

# SATURATION FRAMEWORK RESULTS ON MULTIPLICITY BIASED $P_A$ COLLISIONS

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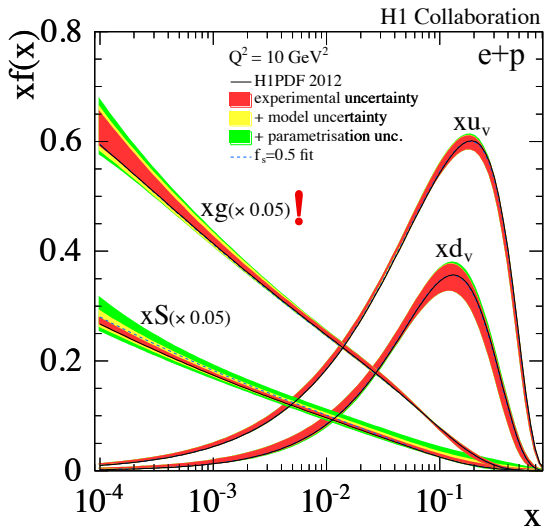


- Introduction: saturation framework/CGC and why theorists like it
- Phenomenological challenges or where CGC fails to describe data. Does it?!

CGC inspired models  $\neq$  CGC

- What do we know from first principle QCD?
- Fluctuations in CGC
- Application:  $Q_{pA}$
- Conclusion and Outlook

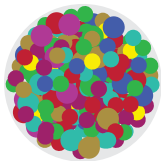
# HIGH ENERGY LIMIT OF QCD



High energy limit (small  $x \propto 1/\sqrt{s}$ )  $\equiv$  high gluon density

# HIGH ENERGY LIMIT OF QCD

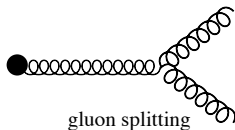
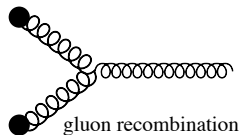
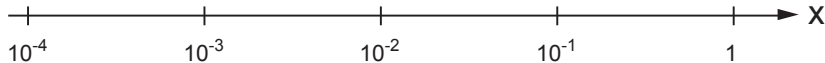
Non-Linear Dynamics  
Regime



Radiation Dominated  
Regime

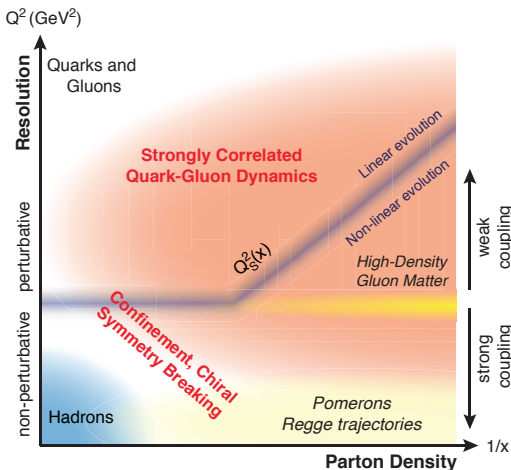


Valence Quark  
Regime

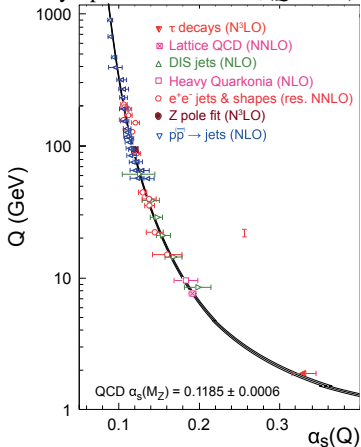


Emerging dynamical scale: saturation momentum,  $Q_s$ .  
Classical Yang-Mills fields at scale  $\lambda$ :  $R_{\text{proton}} > \lambda > 1/Q_s$ .

# HIGH ENERGY “PHASE DIAGRAM” OF QCD

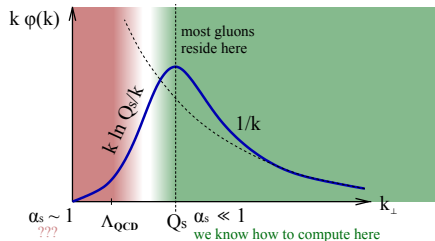


Asymptotic freedom:  $\alpha_s(Q \rightarrow \infty) \rightarrow 0$ .



Only left corners of the diagram are well studied.

- Particle production is dominated by  $k_{\perp} \sim Q_s$
- Weak coupling methods can be applied  $\alpha_s(Q_s) \ll 1$



- Still non-perturbative, as fields are strong,  $A \sim \frac{1}{g} \rightsquigarrow$  non-linearities are important

Other examples of non-perturbative weak coupling regimes:

- instantons, monopoles, ...
- holonomy (Polyakov Loop) of Yang-Mills at finite  $T$ ,  $A_0 \propto T/g$
- phase transitions,  $\sigma^2 \propto |m^2|/\lambda$
- resurgence program and plethora of QCD-like theories
- gravitational (and other non-linear FT) memory effect

Common futures:

- calculable!
- non-trivial as go beyond perturbative

Saturation framework/CGC provides an opportunity to study regions of  
high parton density in the small coupling regime,  
where calculations are still under control!

There are a lot of successes. Nonetheless challenges remain.

- Thermalization?! Indications: requires non-perturbative physics...
- Rare high multiplicity events?! Indications: requires non-perturbative physics...
- Odd harmonics of two particle azimuthal correlations,  $v_3\{2\}$  ✓
  - CGC inspired  $k_T$  factorization approach  $v_3\{2\} = 0$
  - About 7 years of the community effort
  - Solution: original idea in L. McLerran & V.S., arXiv:1611.09870;  
cross-check and evaluation in Yu. Kovchegov & V.S., arXiv:1802.08166
- Higher order correlations,  $v_2\{n > 2\}$ 
  - CGC inspired  $k_T$  factorization approach  $v_2^4\{4\} < 0$ 

*V.S., arXiv:1412.5191*
  - We know what has to be done but it requires analytical and numerical effort
- Centrality dependent nuclear modification factor  $R_{pA}$  ( $Q_{pA}$ )

*this talk and A. Dumitru, G. Kapilevich & V.S., arXiv:1802.06111*



# WHAT DO WE KNOW ANALYTICALLY?

Asymmetric collisions, when  $Q_s$  of the projectile  $\neq Q_s$  of the target, is the easiest case.



Single inclusive production

- In general

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} f\left(\frac{Q_{sp}}{k_{\perp}^2}, \frac{Q_{sA}}{k_{\perp}^2}\right)$$

$f\left(\frac{Q_{sp}}{k_{\perp}^2}, \frac{Q_{sA}}{k_{\perp}^2}\right)$  is known only numerically

- If  $k_{\perp} > Q_{sp}$ ,

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} \frac{Q_{sp}}{k_{\perp}^2} f^{(1)}\left(\frac{Q_{sA}}{k_{\perp}^2}\right) + \frac{1}{\alpha_s} \left(\frac{Q_{sp}}{k_{\perp}^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}}{k_{\perp}^2}\right) + \dots$$

Functions  $f^{(n)}$  are calculable!

Asymmetric collisions, when  $Q_s$  of the projectile  $\neq Q_s$  of the target, is the easiest case.



$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} \frac{Q_{sp}}{k_{\perp}^2} f^{(1)}\left(\frac{Q_{sA}}{k_{\perp}}\right) + \frac{1}{\alpha_s} \left(\frac{Q_{sp}}{k_{\perp}}\right)^2 f^{(2)}\left(\frac{Q_{sA}}{k_{\perp}}\right) + \dots$$

- $f^{(1)}$  is known since '98

*Y. V. Kovchegov and A. H. Mueller, Nucl. Phys. B529, 451 (1998), hep-ph/9802440*  
*A. Dumitru and L. D. McLerran, Nucl. Phys. A700, 492 (2002), hep-ph/0105268*

- $f^{(2)}$ : no complete result yet, attempted in '15

*G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, JHEP 03, 015 (2015), 1501.03106*

# DOUBLE INCLUSIVE PRODUCTION

Asymmetric collisions, when  $Q_s$  of the projectile  $\neq Q_s$  of the target, is the easiest case.



$$\frac{d^2N}{d^3kd^3p} = \frac{1}{\alpha_s^2} Q_{sp}^2 h^{(1)}(Q_{sA}) + \frac{1}{\alpha_s^2} Q_{sp}^4 h^{(2)}(Q_{sA}) + \dots$$

- $h^{(1)}$  is known since '12; invariant under  $(k_{\perp} \rightarrow -k_{\perp}) \rightsquigarrow$  no odd harmonics

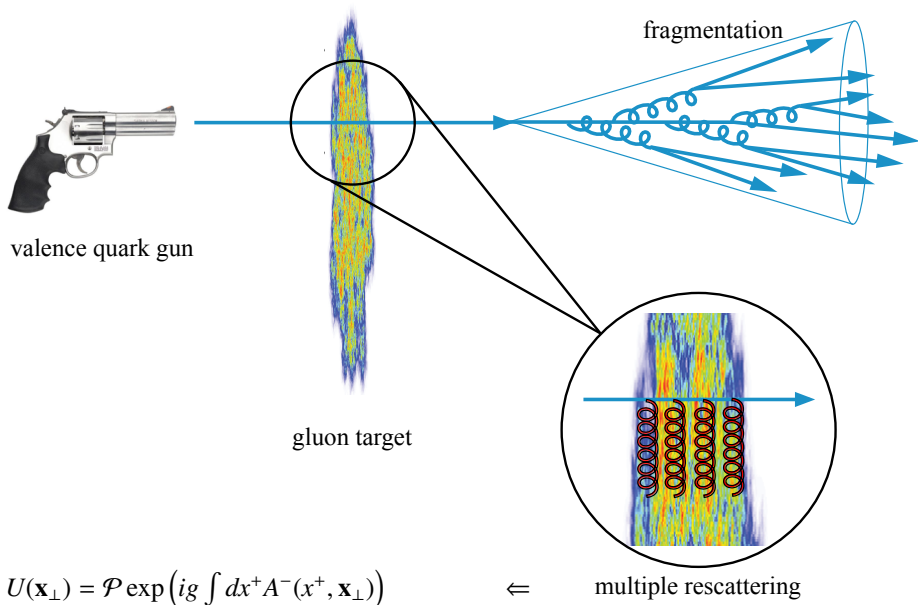
*A. Kovner and M. Lublinsky, Int. J. Mod. Phys. E22, 1330001 (2013), 1211.1928  
Y. V. Kovchegov and D. E. Wertheim, Nucl. Phys. A906, 50 (2013), 1212.1195*

- $h^{(2)}$ : no complete result yet, but odd part under  $(k_{\perp} \rightarrow -k_{\perp})$  was found

*L. McLerran and V. S., Nucl. Phys. A959, 83 (2017), 1611.09870;  
Yu. Kovchegov and V. S., arXiv:1802.08166*

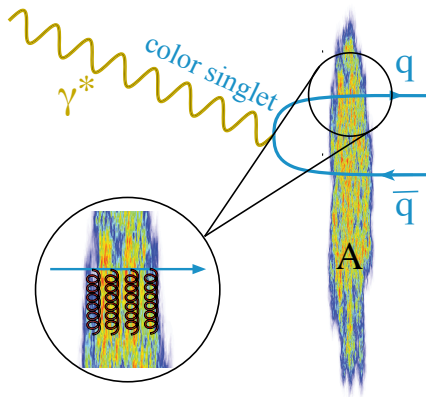
“... we conclude that the odd azimuthal harmonics are an inherent property of particle production in the saturation framework...”

# MULTIPLE RESCATTERING; SHOOTING QUARKS THROUGH NUCLEAR TARGET



# COLOR MEMORY

- Gravitational wave memory: permanent shift of two inertial observers after the passage of gravitational memory



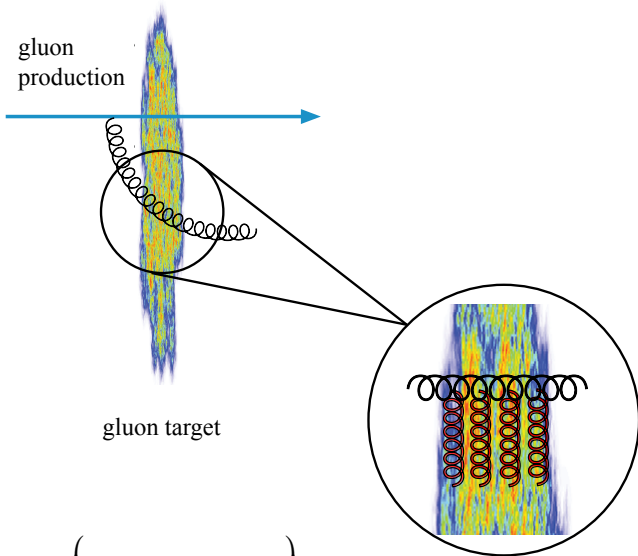
- Analogous effect in YM: two color charges in color singlet state will not be in color singlet state after the passage of color “wave”

- A. Strominger and coauthors:  
“Color Memory: a Yang Mills analog of gravitational wave memory”;

*M. Pate, A. Raclariu & A. Strominger, PRL 119, 261602 (2017)*

- Most of observables in saturation framework are related to “color memory”.

# GLUON PRODUCTION



$$W(\mathbf{x}_\perp) = \mathcal{P} \exp \left( ig \int dx^+ \underline{A}_{\text{adj.}}^-(x^+, \mathbf{x}_\perp) \right)$$

$\Leftarrow$  multiple rescattering

- Computing analytically

$$E_k \frac{dN}{d^3k} \{ \rho_p, \rho_T \} = \frac{1}{2(2\pi)^3 k_\perp^2} (\delta_{ij} \delta_{lm} + \epsilon_{ij} \epsilon_{lm}) \Omega_{ij}^b(\mathbf{k}_\perp) \left[ \Omega_{lm}^b(\mathbf{k}_\perp) \right]^*$$

Here  $\delta_{ij} \Omega_{ij} = \Omega_{xx} + \Omega_{yy}$  and  $\epsilon_{ij} \Omega_{ij} = \Omega_{xy} - \Omega_{yx}$  and

$$\Omega_{ij}^a(\mathbf{x}_\perp) = g \left[ \frac{\partial_i}{\partial^2} \overbrace{\rho^b(\mathbf{x}_\perp)}^{\text{val. sour.}} \right] \partial_j \overbrace{W^{ab}(\mathbf{x}_\perp)}^{\text{target W line}}$$

valence sources rotated by the target

- Note that  $W(\mathbf{x}_\perp)$  depends on positions of color charges in the target. Glauber  $N_{\text{part}}$  fluctuations are also in  $W(\mathbf{x}_\perp)$ .
- Min. bias:

$$E_k \frac{dN}{d^3k} = \left\langle E_k \frac{dN}{d^3k} \{ \rho_p, \rho_T \} \right\rangle_{\rho_p, \rho_T}$$

A. Dumitru and L. D. McLerran, Nucl. Phys. A700, 492 (2002), hep-ph/0105268

L. McLerran and V. S., Nucl. Phys. A959, 83 (2017), 1611.09870

Yu. Kovchegov and V. S., arXiv:1802.08166

# “GLITTERING GLASMAS”

- For fixed  $N_{\text{part}}$ ,  $E_k \frac{dN}{d^3k} \{\rho_p, \rho_T\}$  fluctuates on configuration-by-configuration basis  
event-by-event

↪ Larry’s “Glittering glasma”; in plain words: color density fluctuations

- These fluctuations are negative binomial:  
the fact derived from first principles at momentum  $\gg Q_{sT}$ !

*F. Gelis, T. Lappi and L. McLerran, Nucl. Phys. A **828**, 149 (2009), arXiv:0905.3234*

- Numerical calculations in whole range of  $k_{\perp}$  in satur. framework support the above

*M. Mace & V.S., work in progress*



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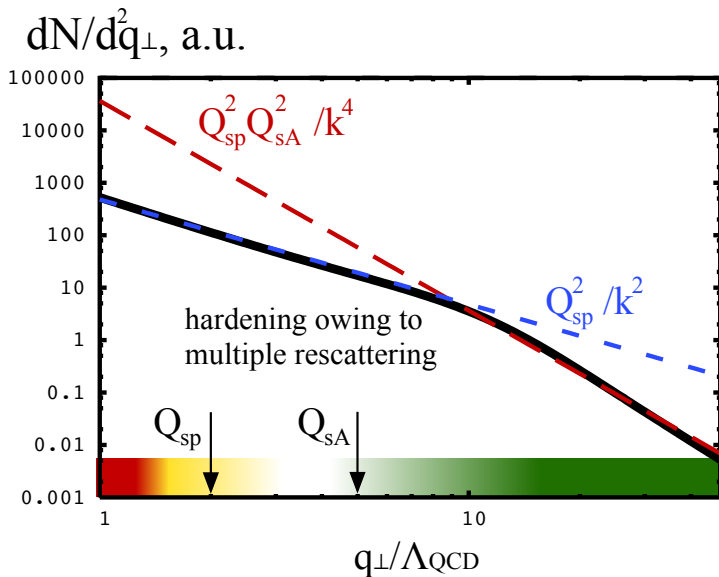
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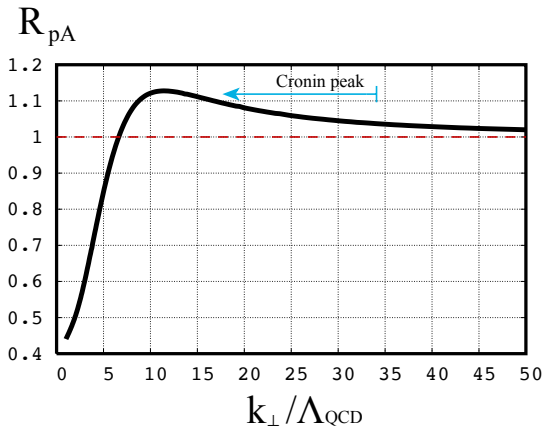
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# NUMERICAL RESULT FOR SINGLE INCLUSIVE PRODUCTION IN pA



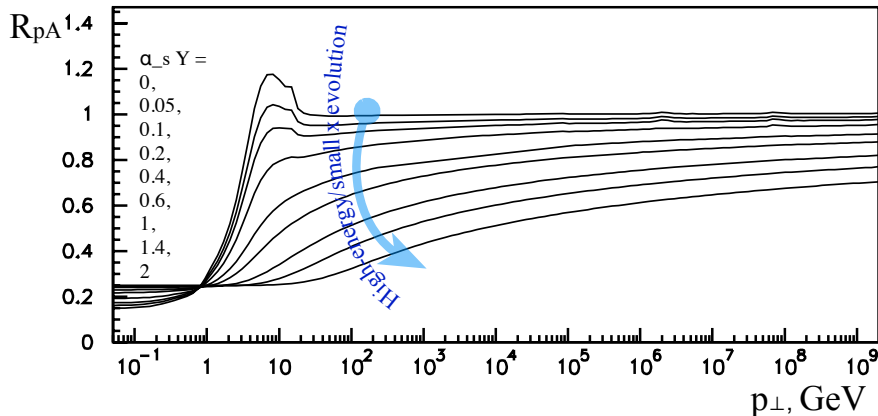
# MIN. BIAS $R_{pA}$ : SCHEMATICALLY

- Nuclear modification factor  $R_{pA} = \frac{\frac{dN_{pA}}{d^3k}}{N_{\text{part}}^{\text{min.bias}} \frac{dN_{pp}}{d^3k}}$
- Minimum bias corresponds to an average over color fluctuations,  $N_{\text{part}}$  etc.



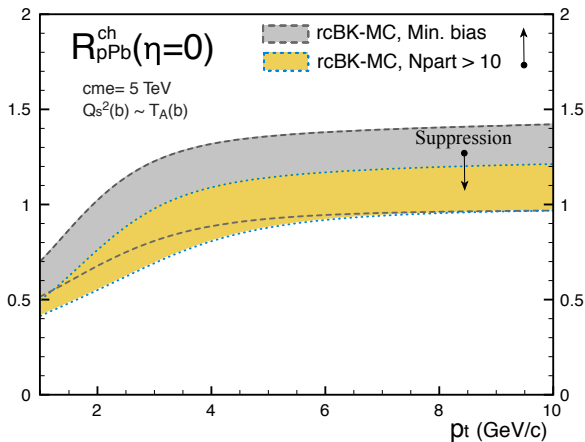
# MIN. BIAS $R_{pA}$ : NUMERICAL RESULTS

- High energy evolution



*J. L. Albacete, N. Armesto, A. Kovner, C. A. Salgado and U. A. Wiedemann, "Energy dependence of the Cronin effect from nonlinear QCD evolution," Phys. Rev. Lett. **92**, 082001 (2004)*

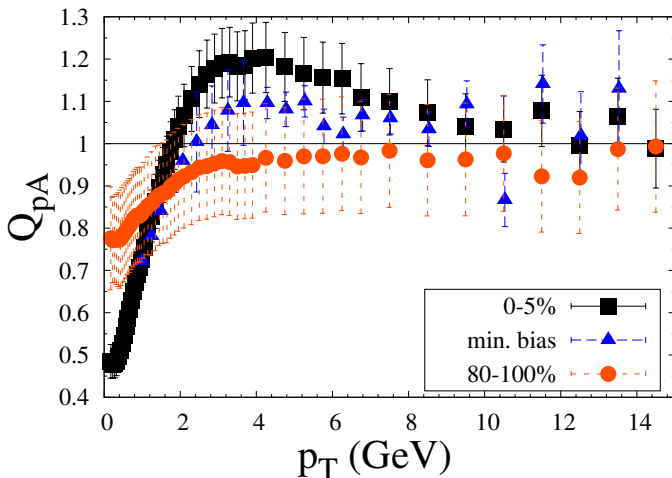
- $R_{pA}$  goes down with  $N_{\text{part}}$ -bias towards larger values



*J. L. Albacete, A. Dumitru, H. Fujii and Y. Nara, Nucl. Phys. A 897, 1 (2013)*

- Naively, central collisions correspond to those where the projectile proton suffers an inelastic collision with a greater than average number of target nucleons.
  - This is analogous to minimum bias pA collisions with a target nucleus with many more than  $\sim 200$  nucleons.
- $\rightsquigarrow$  a stronger suppression of  $R_{pA}(k)$  for central versus minimum bias events.
- $N_{\text{part}}$  or  $N_{\text{coll}}$  can not be measured directly.  
Experimentally, one therefore employs a variety of different centrality measures

- Loosely speaking  $Q_{pA}$  is  $R_{pA}$  in collisions of different centrality  
(details: How to define “centrality”?! See thorough investigation by ALICE)



*J. Adam et al. [ALICE Collaboration], Phys. Rev. C **91**, no. 6, 064905 (2015)*

- Re-emerging Cronin peak  $\uparrow$  in central collisions

↪ Something is missing and we know what it is:

color fluctuations!



- Distribution of classical field

$$\mathcal{W}[A^+] = \exp(-S[A^+])$$

- Expectation values

$$\langle O[A^+] \rangle = \frac{1}{Z} \int \mathcal{D}A^+ \mathcal{W}[A^+] O[A^+]; \quad Z = \int \mathcal{D}A^+ \mathcal{W}[A^+] \quad k^2 A^+(k) = g\rho_T(k)$$

*L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994), hep-ph/9309289*

*L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 3352 (1994), hep-ph/9311205.*

- Example: McLerran and Venugopalan (MV) model

$$S_{\text{MV}} = \int dx^- \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{\text{tr} \left[ k^2 A^+(x^-, \mathbf{k}_\perp) k^2 A^+(x^-, -\mathbf{k}_\perp) \right]}{g^2 \mu^2(x^-)}$$

- $S[A^+]$  contains a plethora of possible excitations/fluctuations
- Integrate out fluctuations which do not change the observable of interest  $O[A^+]$   
 $\leadsto$  effective action/potential for  $X(q) = O[A^+(q)]$ .

$$e^{-V_{\text{eff}}[X(q)]} = \frac{1}{Z} \int \underbrace{\mathcal{D}A^+}_{\text{integrate the rest}} \mathcal{W}[A^+] \underbrace{\delta(X(q) - O[A^+(q)])}_{\text{keep interesting}}$$

- The normalization

$$\int \mathcal{D}X e^{-V_{\text{eff}}[X(q)]} = 1$$

- Consider  $O[A^+] = g^2 \text{tr}[A^+(q)]^2$  – simplest but still non-trivial

*L. O'Raifeartaigh, A. Wipf and H. Yoneyama, Nucl. Phys. B 271, 653 (1986);  
 YM with non-trivial holonomy: C. P. Korthals Altes, Nucl. Phys. B 420, 637 (1994)*

$$O[A^+] = g^2 \text{tr}|A^+(q)|^2$$

- The effective action

$$V_{\text{eff}} = \int \frac{d^2 q}{(2\pi)^2} \left[ \frac{q^4}{g^4 \mu^2} X(q) - \frac{1}{2} A_{\perp} N_c^2 \log X(q) \right]$$

with redefinition  $\exp[\Phi(q)] = X(q)/X_{\text{saddle}}(q)$

$\leadsto$  Liouville action/potential

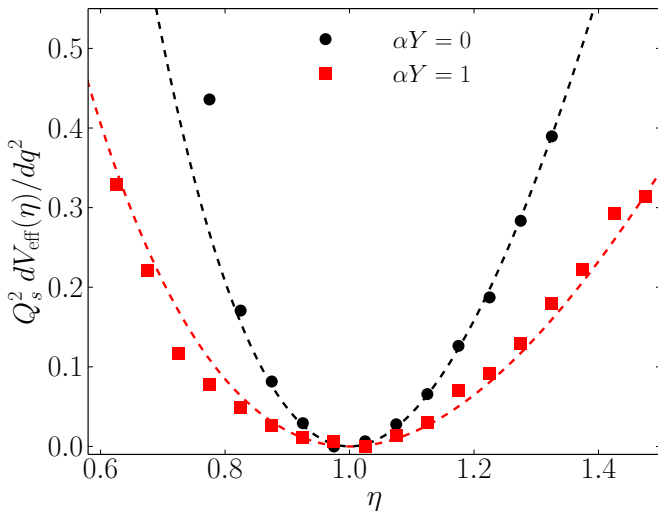
$\leadsto$  2-d gravity results can be used for  $V_{\text{eff}}$

$$V_{\text{eff}} = A_{\perp} N_c^2 \int \frac{d^2 q}{(2\pi)^2} \left[ e^{\Phi(q)} - \Phi(q) - 1 \right]$$

Negative Ricci scalar  $\uparrow$

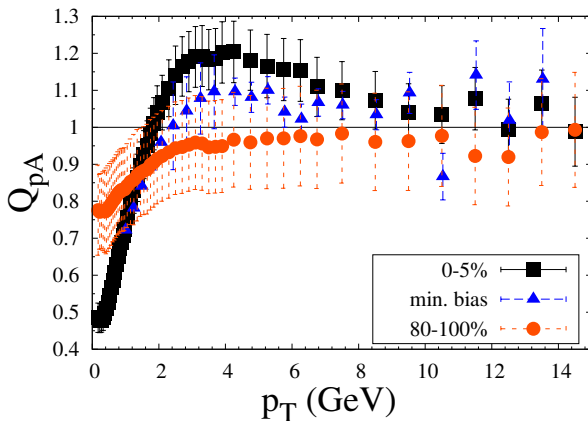
$\times$  Kinetic term: work in progress

# $O[A^+] = g^2 \text{tr}|A^+(q)|^2$ : NUMERICAL RESULT



$$\eta = X/X_{\text{saddle}}$$

- Centrality classes based on signal in zero degree calorimeter in nucleus hemisphere
- $N_{\text{coll}}$  for each centrality class is defined through  $N_{\text{ch}}$  at mid-rapidity



*J. Adam et al. [ALICE Collaboration], Phys. Rev. C **91**, no. 6, 064905 (2015)*

# APPLICATION

- We cannot replicate ALICE centrality selection  
(a theorist's question: what is the zero degree calorimeter?)

- We can reweight towards configuration with more gluons at

$$p_{\perp} > Q_{\text{geom. scal.}} \sim Q_{sT}^2(Y)/\Lambda$$

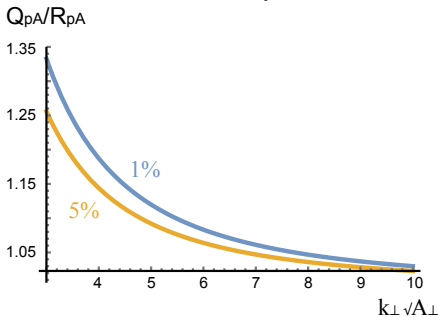
(where the anomalous dimensions  $\sim 1$ , close to DGLAP limit)

- Take 
$$N_{\text{coll}} = \frac{\int_{Q_{\text{geom. scal.}}} \left\langle \frac{d^2 N_{pA}}{d^2 p dy} \right\rangle_{\text{rw}}}{\int_{Q_{\text{geom. scal.}}} \left\langle \frac{d^2 N_{pp}}{d^2 p dy} \right\rangle}$$

- Using formalism of constraint action

$$Q_{pA}(k_{\perp}) \approx \left( 1 + \frac{8\pi \log p_r^{-1}}{N_c^2 A_{\perp} k_{\perp}^2 \log \frac{Q_{UV}^2}{Q_{sT}^2}} \right) R_{pA}(k_{\perp})$$

Schematically  $\Downarrow$



- JIMWLK – functional renormalization group equation describing evolution of ensemble of Wilson lines at small  $x$  ( $Y = \ln x_0/x$ ).

$$\frac{\partial}{\partial Y} \mathcal{W}[U(\mathbf{x}_\perp)] = -H \left[ U, \frac{\partial}{\partial U} \right] \mathcal{W}[U]$$

- Analogous to Fokker-Plank equation.

↪ Langevin form: stochastic evolution of  $U(\mathbf{x}_\perp)$  in  $Y$ .

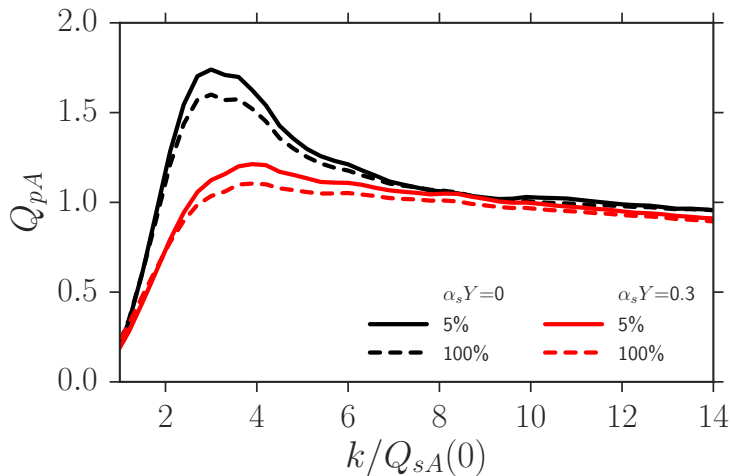
“Random walk” in space of Wilson lines

$$\partial_Y U(\mathbf{x}) = U(\mathbf{x}) \frac{i}{\pi} \int \mathbf{d}^2 \mathbf{u} \frac{(\mathbf{x} - \mathbf{u})^i \eta^i(\mathbf{u})}{(\mathbf{x} - \mathbf{u})^2} - \frac{i}{\pi} \int \mathbf{d}^2 \mathbf{v} U(\mathbf{v}) \frac{(\mathbf{x} - \mathbf{v})^i \eta^i(\mathbf{v})}{(\mathbf{x} - \mathbf{v})^2} U^\dagger(\mathbf{v}) U(\mathbf{x})$$

Gaussian white noise  $\eta^i = \eta_a^i t^a$ :  $\langle \eta_i^a(\mathbf{x}) \eta_j^b(\mathbf{y}) \rangle = \alpha_s \delta^{ab} \delta_{ij} \delta^{(2)}(\mathbf{x} - \mathbf{y})$ .

Noise describes gluon emission/absorption.

In mean-field approximation JIMWLK reproduces BK (Balitsky-Kovchegov) equation.



MV initial condition with  $(Q_{sT}(0)/Q_{sp}(0))^2 \approx 6$



$$Q_{pA}(k) \approx \left( 1 + \frac{8\pi \log p_r^{-1}}{N_c^2 A_{\perp} k^2 \log \frac{Q_{UV}^2}{Q_{sT}^2}} \right) R_{pA}(k)$$

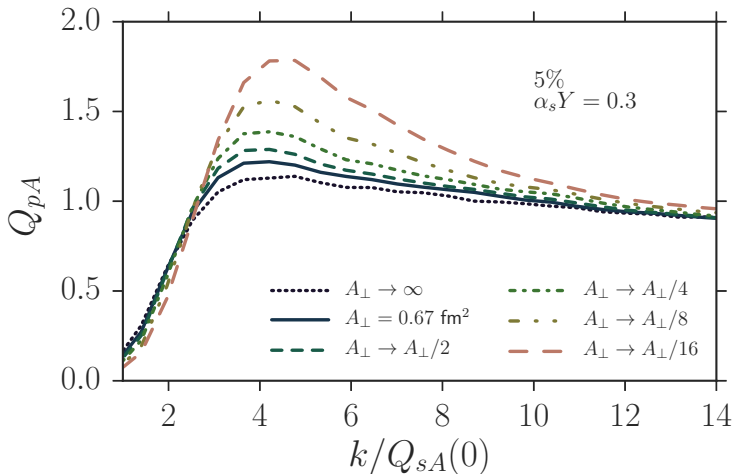
Area of projectile  $\uparrow$

Smaller projectile

$\rightsquigarrow$  larger fluctuations

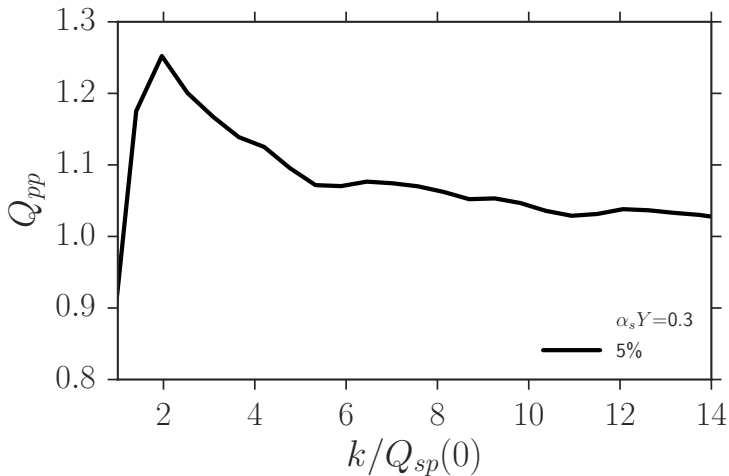
$\rightsquigarrow$  larger deviation from  $R_{pA}$

# NUMERICS: $Q_{pA}$ AND $A_{\perp}$ DEPENDENCE



MV initial condition with  $(Q_{sT}(0)/Q_{sp}(0))^2 \approx 6$

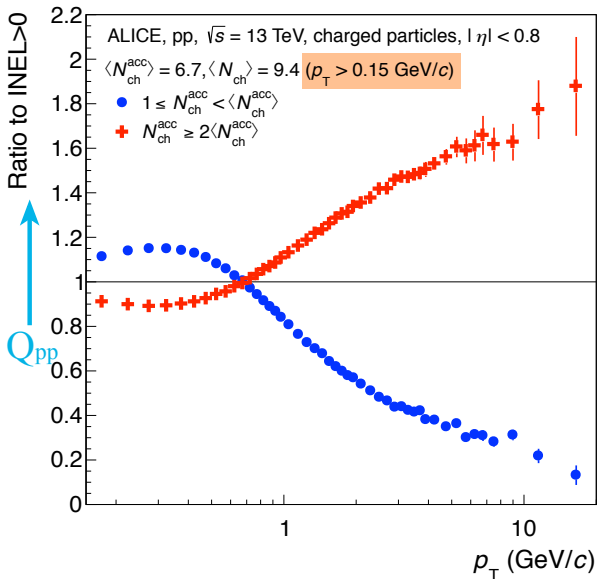
Varying  $A_{\perp}$  through dijet/Z-boson trigger?!



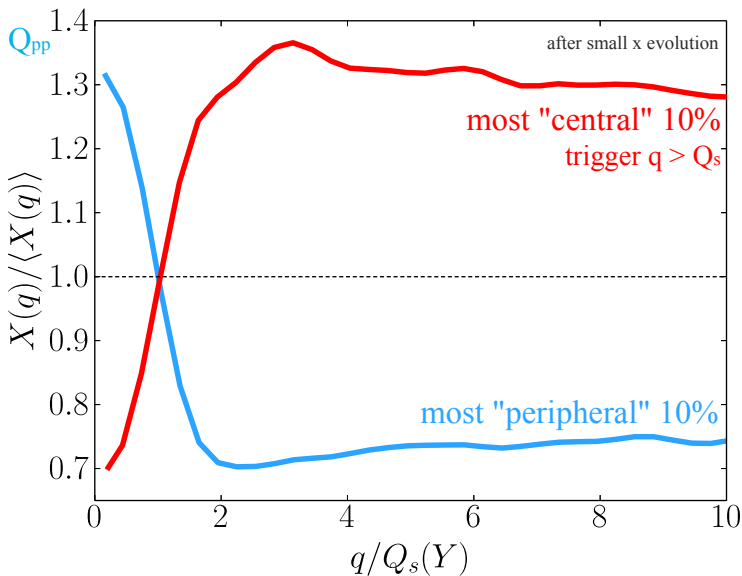
MV initial condition with  $(Q_{sT}(0)/Q_{sp}(0))^2 \approx 1$

- Naive implementation: tiny number of particles with  $q > Q_{gs}$

# DIFFERENT “CENTRALITY” DEFINITION, $Q_{pp}$



# DIFFERENT "CENTRALITY" DEFINITION, $Q_{pp}$



A. Dumitru & V. S., *Phys. Rev. D* **96**, no. 5, 056029 (2017), arXiv:1704.05917

- Saturation framework/CGC provides an opportunity to study regions of high parton density from first principles.
- CGC inspired models  $\neq$  CGC
- Naive handwaving arguments  $\neq$  CGC
- “Shut up and compute!”
- Nuclear modification factor: promising if color fluctuations are accounted for.  
In this talk: proof of the concept. Resolves the conundrum  $Q_{pA} > R_{pA}$ ;  
 $\leadsto$  re-appearance of Cronin peak
- Quantitative comparison: in progress