Growth of Beam Sizes in the Heavy Ion Accumulator Ring AR due to Intrabeam Scattering and Interaction with the Foil

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Abstract

It is proposed that several bunches of 10^{11} ions of Xe_{131}^{47+} or Ca_{40}^{11+} can be injected into an accumulator ring by means of non-Liouvillean injection with full stripping at the foil to the state of Xe_{131}^{54+} or Ca_{40}^{20+} respectively. Growth of beam sizes due to intrabeam scattering (IBS) and interaction with the foil during the accumulation will be considered in the present paper. For this purpose we have developed a special code taking into account IBS and foil interaction.

1 Scheme of Accumulation

The proposed scheme considers that bunches of ions accelerated in the SIS are injected into the accumulator ring with a time interval of 1 s, the last one is defined by the (upgraded) accelerating cycle of SIS [1]. During the injection a local bump of closed orbit in the accumulator will be produced by means of pulsed magnets in order to merge the incoming beam into the already stored one. This allows the accumulated beam to hit the foil only at the injection time and prevents it from interacting with the foil at every revolution and permanently loose energy and particles and eventually destroying the foil. After stripping all bunches will be captured in one RF bucket in the accumulator. Progressive growth of stored bunch intensity and relatively long time of accumulation may lead to noticeable growth of the beam phase volume due to IBS and interaction with the foil, especially in the longitudinal direction (namely growth of the momentum spread). It is worth to mention here that the chosen scheme of non-Liouvillean injection is a "central" one: injected bunches will be merged to the center of the accumulated beam (in contrast with the TWAC scheme of lateral injection and painting into the full acceptance).

Parameters of the storage ring are listed in Table 1 [2]. The accumulator

Projectile	Xe_{131}	Ca_{40}
Ring circumference, [m]	278.6	278.6
Momentum compaction factor η	0.188	0.244
β_x at the foil, [m]	11.2	11.2
β_y at the foil, [m]	25.2	25.2
D at the foil, [m]	0.40	0.40
$<\beta_x>$ average along the ring, [m]	12.0	12.0
$<\beta_y>$ average along the ring, [m]	13.3	13.3
$<D>$ average along the ring, [m]	1.75	1.75
Stationary RF bucket length, [m]	139.3	139.3
Stationary RF bucket height $\Delta p/p$	10^{-3}	10^{-3}
Amplitude of RF voltage, [kV]	1.06	0.93
Revolution time T_{rev}, [μs]	1.05	1.09
Synchrotron oscillations T_{syn}, [ms]	11.6	9.28

Table 1: Parameters of the accumulator ring

lattice is designed in such a way that all derivatives of Twiss functions are equal to zero ($\alpha_x = \alpha_y = D' = 0$) at the stripping foil position in the long straight section of the superperiod, and the dispersion D is fairly small at this point.

Parameters of injected bunches and interaction with the stripping foil are presented in Table 2. The r.m.s. value of the momentum spread may be chosen as:

$$(\Delta p/p)_{rms} = (\Delta p/p)_{max}/\sqrt{5}$$

assuming a parabolic beam density profile of the momentum distribution.

The Coulomb tune shift is calculated for the stored beam of final intensity of $N_b = 10^{12}$ particles for xenon ions and $N_b = 1.5 \cdot 10^{12}$ for calcium.

Projectile from injector	Xe_{131}^{+47}	Ca_{40}^{+11}
Charge state of stored ions, Z	54	20
Atomic number of stored ions, A	131	40
Number of ions per bunch	10^{11}	10^{11}
Number of injected bunches	10	15
Repetition rate, [s]	1	1
Velocity of ions, β	0.887	0.856
Transverse emittance ϵ, [m·rad]	10^{-5}	10^{-5}
Bunch length, [m]	92.9	92.9
Bunch r.m.s. half length, [m]	15.5	15.5
Bunch factor	0.222	0.222
$(\Delta p/p)_{max}$	$6.8 \cdot 10^{-4}$	$6.8 \cdot 10^{-4}$
$(\Delta p/p)_{rms}$	$3 \cdot 10^{-4}$	$3 \cdot 10^{-4}$
Coulomb tune shift ΔQ	-0.303	-0.312
Foil material	Gold	Gold
Mass density ρ, [mg/cm^2]	15	3
Heat density C_{Au},[J/(g·K)]	0.127	0.127
Energy loss E_{loss}, [MeV/u]	0.426	0.038
Momentum loss $(\Delta p/p)_{loss}$	$2.7 \cdot 10^{-4}$	$2.9 \cdot 10^{-5}$
Foil temperature gain ΔT, [K]	900	200
Energy straggling E_{str}, [MeV]	3.68	0.457
Angular straggling θ_{str}, [mrad]	0.173	0.115
Losses in newly coming bunch, [%]	2	0
Losses in stored beam, [%]	0.6	0

Table 2: Parameters of the beam in the accumulator

Energy loss, energy straggling and angular straggling values of ion interaction with a gold target are obtained by the code ATIMA [3]. Loss constants are extracted by the code GLOBAL [3].

The loss of ion energy in the foil leads to heating of the foil. For most metals in the temperature range above 400^0K the specific heat may be evaluated as C=25 J/(mole·K), which gives the value of C_{Au} presented in the table and the increase of the foil temperature by interaction with the beam of final intensity may be estimated by the following formula:

$$\Delta T = \frac{E_{loss} A N_b}{\rho \pi \epsilon \sqrt{\beta_x \beta_y} C_{Au}}, \tag{1}$$

where Twiss parameters should be taken at the foil position in the ring from Table 1.

The loss of momentum is calculated by the following formula:

$$\left(\frac{\Delta p}{p}\right)_{loss} = \frac{1}{\beta^2\gamma}\frac{E_{loss}}{E_0}, \tag{2}$$

where $E_0 = 938$ MeV is the rest energy of nucleon.

2 Interaction with the foil and IBS

Let us describe the beam sizes in terms of the invariants of motion. For the transverse plane they are just the emittances:

$$\epsilon_{x,y} = \gamma_{x,y}z^2 + 2\alpha_{x,y}zz' + \beta_{x,y}z'^2, \tag{3}$$

where z stands for the transverse coordinates: $z = x - D\Delta p/p$ for horizontal plane and $z = y$ for the vertical one. The invariant of the longitudinal motion for a bunched beam is

$$\epsilon_{||} = \left(\frac{\Delta p}{p}\right)^2 + \frac{1}{\Omega_s^2}\left(\frac{d}{dt}\frac{\Delta p}{p}\right)^2, \tag{4}$$

where Ω_s is the frequency of synchrotron oscillations.

In order to describe the beam sizes we introduce the r.m.s. values of these integrals:

$$\epsilon_1 = \frac{2\sigma_x^2}{\beta_x}, \quad \epsilon_2 = \frac{2\sigma_y^2}{\beta_y}, \quad \epsilon_3 = 2\sigma_p^2, \tag{5}$$

where $\sigma_{x,y} = \sqrt{< z^2 >}$ are r.m.s. transverse beam sizes and $\sigma_p = \sqrt{< (\Delta p/p)^2 >}$ is the r.m.s. value of the momentum spread.

Let us assume that the distribution function of the beams are gaussian in all degrees of freedom. Let $(\epsilon_{1,2,3})_0$ be the initial values of the integrals, i.e. those of a bunch coming from the injector (SIS accelerator in our case), and $(\epsilon_{1,2,3})_k$ are r.m.s integrals of the stored beam just after the injected beam and stored one join to each other at the foil in k-th cycle of injections. Then the influence of the foil on beam size may be described by the following equations:

$$(\epsilon_i)_{k+1} = A_i + \frac{(\epsilon_i)_0 C_0 N_0 + (\epsilon_i)_k C_1 N_k}{N_{k+1}}, \quad N_{k+1} = C_0 N_0 + C_1 N_k. \tag{6}$$

Here $C_0 = (N_0 - N_{loss})/N_0$ is a coefficient of ion losses of injected beam due to interaction with the foil (recombination processes) and C_1 is that for stored beam, N_0 is the number of ions in the injected bunch, N_k – number of ions stored in the accumulator after k progressive injections. Thus the second terms in the Eq.(6) describe the change of the stored beam parameters due to merging a new portion of ions coming from the injector.

Coefficients $A_{1,2}$ describe the increase of beam size due to angular straggling and moreover the longitudinal momentum loss in the foil influences the growth of the horizontal beam size due to nonzero dispersion D at the foil:

$$A_1 = \beta_x\frac{\theta_{str}^2}{2} + C_3\left(\frac{\Delta p}{p}\right)_{loss}^2, C_3 = \gamma_x D^2 + 2\alpha_x DD' + \beta_x D'^2 \tag{7}$$

$$A_2 = \beta_y \frac{\theta_{str}^2}{2} \tag{8}$$

Coefficient A_3 describes momentum straggling and is related to the energy straggling constant (see Table 2):

$$A_3 = 2\left(\frac{1}{A\beta^2\gamma}\frac{E_{str}}{E_0}\right)^2 \tag{9}$$

In between the moments of injection the stored beam will expand (or it may shrink in some degrees of freedom) in the accumulator because of IBS. The evolution of beam size may be described by the following equations:

$$\frac{d(\epsilon_i)_k}{dt} = \frac{2(\epsilon_i)_k}{\tau_i^{IBS}(t)}. \tag{10}$$

where growth time coefficients τ_i^{IBS} are dependent on beam sizes and through this on time. Calculation of τ_i^{IBS} may be done numerically involving the theory of IBS proposed by Piwinski [4] and Martini [5].

3 Numerical calculations and discussion of the results

A special code has been developed to calculate the growth of beam sizes due to interaction with the stripping foil and IBS according to Eqs.(6-10). In this section we show numerical results for a xenon ion beam: comparison of IBS growth time coefficients based on the original Piwinski's theory in approximation of a "smooth focusing machine" with Martini's equations which take into account variation of the Twiss functions along the ring. Also the importance of IBS during the accumulation will be shown.

Here it is worth to mention that we don't take into account any losses of particles during storage except those occurring at the stripping foil. And we do not follow exactly the evolution of the ion distribution, assuming everywhere that the accumulated beam distribution function is gaussian. The latter is not true because the injected beam has smaller size and merging it into the stored one deforms the gaussian distribution of the stored beam distribution function (even if it is gaussian initially). But our assumption allows to describe the evolution of the beam r.m.s. sizes with a fairly good accuracy (and almost negligible CPU time consumption) , because injection of new portions of ions into the center of the accumulated beam gives the distribution of particles close to gaussian.

The results of calculations are presented in Table 3. In the first column the number of injection cycle is shown. Columns number 2, 3 and 4 represent the evolution of the r.m.s. integrals in terms of initial ones, three following columns show the IBS growth time coefficients at the end of each cycle, and in the last column the number of stored particles in terms of 10^{11} ions of the incoming bunches are shown.

One can see that because of the non-equilibrium state of the beam (the beam is "hotter" in the transverse planes than in the longitudinal one) the

k	$\frac{(\epsilon_1)_k}{(\epsilon_1)_0}$	$\frac{(\epsilon_2)_k}{(\epsilon_2)_0}$	$\frac{(\sigma_p)_k}{(\sigma_p)_0}$	τ_x	τ_y	τ_p	$\frac{N_k}{N_0}$
1	1.0197	1.0339	1.0656	615.4628	-845.0901	17.5731	0.9809
2	1.0327	1.0486	1.1508	317.2397	-454.4087	10.4966	1.9558
3	1.0475	1.0621	1.2563	223.0168	-331.9006	8.8168	2.9246
4	1.0640	1.0747	1.3723	175.5695	-275.6315	8.4480	3.8873
5	1.0821	1.0867	1.4923	147.8184	-245.8244	8.6248	4.8442
6	1.1017	1.0983	1.6122	129.8574	-229.0783	9.0777	5.7950
7	1.1225	1.1097	1.7299	117.5055	-219.9379	9.6958	6.7400
8	1.1444	1.1211	1.8442	108.4648	-215.7277	10.4004	7.6792
9	1.1674	1.1326	1.9545	101.9311	-215.0153	11.2084	8.6125
10	1.1913	1.1442	2.0605	96.9535	-216.9211	12.0527	9.5400

Table 3: Evolution of stored xenon beam r.m.s. sizes during accumulation (Martini's equations are applied for IBS calculations)

momentum spread changes most drastically – it increases twice to the end of accumulation.

In Table 4 one can see almost coinciding numbers for the longitudinal plane

k	$\frac{(\epsilon_1)_k}{(\epsilon_1)_0}$	$\frac{(\epsilon_2)_k}{(\epsilon_2)_0}$	$\frac{(\sigma_p)_k}{(\sigma_p)_0}$	τ_x	τ_y	τ_p	$\frac{N_k}{N_0}$
1	1.0069	1.0069	1.0574	-836.4149	-702.2457	19.7321	0.9809
2	1.0092	1.0091	1.1328	-440.9486	-370.7429	11.8738	1.9558
3	1.0109	1.0108	1.2275	-312.4740	-263.2205	9.8495	2.9246
4	1.0120	1.0119	1.3336	-250.2365	-211.2670	9.2974	3.8873
5	1.0127	1.0126	1.4453	-214.1780	-181.2803	9.3454	4.8442
6	1.0131	1.0129	1.5586	-191.0261	-162.1288	9.7028	5.7950
7	1.0132	1.0130	1.6713	-175.1483	-149.0883	10.2452	6.7400
8	1.0131	1.0129	1.7818	-163.7619	-139.8269	10.9106	7.6792
9	1.0129	1.0127	1.8892	-155.3422	-133.0675	11.6647	8.6125
10	1.0125	1.0123	1.9932	-148.9867	-128.0546	12.4871	9.5400

Table 4: Evolution of stored xenon beam r.m.s. sizes during accumulation (Piwinski's "smooth" approximation is applied for IBS calculations)

in the case where Piwinski's "smooth" approximation is applied with use of the average Twiss parameters listed in Table 1, although for the transverse planes the results are different, but not drastically.

Table 5 shows the importance of IBS: if we "switch off" IBS and take into account only interaction with the foil then the beam sizes do not change much. For completeness we show in columns 5-7 the instantaneous IBS growth times (without taking them into account for the evolution).

k	$\frac{(\epsilon_1)_k}{(\epsilon_1)_0}$	$\frac{(\epsilon_2)_k}{(\epsilon_2)_0}$	$\frac{(\sigma_p)_k}{(\sigma_p)_0}$	τ_x	τ_y	τ_p	$\frac{N_k}{N_0}$
1	1.0163	1.0364	1.0034	599.2370	-821.4041	15.7502	0.9809
2	1.0244	1.0546	1.0050	311.7085	-420.8781	8.0983	1.9558
3	1.0325	1.0727	1.0067	212.5810	-289.7731	5.5182	2.9246
4	1.0406	1.0908	1.0084	163.0719	-223.4852	4.2290	3.8873
5	1.0487	1.1088	1.0100	133.4094	-183.7146	3.4564	4.8442
6	1.0568	1.1268	1.0117	113.6698	-157.2627	2.9419	5.7950
7	1.0648	1.1448	1.0133	99.6012	-138.5609	2.5749	6.7400
8	1.0728	1.1627	1.0150	89.0783	-124.5374	2.3002	7.6792
9	1.0809	1.1806	1.0166	80.9128	-113.6716	2.0869	8.6125
10	1.0888	1.1985	1.0182	74.4104	-104.9855	1.9166	9.5400

Table 5: Evolution of stored xenon beam r.m.s. sizes during accumulation with "switched off" IBS

4 Dynamical bunch length

Till now we assumed that the stored bunch length remains constant during accumulation. That means one should increase the RF cavity voltage during storage in order to prevent the bunch to expand in the longitudinal direction.

Moreover, we may suggest some phase gymnastics of the beam with the following purpose: it is worth to increase the momentum spread of the beam as much as possible in order to make the beam temperature to be about the same in all degrees of freedom – then we prevent a transfer of energy from the transverse planes (usually "hotter") to the longitudinal one. Indeed in Fig. 1 one can see that the final growth of the momentum spread increases with bunch length (the example is given for xenon ions) if the longitudinal phase volume remains constant. But the length of the beam has a lower limit determined by tolerable incoherent Coulomb tune shift of betatron oscillations.

If we want to work with the same tune shift over all storage time then we may shrink the length of the first injected bunch by a factor of ten (for xenon ions) compare to the last incoming bunch. Then we increase the length of bunches (both the stored and a newly injected) proportionally to the number of injected bunches which permits us to keep the same Coulomb tune shift during the whole accumulation process. The results of simulation of such a process are presented in Fig. 2. One may see that this procedure gives some profit in saving the momentum spread but not too much – only of about 10%. Taking into account that very short bunches require high RF voltage (about 100 MeV for the first bunch in our case) and all the procedure requires not very simple beam gymnastics and one needs to survive with a big Coulomb tune shift ($\Delta Q = -0.3$) in as long as 10 seconds it seems that the suggested procedure is not attractive for this particular set of accumulation parameters.

The similar picture for the calcium ions is given in Fig. 3. In both cases the transverse emittances changes only by 10÷20% which is tolerable.

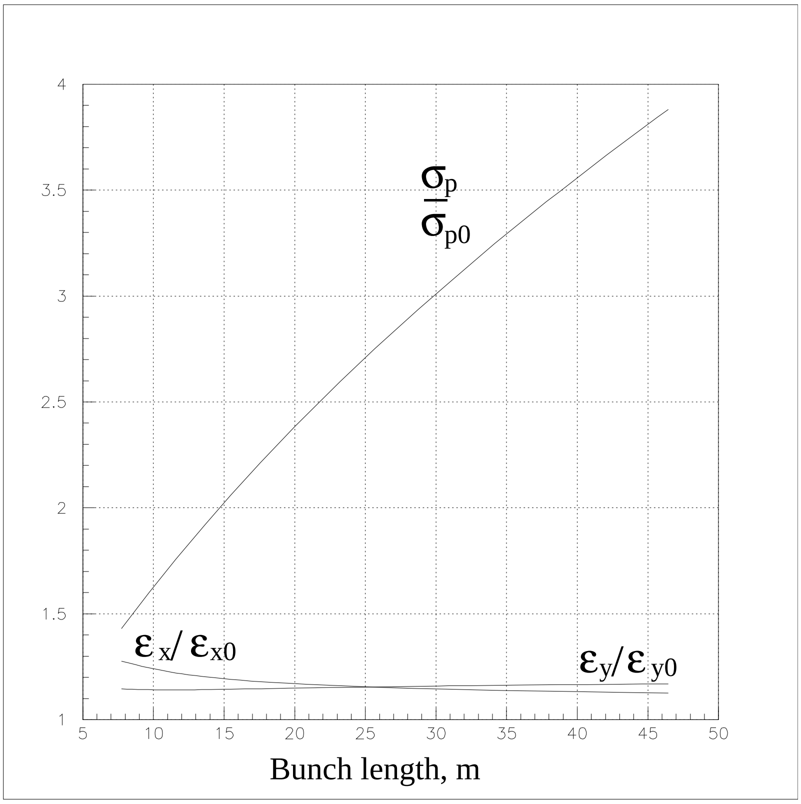

Figure 1: Dependence of growth rates on bunch length for Xe_{131}^{54+}

5 Barrier bucket

In order to calculate IBS growth rates we may consider a "rectangular" bunch as a coasting beam with the ring circumference reduced by the length of the bucket.

Let's assume that a conventional harmonic bucket will be adiabatically transformed to a rectangular one. To keep the same Coulomb tune shift the length of the barrier bucket should be taken according to the following formula:

$$L_{barrier} = B_f L_{ring} \tag{11}$$

Momentum spread of the beam may be calculated from the condition that the barrier bucket should have the same longitudinal phase volume as the original harmonic one:

$$\sigma_{p,barrier} L_{barrier} = \pi \sigma_{p,garmonic} L_{garmonic}/2, \tag{12}$$

where $\sigma_{p,}$ are the corresponding r.m.s. momentum spread values.

Assuming $L_{garmonic}/2 = m\sigma_{s,garmonic}$ and taking into account Eqs. (11-12) one may evaluate the maximum off-momentum for parabolic distribution:

$$(\Delta p/p)_{max} = m\sigma_{p,barrier} = \frac{m^2\pi}{2} \frac{\sigma_{p,garmonic}\sigma_{s,garmonic}}{B_f L_{ring}} \tag{13}$$

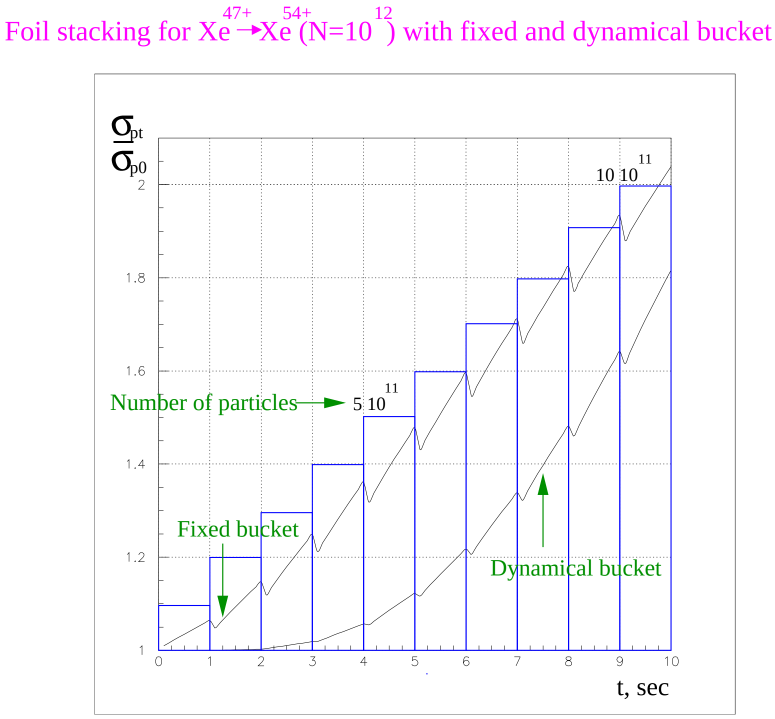

Figure 2: Evolution of stored xenon beam r.m.s. momentum spread during accumulation

The usual conversion factor from r.m.s. size of gaussian distribution to maximal size of parabolic distribution is $m = \sqrt{5}$. Taking from Table 2 $L_{ring} = 278.6$ m, $B_f = 0.222$, $\sigma_{s,garmonic} = 15.5$ m and $\sigma_{p,garmonic} = 3 \cdot 10^{-4}$ one may calculate:

$$L_{barrier} = 62 \quad \text{m}, \qquad (\Delta p/p)_{max} = 6 \cdot 10^{-4}$$

It is seen from Table 6 that like in the case of conventional harmonic bucket

Projectile	Xe^{54+}_{131}			Ca^{20+}_{40}		
Final/initial sizes	$\frac{(\epsilon_1)_{10}}{(\epsilon_1)_0}$	$\frac{(\epsilon_2)_{10}}{(\epsilon_2)_0}$	$\frac{(\sigma_p)_{10}}{(\sigma_p)_0}$	$\frac{(\epsilon_1)_{15}}{(\epsilon_1)_0}$	$\frac{(\epsilon_2)_{15}}{(\epsilon_2)_0}$	$\frac{(\sigma_p)_{15}}{(\sigma_p)_0}$
Fixed bucket	1.18	1.16	2.71	1.12	1.10	2.40
Dynamical bucket	1.21	1.15	2.40	1.15	1.09	2.12

Table 6: Evolution of "barrier bucket" beam sizes

the most crucial growth is in the longitudinal temperature by about a factor slightly bigger than two.

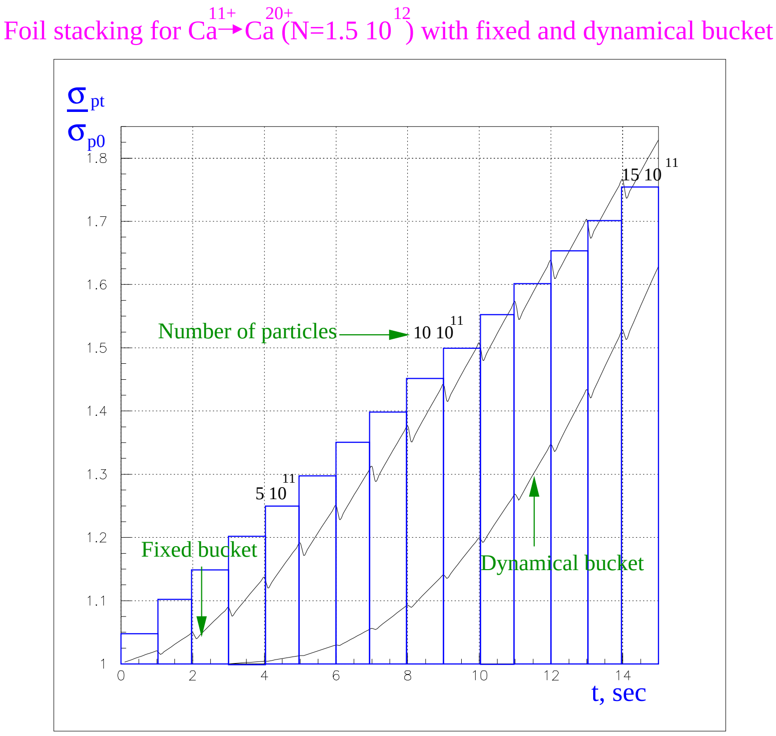

Figure 3: Evolution of stored calcium beam r.m.s. momentum spread during accumulation

6 Foil matter heating

Up to now gold foil was under consideration because gold is in the list of the elements available in ATIMA [3] code to obtain relevant input data for calculation of interaction of the beam with the foil.

But as it is seen from Table 2 foil heating by the xenon beam may increase the temperature of the gold foil up to about melting temperature 1063^0C. More accurate calculations [6] show that it might be even higher – up to 1500^0C, thus gold is not a good choice for the foil matter.

More acceptable element for foil matter is tantalum with melting temperature of about 3000^0C. Because of higher emittivity tantalum foil will be heated only up to 1100^0C [6] which is far below the melting temperature and does not affect strongly mechanical properties of the foil. From the other side Ta_{73} close to Au_{79} in the periodic table and parameters of beam interaction with the foil has close values for gold and tantalum matter.

7 Conclusion

During accumulation the longitudinal phase volume of the beam increases twice because of interaction with the foil and IBS.

In order to prevent even bigger expansion of the phase volume one has to keep the stored bunch length constant, which requires to increase the RF voltage by a factor of 4 during accumulation. Moreover, the stored beam looses its energy when it hits the foil which means it gets out from synchronism with the RF field and changes the closed orbit in the ring. Thus the cavity should be tuned to the new corresponding frequency (and the phase) in order to prevent coherent synchrotron oscillations of the beam after each pass through the foil at injection time. The latter may result in irreversible growth of the beam longitudinal phase volume because of nonlinearity of the longitudinal focusing field. With ferrite loaded cavities with relatively low quality factor correction of the RF frequency to the right phase may be done in few microseconds. It is especially important if one works with "barrier bucket" bunches.

A correction of the closed orbit may be done either by decreasing the magnetic field of the lattice magnets or by acceleration of the stored beam up to its original energy. In the last case one should note that the total loss of energy in the foil is about $AE_{loss}/Z = 1$ MeV for xenon ions. With a cavity voltage of about 10 kV it takes a hundred revolutions of the ions to compensate the loss.

The growth of the horizontal emittance because of the dispersion is inevitable in both cases and it is already taken into account in the calculations (by means of the coefficient A_1 in Eq. 6). Growth of the transverse emittances in all cases is not more than 20% i.e. may be considered as tolerable.

Although calculations show that tantalum foil is fairly good, optimization of the foil matter and design is still a subject for further studies.

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