# Extracting the Diffusivity and Charge Susceptibilities of the QGP from Experiment

*Scott Pratt*
*Department of Physics & Astronomy …*

S.P., J. Kim & C.Plumberg PRC(2018)
S.P. & C.Plumberg PRC(2019)
S.P. PRC (2020)
S.P. & R.Steinhorst, PRC (2020)
S.P. & C.Plumberg, PRC (2020)
S.P. & K.Martirosova (arrive 2022)



# Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

## Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

# Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Charge balance functions are principal tool

## Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Charge balance functions also important for:

- CME background
- Background for fluctuations for phase transitions

# I. Theory of Correlations and Balance Functions

# Charge Correlations

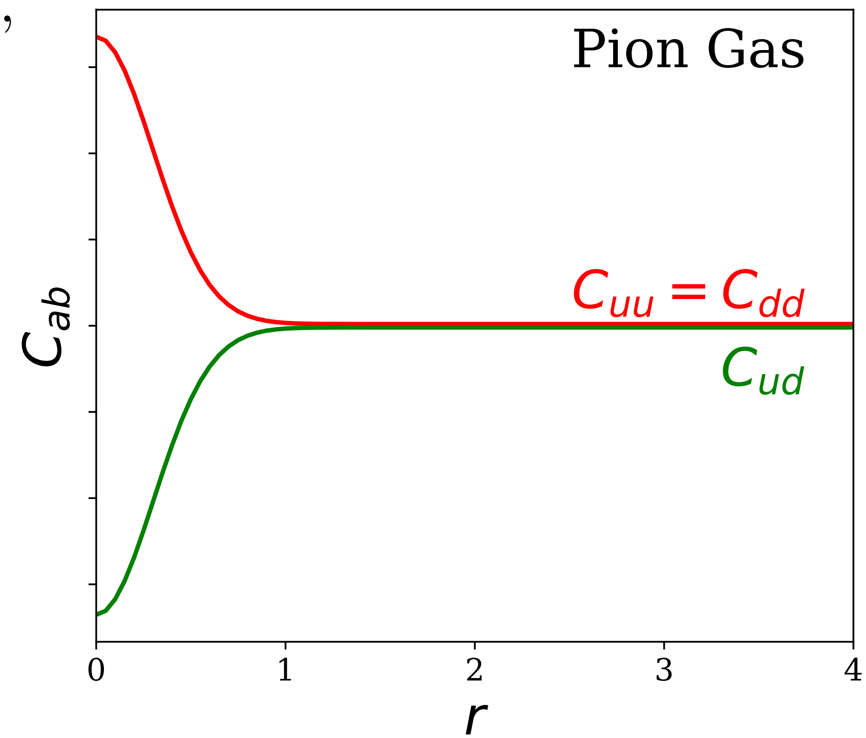
## (Equilibrated System)

$$C_{ab}(\vec{r}_1,\vec{r}_2) \equiv \langle \delta\rho_a(\vec{r}_1)\delta\rho_b(\vec{r}_1)\rangle = \chi_{ab}\delta(\vec{r}_1-\vec{r}_2),$$

$$\chi_{ab} = \frac{1}{V}\langle \delta Q_a \delta Q_b\rangle = \frac{T^2}{Z}\frac{\partial^2}{\partial\mu_a\partial\mu_b}\ln Z,$$

$$= \int d^3r\ C_{ab}(0,\vec{r}).$$

3x3 matrix




**Quark Gas:**

$$\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$$

**Hadron Gas:**

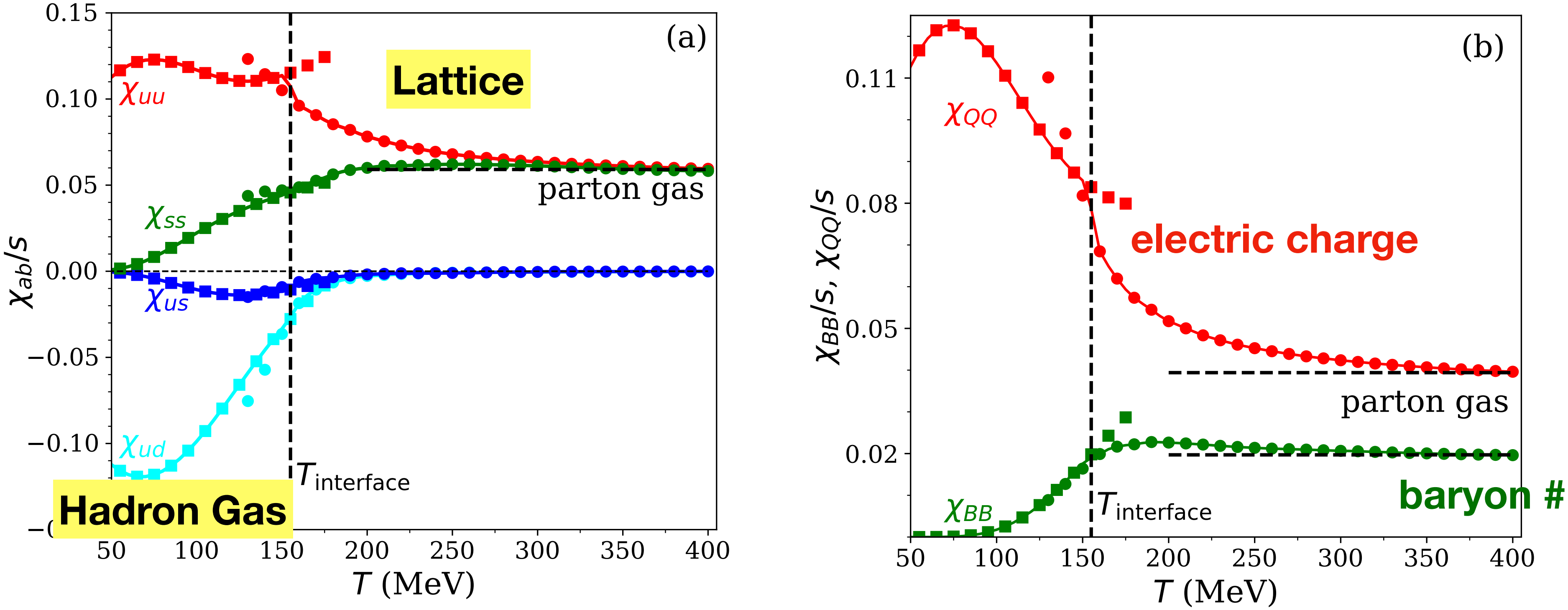
$$\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$$

# Susceptibility
## (Lattice, BW-Claudia Ratti)

**For hadron gas:** $\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$

**a=(u,d,s)**

**For parton gas:** $\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$



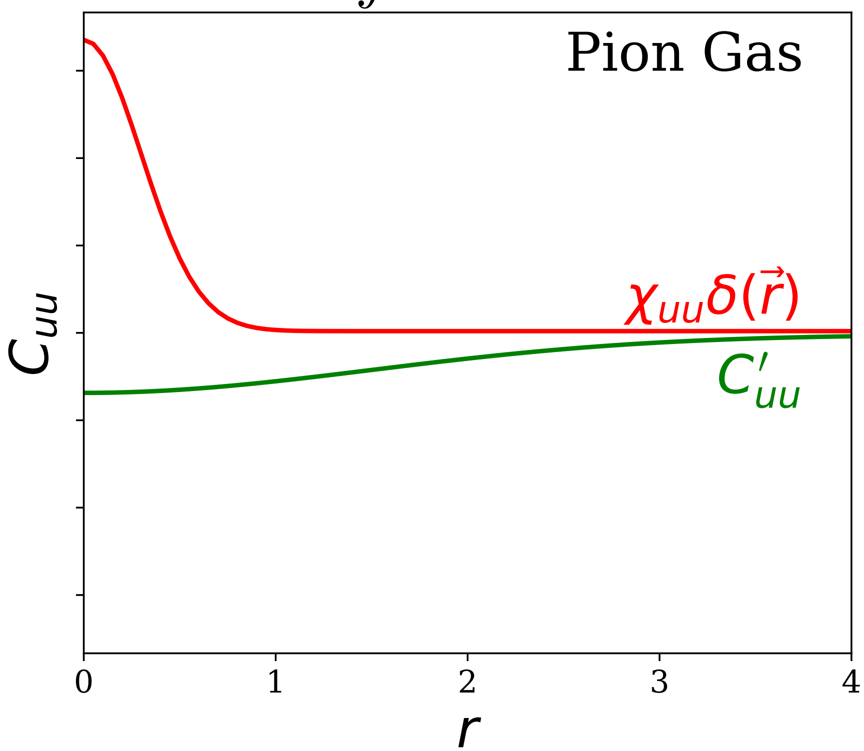
# Charge Correlations

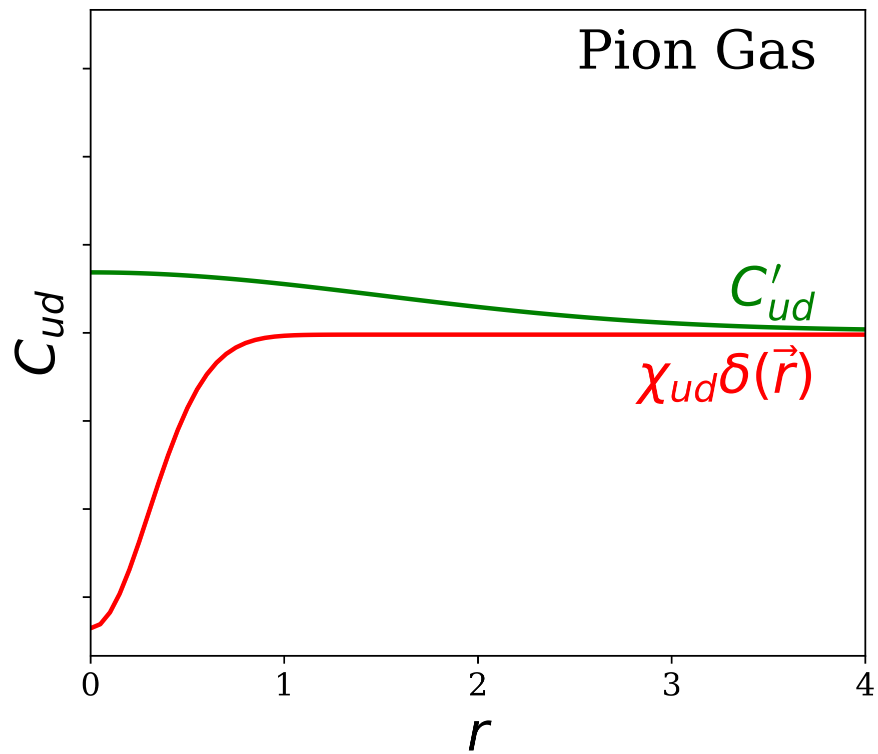
## (Dynamic System)

$$C_{ab}(\vec{r}_1, \vec{r}_2) = \chi_{ab}\delta(\vec{r}_1 - \vec{r}_2) + C'_{ab}(\vec{r}_1, \vec{r}_2)$$

$$\int d^3r C'_{ab}(\vec{r}) = -\chi_{ab}$$

***Balancing*** **correlation**

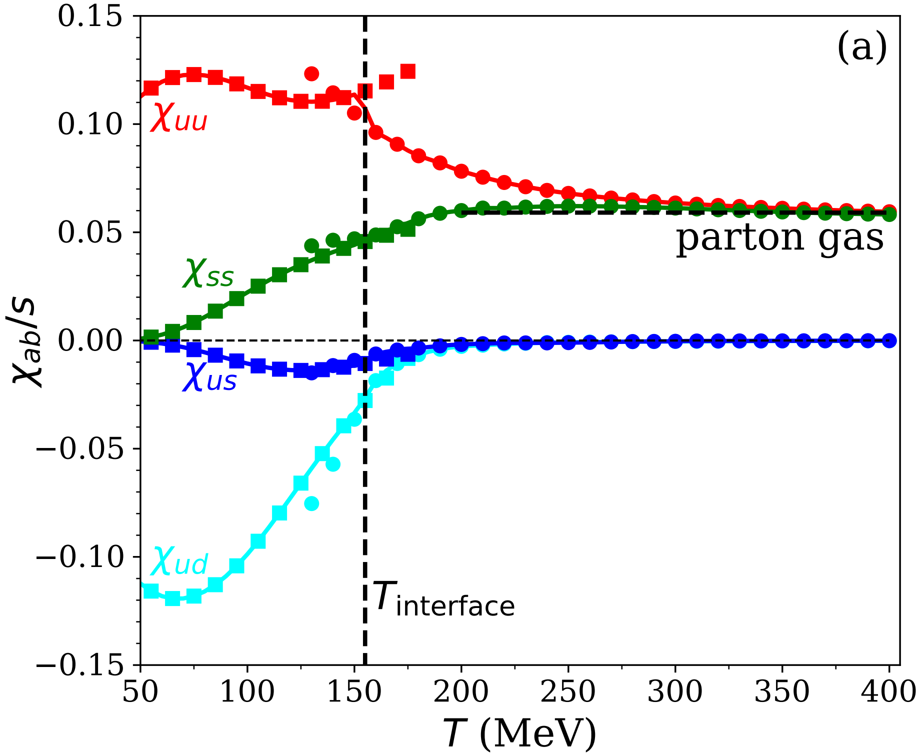


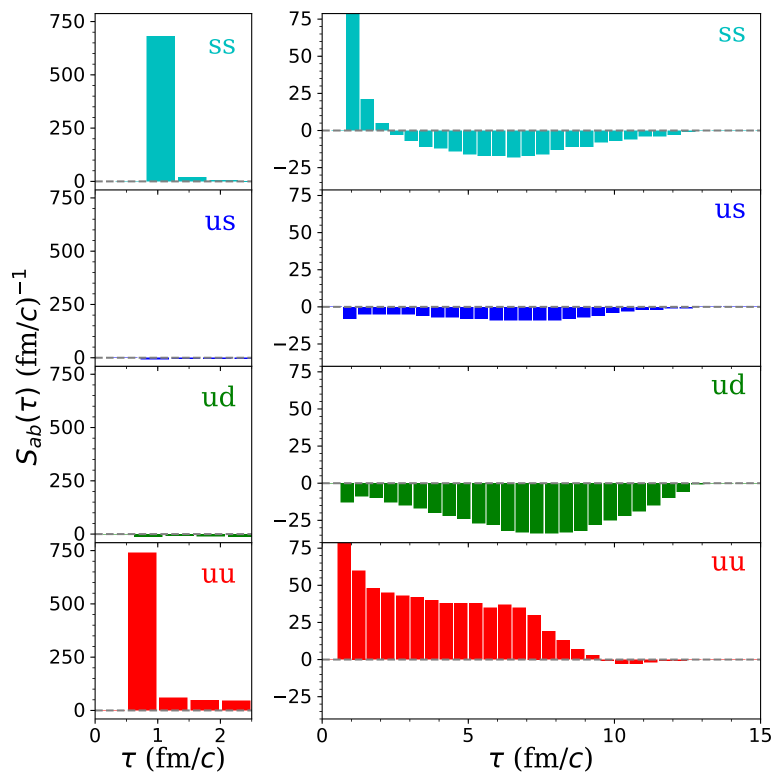



# Eq.s of motion

$$\partial_t C'_{ab}(\vec{r}_1 - \vec{r}_2) - D_{ab}\nabla_1^2 C'_{ab}(\vec{r}_1, \vec{r}_2) - D_{ab}\nabla_2^2 C'_{ab}(\vec{r}_1, \vec{r}_2) = -S_{ab}(\vec{r}_1, t)\delta(\vec{r}_1 - \vec{r}_2),$$

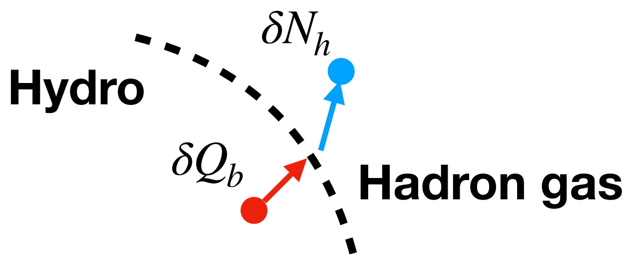
$$S_{ab}(\vec{r}, t) = \left[\partial_t + \vec{v}\cdot\vec{\nabla} + (\nabla\cdot\vec{v})\right]\chi_{ab}(\vec{r}, t) \approx s(\vec{r}, t)\left[\partial_t + \vec{v}\cdot\vec{\nabla}\right]\frac{\chi_{ab}(\vec{r}, t)}{s(\vec{r}, t)}$$



# Translate $C_{ab}$ into $C_{hh'}$

$$\delta N_h = \chi_{ab}^{-1}(T_{\rm interface}) q_{ha} n_h \delta Q_b$$



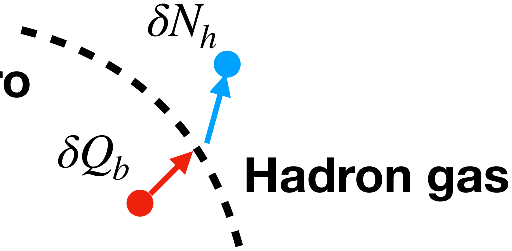

# Charge Balance Function

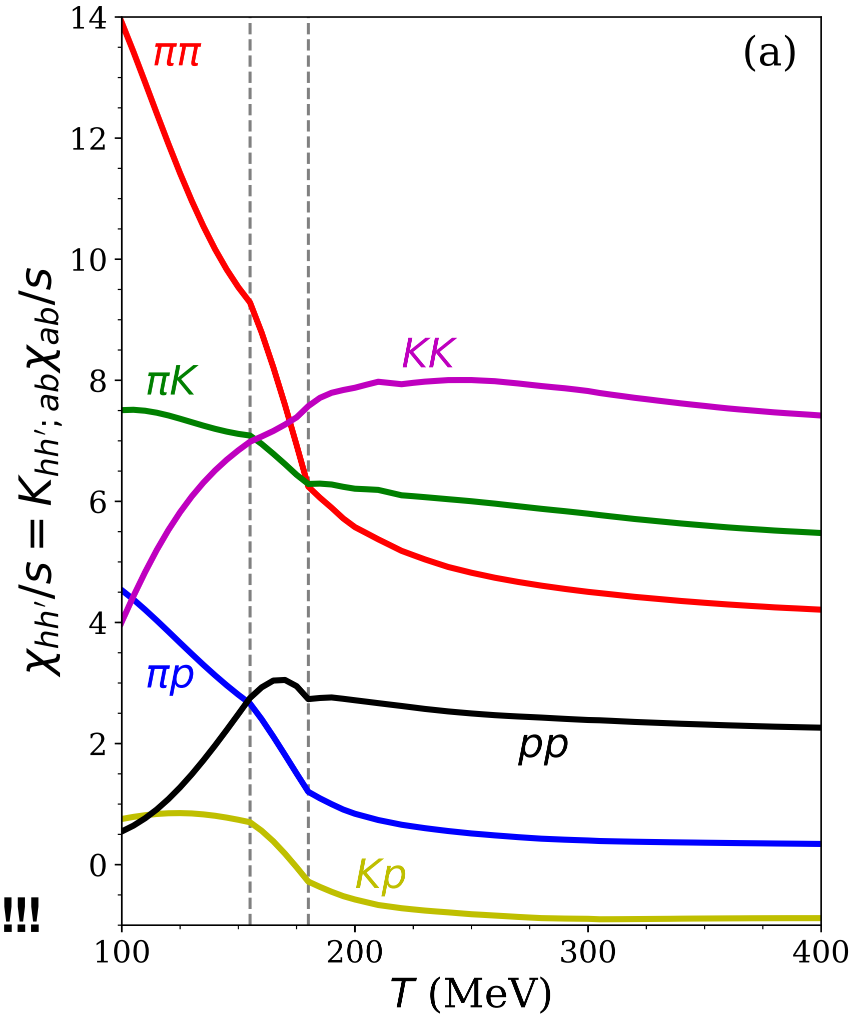
$$C_{hh'}(\vec{r}_1, \vec{r}_2) = \langle [\delta n_h(\vec{r}_1) - \delta n_{\bar{h}}(\vec{r}_1)][\delta n_{\bar{h}'}(\vec{r}_2) - \delta n_{h'}(\vec{r}_2)] \rangle,$$

$$B_{h|h'}(\vec{p}_1|\vec{p}_2) = \frac{1}{N_{h'}(\vec{p}_2) + N_{\bar{h}'}(\vec{p}_2)} \langle [\delta N_h(\vec{p}_1) - \delta N_{\bar{h}}(\vec{p}_1)][\delta N_{\bar{h}'}(\vec{p}_2) - \delta N_{h'}(\vec{p}_2)] \rangle$$

# Translate $C_{ab}$ into $C_{hh'}$

$$\delta N_h = \chi^{-1}_{ab}(T_{\text{interface}}) q_{ha} n_h \delta Q_b$$




**$B_{\pi\pi}$ has contribution from hadronization stage**

**$B_{pp}$ and $B_{KK}$ are sourced at thermalization**

**$B_{\pi\pi}(\Delta y)$ should be narrower than $B_{KK}(\Delta y)$ or $B_{pp}(\Delta y)$!!!**

# Diffusivity

$$\vec{j}_a = -D_{ab}\nabla\rho_b,$$

3x3 matrix (colors)

$$= -\sigma_{ab}\nabla(\mu_b/T),$$

$$\sigma = \chi D,$$

$$\chi_{ab} = \langle\delta Q_a \delta Q_b\rangle/V = \partial\rho_a/\partial(\mu_b/T)$$

susceptibility

# Kubo Relation

$$\sigma_{ab} = \frac{1}{2T}\int d^4x\ \langle\{j_a(0), j_b(x)\}\rangle$$

difficult (not impossible) for lattice gauge theory

# II. The Calculation

Diffusion = Random walk

Monte Carlo procedure:

# ALGORITHM

A) Overlay with hydro evolution to create $S_{ab}(t, \vec{r})$

B) Generate partners (uu,dd,ss,ud,us,ss) proportional to $S_{ab}(t, \vec{r})$ with weights

C) Move particles in random directions punctuated by re-directioning according to $\tau_{\rm coll}$

D) Translate $\delta Q_a$ to $\delta N_h$ at hyper surface

E) Collide (fixed $\sigma$) and decay particles

F) Combine decay products with those from partner

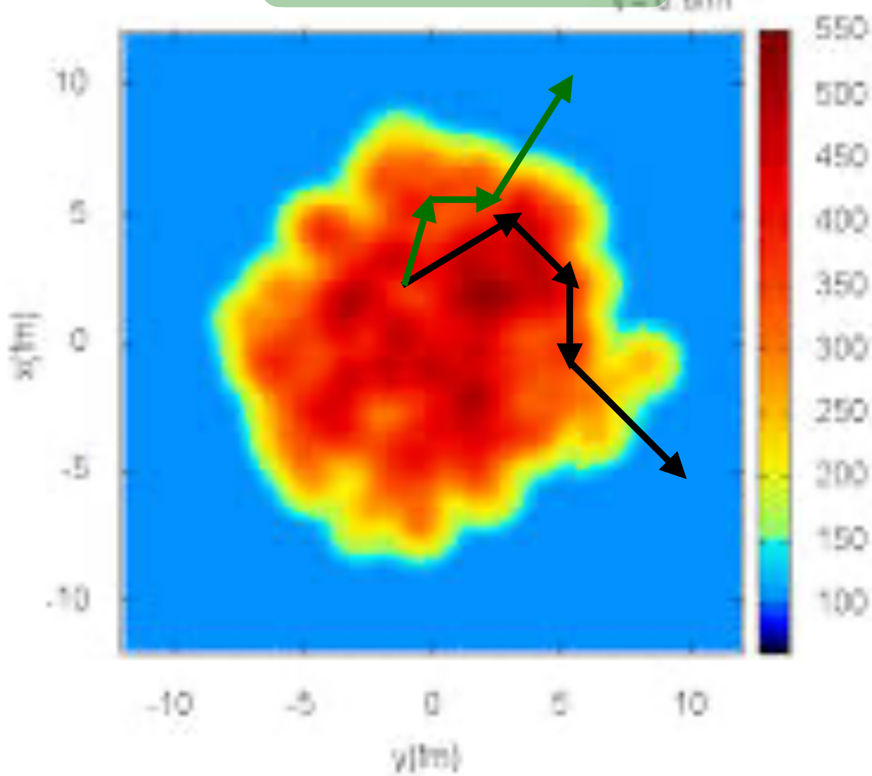
G) Correlations created during hadronic phase: create uncorrelated hadrons, run through cascade, combine ALL particles to create BF

H) Add contributions from (E) and (F)

I) Fold with acceptance/efficiency

J) Test sum rules

$$\tau_{\rm coll} = 6D$$



# ALGORITHM

TYPE 1:

Correlations from Hydro:

— Depends of D and $\sigma_0$

— Only a few hours of CPU

— track charges from same source point

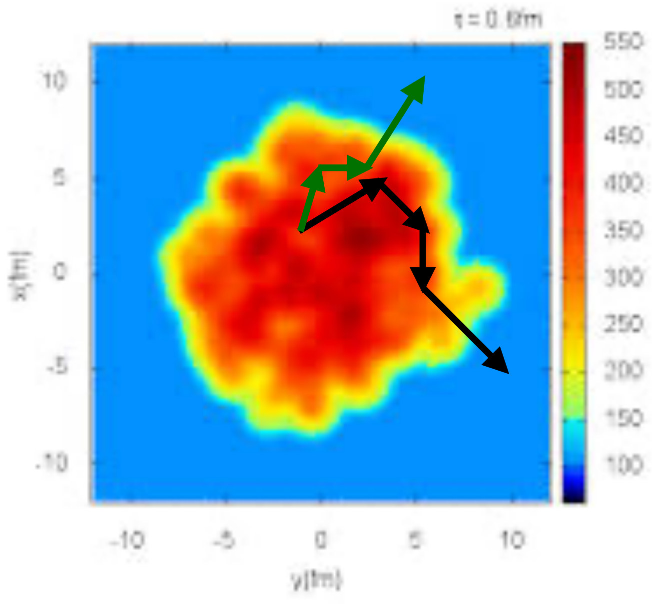
TYPE 2:

Correlations from Cascade

— Weeks of CPU

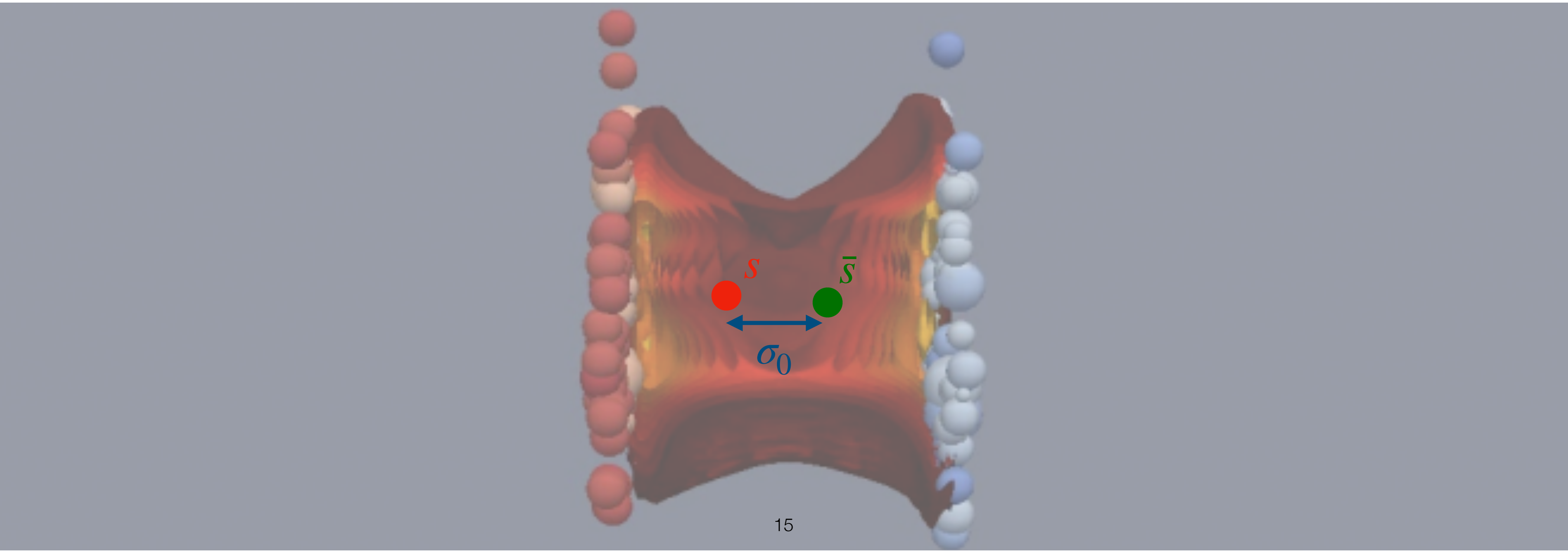
— One hydro event (independent of D, $\sigma_0$)

— Millions of cascade events

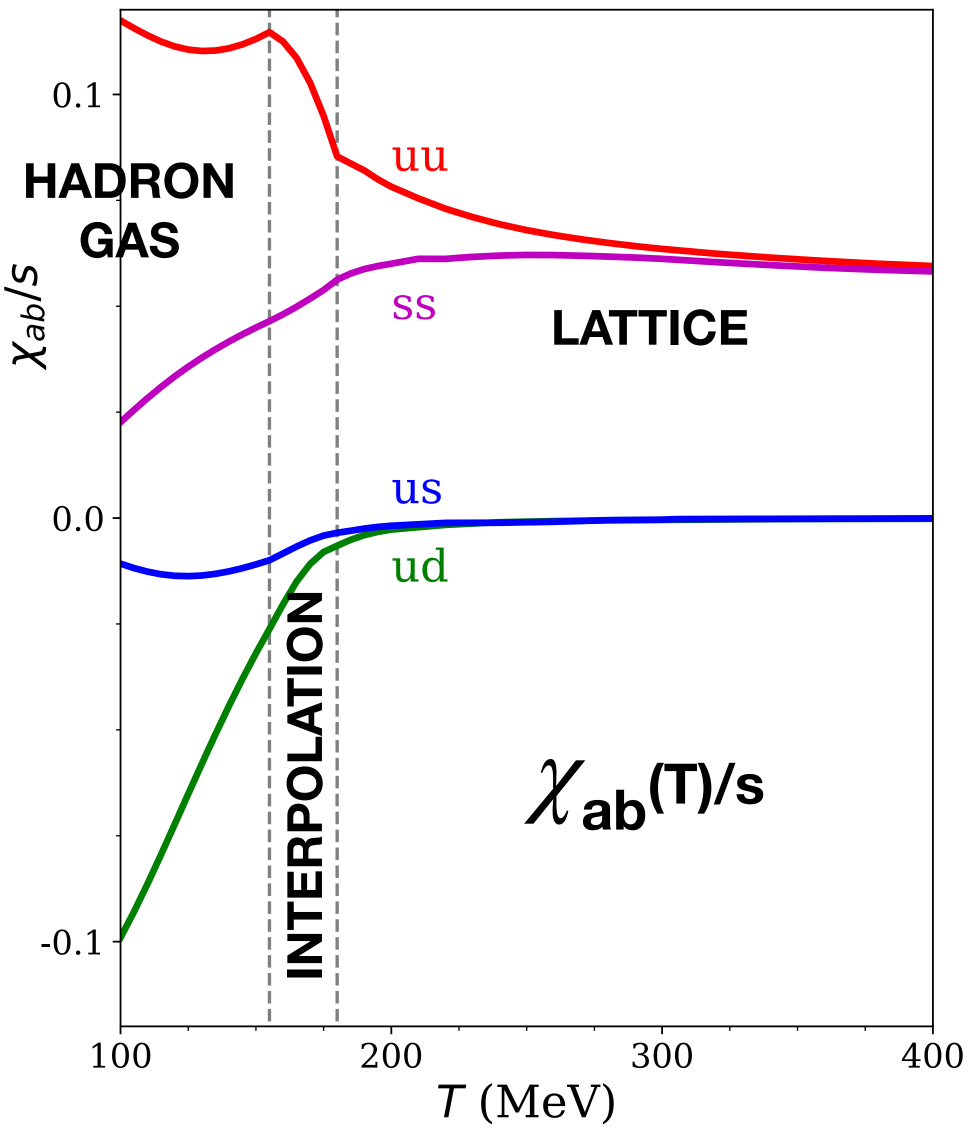


# Adjustable Parameters

1. **Diffusion Constant $D(T)$ (multiples of lattice values)**
2. **$T_h$ = 155 MeV**
3. **$\sigma_0$ = spread in spatial rapidity at $\tau_0$ = 0.6 fm/c**
   **— creation before $\tau_0$ or tunneling (flux tubes)**

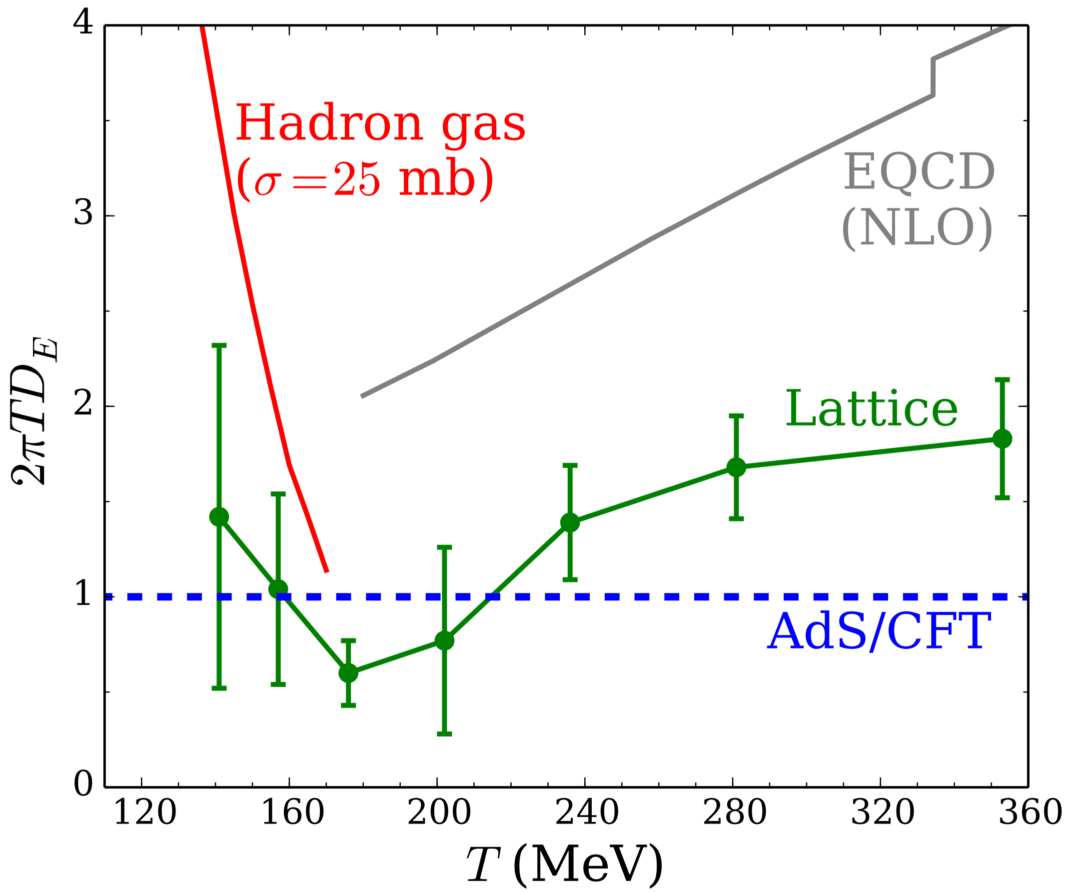


# Model input Susceptibility




**Claudia Ratti**
**BW Collaboration**

# $D(T)$ — No Clear Consensus

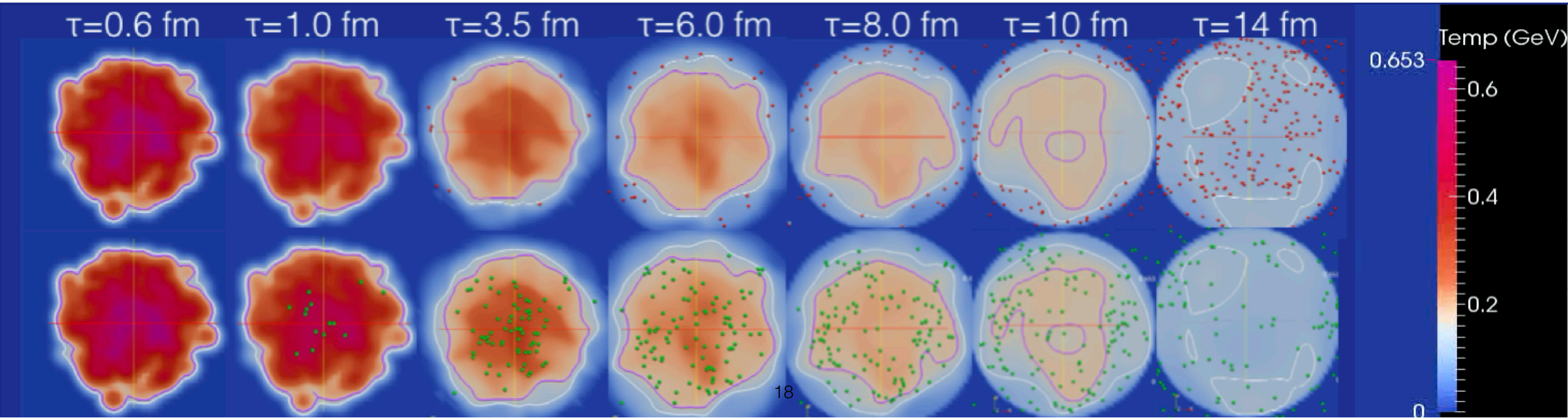



G.Aarts et al, JHEP (2015)
J.Ghiglieri et al, JHEP (2018)
G.Policastro et al, JHEP (2002)

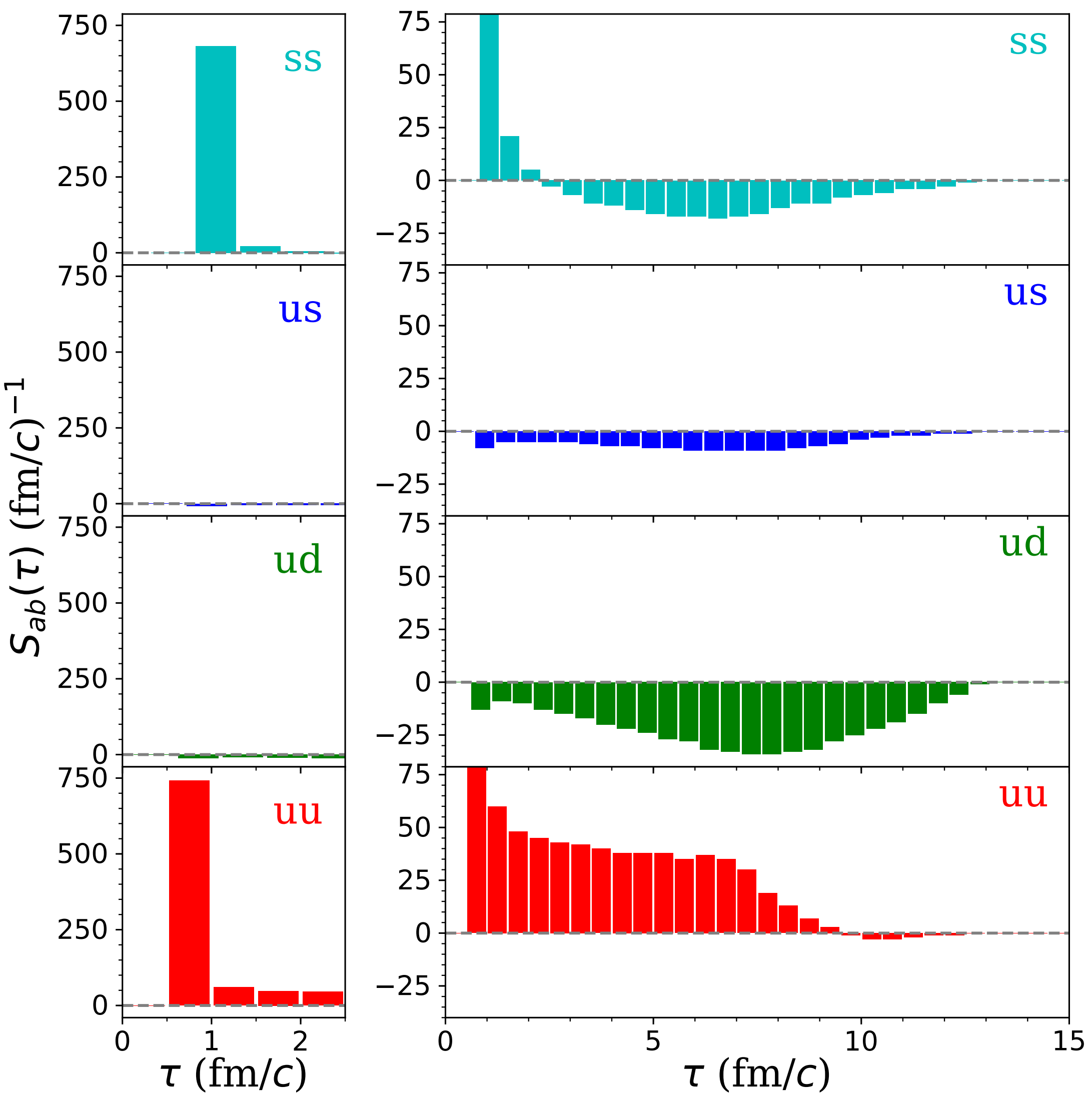
# Model input
# Hydro history

Chris Plumberg

VISHNU Hydro, Au+Au (200A GeV)

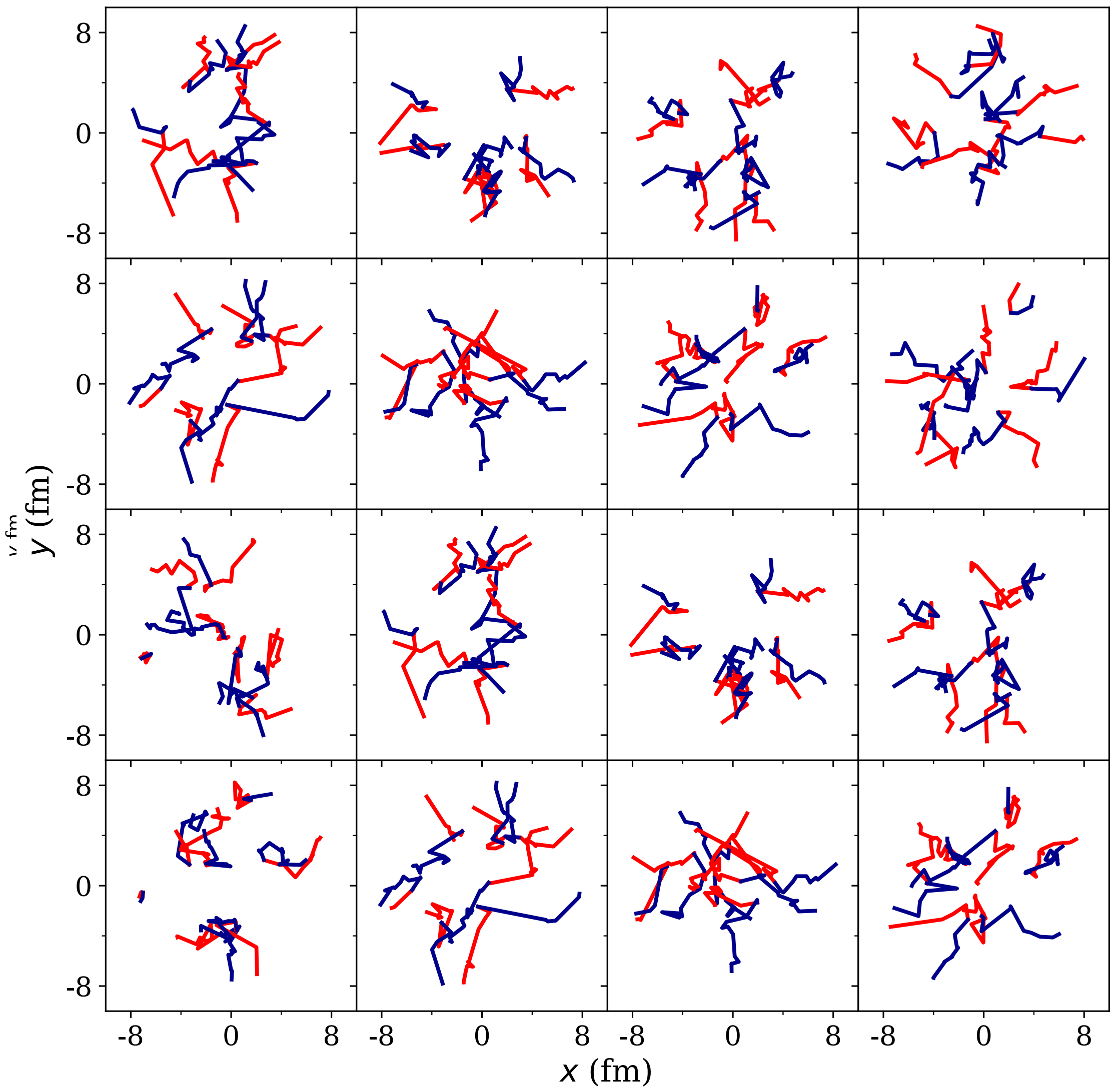


# Source Function

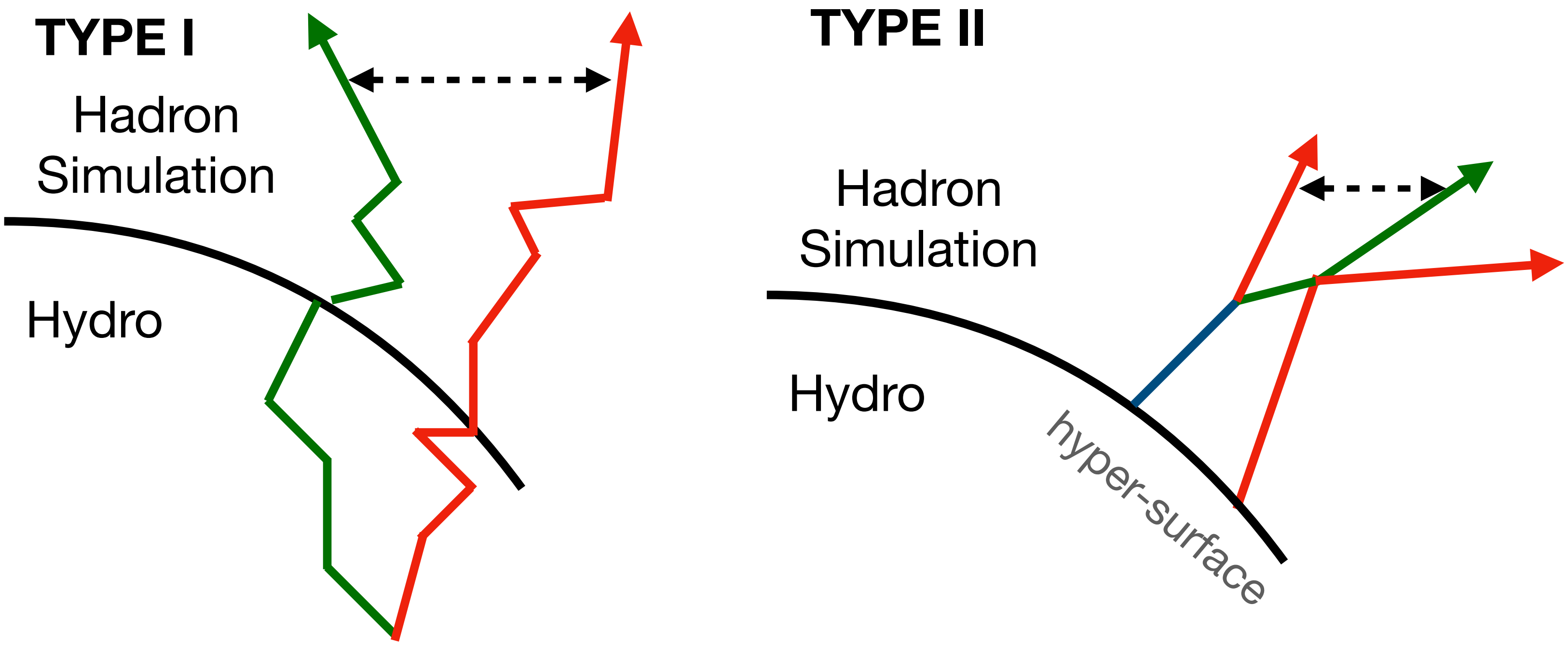



- First surge when QGP is created
- uu,dd continuously created
- ss nearly steady
- ud,us,ds at hadronization

# Diffusive Trajectories

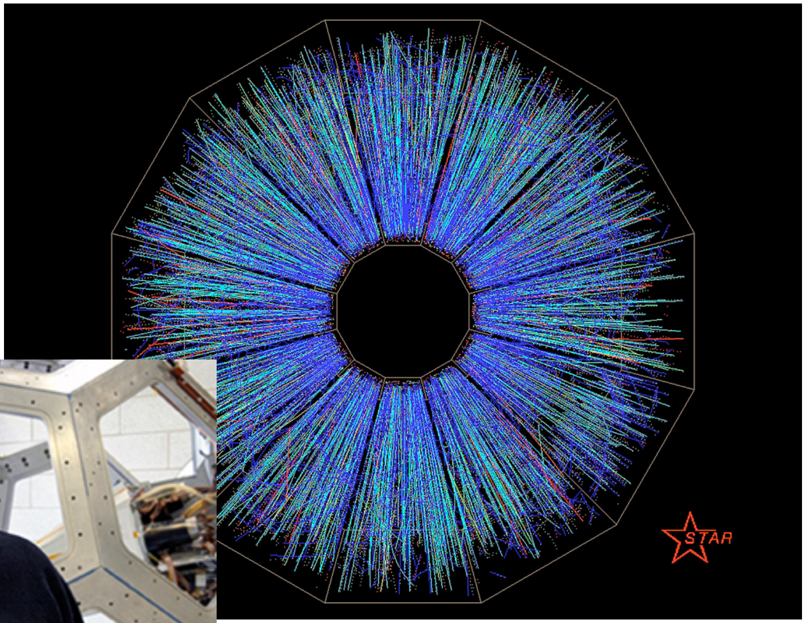


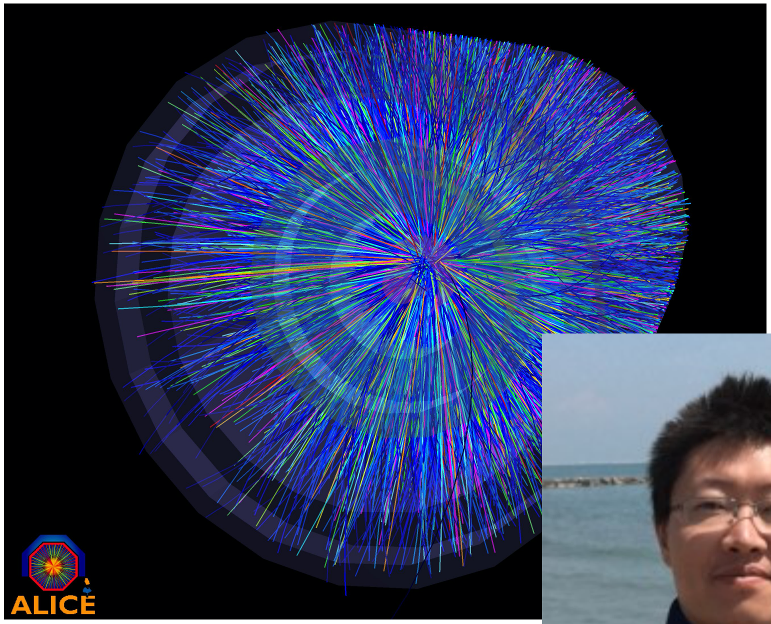
# ALGORITHM

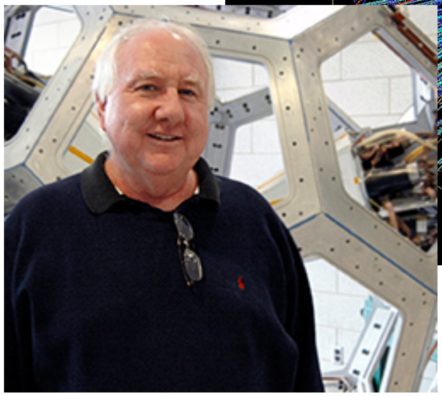

TYPE I
Hadron Simulation
Hydro
TYPE II
Hadron Simulation
Hydro
hyper-surface

# III. Results vs. Data

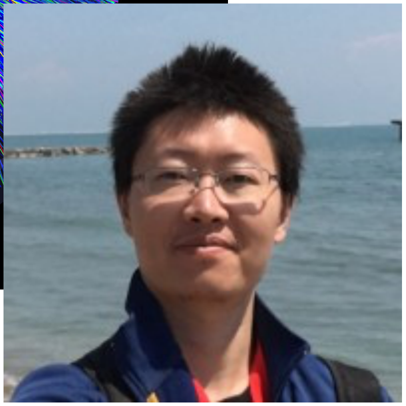
# Experimental Acceptance/Efficiency



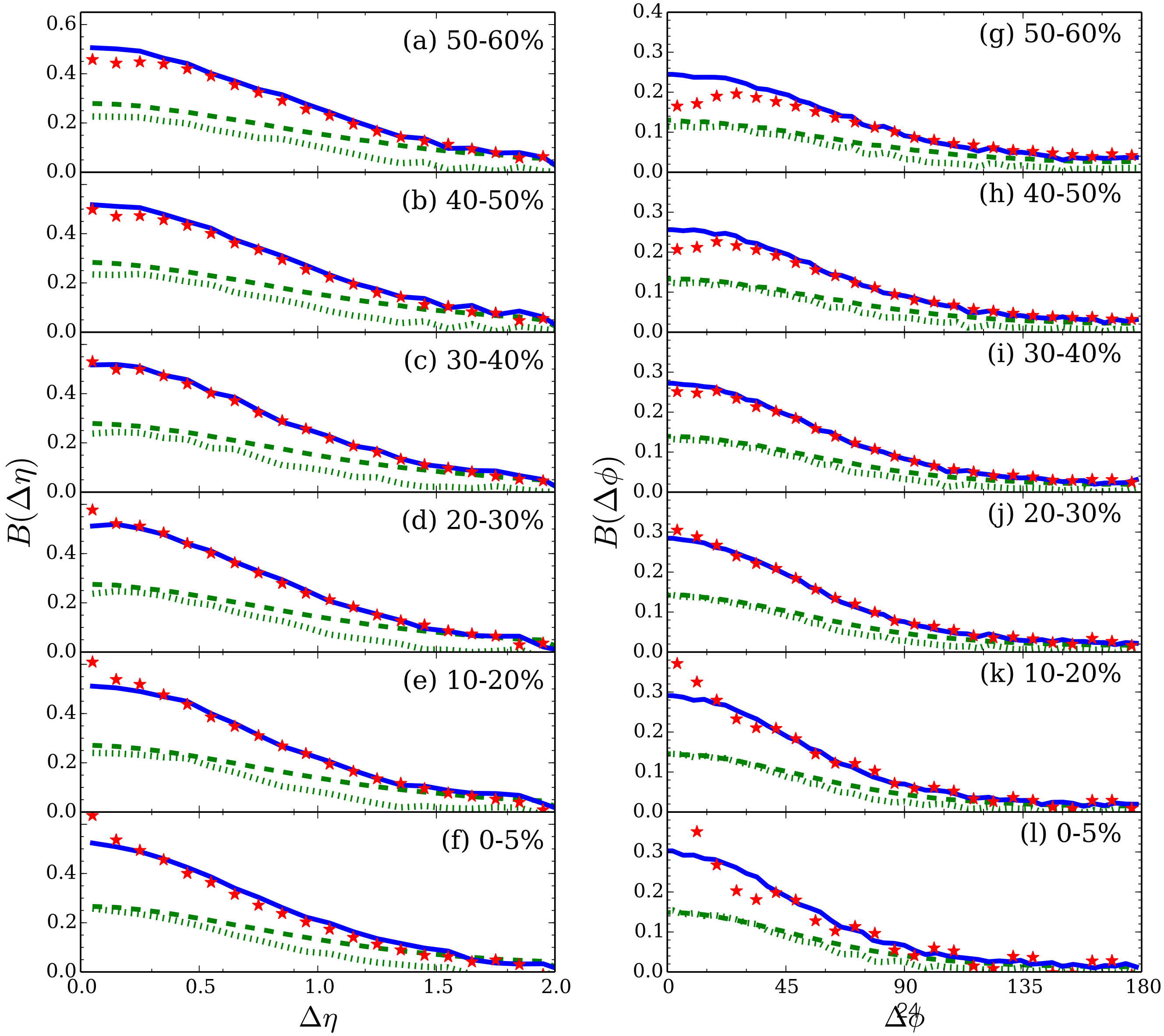



Gary Westfall
MSU



Jinjin Pan
Wayne State




# Model vs. STAR

## Unidentified Particles

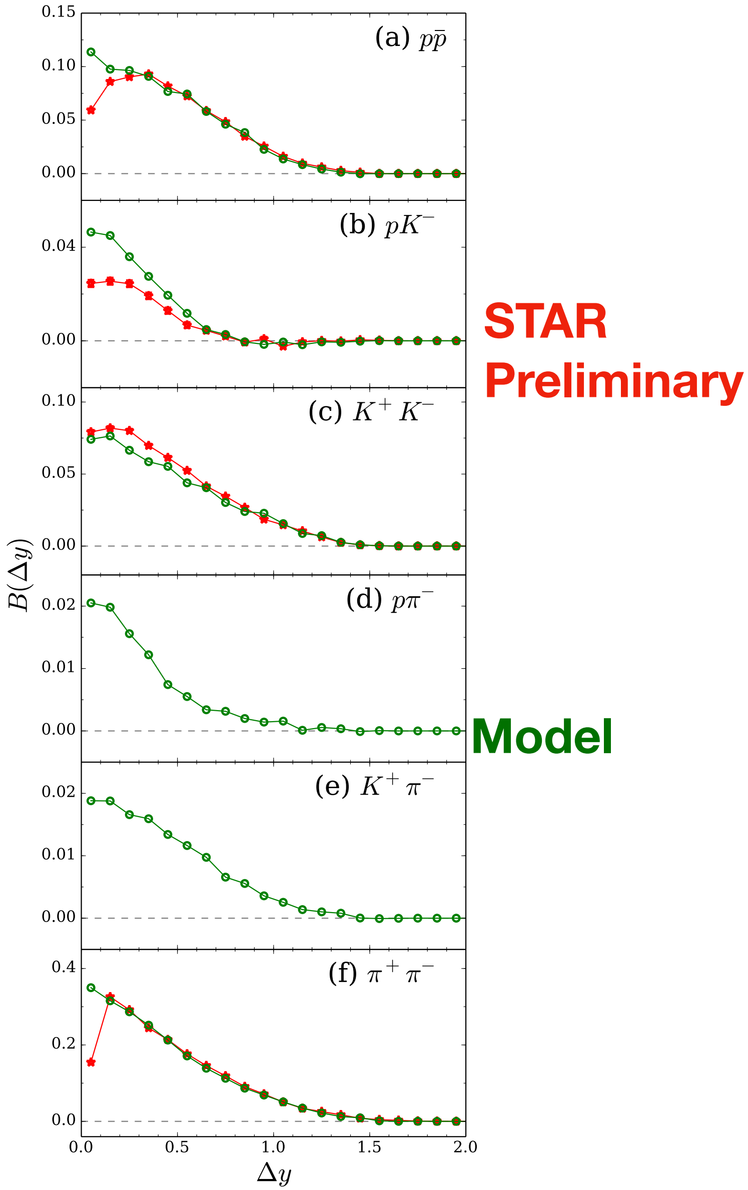
★ STAR Preliminary

**Model, Type 1 + Type 2**

**Type 1 (dashes, hydro)**

**Type 2 (dots, cascade)**

# Model vs. STAR

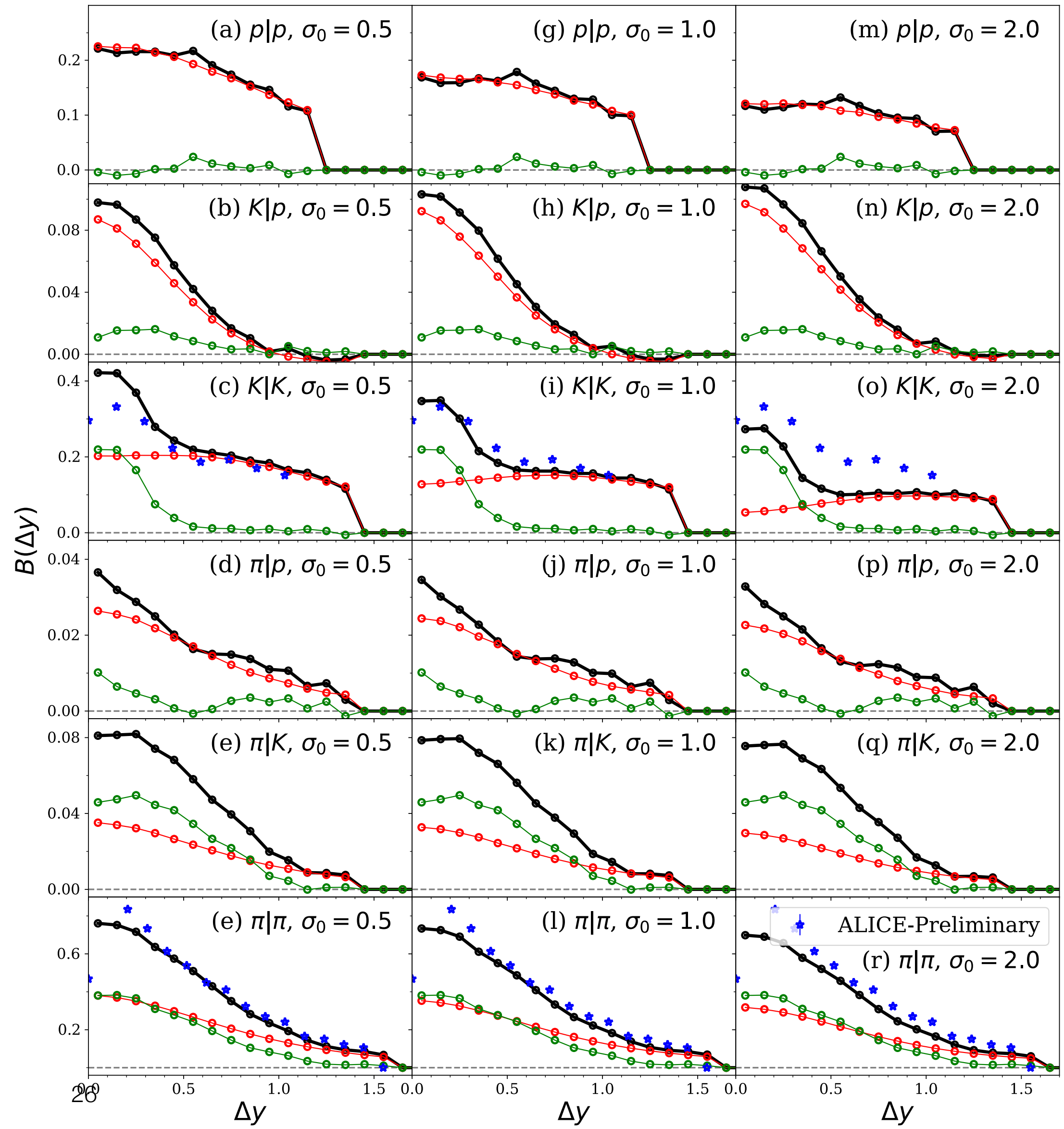



- Identified particles (vs. $\Delta y$)
- pK is off
- pp is off (annihilation missing)

# Model vs ALICE

## Thesis of Jin-Jin Pan

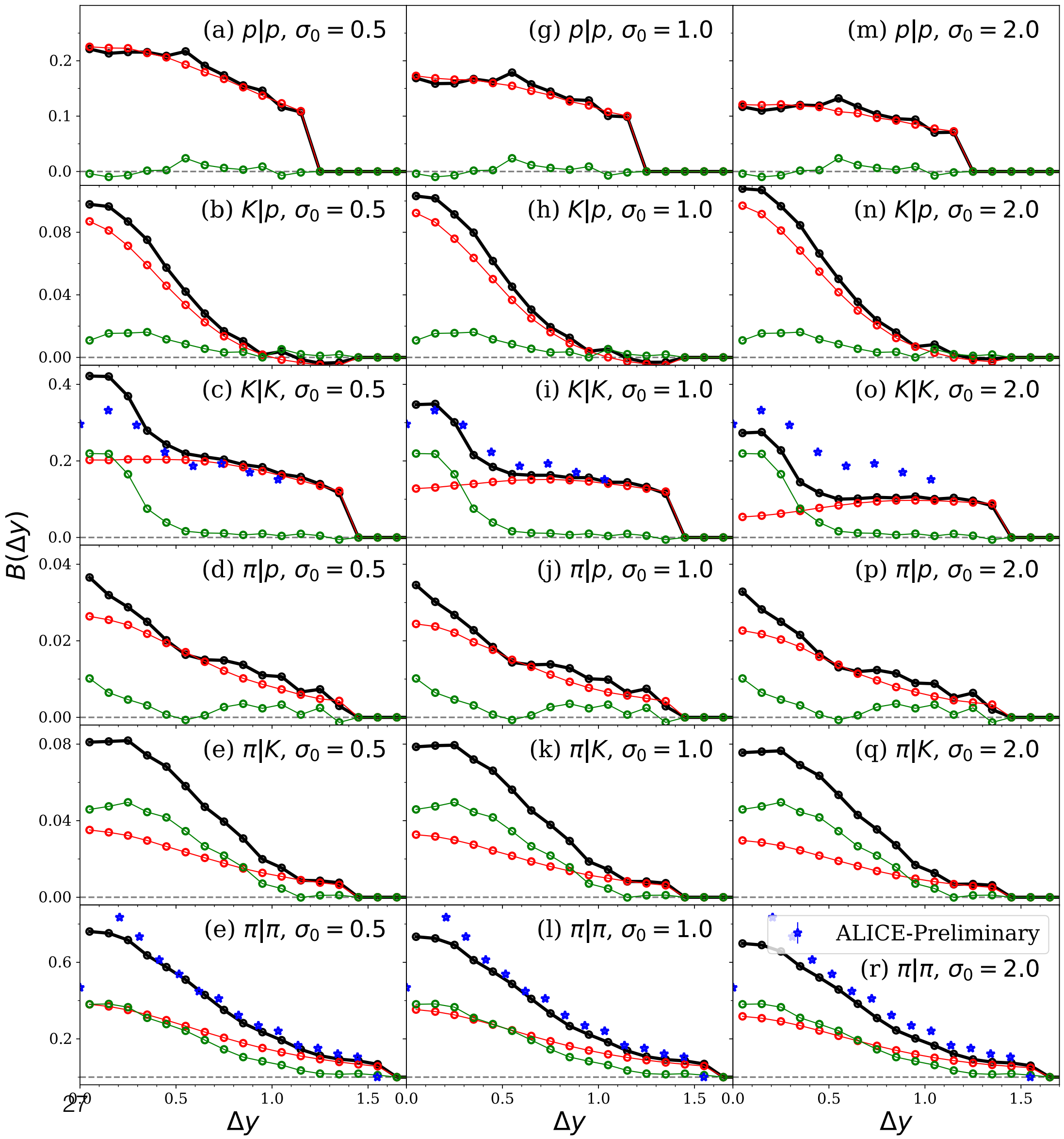
Binned by $\Delta y$




**Type 1 + Type 2**
**Type 1 (hydro)**
**Type 2 (cascade)**

# Evidence of early chemical equilibrium

- $p\bar{p}$, $K^+K^-$
  **BFs broader than $\pi^+\pi^-$ BFs!!**
- $\sigma_0 > 0$



# First Conclusion:

$KK$ **and** $pp$ **BFs are wider than** $\pi\pi$ **BFs**

- **Strong evidence of early charge creation!**

$\sigma_0 \gtrsim 0$

- **Charge (quarks) must have been created (and separated) by** $\tau_0 = 0.6$ **fm/c!**

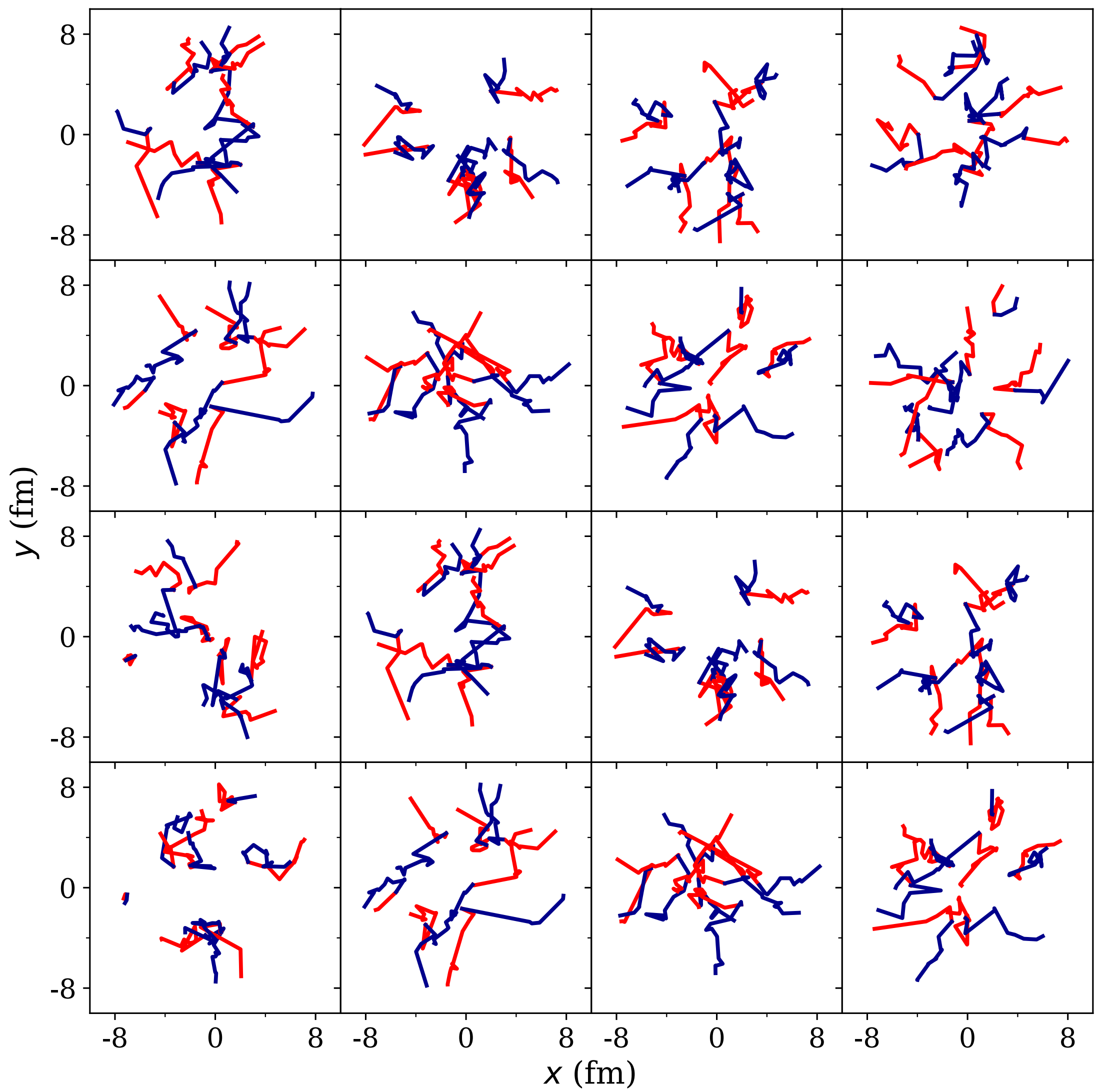
# BUT if diffusion underestimated:

**Charge production could have been later**

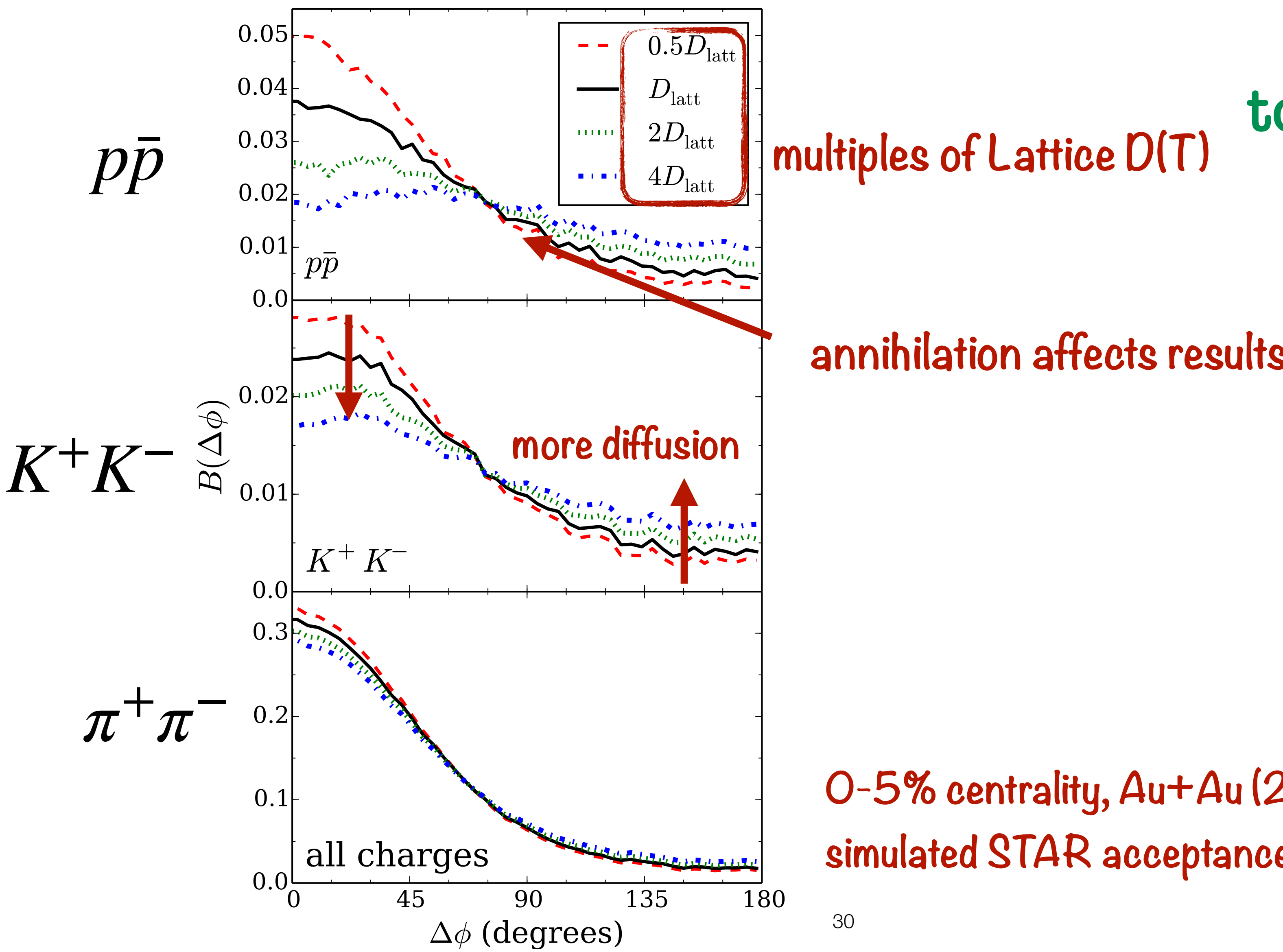
# Look at $B(\Delta\phi)$

- **Width:**
  **Insensitive to pre-thermal separation**

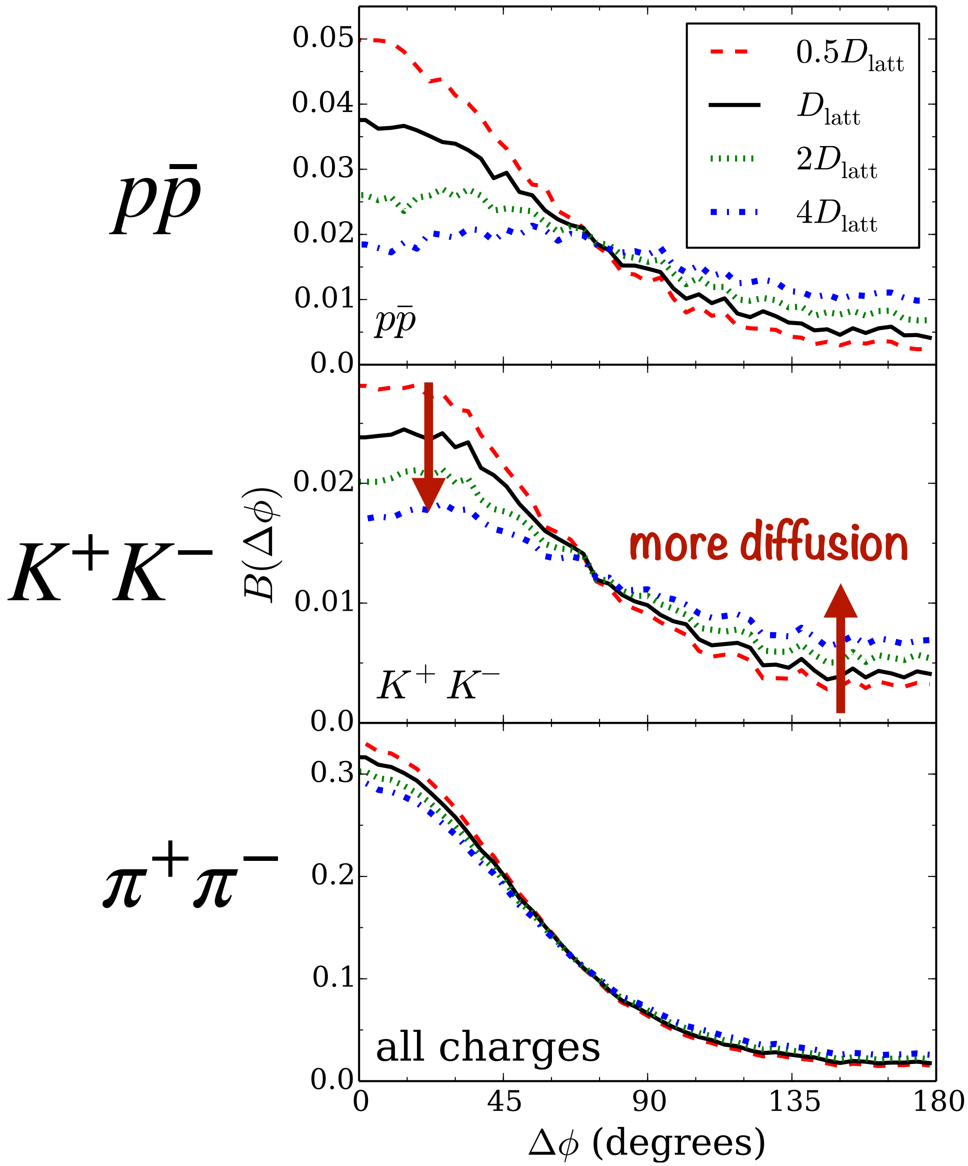
- **Eliminate sensitivity to late-production**
  **—$pp$ or $KK$ BFs**
  **— Only consider $\Delta y \gtrsim 1.0$**



# Sensitivity to Diffusivity

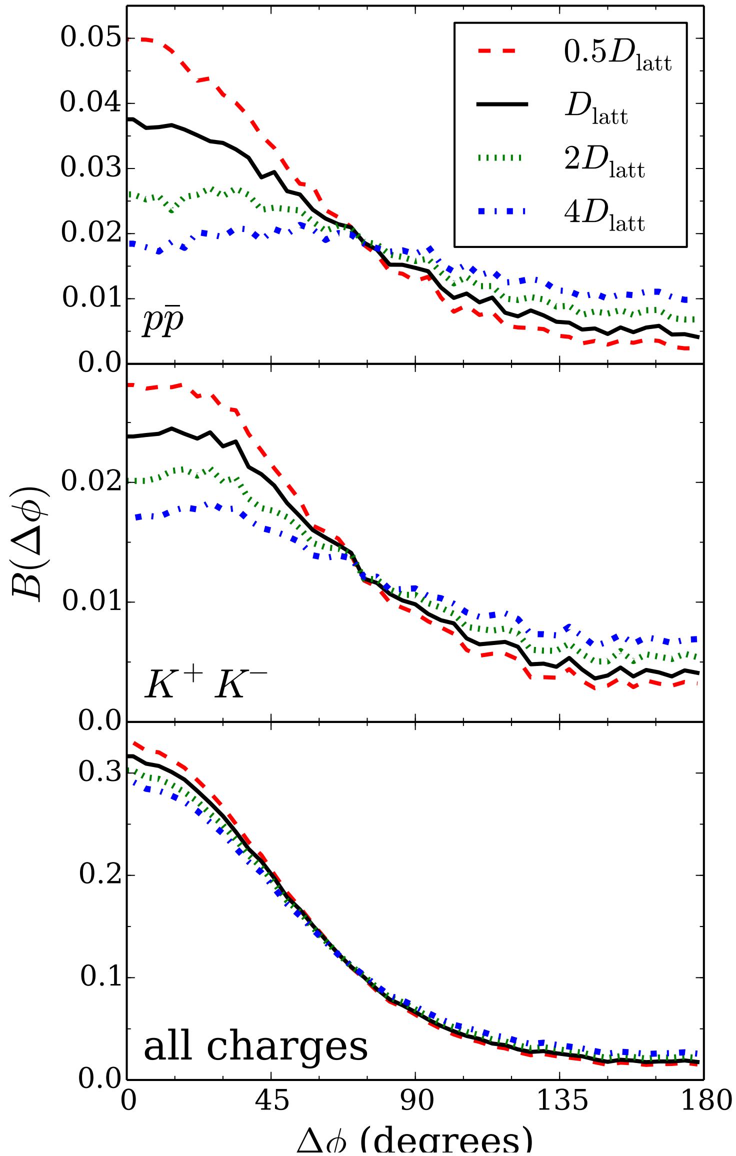



0-5% centrality, Au+Au (200A GeV)
simulated STAR acceptance




# Sensitivity to Diffusivity

- $\Delta\phi$ binning reduces dependence on $\sigma_0$
- kaons or protons best suited:
- $\chi_{ss}/s$ roughly constant
  $\approx$ only phi contributes from final state
- $\chi_{BB}/s$ roughly constant
  annihilation an issue




# Sensitivity to Diffusivity

Extract D ~ ±50 % ?

But work needed:

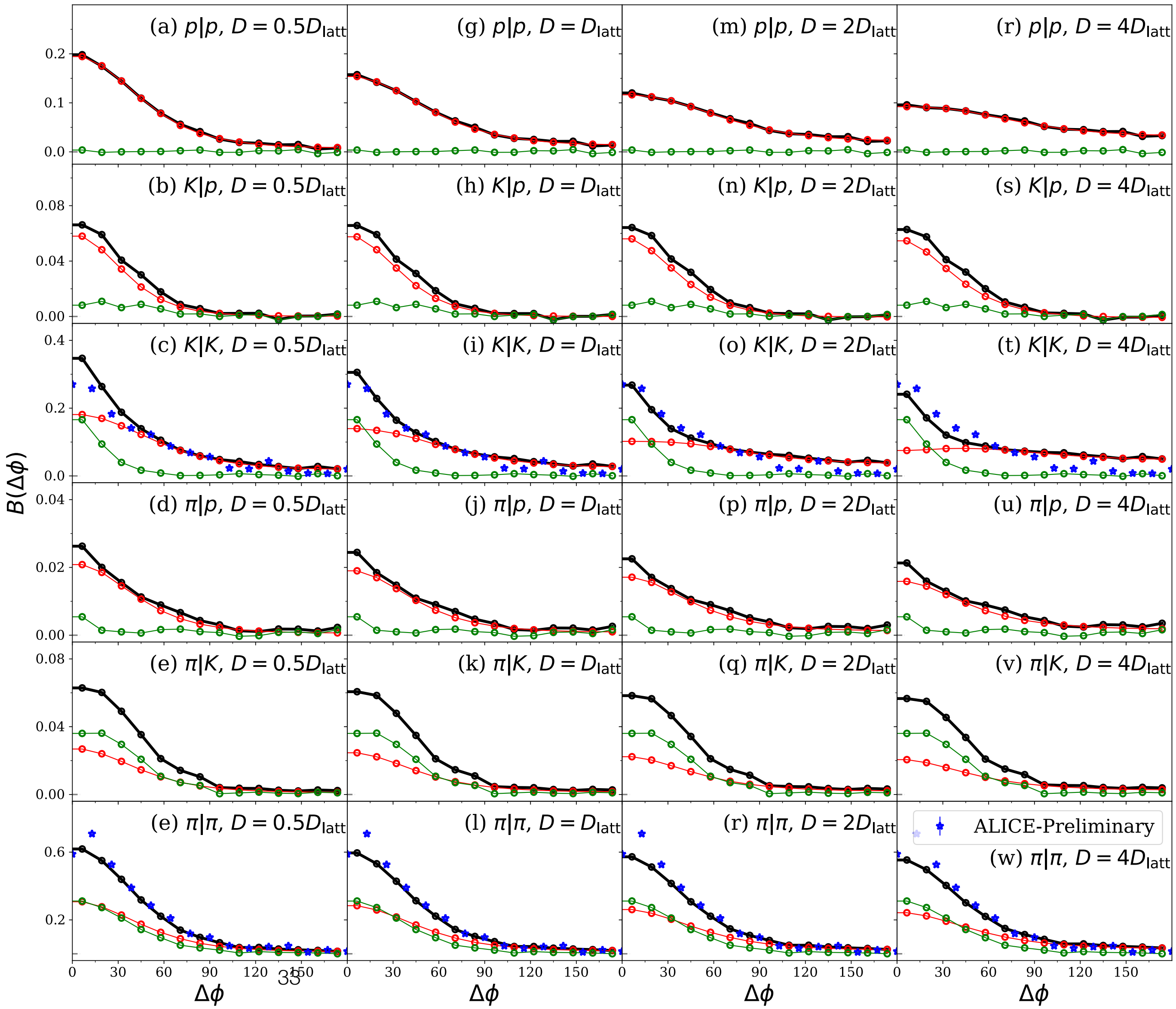
- φ contribution to kaon B.F.
  — BF binned by $Q_{inv}$
- absorption of strangeness into baryons
  — look at pK, KΛ BFs
- strangeness annihilation
  — multiplicities and BF vs Δy

# Model vs. ALICE

Binned by $\Delta\phi$

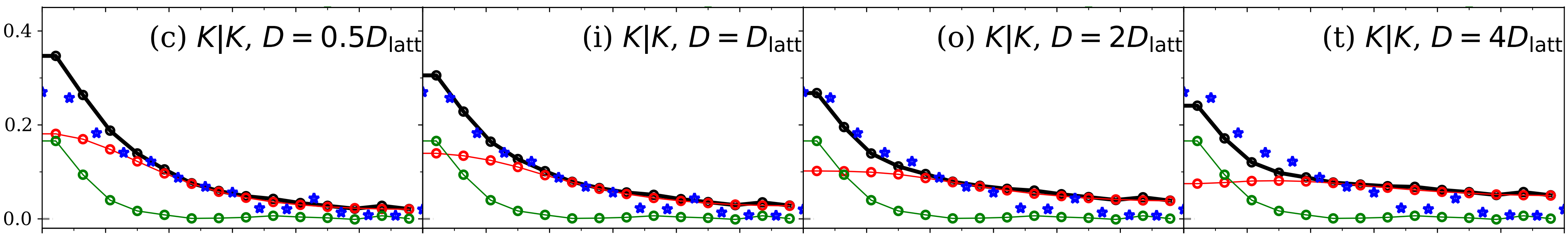
Lattice diffusion looks OK

**Type 1 + Type 2**
**Type 1 (hydro)**
**Type 2 (cascade)**



# Model vs. ALICE

**Type 1 + Type 2**
**Type 1 (hydro)**
**Type 2 (cascade)**



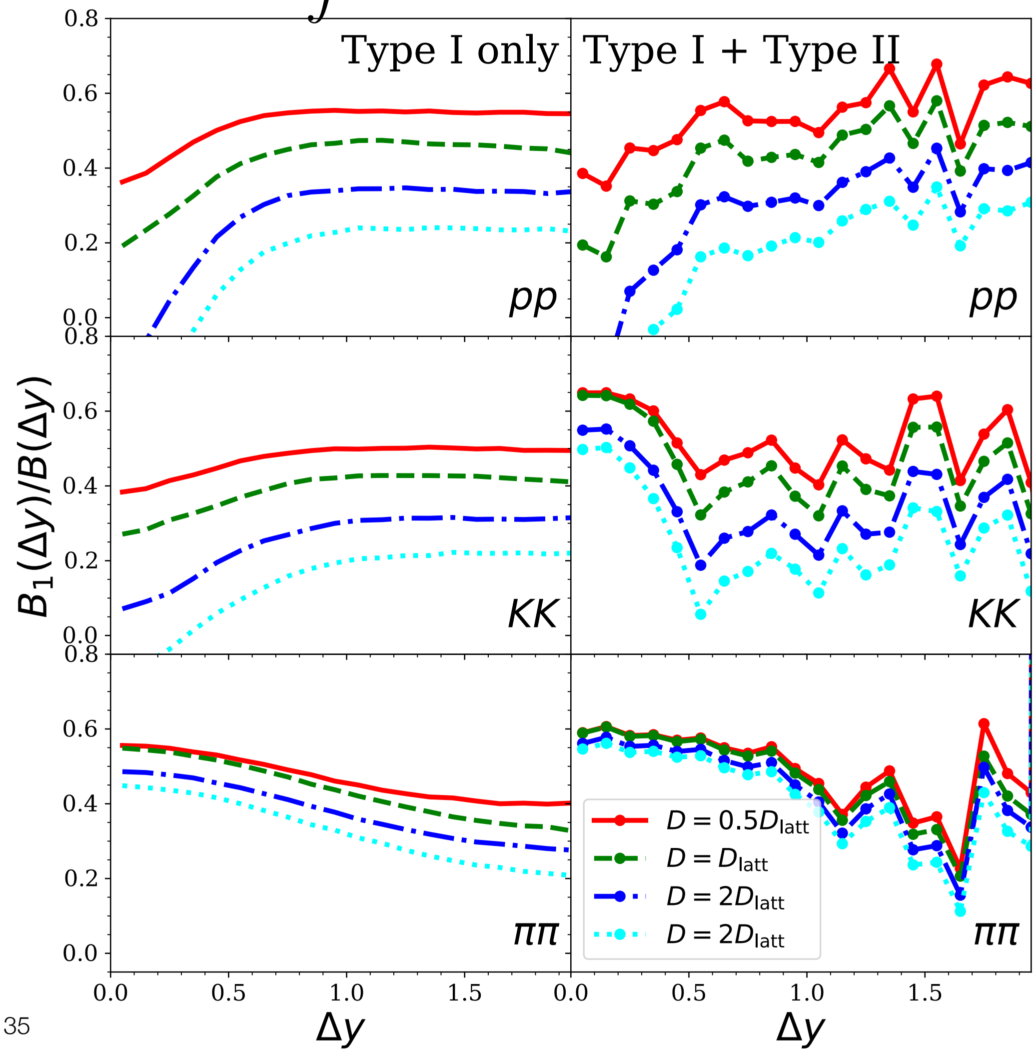

Lower diffusivities look better

# Better Focus on Diffusivity
# Analyze $B(\Delta\phi)$,
# Cutting on large $\Delta y$

Eliminate Effects from:

- HBT
- Resonant Decays
- Annihilation
- Experimental 2-track resolution
- $\Delta y \gtrsim 0.75$ should be good enough

$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi) \cos(\Delta\phi)$$



# Analyze $B(\Delta\phi)$
# Cutting on large $\Delta y$

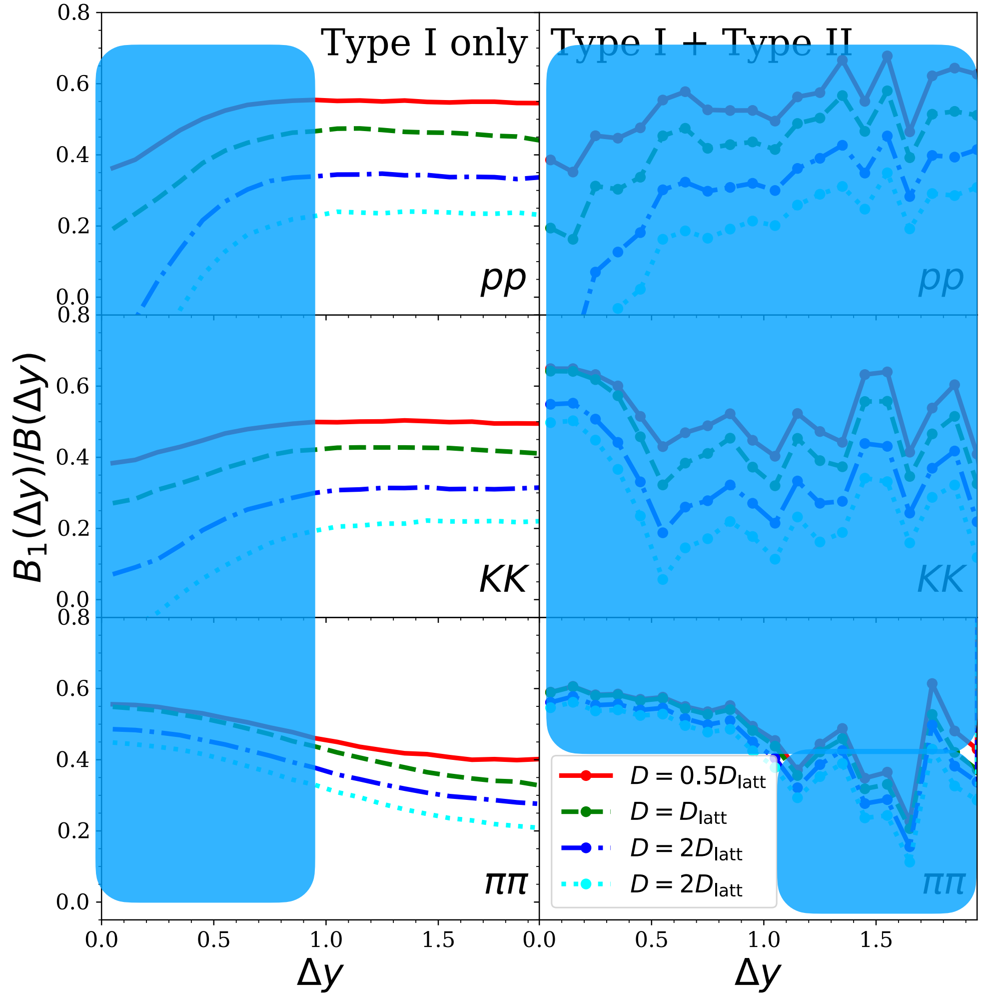
$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi),$$

$$B(\Delta y) = \int d\Delta\phi B(\Delta y, \Delta\phi)$$

Type II only provides noise for $\Delta y \gtrsim 1$

Robust extraction of diffusivity

for this window



# IV. Plans & Summary

# Plans

- Understand background from final-state interactions (HBT)
  S.P. and K.Martirosova ( PRC 2022)
- Understand baryon annihilation
  ongoing (with Dima Oliinchenko, being submitted)
- Pollution from charge-independent correlations (when $\mu_B \neq 0$)
- Push experiments to publish results:
  STAR, ALICE — but also CMS and ATLAS
- Perform Bayesian Analysis
  New emulator built (BAND Collaboration 2022)

# Summary

- Charge correlations (order $Q^2$) calculated in "standard model"
- STAR/ALICE BFs vs $\Delta y$ suggest early chemical equilibration
  $K^+K^-, p\bar{p}, \pi^+\pi^-$ systematics reproduced(STAR $pK$ normalization off)
- Diffusivity can be extracted from BFs binned by $\Delta\phi$ cut on large $\Delta y$
  High statistics STAR & ALICE data coming
- Many opportunities for progress
  — Both theoretical and experimental
  — Both for diffusivity and for chemistry
  — Similar to femtoscopy

## Bonus Slides

- CME background
- Skewness/kurtosis background
- Theory for higher-order charge fluctuations

U.S. DEPARTMENT OF ENERGY
Office of Science
NSF

# Bonus Slides

- **CME background**
- **Baryon Annihilation**
- **Higher Moments**

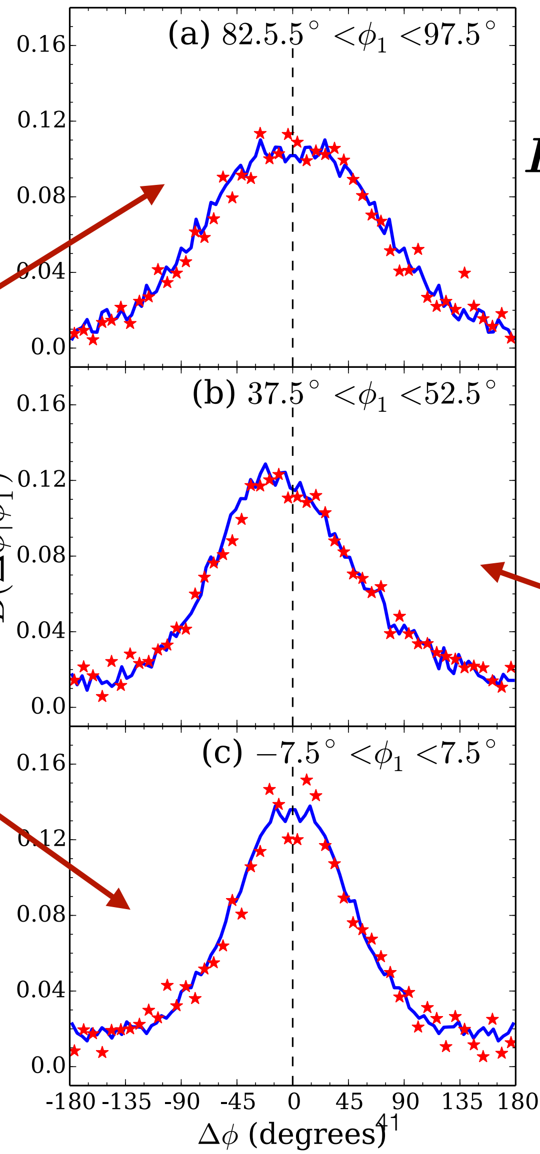
## Model vs. STAR

$\phi_1 \sim 90^\circ$

correlation
tighter in-plane
due to elliptic flow

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$




## Effect of Elliptic Flow

$$B(\Delta\phi|\phi_1) = \int d\phi_2\ B(\phi_1,\phi_2)\delta(\Delta\phi - \phi_1 + \phi_2)$$

balancing charge
more likely to be
in-plane

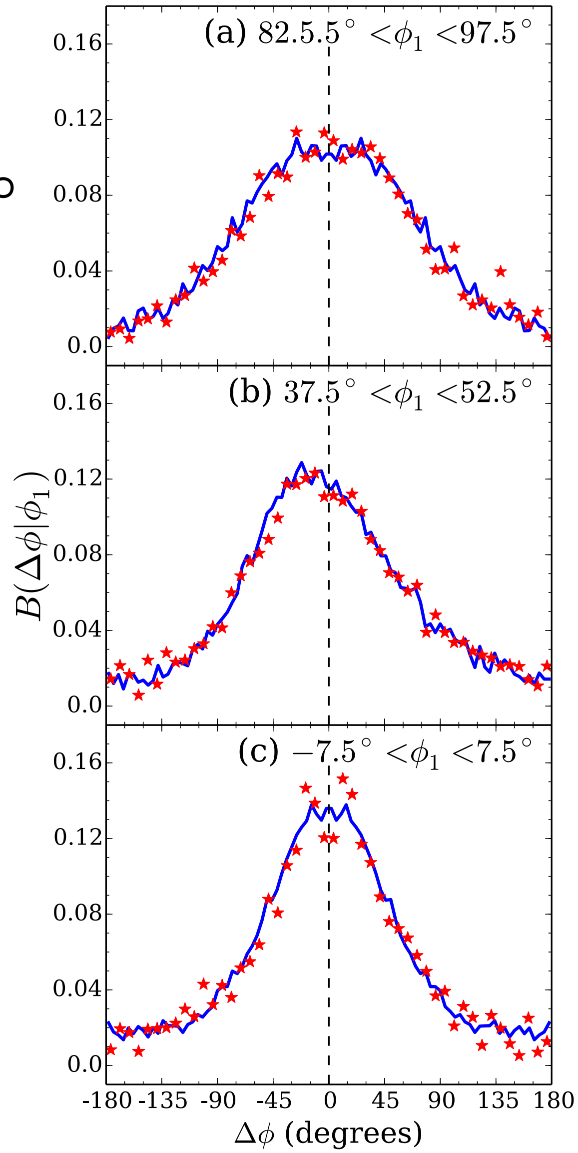
# ASIDE: CME correlator

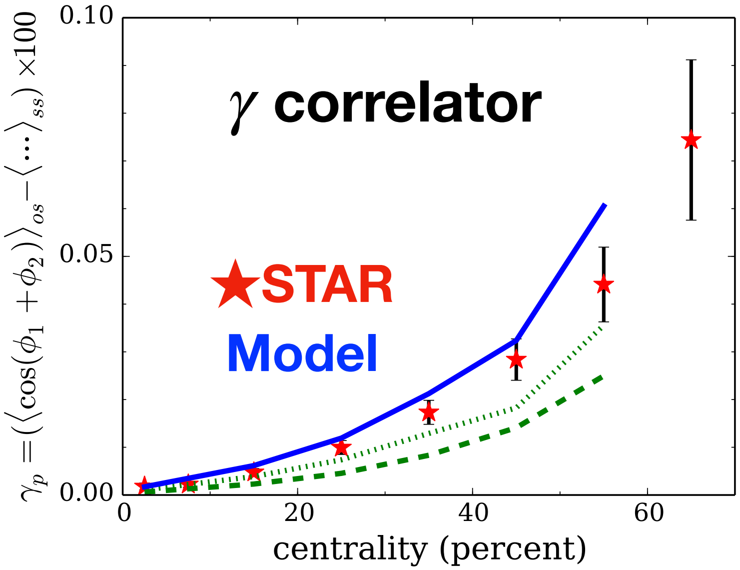
$$\gamma = \frac{1}{2}\{\langle \cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\text{opp.sign}}\}$$
$$- \frac{1}{2}\{\langle \cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\text{same.sign}}\}$$
$$= \frac{1}{M^2}\int d\phi_1 d\Delta\phi\ \frac{dM}{d\phi_1} B(\Delta\phi|\phi_1)\cos(2\phi_1 + \Delta\phi)$$

$\phi_1 \sim 90^\circ$

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$

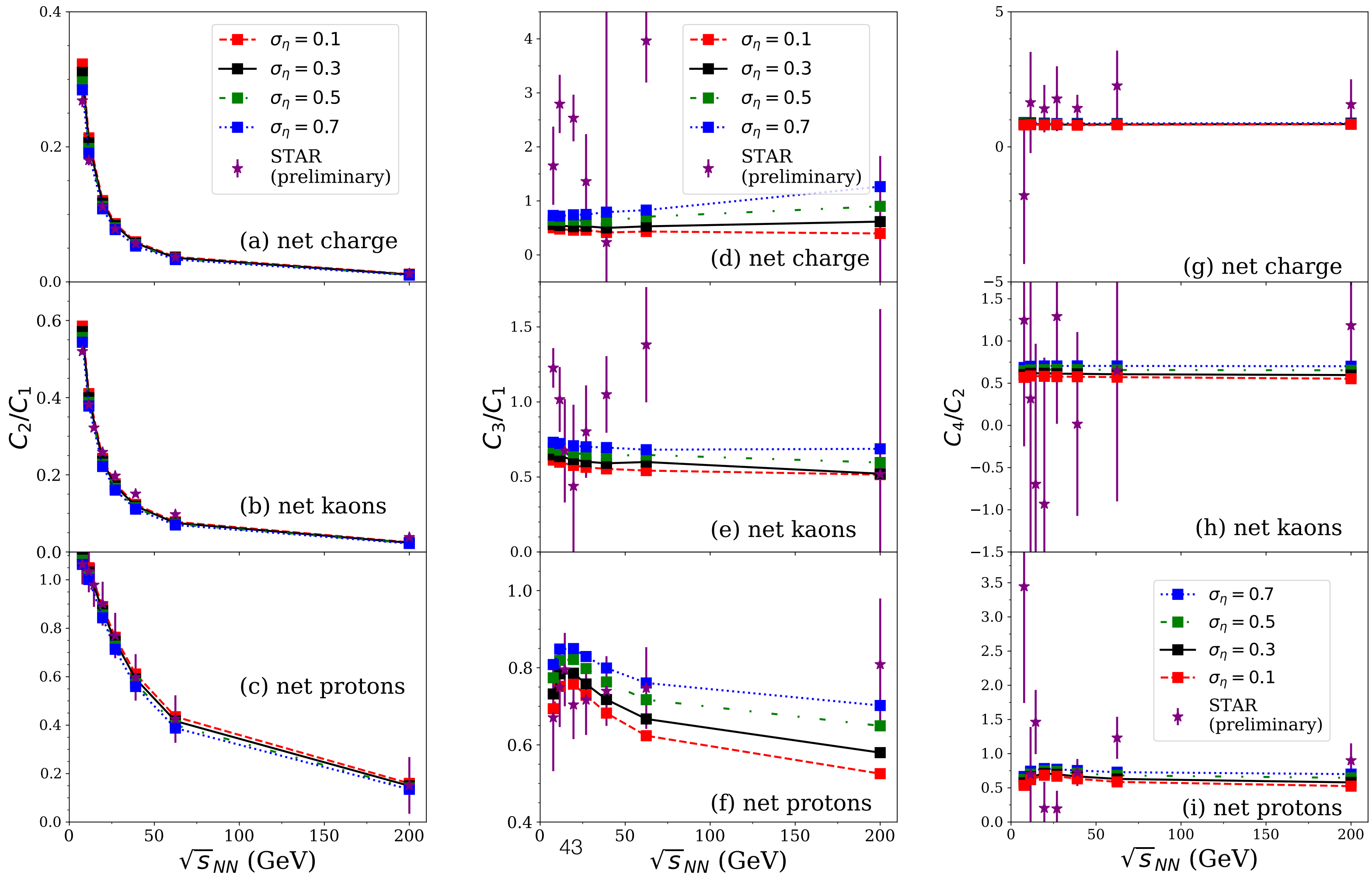



Model predicts ~ 115% of signal

# Charge Conservation and $Q^3, Q^4$ correlations

## b) Perform canonical ensemble on sub-volumes & superimpose on blast wave (crude)

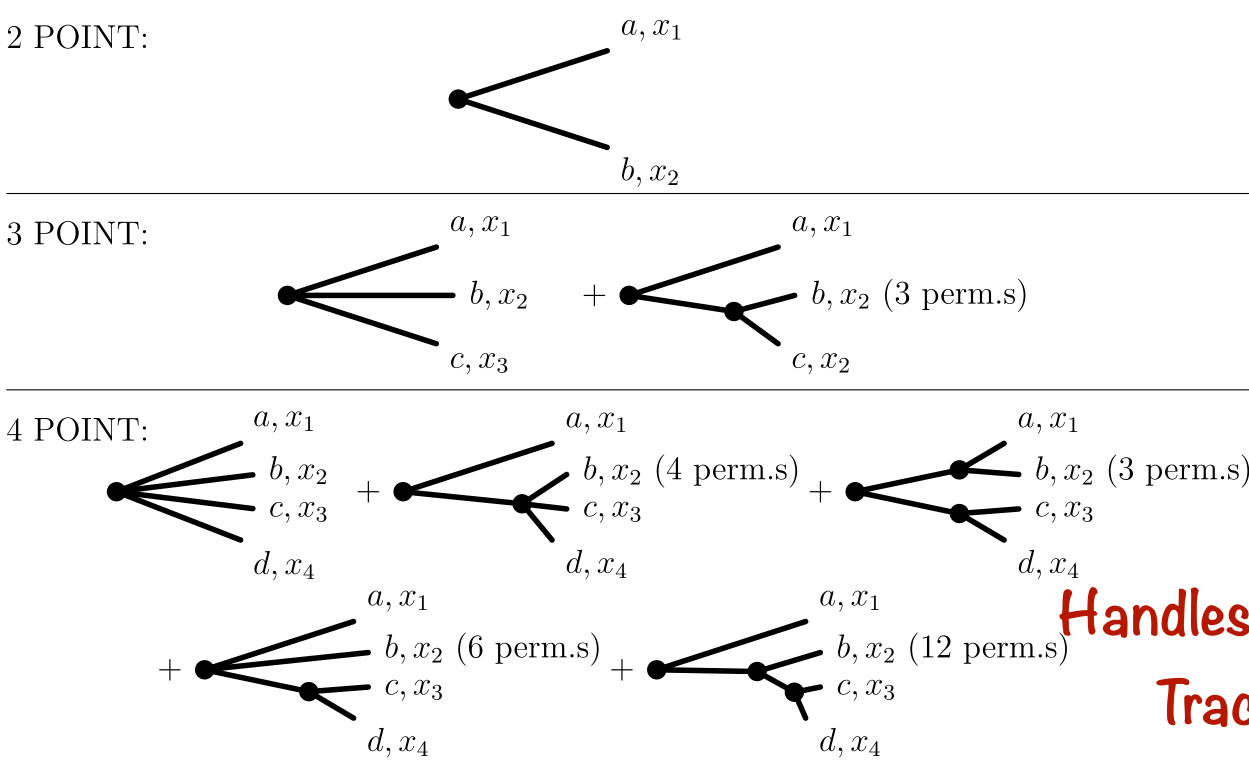



**S.P. and R.Steinhorst PRC (2020)**

# BONUS: Charge conservation and $Q^3, Q^4$ correlations (formalism)

**a) Integrate n-point correlations to obtain skewness & kurtosis**

**S.P., PRC (2020)**




Handles full 3x3 flavor dynamics

Tractable, but DIFFICULT

# BONUS: baryon annihilation

- $p - \bar{p}$ **BF vs** $q_{\rm inv}$
- **Includes recombination**
- **Great way to constrain baryon annihilation in final-state**