ... *transverse system size*

$L_{\rm L}$ ... *longitudinal system size*

V ... *system volume*

Universal magnetoresponse in QCD ...

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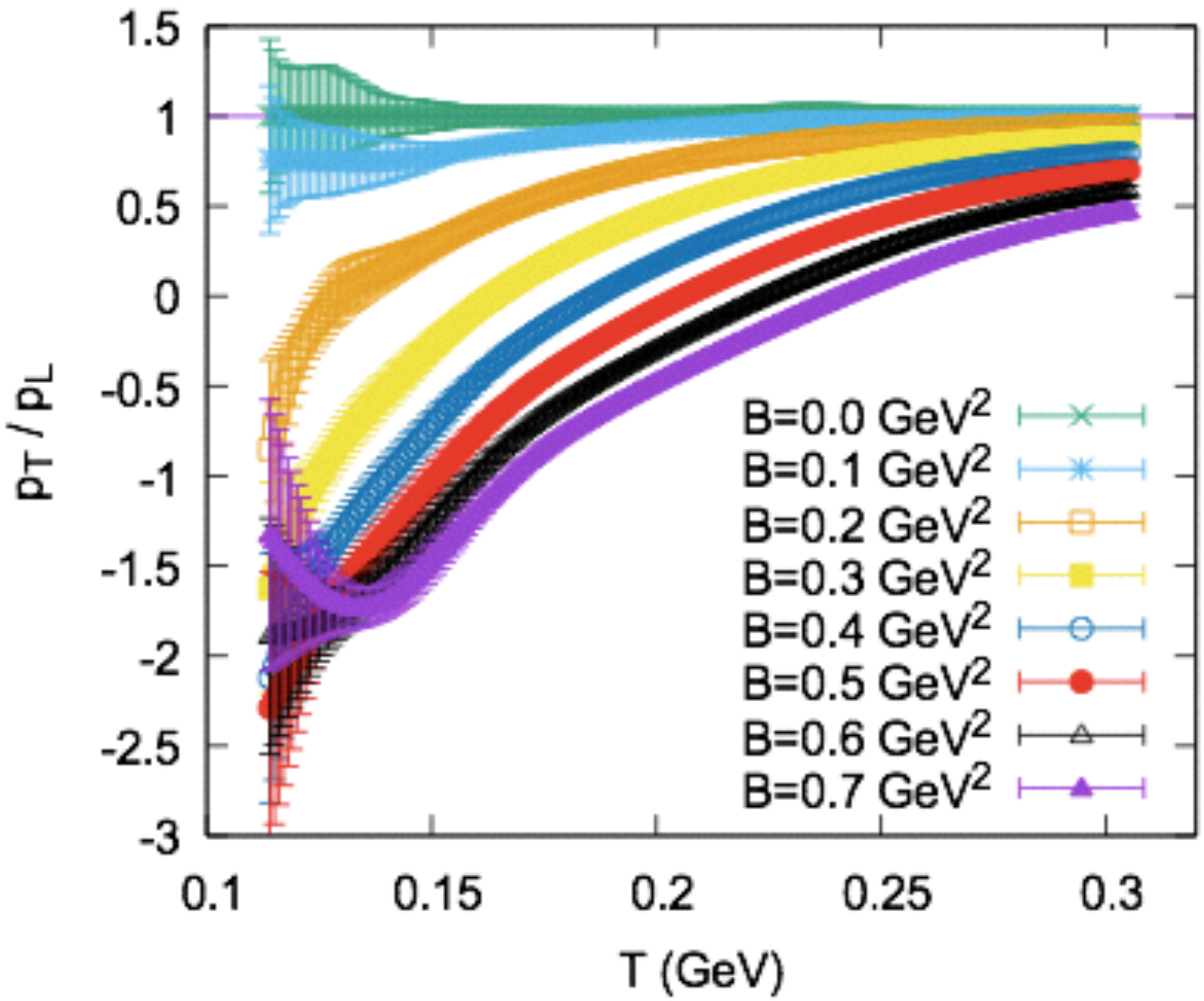
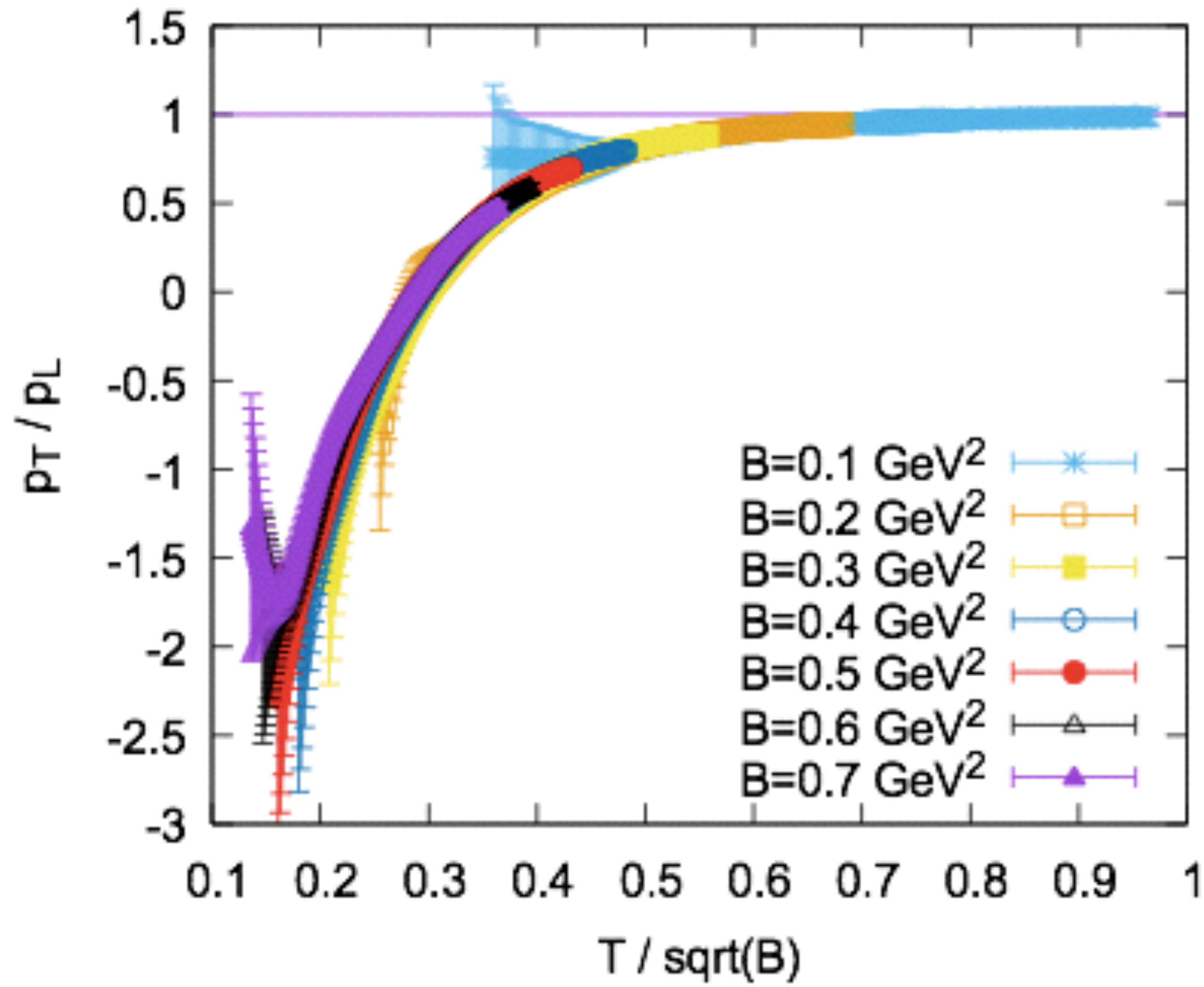




Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$$

longitudinal pressure: $$p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$$

F_{QCD} ... *free energy*

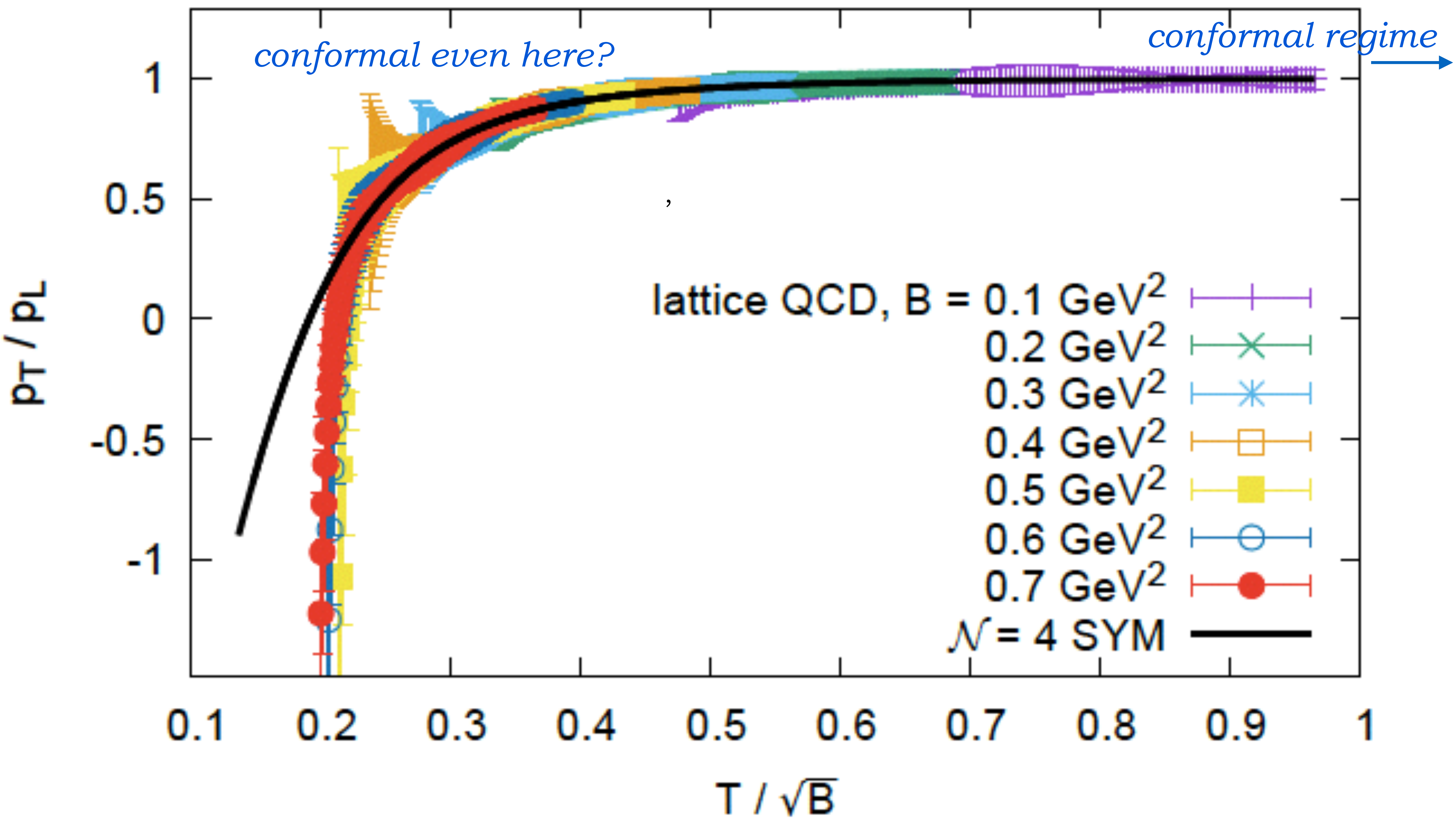
L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

… and $\mathcal{N}$=4 Super-Yang-Mills theory

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Polarized matter in strong magnetic field

Generating functionals $W \sim P$ (pressure) for thermodynamics $\boxed{B \sim \mathcal{O}(1)}$

[Kovtun; JHEP (2016)]

$$T^{\mu\nu} = Pg^{\mu\nu} + (Ts + \mu\rho)u^\mu u^\nu + T^{\mu\nu}_{\text{EM}}$$

$$J^\alpha = \rho u^\alpha - \underbrace{\nabla_\lambda M^{\lambda\alpha}}_{\text{bound current}}$$

$$T^{\mu\nu}_{\text{EM}} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu} \right)$$

[Israel; Gen.Rel.Grav. (1978)]

Polarization tensor:

$$M_{\mu\nu} = p_\mu u_\nu - p_\nu u_\mu - \epsilon_{\mu\nu\rho\sigma} u^\rho m^\sigma \qquad M^{\mu\nu} = 2\frac{\partial P}{\partial F_{\mu\nu}}$$

Including vorticity:
$W \sim M_\omega \, B \cdot \omega$
[Kovtun, Hernandez; JHEP (2017)]

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Including vorticity:
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[Kovtun, Hernandez; JHEP (2017)]

Leads to **(anisotropic) hydrodynamics** and new transport effects:
(incomplete list)

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

$\eta_\perp$	perpendicular shear viscosity
$\eta_{\|\|}$	parallel shear viscosity
$\tilde{\eta}_\perp$	perpendicular Hall viscosity
$\tilde{\eta}_{\|\|}$	parallel Hall viscosity
ζ_1	bulk viscosity
ζ_2	bulk viscosity
η_1	bulk viscosity
η_2	bulk viscosity
$\sigma_\perp$	perpendicular conductivity
$\sigma_{\|\|}$	parallel conductivity
$\tilde{\sigma}$	Hall conductivity

(Anisotropic) hydrodynamics in strong B

$$\boxed{B \sim \mathcal{O}(1)}$$

➡ (anisotropic) hydrodynamics as single framework allows for polarization, magnetization, external vorticity, E, B *(and* chiral anomaly)

[Kovtun, Hernandez; JHEP (2017)]

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

➡ new transport effects (and Kubo formulae)

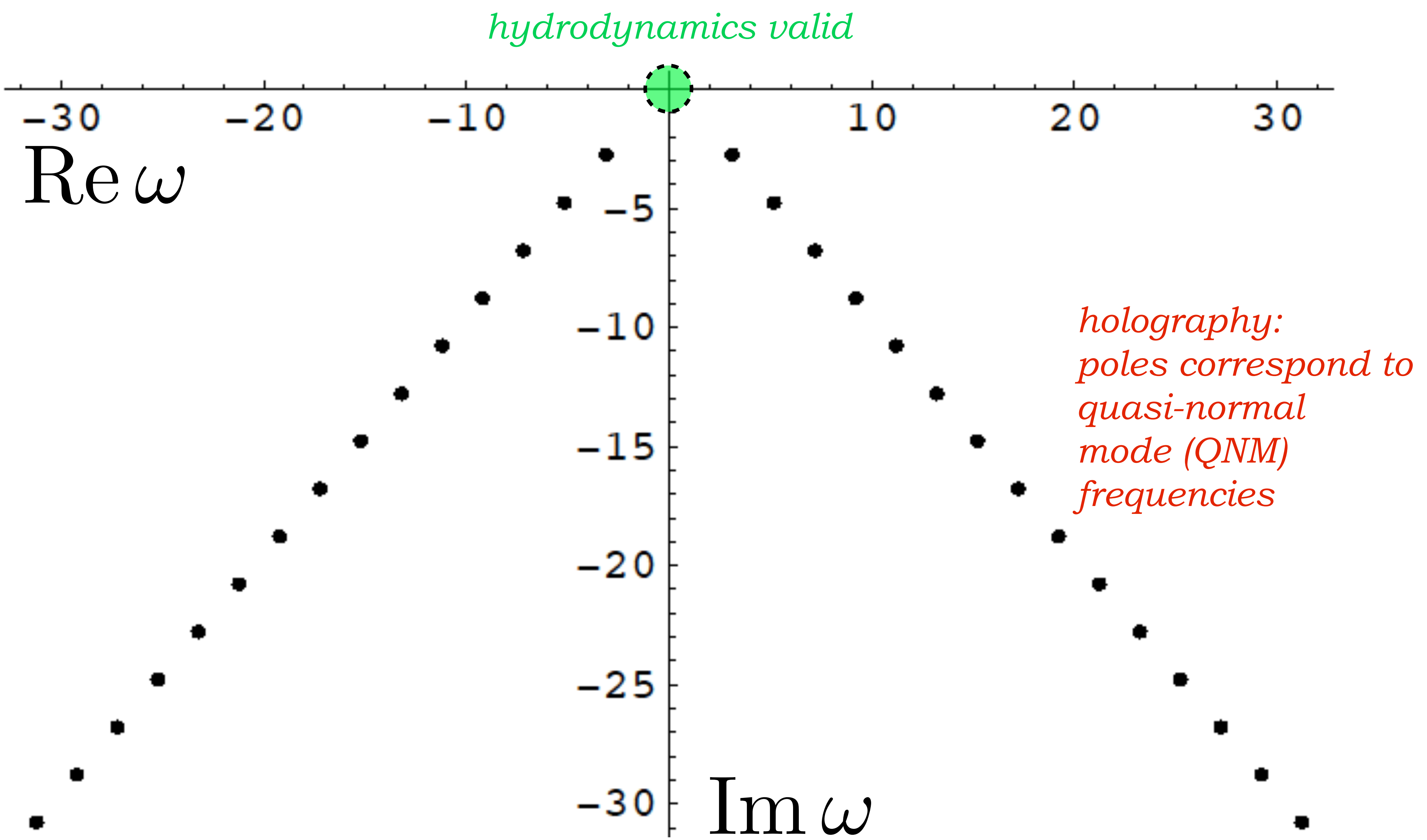
➡ opportunity: dynamical E and B; magnetohydrodynamics

[Kovtun, Hernandez; JHEP (2017)]

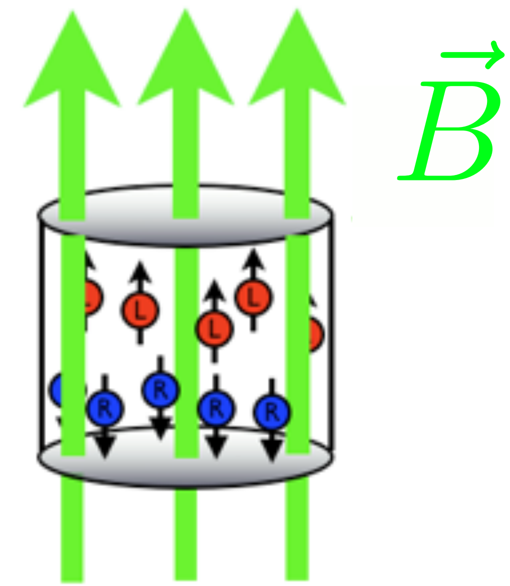
➡ opportunity: study equilibrium and near-equilibrium transport

Reminder: holography far beyond hydrodynamics

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of shear correlation function



[Starinets; JHEP (2002)]



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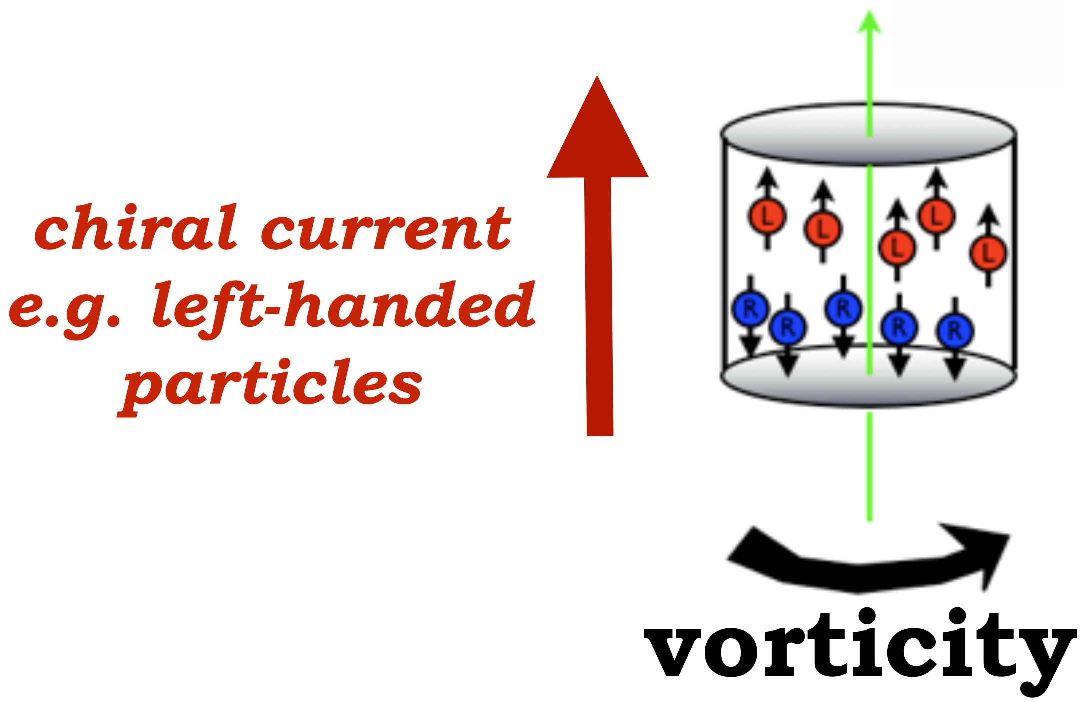
String theory prediction of new transport in hydrodynamics

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]



chiral vortical effect

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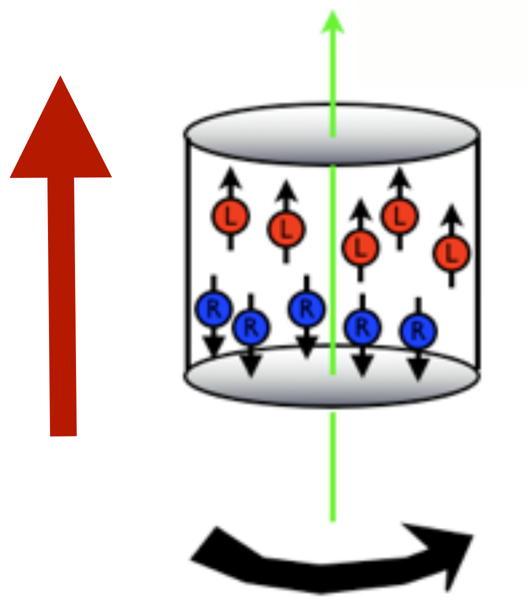
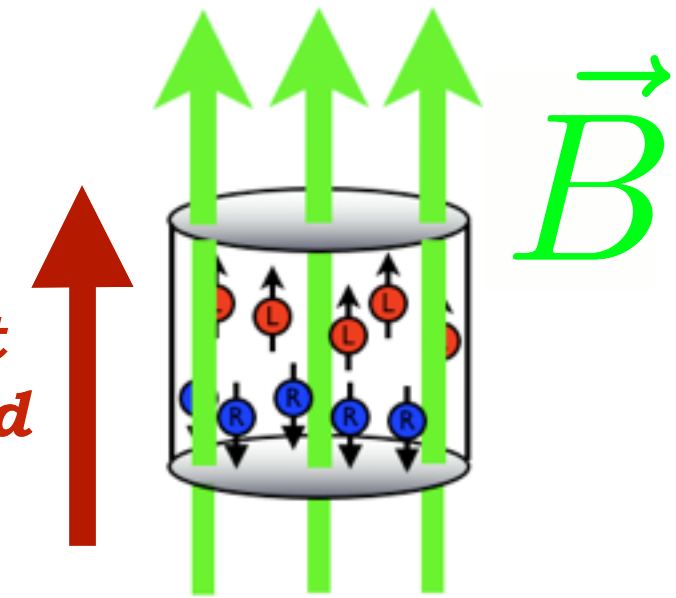
chiral current e.g. left-handed particles



vorticity

chiral vortical effect

chiral current e.g. left-handed particles



magnetic field

chiral magnetic effect

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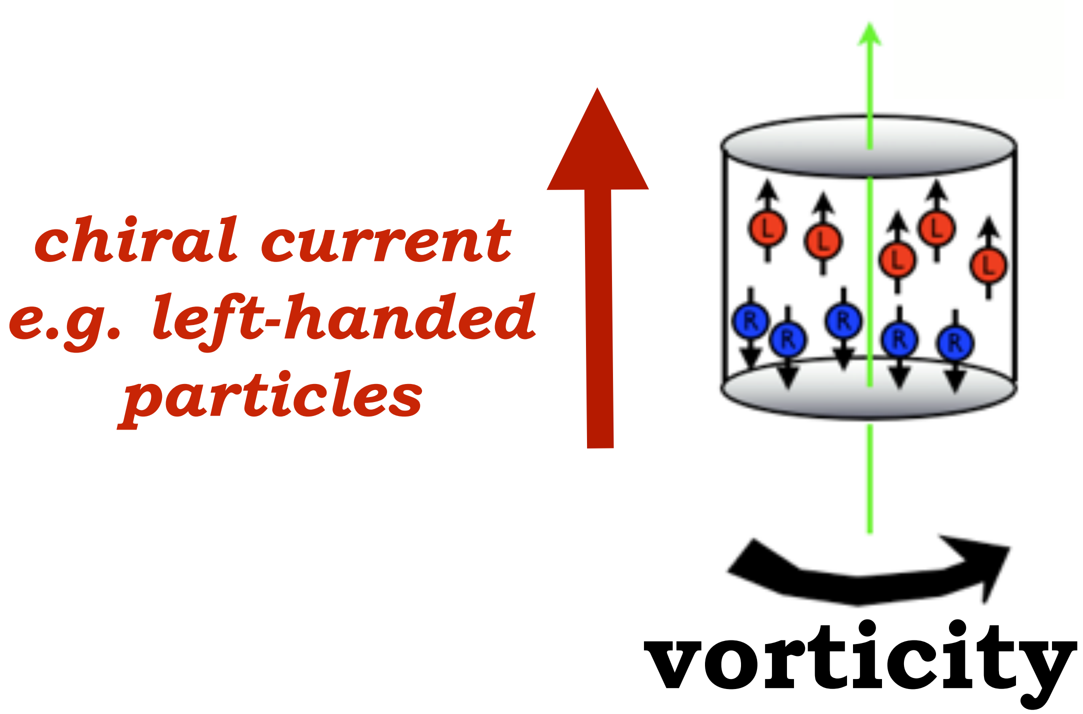
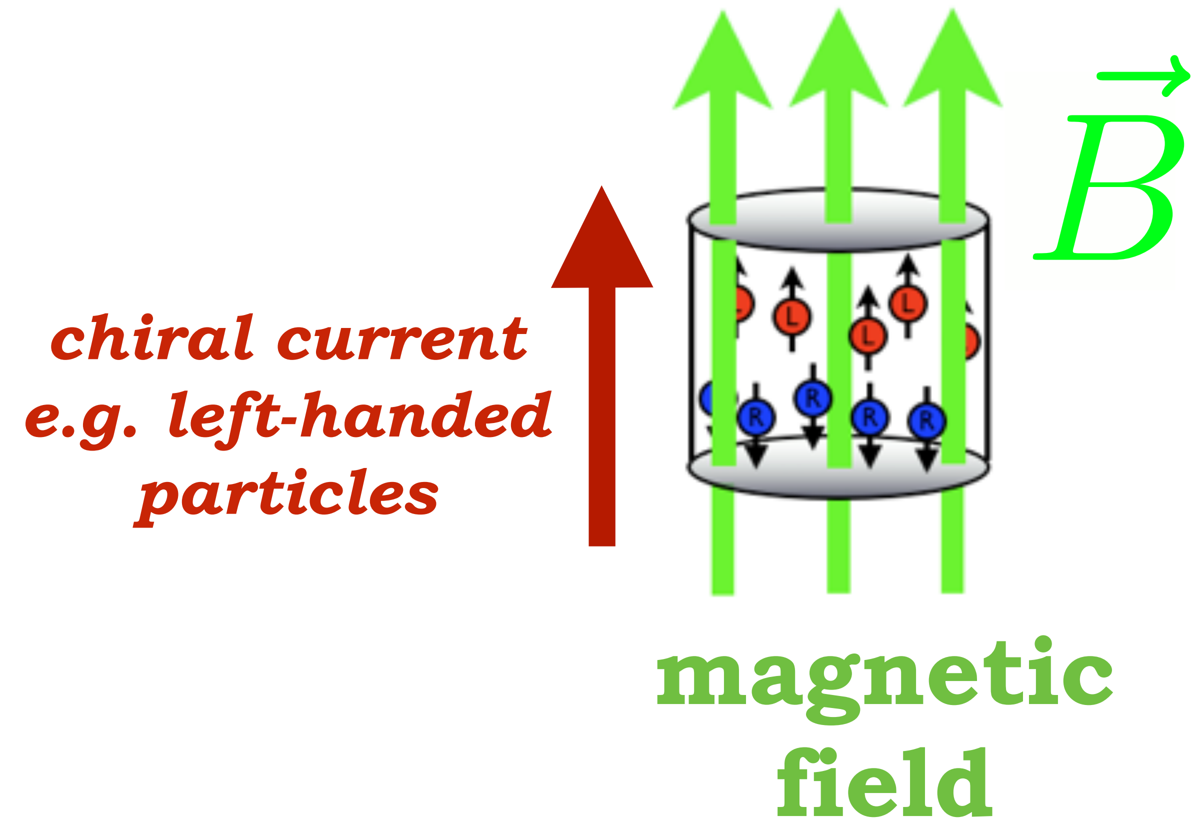


chiral vortical effect



chiral magnetic effect

Condensed matter experiments with Weyl-semi-metals confirm:

[Landsteiner; (2014)]
[Li et al; (2014)]
[Zhang et al; (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano; (2015)]

Hydrodynamic proof of new current contributions

$$\boxed{B \sim \mathcal{O}(\partial)}$$

New contributions in electric current discovered:

fluid velocity

electric field

magnetic field

vorticity

$$\vec{J} = n\vec{u} + \sigma\vec{E} + \xi\,\vec{B} + \xi_{\mathrm{V}}\vec{\Omega} + \ldots$$

charge density

conduc- tivity

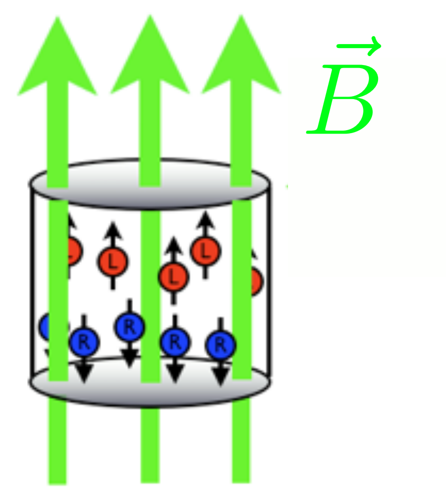
chiral magnetic effect

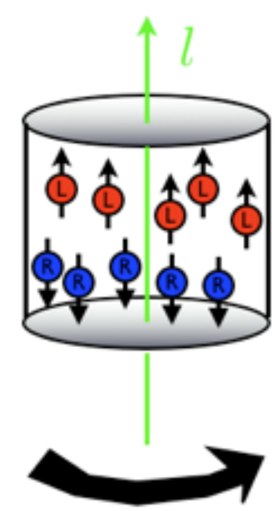
chiral vortical effect

Analytic result: chemical potential important!

$$\xi = C\mu$$

C ... anomaly coefficient





Hydrodynamic proof of new current contributions

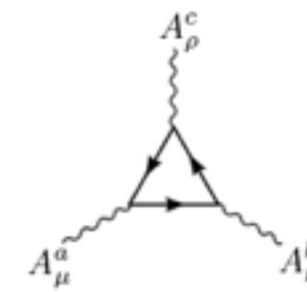
$\boxed{B \sim \mathcal{O}(\partial)}$

New contributions in electric current discovered:

fluid velocity · *electric field* · *magnetic field* · *vorticity*

$$\vec{J} = n\vec{u} + \sigma\vec{E} + \xi\,\vec{B} + \xi_{\mathrm{V}}\vec{\Omega} + \dots$$

charge density · *conductivity* · *chiral magnetic effect* · *chiral vortical effect*

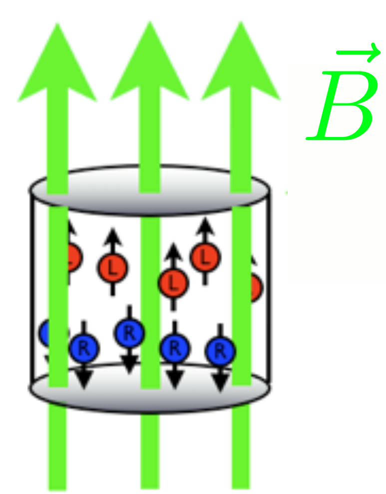


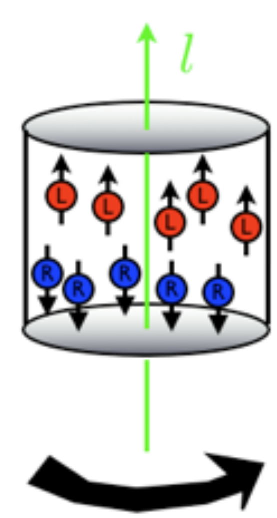
Analytic result: chemical potential important!

$$\xi = C\mu$$

C … anomaly coefficient

Proof in hydrodynamics: [Son,Surowka PRL 2009]





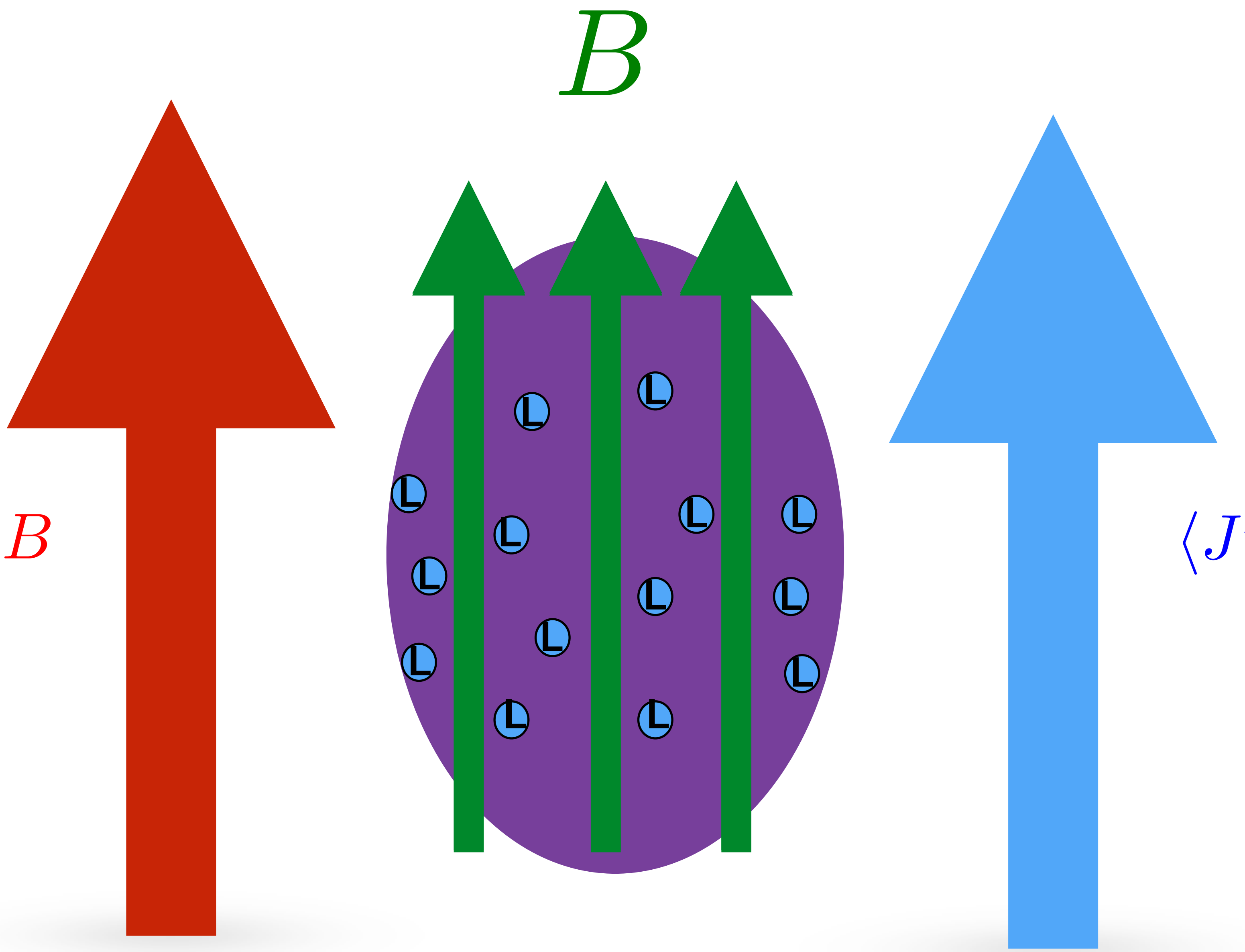
Confirms string model discovery.

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Zeroth order chiral magnetic effect -thermodynamic chiral currents

$\boxed{B \sim \mathcal{O}(1)}$

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]



$\langle T^{0z} \rangle = \xi_V^{(0)} B$

heat current

$\langle J^z \rangle = \xi_B^{(0)} B$

axial current

EFT result: strong B thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic frame:**

$\boxed{B \sim \mathcal{O}(1)}$

Energy momentum tensor:

equilibrium heat current

$$\langle T^{\mu\nu}_{\mathrm{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^{\mu}_{\mathrm{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

equilibrium charge current

building on:
[Kovtun; JHEP (2016)]
[Jensen, Loganayagam, Yarom; JHEP (2014)]

Holographic result: thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]
[Ammon, Leiber, Macedo; JHEP (2016)]

$$\boxed{B \sim \mathcal{O}(1)}$$

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Holographic thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^{\mu}\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

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with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

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with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

➡ **chiral thermodynamic equilibrium observables (currents)**
➡ **confirmed by holographic model**

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 1 modes under SO(2) rotations around *B*

$$\omega = \quad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$$

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$$\omega = \mp \underline{\frac{B n_0}{\epsilon_0 + P_0}} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + \underline{k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}}$$

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spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

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former sound modes

➡ **a chiral magnetic wave**
[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B\, T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

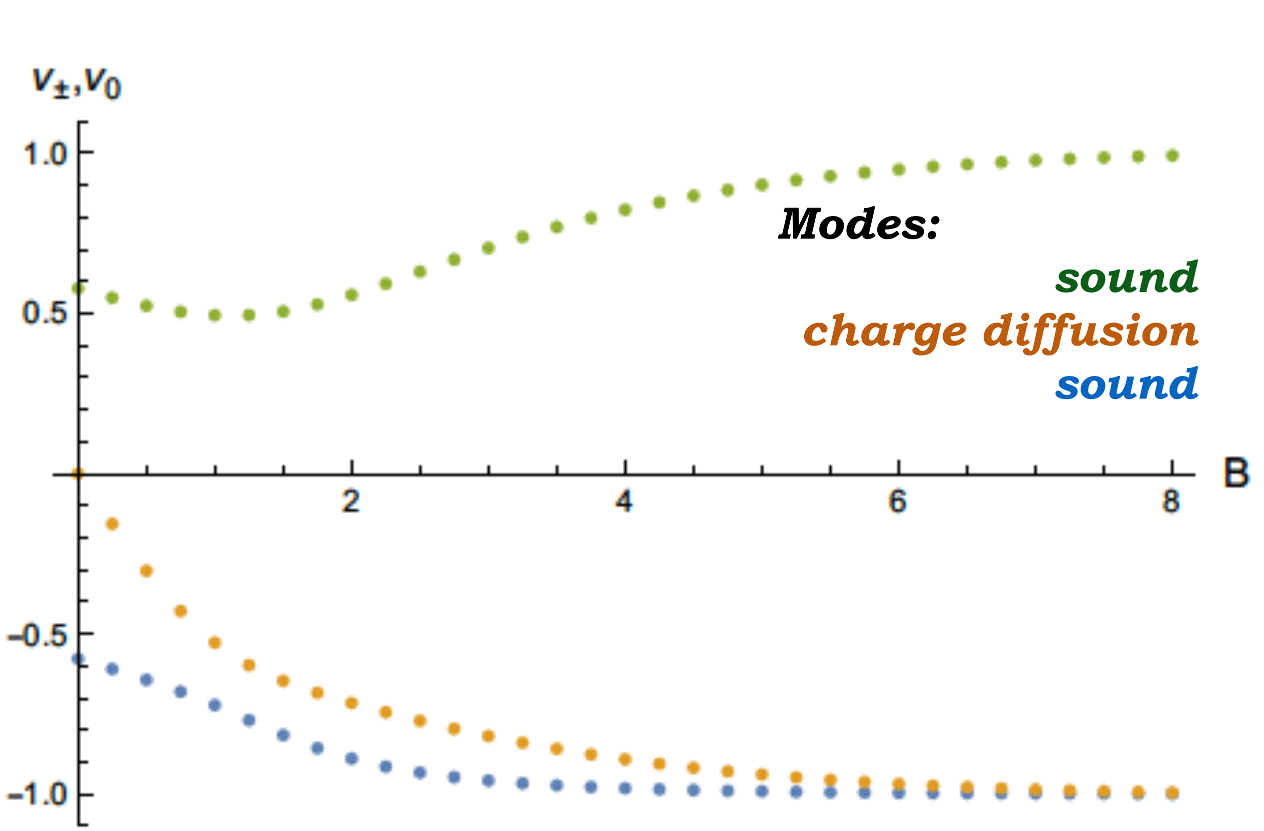
➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

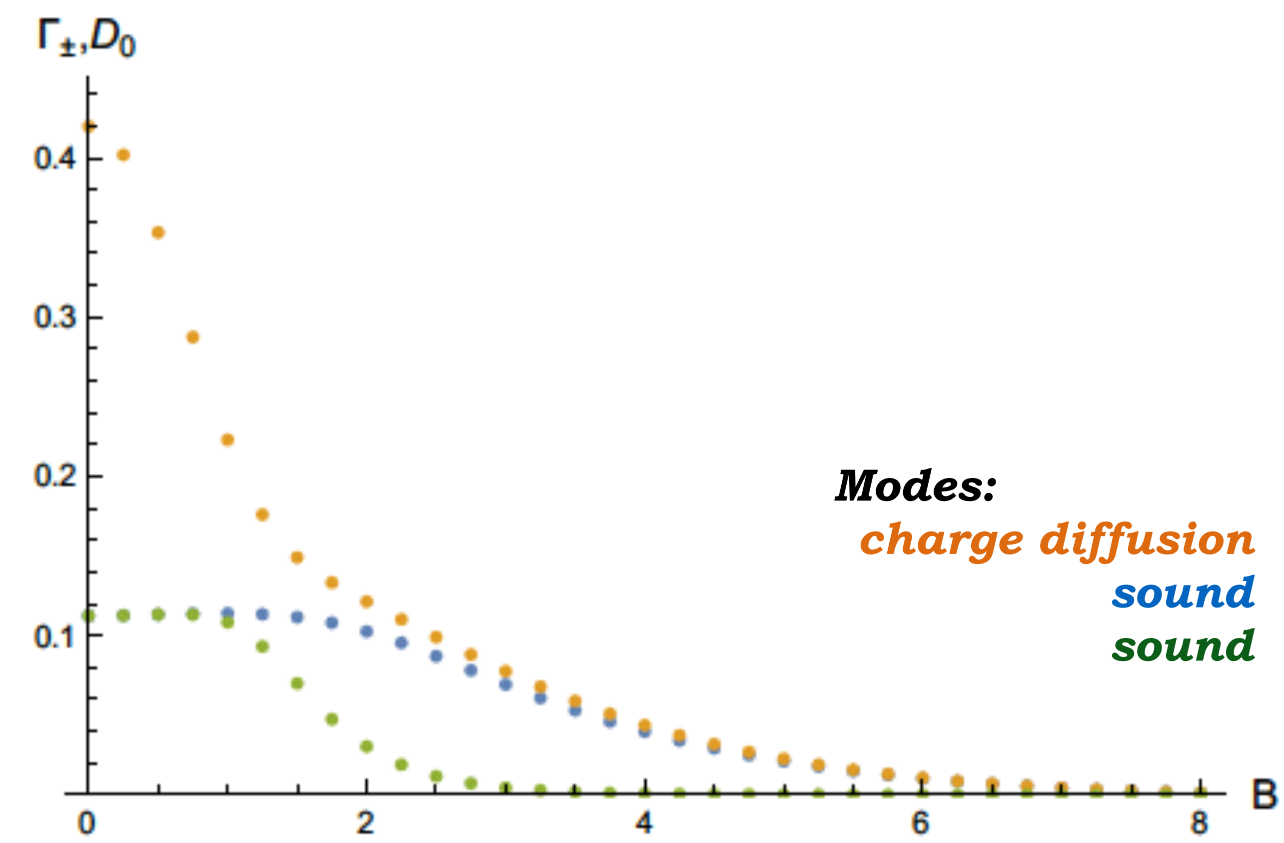
Holographic result II: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak B: **holographic results are in "agreement" with hydrodynamics.**
- Strong B: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

the speed of light **and without attenuation**





confirming conjectures and results in probe brane approach *[Kharzeev, Yee; PRD (2011)]*

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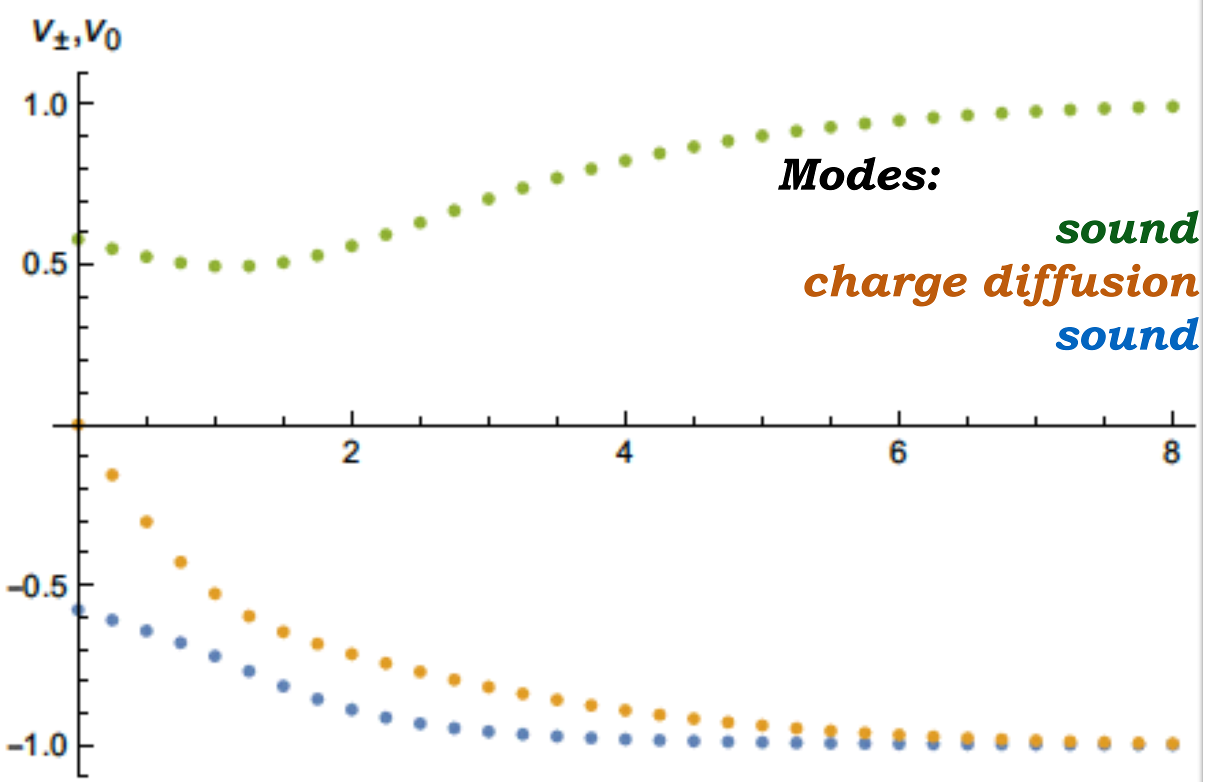

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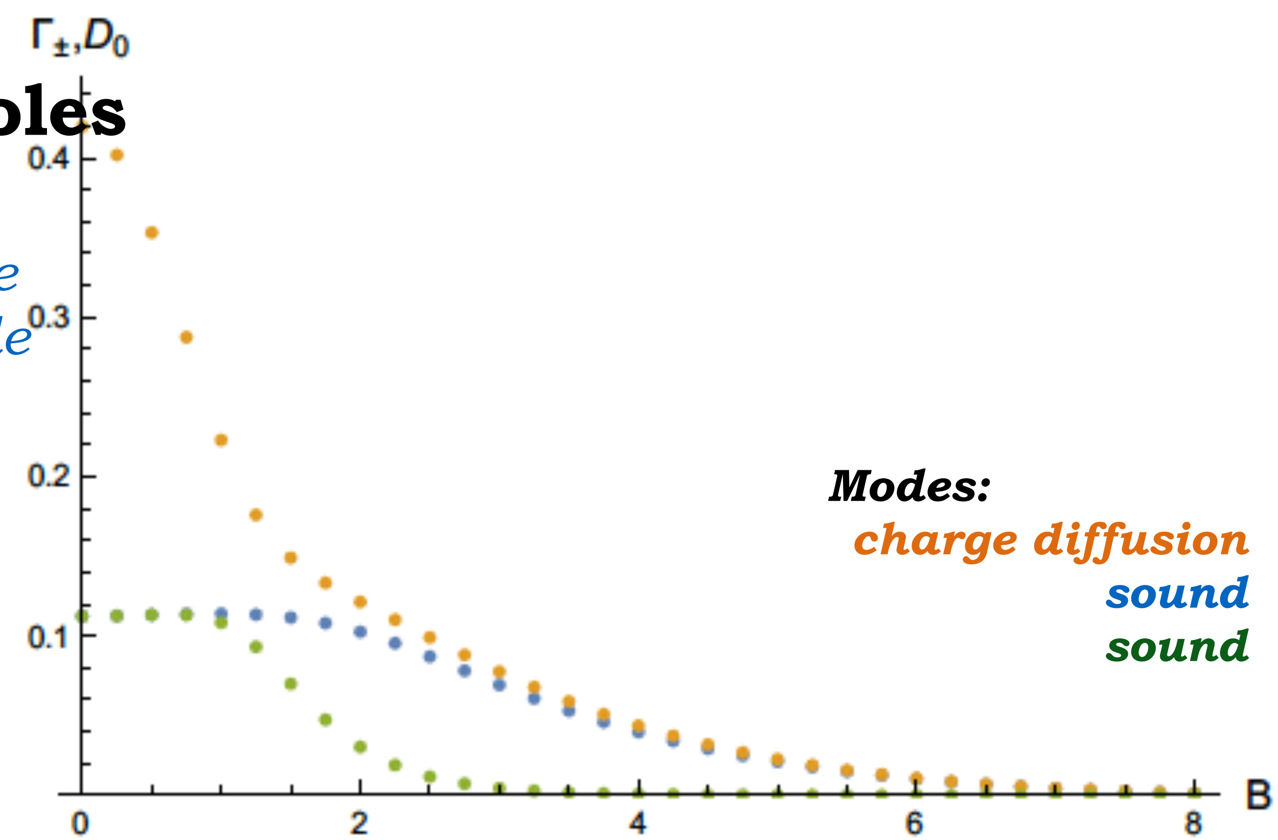

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Application: hydrodynamics & neutron star kicks

observation: neutron stars undergo a large momentum change (a kick)

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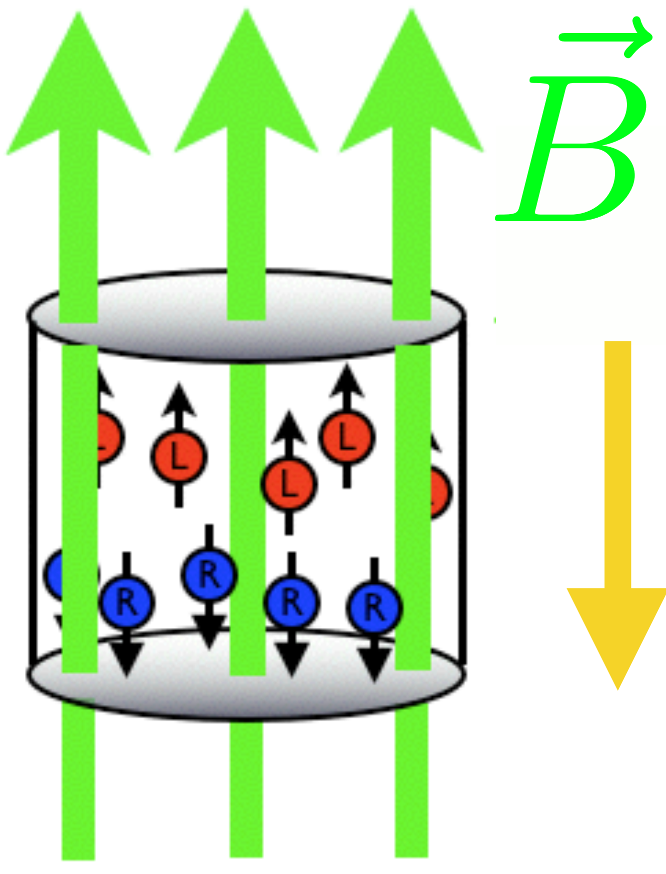

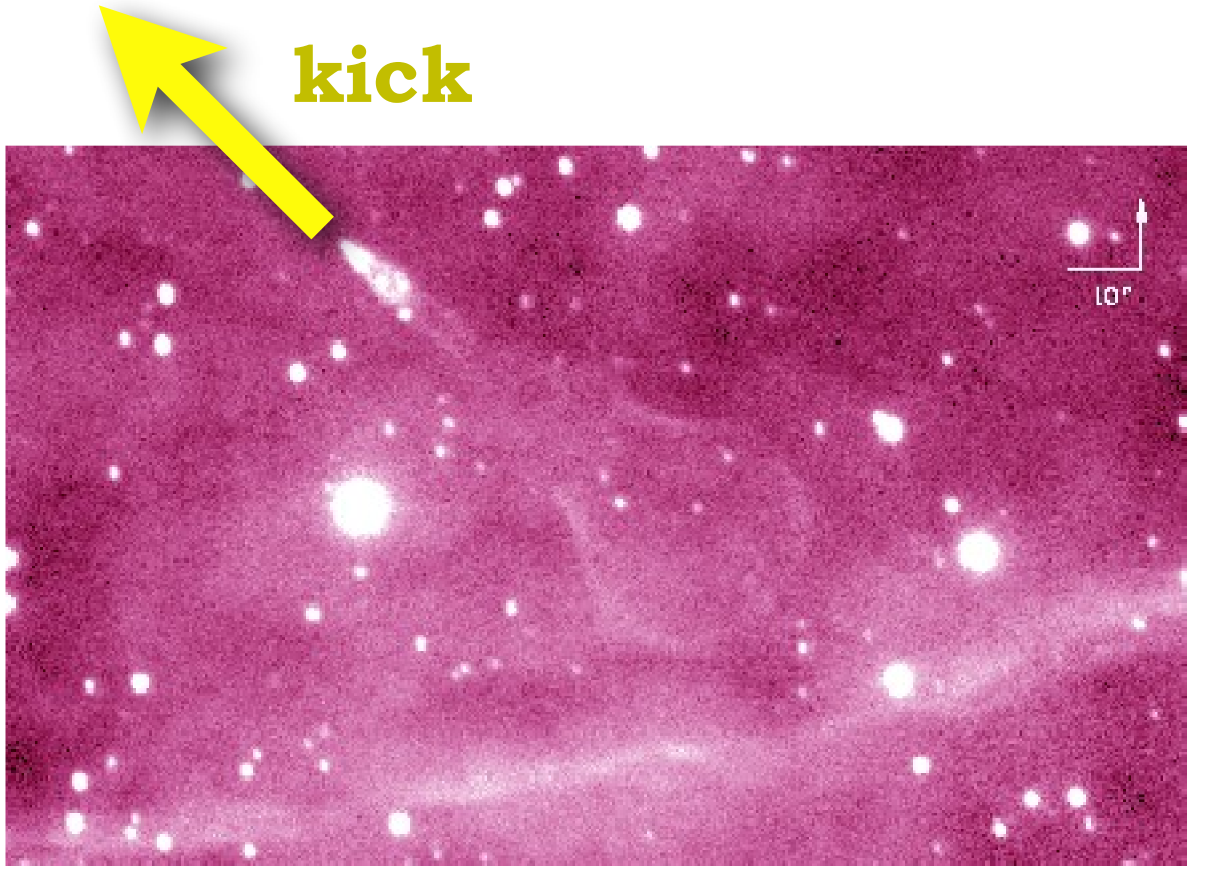

hydrodynamics: fluids with left-handed and right-handed particles produce a **current** along magnetic field

e.g. right/left-handed electrons, neutrinos, ...

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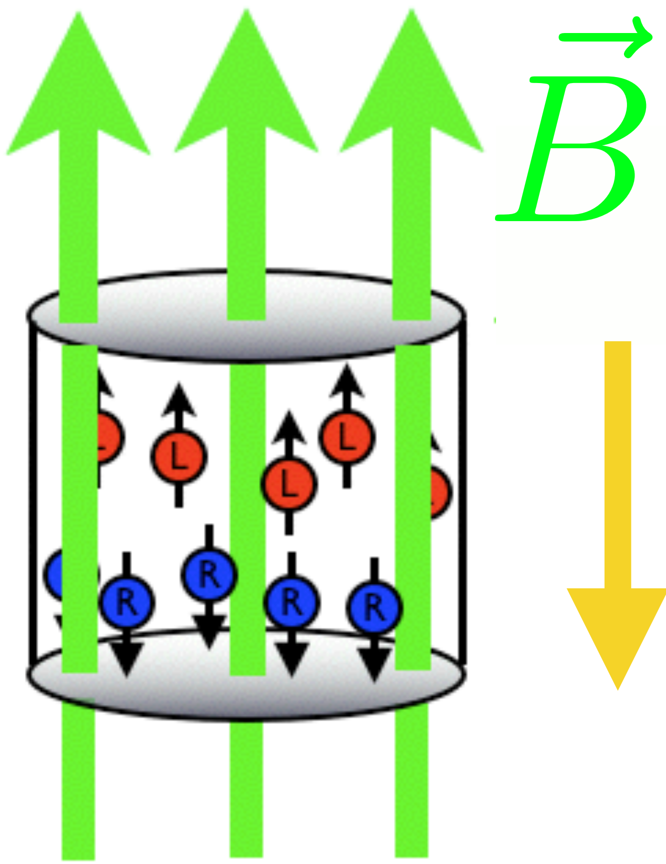

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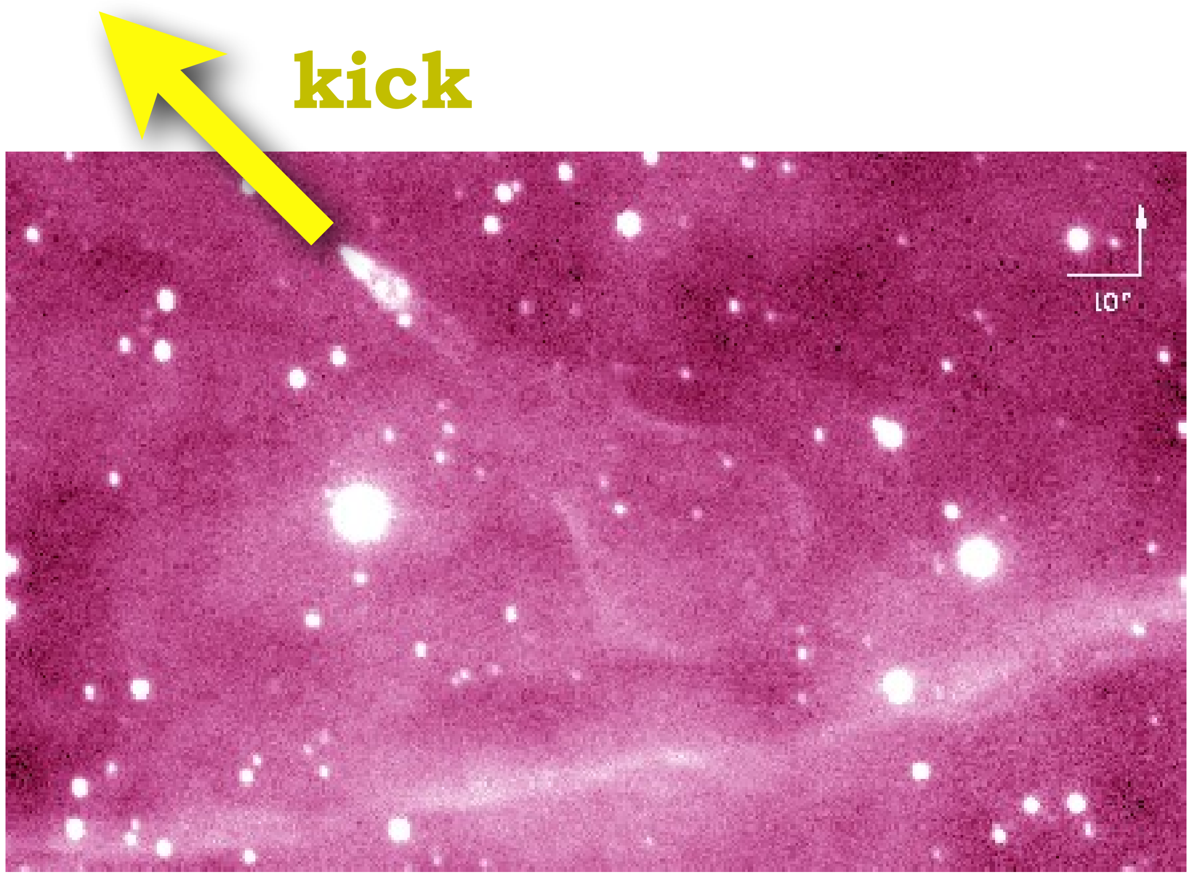

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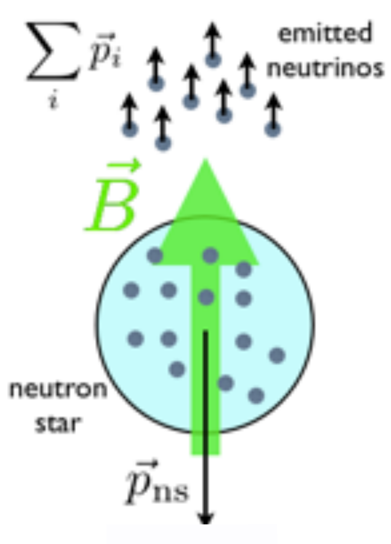


Chiral hydrodynamics leads to neutron star kicks

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

2. Results

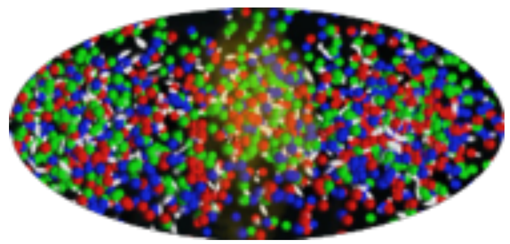
2.1 Phase transitions and critical points
2.2 Mesons at finite densities
2.3 Magnetic field — anisotropic hydrodynamics
2.4 Chiral transport effects
2.5 Holographic heavy ion collision - correlations

Holography far-from equilibrium

thermalization

examples:
heavy ion collision

time

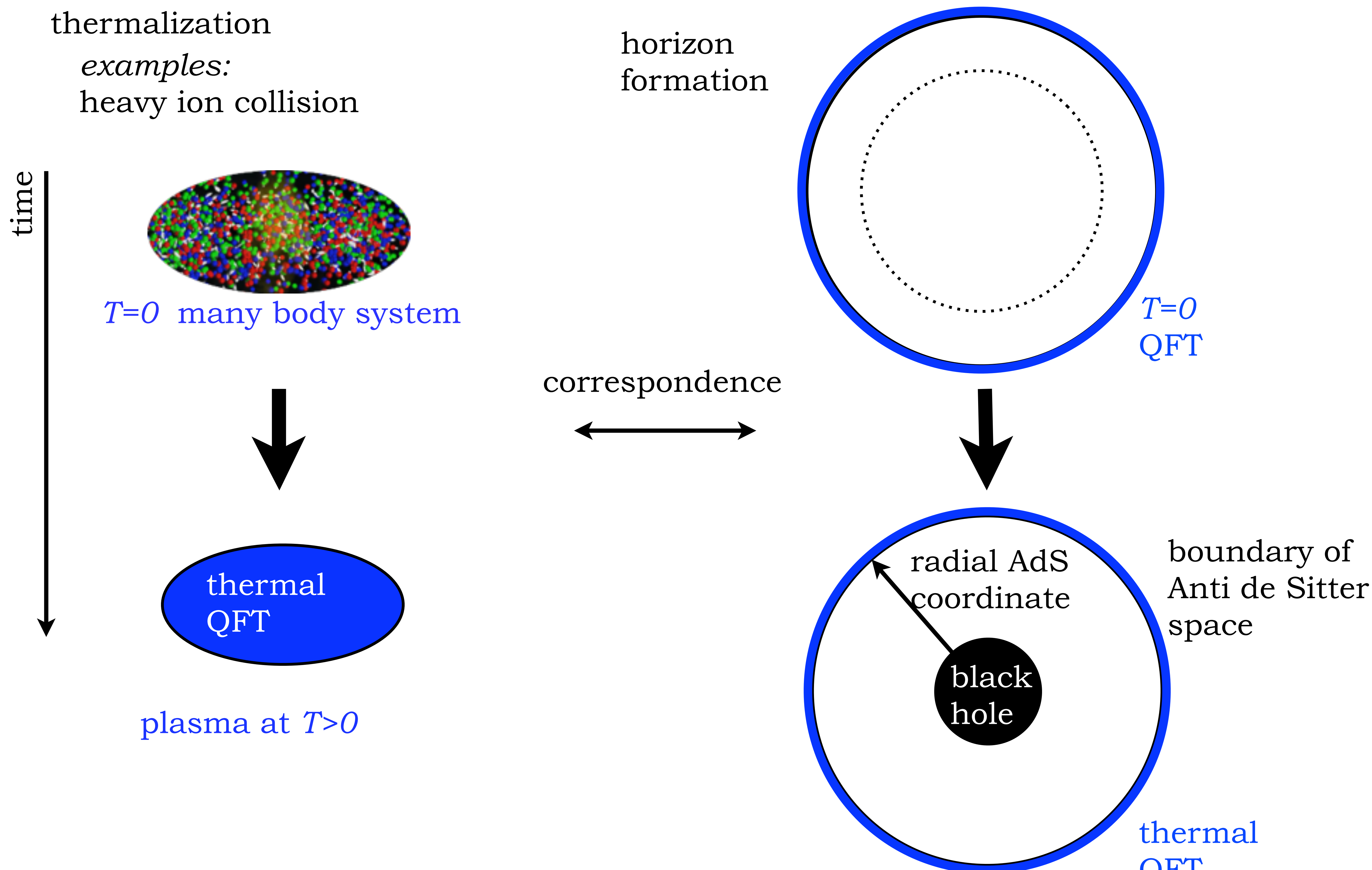


T=0 many body system

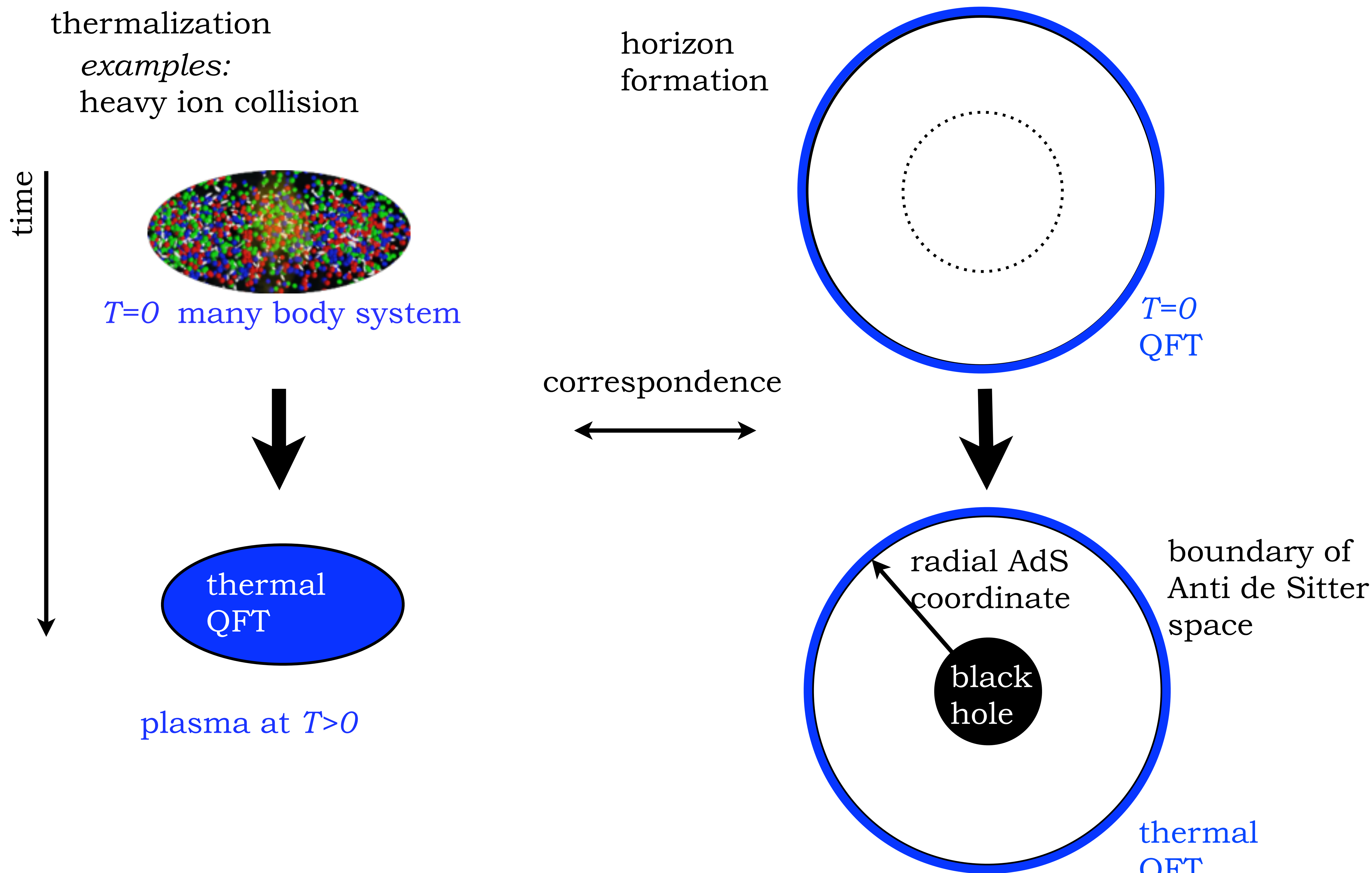
thermal
QFT

plasma at *T>0*

Holography far-from equilibrium

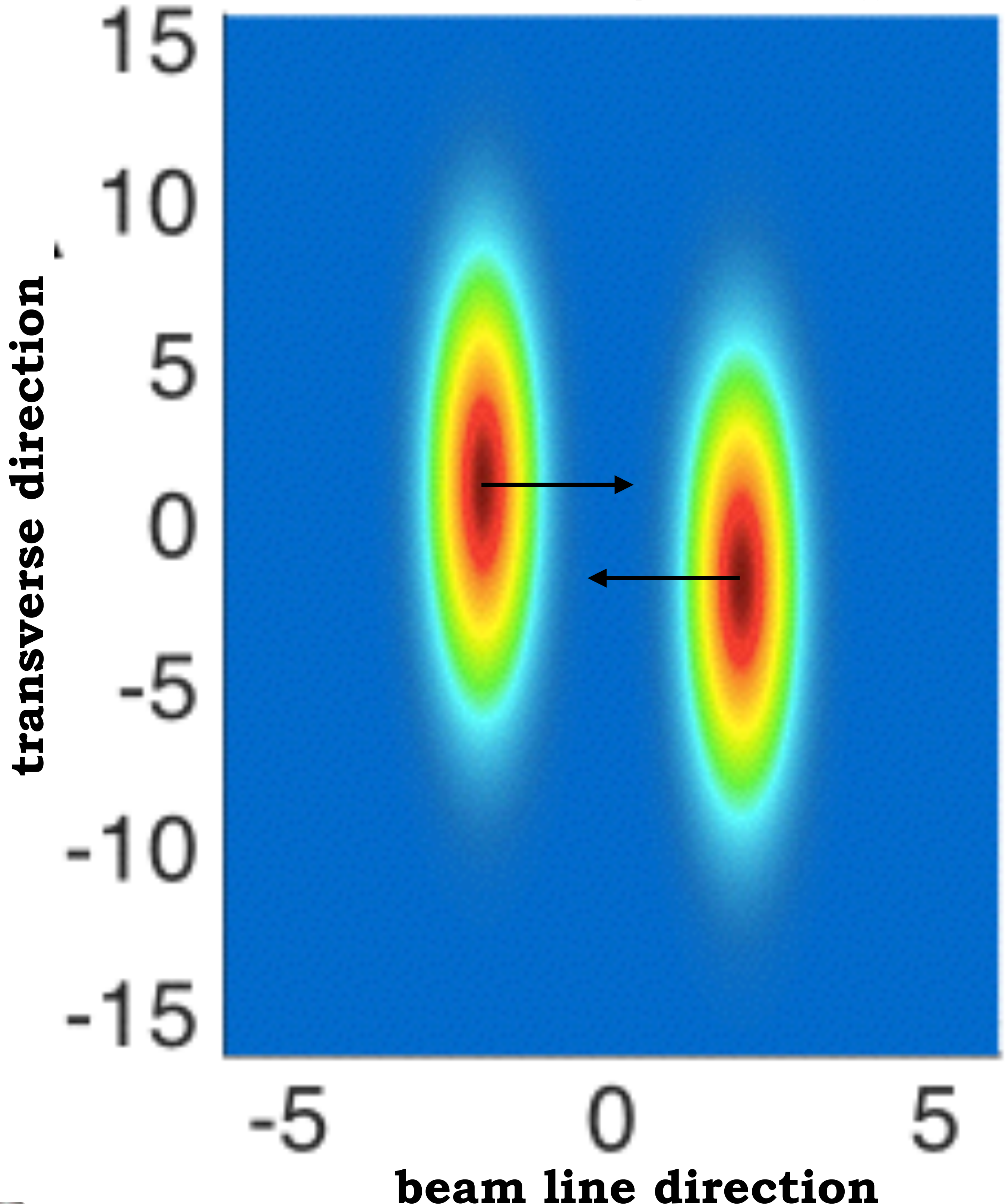


Holography far-from equilibrium



Holographic heavy ion collision

Non-central collision: [Chesler, Yaffe; JHEP (2015)]



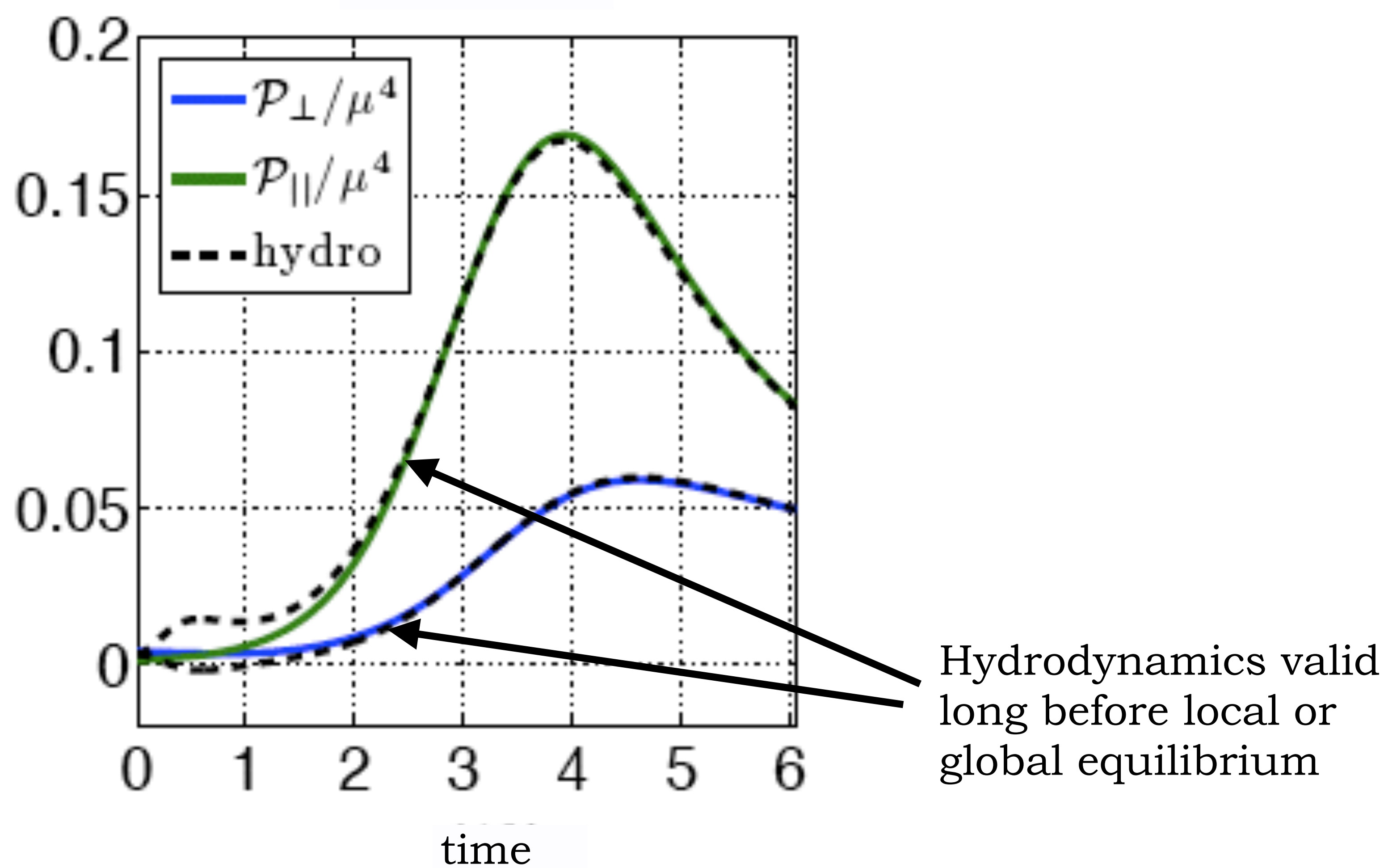
Gravitational dual

shock wave collision in
Anti de Sitter space
[Janik; PRD (2006)]

color coding:
energy density
red = high density
blue = low density

Holographic plasma "hydrodynamizes" quickly

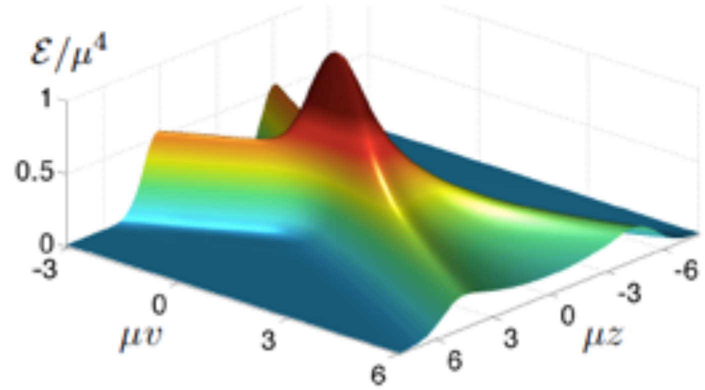
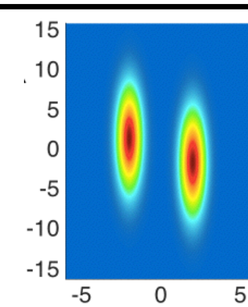
[Chesler, Yaffe; PRL (2011)]



Suggests existence of a new formulation of "hydrodynamics far from equilibrium". ⟷ *Resurgence in QFT*

confirmed by field theory: [Romatschke; (2016), (2017)]

Holographic plasma thermalization results

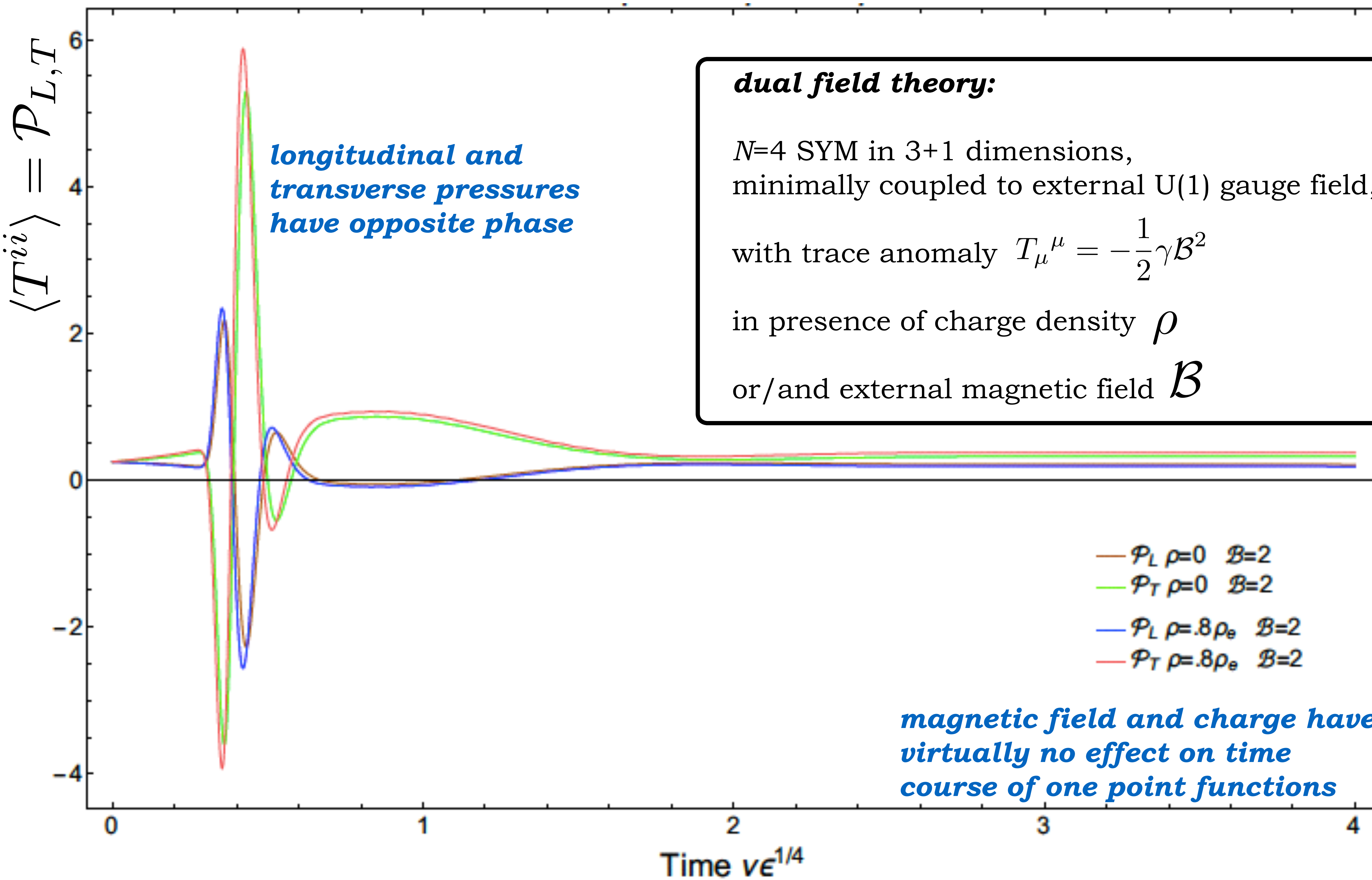
Model	Equilibration time
Central collision of two energy lumps in N=4 Super-Yang-Mills.  *[Chesler, Yaffe; PRL (2011)]*	~ 0.35 fm/c
[Fuini, Yaffe; (2015)] Initial anisotropy in N=4 Super-Yang-Mills, with charges / magnetic field. Confirmed by non-conformal study. *[Buchel, Heller, Myers; (2015)]*	~ 0.35 fm/c largely unaffected by charges/magnetic field
Off-center collision of two energy lumps in N=4 Super-Yang-Mills. *[Chesler, Yaffe; JHEP (2015)]* 	~ 0.25 fm/c

Equilibration happens very fast, in agreement with experiment.
Hydrodynamics works way before that.

cf. [Romatschke; (2016), (2017)]

Background - One Point Functions

[Fuini, Yaffe; JHEP (2015)]



longitudinal and transverse pressures have opposite phase

dual field theory:

N=4 SYM in 3+1 dimensions,
minimally coupled to external U(1) gauge field,

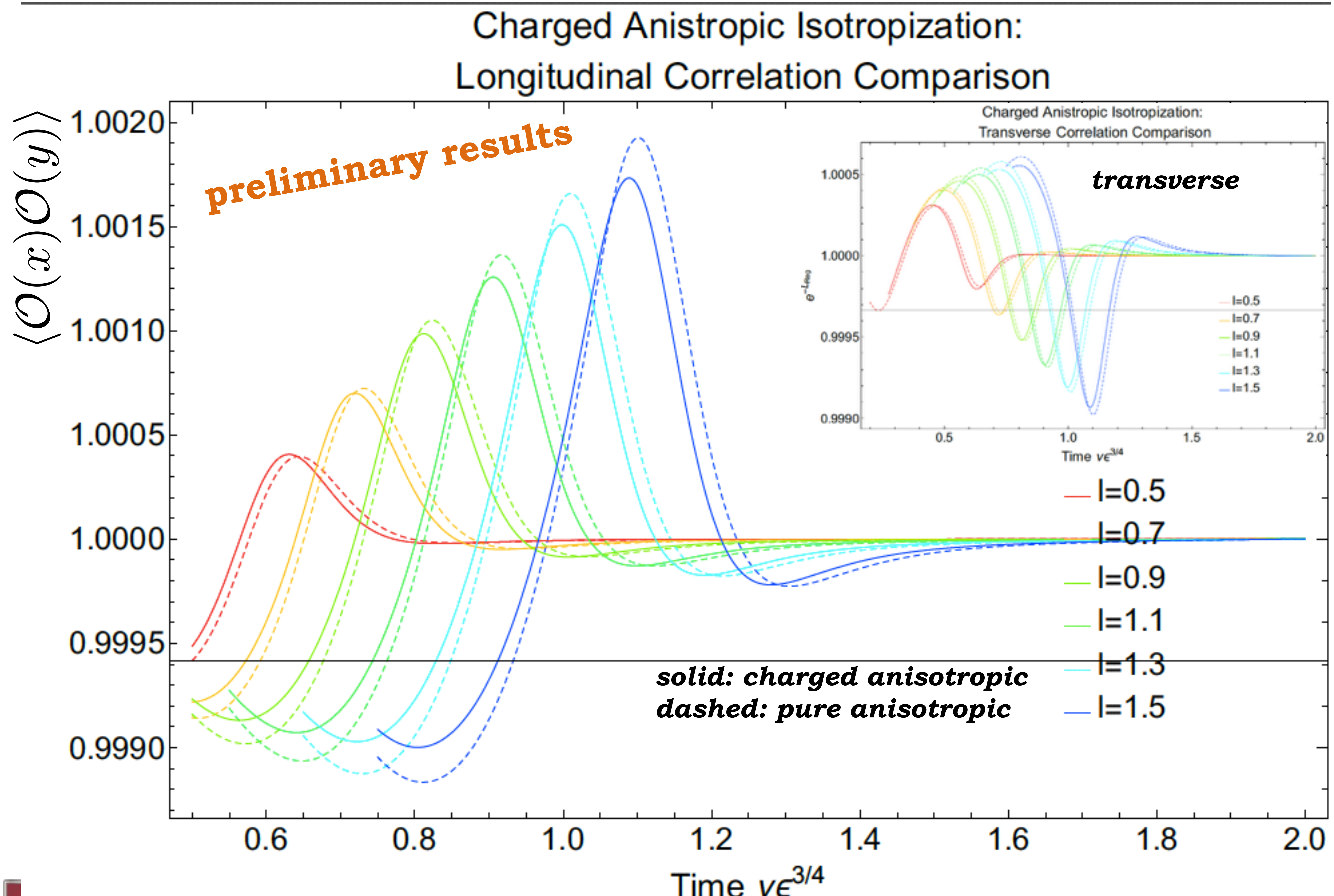
with trace anomaly $T_\mu{}^\mu = -\frac{1}{2}\gamma\mathcal{B}^2$

in presence of charge density ρ

or/and external magnetic field $\mathcal{B}$

magnetic field and charge have virtually no effect on time course of one point functions

Correlations - finite charge, zero B



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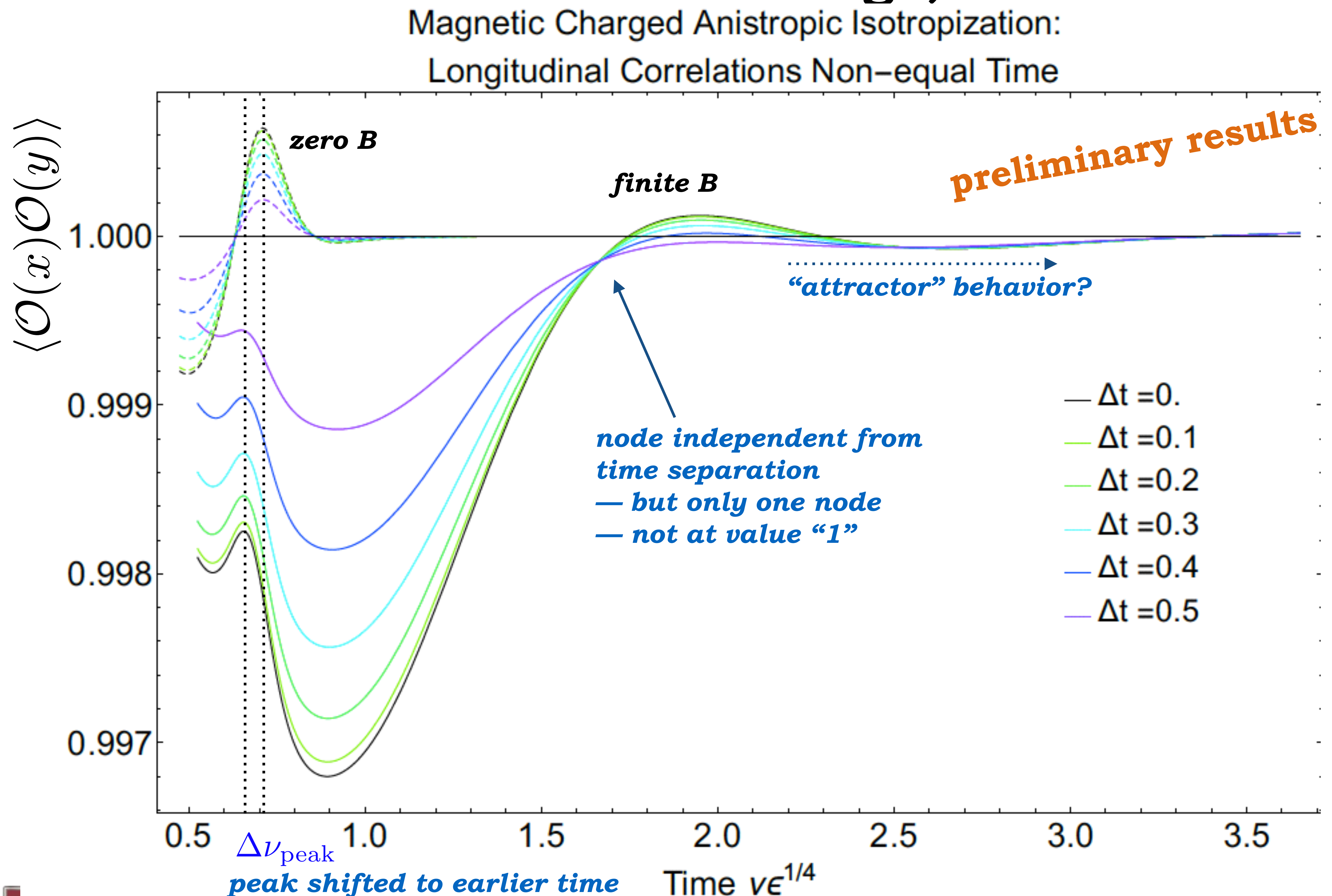

Method summary: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory, for example **QCD**

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model

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holography (gauge/gravity correspondence)

Simple problem in a particular gravitational theory

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➡ **Understand holography as an effective description.**

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CONSISTENCY CHECK

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REALITY CHECK: MODEL APPROPRIATE?

- hydrodynamic approximation of model theory

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3. Discussion & Outlook

- Holographic methods allow calculation of observables at high densities in strongly coupled quantum field theories

  → *phase diagrams, critical points, superconducting phase, …*

- strong combination:
  effective theories, e.g. hydrodynamics, together with holography

  → *discovered new transport effect(s): chiral vortical effect*

- Effective quantitative results

  → *shear viscosity over entropy density,*
  universal pressure ratio at nonzero B

- Qualitative results

  → *(chiral) equilibrium currents at nonzero B,* μ, *C*
  Kubo transport formulae, ballpark values

- Introduction to holography

  → *generic principle, discovery tool, model building*

3. Discussion & Outlook

- Outlook:

➡ transport coefficients (Kubo formulae, values) at strong magnetic field (anisotropic hydrodynamics)
[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

➡ correlations far from equilibrium at high density and magnetic field with chiral anomaly
[Cartwright, Kaminski; to appear] see my ***talk at HoloQuark2018***

➡ dynamical electromagnetic fields - magnetohydrodynamics

➡ comparison to experimental data

Collaborators

Friedrich-Schiller University of Jena, Germany

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Dr. Julian Leiber

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Regensburg University, Germany

Prof. Dr. Andreas Schäfer

Goethe University, Frankfurt, Germany

Dr. Gergely Endrödi

University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

University of Alabama, Tuscaloosa, USA

Dr. Jackson Wu

Roshan Koirala

Casey Cartwright

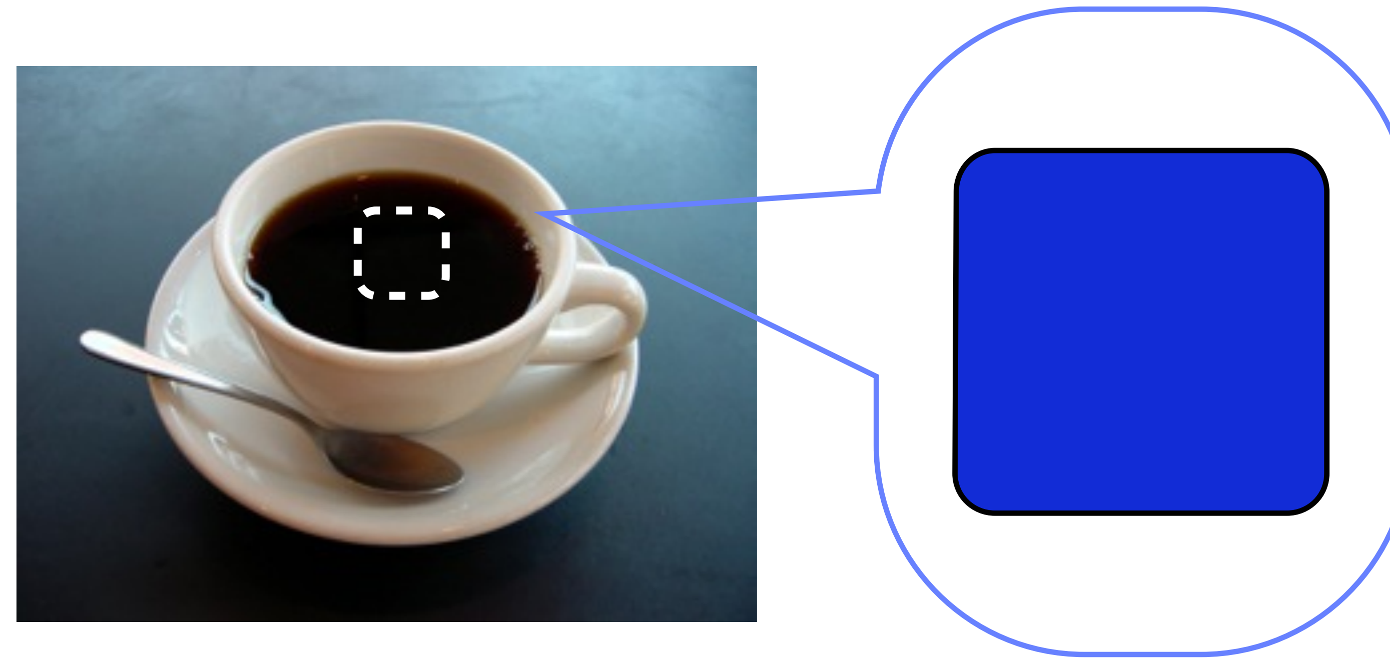
APPENDIX

Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$

thermodynamic variables:
temperature, chemical potential,
fluid velocity



Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$

thermodynamic variables:
temperature, chemical potential,
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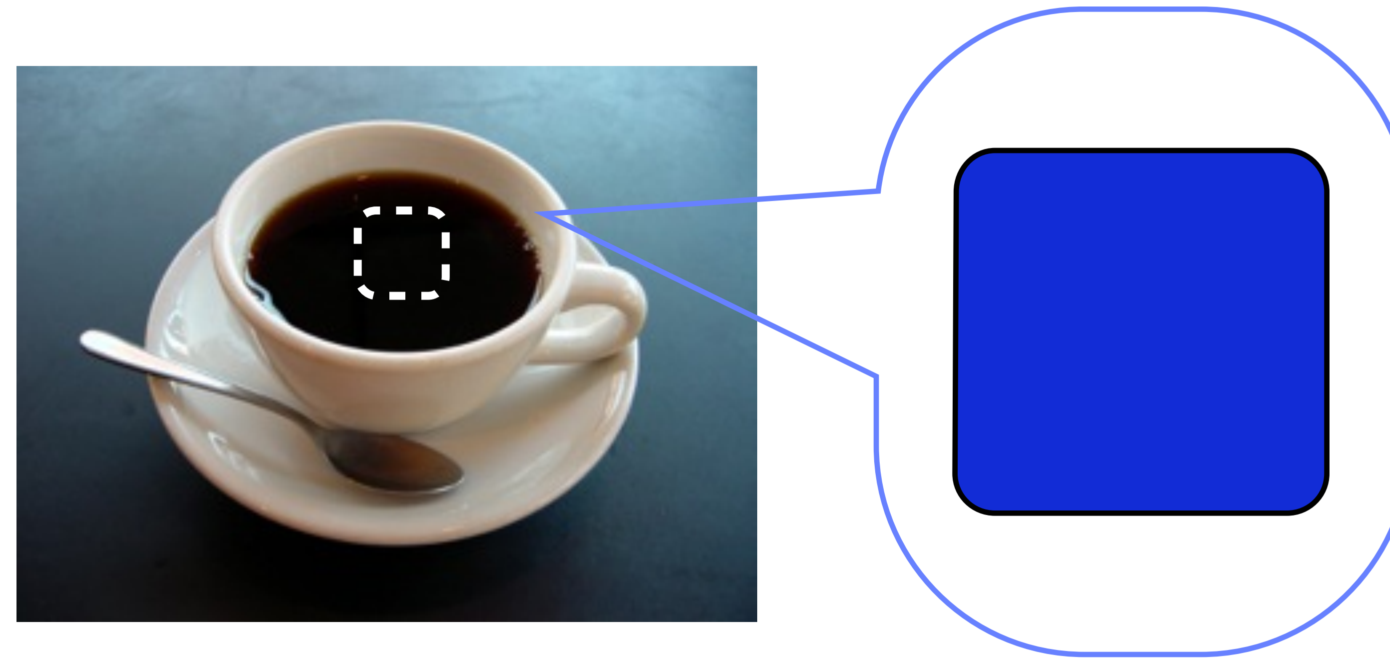
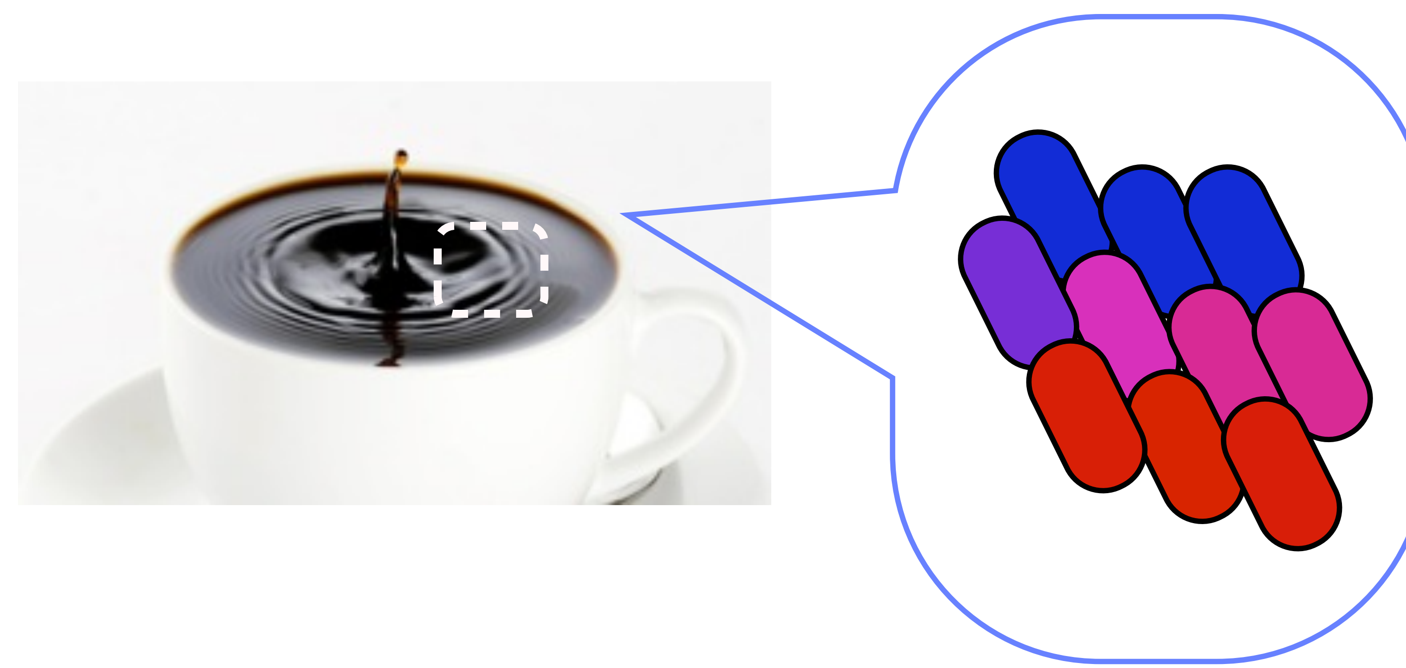


Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu\overbrace{(t,\vec{x})}^{x}$$

hydrodynamic fields
-protected by symmetry



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temperature, chemical potential,
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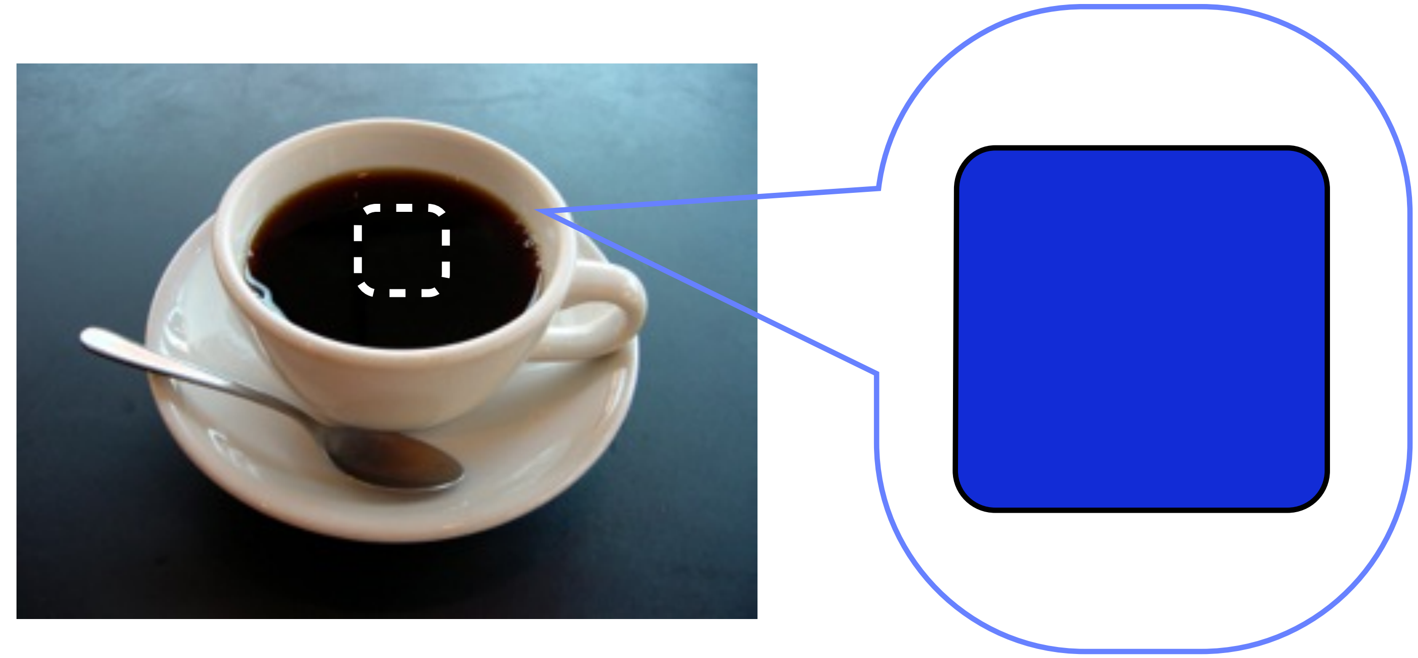
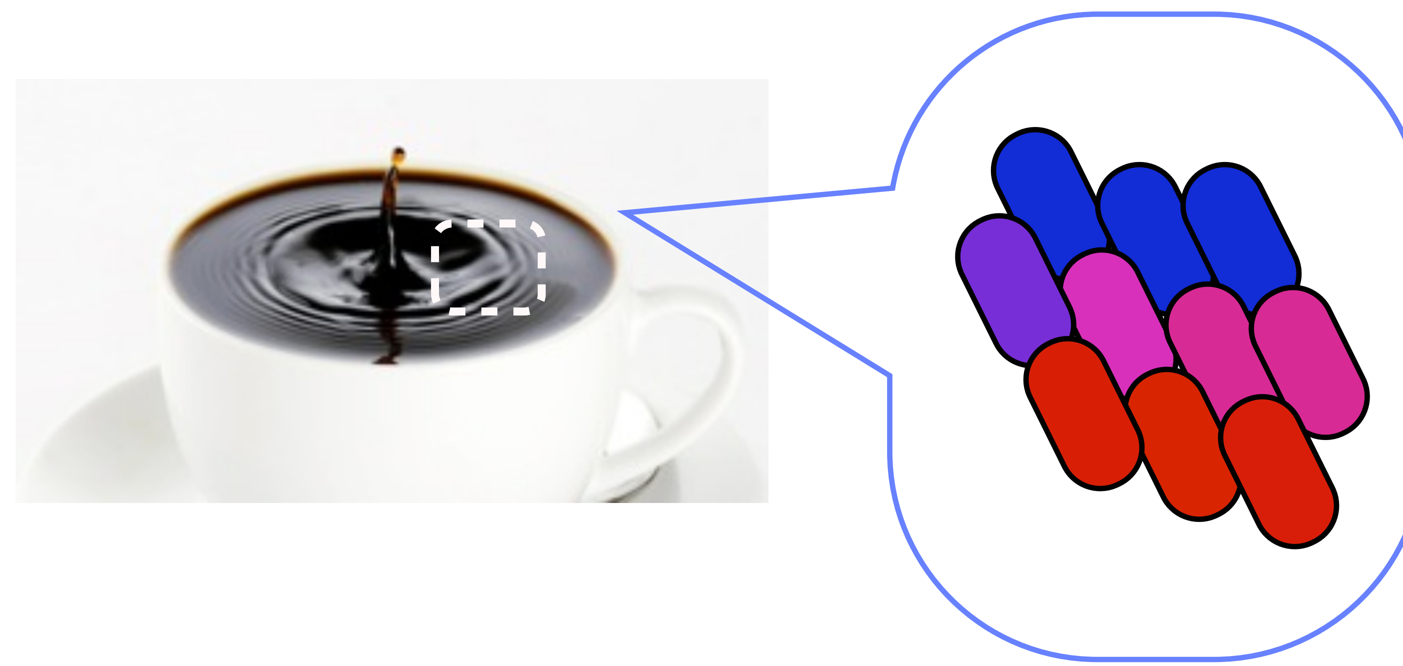


Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu\overbrace{(t,\vec{x})}^{x}$$

hydrodynamic fields
-protected by symmetry



Hydrodynamics

effective field theory, expansion in gradients of fields

- fields $T(x),\ \mu(x),\ u^\nu(x)$

  temperature *chemical potential* *fluid velocity*
- sources $A_\mu(x), \ldots$

  gauge field
- conservation equation $\nabla_\nu j^\nu = 0$
- constitutive equation (Landau frame)

[Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]

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  Conserved current

  $$j^\mu = n u^\mu + \nu^\mu$$

  gradient terms

  Form can be derived and restricted from first principles.

  [Landau, Lifshitz]

  [Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]

$$\nabla_\nu j^\nu = 0$$ classical theory

Chiral hydrodynamics

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Derived for any theory with chiral anomaly

(e.g. the standard model of particle physics)

$$\nabla_\mu j^\mu = \boxed{C\,\epsilon^{\nu\rho\sigma\lambda} F_{\nu\rho} F_{\sigma\lambda}}$$ quantum theory

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Generalized constitutive equation with external fields:

$$j^\mu = n u^\mu + \sigma E^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu + \dots$$

(non) conserved current

(ideal) charge flow

conductivity term

chiral magnetic conductivity

Agrees with gauge/gravity prediction

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]

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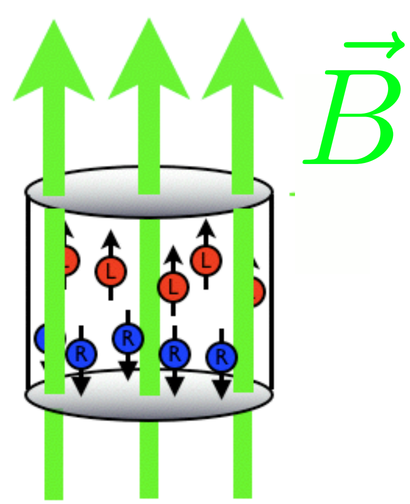
Chiral magnetic conductivity:

$$\sigma^B = C\,\mu$$

anomaly-coefficient C



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- j^μ: *(non) conserved current*
- $n u^\mu$: *(ideal) charge flow*
- σE^μ: *conductivity term*
- $\sigma^B B^\mu$: *chiral magnetic conductivity*
- $\sigma^V \omega^\mu$: *chiral vortical conductivity*

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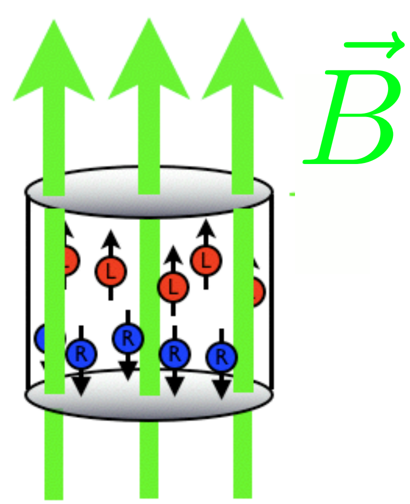

Measured in Weyl semi metals !

e.g. [Huang et al; PRX (2015)]
[Landsteiner] various others …

energy extraction ?

neutron stars ?

Dirty details: chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]
[Neiman, Oz; JHEP (2010)]

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \underbrace{\xi_{VV} B^\mu}_{\text{chiral magnetic effect}} + \xi_{VA} B_A^\mu$$

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Note:

* hydrodynamic frame choice
[Ammon, Kaminski et al.; JHEP (2017)]
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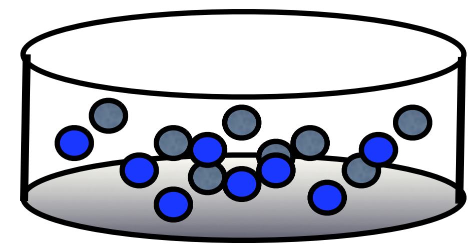
Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; PLB (2016)]

A bucket full of electrons and electron neutrinos with short mean free path



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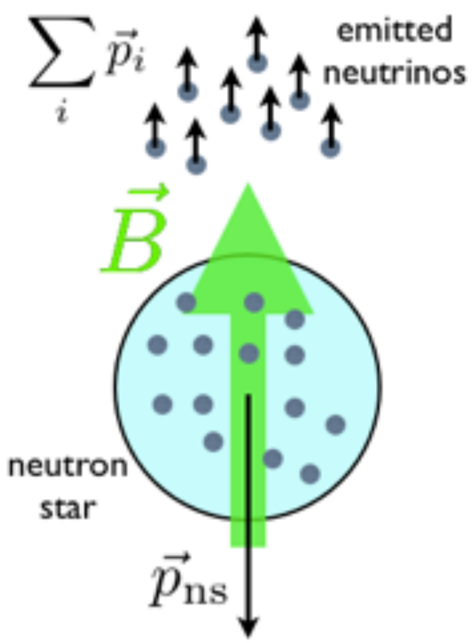

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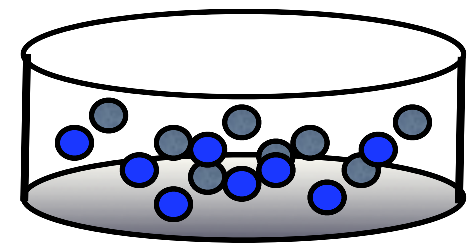

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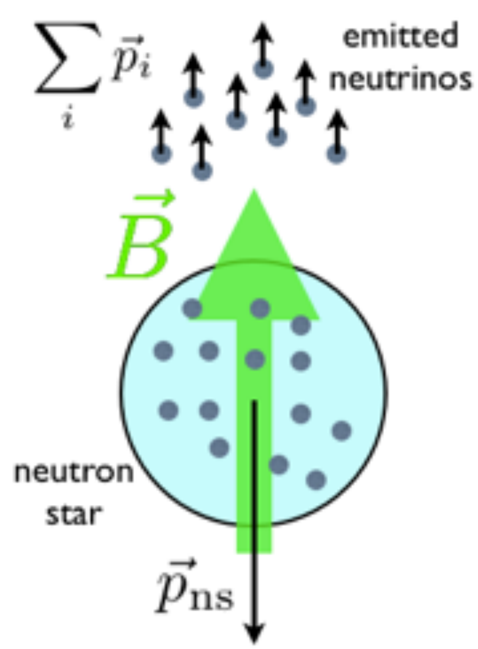

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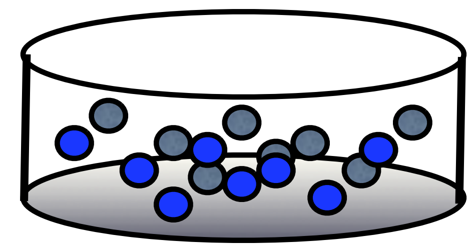

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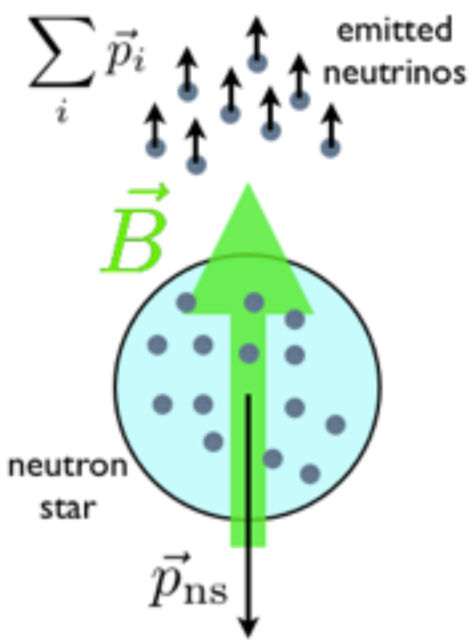

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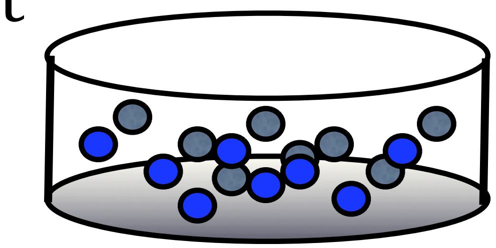

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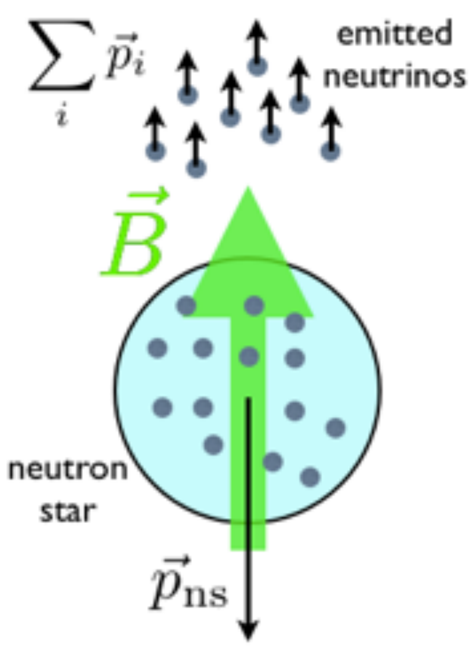

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$$\Delta P_{\mathrm{NS}} = \Delta t \dot{N}_\nu \langle p_\nu \rangle$$
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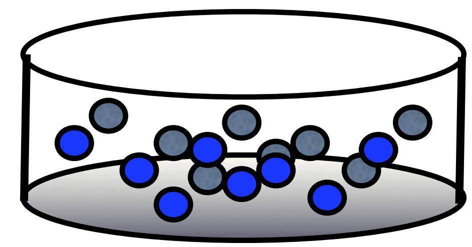

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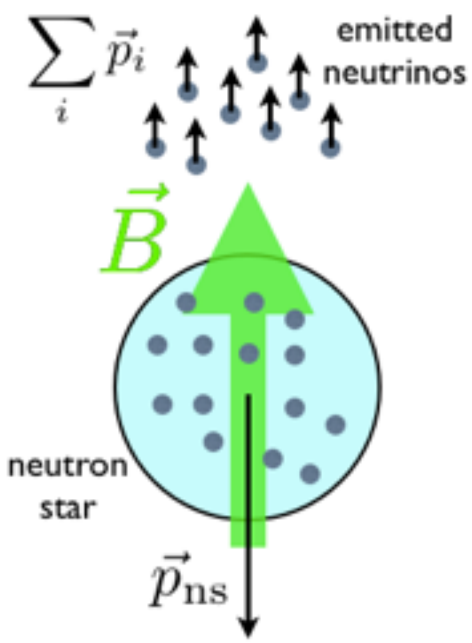

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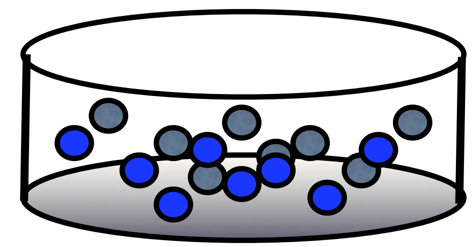

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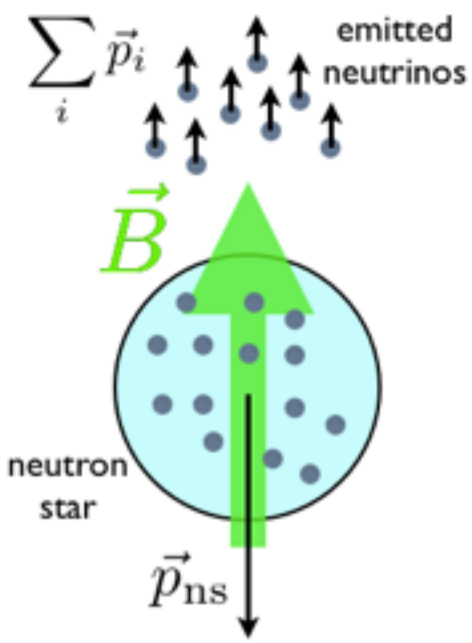

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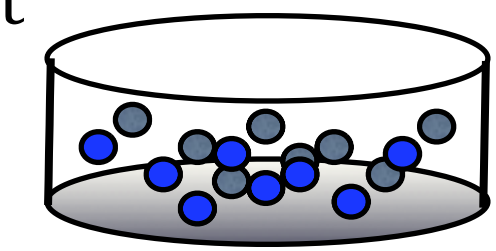

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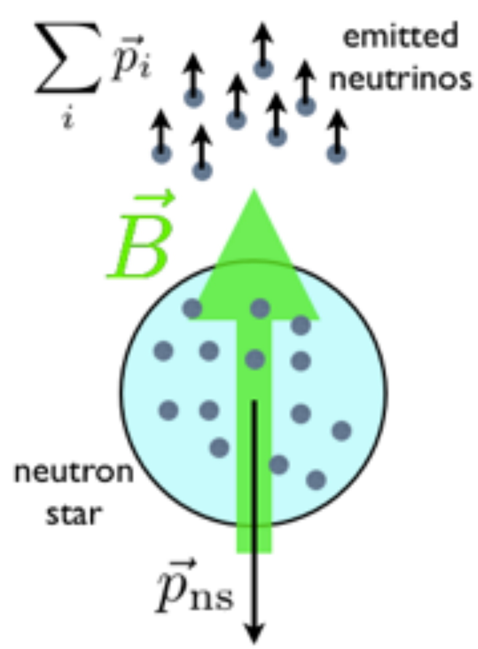

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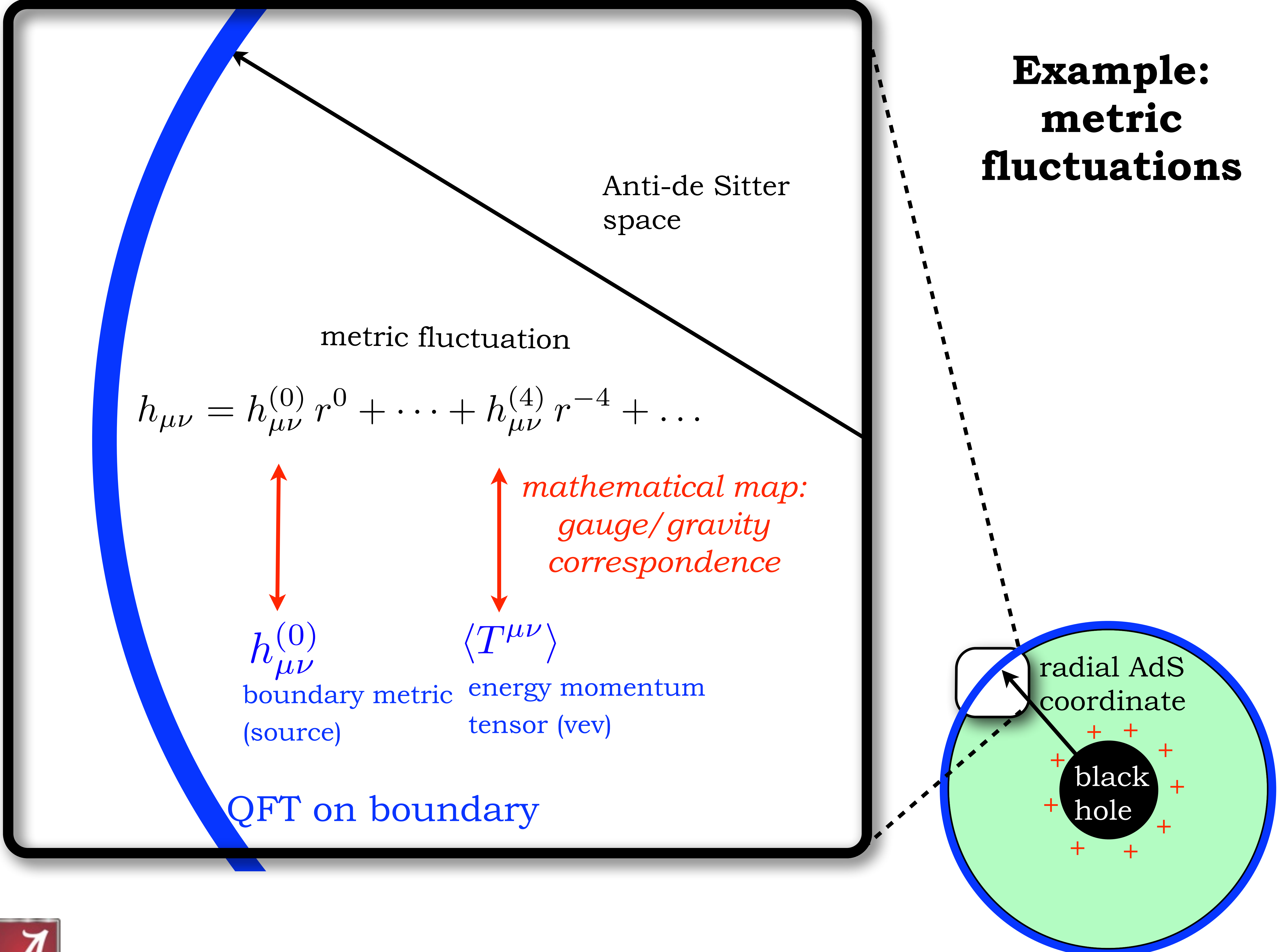
Example: metric fluctuations



Correlations - geodesic approximation

[Balasubramanian, Ross; PRD(2000)]

Correlator as a sum over geodesics: $\Delta L = L - L_{\text{thermalized}}$

$$\langle \mathcal{O}(t,\vec{x}_1)\mathcal{O}(t,\vec{x}_2)\rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L}$$

Geodesic length (Lagrangian):

$$L = \int \mathrm{d}\lambda \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \quad \Rightarrow \quad \frac{d^2x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0$$

geodesic equation

($L = m\int \mathrm{d}\lambda \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + \frac{q}{m}A_\mu \dot{x}^\mu$ charged probe particle)

Lorentz force term

Numerical implementation - relaxation method:

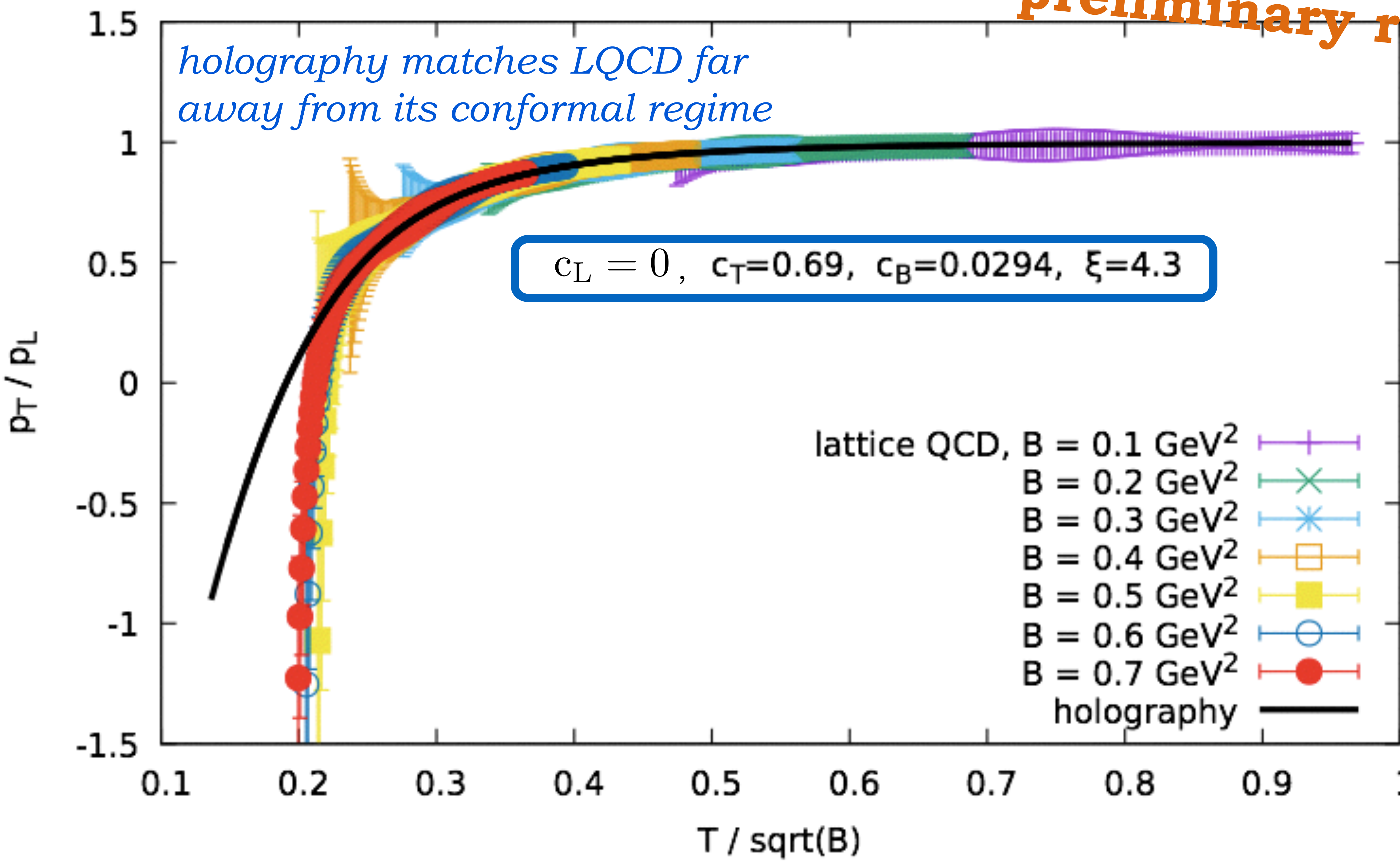
[Ecker, Grumiller, Stricker; JHEP (2015)]

1. Generate the dynamic background
2. Generate interpolations of the metric functions
3. Discretize the geodesic equations using a relaxation scheme
4. Approximate the proper length using a Riemann sum

Good agreement with N=4 Super-Yang-Mills (from holography)

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]

preliminary results



"ideal" renormalization scale:

- ***maximum overlap in LQCD data***
- ***maximum overlap with holography***
- ***minimum number of "fit parameters"***

$$\mu = \sqrt{c_T T^2 + c_L \Lambda_{\rm H}^2 + c_B |B|}$$

How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $$S = S_{\mathrm{QCD}}(e, B) + S_{\mathrm{EM}}(e, B)$$

QCD action coupled to external magnetic field (through covariant derivative)

action for external magnetic field; not included in code (not part of the dynamics)

Electric charge is renormalization scale dependent: $e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log\frac{\mu}{\Lambda}, \qquad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\mathrm{H}}^2 + c_B|B|}$

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Electric charge is renormalization scale dependent: $e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log \frac{\mu}{\Lambda}, \qquad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\rm H}^2 + c_B |B|}$

How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $S = S_{\mathrm{QCD}}(e, B) + S_{\mathrm{EM}}(e, B)$

QCD action coupled to external magnetic field (through covariant derivative)

action for external magnetic field; not included in code (not part of the dynamics)

Free energy: $F = -T \log \mathcal{Z}[S]$

$$= \boxed{F_{\mathrm{QCD}}(e, B)} + F_{\mathrm{EM}}(e, B) \qquad F_{\mathrm{EM}}(e, B) = -V \frac{B^2}{2e^2}$$

Transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V} \frac{\partial F_{\mathrm{QCD}}(e, B)}{\partial L_{\mathrm{T}}}$$

this free energy is renormalization scale dependent

hence this pressure is renormalization scale dependent

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How to compare QCD to Super-Yang-Mills

SYM action: $$S = S_{\mathrm{SYM}}(e, \mathcal{B}) + S_{\mathrm{EM}}(e, \mathcal{B})$$

SYM field content: fermions, scalar particles, vector field

SYM properties: conformal symmetry, supersymmetry, ...

SYM appears to be entirely different from QCD!

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Strategy:

- compare thermodynamic quantities (macroscopic / effective); e.g. pressure
- match divergencies in the two theories, i.e. match beta functions
- measure magnetic fields in "same units"
- compare two theories at same renormalization scale

SYM magnetic field $\mathcal{B}$ vs. QCD magnetic field B: $$B = \boxed{\xi}\mathcal{B}$$

How to choose a holographic model?

The same way, we choose a hydrodynamic model:

- match symmetries (and anomalies)
- include interesting operators

depends on the physical question

- match magnetic properties

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Einstein-Maxwell-Chern-Simons gravity has dual with:
cf. talk by K. Landsteiner

- chiral anomaly, breaking a U(1) axial symmetry
- axial current and energy momentum tensor
  chiral magnetic transport
- thermodynamics match well (in external B field)

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} \mathrm{d}^5x \sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

dual to $\mathcal{N}$=4 Super-Yang-Mills theory coupled to U(1)

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Successful example: holographic model discovering chiral vortical effect (2008)

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

[Banerjee et al; JHEP (2011)]

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…gy momentum tensor

chiral magnetic transport

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dual to $\mathcal{N}$=4 Super-Yang-Mills theory coupled to U(1)

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]

[Abbasi et al.; PLB (2016)]

[Kalaydzhyan, Murchikova; NPB (2016)]

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial \epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6 w_0} + c_s^2\, \frac{w_0\, \sigma}{2 n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\, \frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3 C T_0 \mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\, \frac{1 - c_s^2}{w_0}\, \xi_V^{(0)}\,, \qquad v_0 = \frac{2 B\, T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument

[Son,Surowka; PRL (2009)]

[Neiman,Oz; JHEP (2010)]