Evolution of charm-meson ratios in an expanding hadron gas

Kevin Ingles, Ohio State University

Collaborators:
Eric Braaten, Ohio State University
Roberto Bruschini, University of Valencia
Li-Ping He, University of Bonn
Jun Jiang, Shandong University

Based on arXiv:2209.04972

Seminar at GSI
October 5$^{\text{th}}$, 2022

THE OHIO STATE UNIVERSITY

Outline

Outline

t-channel singularities

Definition

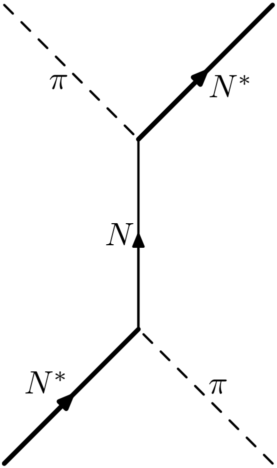
A t-channel singularity is a divergence in the rate of a reaction in which an unstable particle decays and one of its decay products scatters. The divergence arises if the exchanged particle can be on-shell.

t-channel singularities

t-channel singularities were first discussed by Peierls in 1961 for πN^* scattering

In the diagram, the exchanged nucleon N can be on shell because the N^* can decay into $N\pi$. This leads to a divergence in the cross section

Peierls suggested that the N^* width be inserted into N propagator
However, this still leads to unphysically large cross sections



Peierls, PRL **6**, 641-643 (1961)

t-channel singularities

Simplest examples in Standard Model

Reaction $\nu Z \to \nu Z$ proceeds through exchange of $\bar{\nu}$

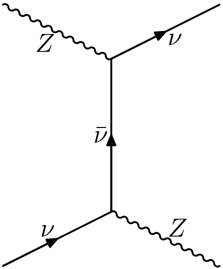
Cross section diverges for

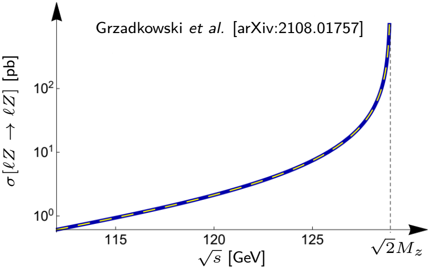
$$\sqrt{2}M_Z < \sqrt{s}$$

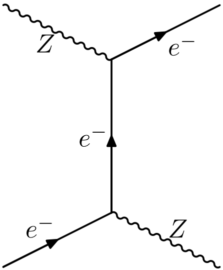
Weak Compton scattering $e^- Z \to e^- Z$

Cross section diverges for

$$\sqrt{2}M_Z < \sqrt{s} < M_Z^2/m_e$$







t-channel singularities

For muon colliders, the reaction $\mu^+\mu^- \to e^-\bar{\nu}_e W^+$ can proceed through the exchange of a ν_μ
The exchanged ν_μ can be on-shell

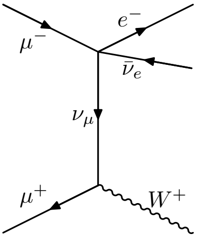
Melnikov and Serbo proposed to regulate divergence using finite beam width

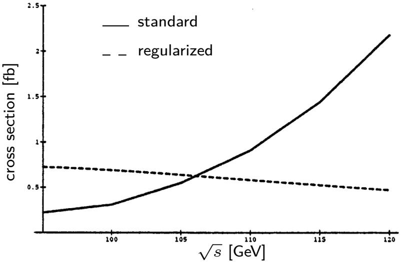
Scattering cross section linear in transverse beam size $\sigma_\perp$

$$\text{cross section} \propto \frac{\sigma_\perp}{q_\perp}$$

$q_\perp$ - transverse momentum component of beam

Melnikov and Serbo, PRL **76**, 3263-3266 (1996)



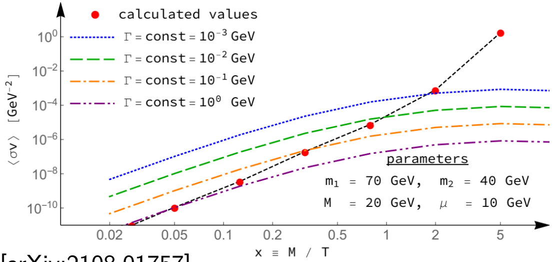


t-channel singularities

Grzadkowski *et al.* reviewed t-channel singularities and their proposed regularization methods in 2021

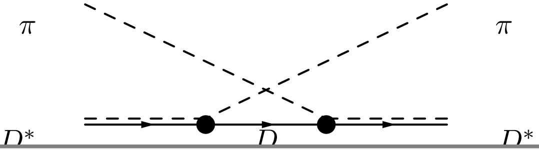
For in-medium reaction, they noted that t-channel singularities are regularized by thermal widths

$$\frac{1}{t - M^2} \longrightarrow \frac{1}{t - M^2 - \Sigma}, \quad \Sigma \approx M\delta M - iM\Gamma$$



Grzadkowski *et al.* [arXiv:2108.01757]

t-channel singularities



Charm meson have remarkable property

$$\underbrace{M_{D^*} - M_D \approx m_\pi}_{\text{Can be used to simplify calculations}}$$

Charm-meson reaction $\pi D^* \to \pi D^*$ can have t-channel singularity because exchanged D can be on-shell

For production in heavy-ion collisions, t-channel singularities are regularized by thermal width of D

$$\text{cross section} \propto \frac{1}{\text{thermal width}}$$

Outline

Heavy-ion collisions: Overview

The standard model of heavy-ion collisions is a multi-stage model

S1 Initial Collision

- Impact parameter/centrality, energy deposition

S2 Thermalization

- Hydrodynamic/transport modeling

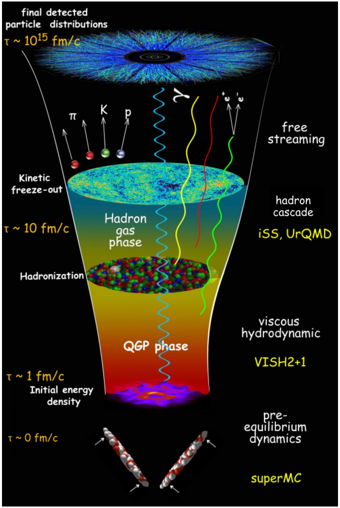
S3 Hadronization

- Particle production at phase transition

S4 Kinetic freeze-out

- Particles stop interacting, momentum distributions frozen

State of the art models consist of complex numerical simulations for each stage



Credit: Chun Shen, Wayne State University

Heavy-ion collisions: Hadronization

After quark gluon plasma expands and cools to the hadronization temperature T_H, quarks become confined to hadrons in process called "hadronization"

Common models that exist for hadronization are:

(a) Statistical Hadronization Model
(b) Cooper-Frye Particlization
(c) Quark-Coalescence Model

Heavy-ion collisions: Hadronization

After quark gluon plasma expands and cools to the hadronization temperature T_H, quarks become confined to hadrons in process called “hadronization”

Common models that exist for hadronization are:

(a) Statistical Hadronization Model

(b) Cooper-Frye Particlization

(c) Quark-Coalescence Model, and more

The hadrons produced then undergo collisions within the hadron resonance gas until interactions stop (kinetic freeze-out)

The evolution of the hadron resonance gas can be modelled using hydrodynamics as well

Heavy-ion collisions: Hadronization

We assume the hadronic resonance gas can be modeled as a highly viscous fluid whose volume and temperature depend on proper time[1]

$$V(\tau) = \left[R + v(\tau - \tau_H) + \frac{a}{2}(\tau - \tau_H)^2\right]^2 c\tau$$

$$T(\tau) = T_H - (T_H - T_F)\left(\frac{\tau - \tau_H}{\tau_F - \tau_H}\right)^{4/5}$$

which are fit to hydrodynamics simulations for Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV[2]

T_H [MeV]	T_F [MeV]	τ_H [fm/c]	τ_F [fm/c]	R [fm]	v [c]	a [c^2/fm]
156	115	7.1	21.5	11	0.5	0.09

[1] Hong *et al.*, PRC **98**, 014913 (2018)
[2] Abreu, PRD **103**, 036013 (2021)

Heavy-ion collisions: Kinetic freeze-out

At kinetic freeze-out, all interactions are assumed to stop and particle spectra are determine by feed-down

The t-channel singularity in the reaction $\pi D^* \to \pi D^*$ allow charm meson to continue interacting well after kinetic freeze-out

Heavy-ion collisions: Kinetic freeze-out

In our model we assume that:

- charm mesons can continue to interact after kinetic freeze-out
- the pion momentum distributions are frozen and given by

$$\mathfrak{f}_\pi = \frac{\mathfrak{n}_\pi}{\mathfrak{n}_\pi^{(\mathrm{eq})}} \frac{1}{e^{\omega_q/T_F} - 1}, \; \mathfrak{n}_\pi^{(\mathrm{eq})} = \int \frac{d^q}{(2\pi)^3} \frac{1}{e^{\omega_q/T_F} - 1}, \; \omega_q = \sqrt{q^2 + m_\pi^2}$$

- the temperature of the hadron gas of fixed to the kinetic freeze-out temperature $T_F = 115$ MeV
- the volume continues to expand with

$$V(\tau) = \pi \left[R_F + v_F(\tau - \tau_F)\right]^2 c\tau$$

$R_F = 24.0$ fm, $v_F = 1.00c$, $\tau_F = 21.5$ fm/c

- the number density is given by

$$\mathfrak{n}_\pi(\tau) = \frac{V(\tau_F)}{V(\tau)} \mathfrak{n}_\pi(\tau_F)$$

Outline

THE OHIO STATE UNIVERSITY

Statistical Hadronization Model

A simple model for particle production at heavy-ion collisions is the statistical hadronization model (SHM)

Light hadrons are produced during phase transition of QGP and are assumed to be in thermal and chemical equilibrium with QGP

Abundance for a light hadron h given by

$$N_h = \nu_h V \int \frac{d^3k}{(2\pi)^3} f_{\rm eq}(E_k/T_H), \quad E_k = \sqrt{k^2 + M_h^2}$$

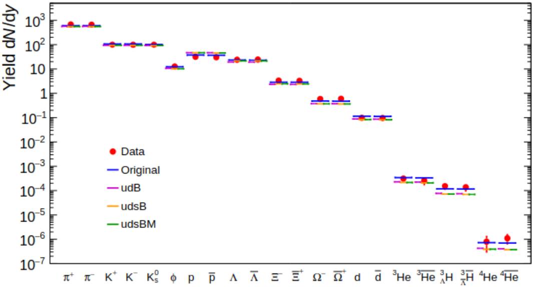
only depends on particle spin, mass and temperature of QGP

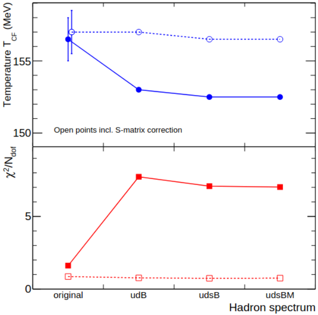
Model used to fit data from heavy-ion colliders and determine hadronization temperature $T_H = 156$ MeV

Statistical Hadronization Model

Statistical hadronization model does a very good job at reproducing the data

Andronic *et al*. Nucl. Phys. A **1010**, 122176 (2021)





Statistical hadronization of charm quarks: SHMc

Charm quarks are primarily produced from hard collisions and remain out of chemical equilibrium with the QGP since $m_c \gg T_{\mathrm{QGP}}$

Conservation of charm quarks leads to charm fugacity $g_c = 29.6 \pm 5.2$

Abundances of charm hadron H with n charm and anti-charm quarks is given by Boltzmann distribution

$$N_H = \nu_H g_c^n V \frac{M_H^2 T_H^2}{2\pi^2} K_2(M_H/T_H)$$

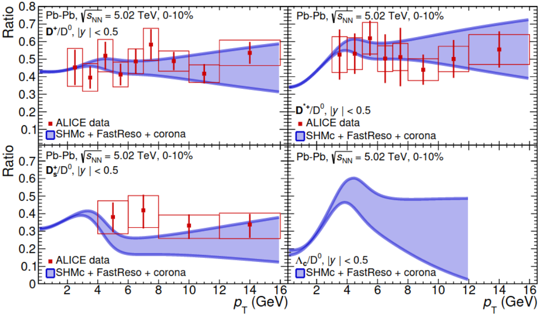
Final abundances at heavy ion colliders is calculated as sum SHMc + `FastReso` + corona

- `FastReso` - numerical method for calculating p_T-distributions based on particle decays
- corona - region of heavy-ion collision described by colliding protons

Andronic *et al.* JHEP **07**, 035 (2021)

Statistical Hadronization of charm quarks: SHMc

Comparison with data



Andronic *et al.* JHEP **07**, 035 (2021)

ALICE, JHEP **01**, 174 (2022)

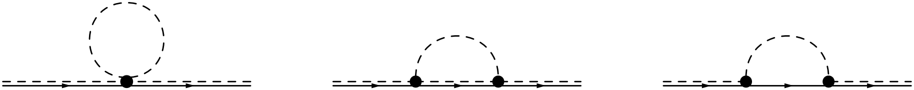
	Measured dN/dy	SHMc dN/dy
	0–10% centrality	
D^0	6.819 ± 0.457 (stat.) $^{+0.912}_{-0.936}$ (syst.) ± 0.054 (BR)	6.42 ± 1.07
D^+	3.041 ± 0.073 (stat.) $^{+0.154}_{-0.155}$ (syst.) ± 0.052 (BR) $^{+0.352}_{-0.618}$ (extrap.)	2.84 ± 0.47
D^{*+}	3.803 ± 0.037 (stat.) $^{+0.084}_{-0.085}$ (syst.) ± 0.041 (BR) $^{+0.854}_{-1.175}$ (extrap.)	2.52 ± 0.42
	30–50% centrality	
D^0	1.275 ± 0.099 (stat.) $^{+0.167}_{-0.173}$ (syst.) ± 0.010 (BR)	1.06 ± 0.15
D^+	0.552 ± 0.008 (stat.) $^{+0.024}_{-0.024}$ (syst.) ± 0.009 (BR) $^{+0.068}_{-0.114}$ (extrap.)	0.471 ± 0.069
D^{*+}	0.663 ± 0.023 (stat.) $^{+0.038}_{-0.039}$ (syst.) ± 0.007 (BR) $^{+0.149}_{-0.165}$ (extrap.)	$0.419\ ^{+0.065}_{-0.061}$

Outline

Thermal mass shifts and widths: Overview

D self-energy diagrams

D^* self-energy diagrams



Thermal mass shifts and widths: Overview

Steps to calculate thermal self-energy for charm mesons from pion interactions

(1) Draw all one-loop diagrams in HHχEFT

(2) Replace the loop integrals with following rule

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_\pi + i\epsilon} \longrightarrow \int \frac{d^3q}{(2\pi)^2 2\omega_q} \mathfrak{f}_\pi(\omega_q), \quad \omega_q = \sqrt{q^2 + m_\pi^2}$$

here $\mathfrak{f}_\pi(\omega_q)$ is the pion's momentum distribution

(3) Evaluate diagrams assuming external lines are off-shell, this gives the self-energy as function of charm meson's 4-momentum $p = (p_0, \boldsymbol{p})$

(4) Mass shift and thermal width obtained from self-energy on its mass shell $(\boldsymbol{p} = 0)$

Thermal mass shifts and widths: Charm mesons

Mass shifts for D and D^* insensitive to mass splitting

D	D^*
$\delta M = \frac{3g_\pi^2}{2f_\pi^2}\mathfrak{n}_\pi \Delta \left\langle \frac{1}{\omega_q} \right\rangle$	$\delta M_* = -\frac{1}{3}\delta M$

Thermal widths are sensitive to mass splitting

D	D^*
$\Gamma_a = 3\mathfrak{f}_\pi(\Delta)\sum_c \Gamma_{*c,a}$	$\delta\Gamma_{*a} = [1+\mathfrak{f}_\pi(\Delta)]\sum_c \Gamma_{*a,c}$

Where $\Delta_{ab} = M_{*a} - M_b$ and Δ is average of 4 Δ_{ab}'s

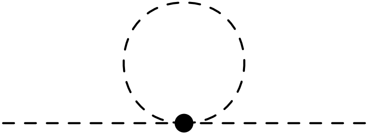
$$\mathfrak{f}_\pi(\Delta) = \frac{\mathfrak{n}_\pi}{\mathfrak{n}^{(\mathrm{eq})}}\frac{1}{e^{\Delta/T_F}-1} \approx 0.414\frac{\mathfrak{n}_\pi}{\mathfrak{n}^{(\mathrm{eq})}}$$

$\Gamma_{*a,b}$ is the partial decay width for $D^{*a} \to D^b\pi$

$$\Gamma_{*a,b} = \frac{(2-\delta_{ab})g_\pi^2}{12\pi f_\pi^2}\left[\Delta_{ab}^2 - m_{\pi ab}^2\right]^{3/2},\ m_{\pi ab} = \begin{cases} m_{\pi 0}, & a=b \\ m_{\pi +}, & a\neq b \end{cases}$$

Thermal mass shifts and widths: Pions

π self-energy diagram



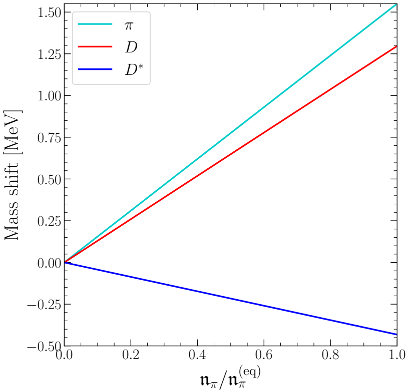
Pion mass shift is insensitive to isospin splitting

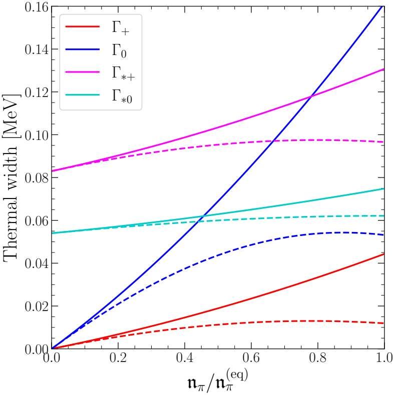
$$\delta m = \frac{m_\pi}{2f_\pi^2}\mathfrak{n}_\pi \left\langle \frac{1}{\omega_q} \right\rangle$$

At leading order, $\Gamma_\pi = 0$ MeV

Thermal mass shifts and widths: Charm mesons

Thermal mass shifts and thermal widths

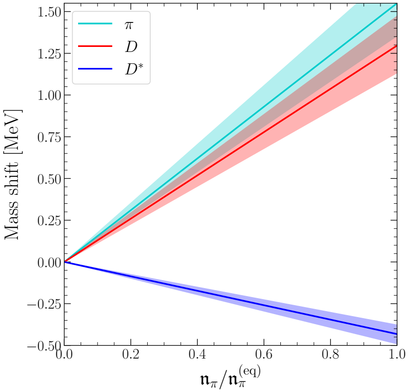


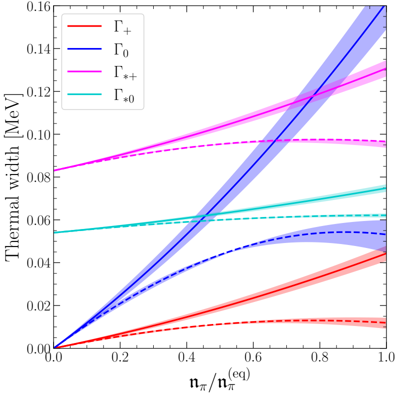


$$\Gamma_{*a,b} = \frac{(2-\delta_{ab})g_\pi^2}{12\pi f_\pi^2}\left[(M_{*a}-M_b)^2 - m_{\pi ab}^2\right]^{3/2},\; m_{\pi ab} = \begin{cases} m_{\pi 0}, & a = b \\ m_{\pi +}, & a \neq b \end{cases}$$

Thermal mass shifts and widths: Charm mesons

Thermal mass shifts and thermal widths for 110 MeV $\leq T_F \leq 120$ MeV





$$\Gamma_{*a,b} = \frac{(2-\delta_{ab})g_\pi^2}{12\pi f_\pi^2}\left[(M_{*a}-M_b)^2 - m_{\pi ab}^2\right]^{3/2}, \; m_{\pi ab} = \begin{cases} m_{\pi 0}, & a = b \\ m_{\pi +}, & a \neq b \end{cases}$$

Outline

Evolution of charm-meson abundances

D^{*0} decays into D^0 at 100%
D^{*+} decays into D^0 at B_{+0} and D^+ at $1 - B_{+0}$
$B_{+0} = \mathrm{Br}[D^{*+} \to D^0 \pi^+] = 0.677 \pm 0.005$

Naïve equations for final charm-meson abundances are

$$N_0 = (N_0)_0 + (N_{*0})_0 + B_{+0}\,(N_{*+})_0$$
$$N_+ = (N_+)_0 + 0\,(N_{*0})_0 + (1 - B_{+0})\,(N_{*+})_0$$

Using the SHMc predictions gives $(N_0/N_+)_{\text{naïave}} = 2.256 + 0.014$, the error-bar is only from B_{+0}

Evolution of charm-meson abundances

All previous studies have assumed that D^* just decay after kinetic freeze-out

In this study, we exploit the fact that charm mesons still interact kinetic freeze-out due to t-channel singularities in the reaction $\pi D^* \to \pi D^*$

Evolution of charm-meson abundances

Full evolution equations

$$\mathfrak{n}_\pi \frac{d}{d\tau}\left(\frac{\mathfrak{n}_a}{\mathfrak{n}_\pi}\right) = [1+f_\pi(\Delta)]\sum_b \Gamma_{*b,a}\mathfrak{n}_{*b} + \Gamma_{*a,\gamma}\mathfrak{n}_{*a} - 3\sum_b \langle v\sigma_{\pi a,*b}\rangle \mathfrak{n}_a\mathfrak{n}_\pi$$
$$+ 3\sum_{a\neq b}\langle v\sigma_{\pi a,\pi b}\rangle(\mathfrak{n}_b - \mathfrak{n}_a)\mathfrak{n}_\pi$$
$$+ 3\sum_b \left(\langle v\sigma_{\pi *b,\pi a}\rangle \mathfrak{n}_{*a} - \langle v\sigma_{\pi a,\pi *b}\rangle \mathfrak{n}_a\right)\mathfrak{n}_\pi + \cdots$$

$$\mathfrak{n}_\pi \frac{d}{d\tau}\left(\frac{\mathfrak{n}_{*a}}{\mathfrak{n}_\pi}\right) = 3\sum_b \langle v\sigma_{\pi b,*a}\rangle \mathfrak{n}_b\mathfrak{n}_\pi - \left([1+\mathfrak{f}_\pi(\Delta)]\sum_b \Gamma_{*b,a} + \Gamma *a,\gamma\right)\mathfrak{n}_{*a}$$
$$+ 3\sum_b \left(\langle v\sigma_{\pi b,\pi *a}\rangle \mathfrak{n}_b - \langle v\sigma_{\pi *a,\pi b}\rangle \mathfrak{n}_{*a}\right)\mathfrak{n}_\pi$$
$$+ 3\sum_{b\neq a}\langle v\sigma_{\pi *b,\pi *a}\rangle(\mathfrak{n}_{*b} - \mathfrak{n}_{*a})\mathfrak{n}_\pi + \cdots$$

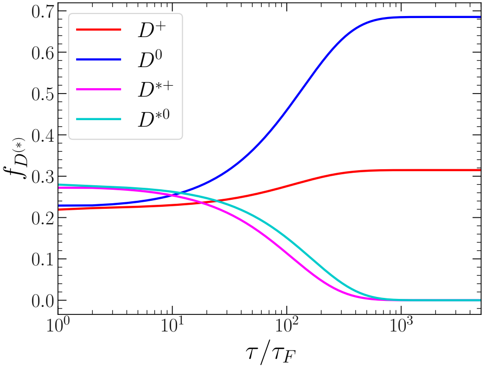
where, for example $\langle v\sigma_{\pi a,*b}\rangle \longleftrightarrow \langle v\sigma[\pi D^a \to D^{*b}]\rangle$

Evolution of charm-meson abundances

Using initial conditions provided by the predictions from SHMc

$$\left(\frac{\mathfrak{n}_0}{\mathfrak{n}_\pi}, \frac{\mathfrak{n}_+}{\mathfrak{n}_\pi}, \frac{\mathfrak{n}_{*0}}{\mathfrak{n}_\pi}, \frac{\mathfrak{n}_{*+}}{\mathfrak{n}_\pi}\right) = 10^{-3}(2.76,\, 2.64,\, 3.37,\, 3.28)$$

Note, $f_{D^a} = \mathfrak{n}_a/\mathfrak{n}_{\rm tot}$, $f_{D^{*a}} = \mathfrak{n}_{*a}/\mathfrak{n}_{\rm tot}$ and $x_{\rm tot} = \sum_b (x_b + x_{*b})$



Evolution of charm-meson abundances

At late times, $\mathfrak{n}_\pi \ll 10^{-3}\mathfrak{n}_\pi^{(\text{eq})}$, the only terms that remain in the evolution equations are 1-body terms:

- decay terms
- t-channel singularities: e.g. reaction rate $\langle v\sigma_{\pi*+,\pi*0}\rangle$ for $\mathfrak{n}_\pi \to 0$ has form
$$\langle v\sigma[\pi D^{*+}, \pi D^{*0}]\rangle = \frac{1}{3\mathfrak{n}_\pi}\frac{\Gamma_{*0,0}\Gamma_{*+,0}}{\Gamma_{*0,0}+\Gamma_{*+,0}}$$
factor $1/\mathfrak{n}_\pi$ cancels with $\mathfrak{n}_\pi$ in evolution equations

Resulting system of differential equations can be solved exactly

Evolution of charm-meson abundances

Simplified evolution equations are

$$\frac{d}{d\tau}\begin{pmatrix}\mathfrak{n}_0/\mathfrak{n}_\pi \\ \mathfrak{n}_+/\mathfrak{n}_\pi \\ \mathfrak{n}_{*0}/\mathfrak{n}_\pi \\ \mathfrak{n}_{*+}/\mathfrak{n}_\pi\end{pmatrix} = \begin{pmatrix}0 & 0 & \Gamma_{*0} & B_{+0}\Gamma_{*+} \\ 0 & 0 & 0 & (1-B_{+0})\Gamma_{*+} \\ 0 & 0 & -(\Gamma_{*0}+\gamma) & \gamma \\ 0 & 0 & \gamma & -(\Gamma_{*+}+\gamma)\end{pmatrix}\begin{pmatrix}\mathfrak{n}_0 \\ \mathfrak{n}_+ \\ \mathfrak{n}_{*0} \\ \mathfrak{n}_{*+}\end{pmatrix}$$

$$\frac{1}{\gamma} = \frac{1}{B_{00}\Gamma_{*0}} + \frac{1}{B_{+0}\Gamma_{*+}}$$

$$\Gamma_{*0} = \Gamma[D^{*0}] = 0.0554 \pm 0.0015 \text{ MeV}$$

$$\Gamma_{*+} = \Gamma[D^{*+}] = 0.0834 \pm 0.0018 \text{ MeV}$$

$$B_{00} = \mathrm{Br}[D^{*0} \to D^0\pi^0] = 0.647 \pm 0.009$$

$$B_{+0} = \mathrm{Br}[D^{*+} \to D^0\pi^+] = 0.677 \pm 0.005$$

Evolution of charm-meson abundances

Difference between naïve prediction and analytic prediction with the t-channel singularity can also be calculate exactly, and is significantly different from 0 at 13σ

$$\left(\frac{N_0}{N_+}\right)_{\text{naïve}} - \left(\frac{N_0}{N_+}\right)_{\text{analytic}} = 0.079 \pm 0.006$$

errors come from $B_{+0}, B_{00}, \Gamma_{*+}$ and Γ_{*0}

Quick review of predictions from models described

Summary	naïve	numerical	analytic
N_0/N_+	2.256 ± 0.014	2.100	2.177 ± 0.016

Outline

1. t-channel singularities
2. Heavy-ion collisions
3. Statistical hadronization model
4. Thermal mass shifts and widths
5. Evolution of charm-meson abundance: after kinetic freeze-out
6. Summary

Summary

We have studied the evolution of charm meson ratios in an expanding hadron gas after the kinetic freeze-out of a hadron resonance gas produced from a heavy-ion collision

(a) We have identified an aspect of charm-meson physics in which the effects of the t-channel singularity is observable

(b) Our findings provide encouragement to study other effects of t-channel singularities, such as in the production of loosely bound charm-meson molecules in heavy-ion collisions

Future studies will be to investigate the effects of t-channel singularities on $X(3872)$ and $T_{cc}^{+}(3875)$