Particle Trapping by Nonlinear Resonances and Space Charge

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Abstract. In the FAIR [1] facility planned at GSI high space charge effects and nonlinear dynamics may play an important role for limiting nominal machine performance. The most relevant interplay of these two effects on the single particle dynamics has been proposed in terms of trapping of particles into stable islands [2]. Subsequent numerical studies and dedicated experiments have followed [3, 4]. We present here the effect of the chromaticity on the mechanisms of halo formation induced by particle trapping into resonances.

Keywords: Beam dynamics;Space Charge;Resonance Trapping
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INTRODUCTION

Beam loss control is an important issue for high-intensity heavy ion rings of the new generation [1]. The new design challenges need to be supported by theoretical/numerical understanding of the basic relevant beam loss mechanisms. PIC simulation would present here a difficulty: the long term storage of a bunched beam, typically 10^6 turns, results in unreasonably large CPU times. Alternatively to the self-consistent approach, a particle-core model of the 3D bunch in a nonlinear lattice can be used. Even if the validity of this model is confined to small beam loss, we expect in this limit to obtain a reliable indication on basic mechanisms of beam loss.

SIMPLIFIED MODEL OF A BUNCH IN A NONLINEAR LATTICE

The linear dynamics is modelled by using the smooth approximation in all three space directions. The motion of one particle is then governed by three harmonic oscillators of strengths $k_x = (Q_{x0}/R)^2, k_y = (Q_{y0}/R)^2$, and $k_z = (Q_{z0}/R)^2$, where Q_{x0}, Q_{y0}, Q_{z0} are the horizontal vertical and longitudinal tunes, R is the radius of the ring. Consistently a stationary Gaussian bunch (with small intensity) can approximately be represented with the distribution

$$\rho \propto \exp\left(-\frac{\varepsilon_x}{2E_x} - \frac{\varepsilon_y}{2E_y} - \frac{\varepsilon_z}{2E_z}\right) \tag{1}$$

where $\varepsilon_i = x^2/\beta_i + x'^2\beta_i$ are the Courant-Snyder invariants, $\beta_i = R/Q_{i0}$ the beta functions, E_i the beam emittances and $i = x, y, z$. Throughout the paper $()' = d/ds$. In the real space the distribution Eq. 1 reads

$$\rho(x,y,z) \propto \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right). \tag{2}$$

Here $\sigma_x, \sigma_y, \sigma_z$ are the x, y, z rms bunch sizes and $\sigma_i = \sqrt{E_i\beta_i}$.

In order to simplify the problem we consider an axi-symmetric beam ($\sigma_x = \sigma_y$) and we will refer throughout this paper to the horizontal plane. In the condition of small aspect ratio $\sigma_x/\sigma_z \ll 1$, common in realistic bunch operation, the transverse electric field $\mathscr{E}_x$ in the point (x,y,z) is obtained in good approximation by the space charge of the local portion of the bunch neighbouring z which resembles a coasting beam [6]. Therefore we can apply the approximation

$$\mathscr{E}_x(x,y,z) = Ke^{-z^2/(2\sigma_z^2)}\frac{x}{r^2}\left[1 - e^{-r^2/(2\sigma_x^2)}\right] \tag{3}$$

where the electric field is normalized in units of the standard equation of motion [m^{-1}]; the perveance is defined as $K = qI/(2\pi\varepsilon_0 mc^3\beta^3\gamma^3)$, and $r = \sqrt{x^2+y^2}$. Here q stands for the charge of the particle transported, I is the bunch peak current, ε_0 the permettiviy, m the mass of the particle, c the speed of light, $\beta = v_z/c$ with v_z the longitudinal velocity of the bunch, γ the relativistic factor. The equation of motion for one particle becomes then

$$\left\{\begin{array}{lcl} x'' + \left(\frac{Q_{x0}}{R}\right)^2 x & = & Ke^{-z^2/(2\sigma_z^2)}\frac{x}{r^2}\left[1 - e^{-r^2/(2\sigma_x^2)}\right] + \\ & & +a_2(s)(x^2 - y^2) \\ z'' + \left(\frac{Q_{z0}}{R}\right)^2 z & = & 0. \end{array}\right. \tag{4}$$

The quadratic term with s dependent strength $a_2(s)$ models the effect of a sextupolar error to the lattice linearity.

For particles with small amplitude in all 3 directions the linearization of Eq. 4 allows to relate the perveance K with the maximum tune shift ΔQ_x and we find

$$K = \frac{2\sigma_x^2}{R^2}(2Q_{x0}\Delta Q_x - \Delta Q_x^2). \tag{5}$$

In order to discuss the interplay of space charge forces with the single particle nonlinear dynamics we take the typical parameters of the SIS18 synchrotron foreseen to become the injector of the SIS100 in the FAIR project [5]: $R = 34.4$ m, $Q_{x0} \sim 4.3$. The maximum tune shift is taken as $\Delta Q_x = 0.1$ and $\sigma_x = 0.01$ m. The numerical studies are performed by transporting a particle via the lattice linear map with 101 space charge kicks per betatron wavelength. The nonlinear lattice is obtained by concentrating all the nonlinear components in a single kick. We take the integrated kick of strength $A_2 = a_2 \times 2\pi R = 0.05$ m^{-2}.

STABLE ISLANDS OF THE FROZEN SYSTEM

We first explore the properties of a “frozen system”, for which we assume that the longitudinal motion is frozen ($Q_{z0} \to 0$).

In this limit Poincare’ sections can be drawn for each z as the longitudinal motion is suppressed. In Fig. 1a we plot at $z = 0$ an example of phase space orbits and in Fig. 1b correspondent tunes obtained by FFT [7]. Note that the position of the islands depends on the distance of the bare tune Q_{x0} from its value at the third order resonance $3Q_{x0} = 13$. In Fig. 1 we take $Q_{x0} = 4.35$. Qualitatively the center of the island (located at $x > 0$) in this longitudinal section is obtained by the interception of the single particle tune curve in the absence of external sextupoles (dotted line) with the horizontal line $Q_x = Q_{x,res}$. In Fig. 1b the horizontal flat indicates that the particle is locked in to the islands, and the fractional tune $1/3$ is obvious.

In Fig. 2 we show how the islands of the frozen system change position for $z = 1.3\sigma_z$. At large z the islands approach the closed orbit and have smaller size. This is explained as result of the weakening of the transverse space charge for increasing z, which is related to the maximum tuneshift of the frozen system at $x = y = 0$:

$$\Delta Q(z) = \Delta Q_x e^{-z^2/(2\sigma_z^2)}. \tag{6}$$

For large z the maximum tune-shift $\Delta Q(z)$ decreases, thus reducing the space charge effectiveness in creating the island. From Eq. 6 follows that if $Q_{x0} - Q_{x,res} < \Delta Q_x$ and $Q_{x0} > Q_{x,res}$, there is a longitudinal coordinate z_t such that $\Delta Q(z_t) = Q_{x0} - Q_{x,res}$. There the islands merge the origin because the resonance condition is met on

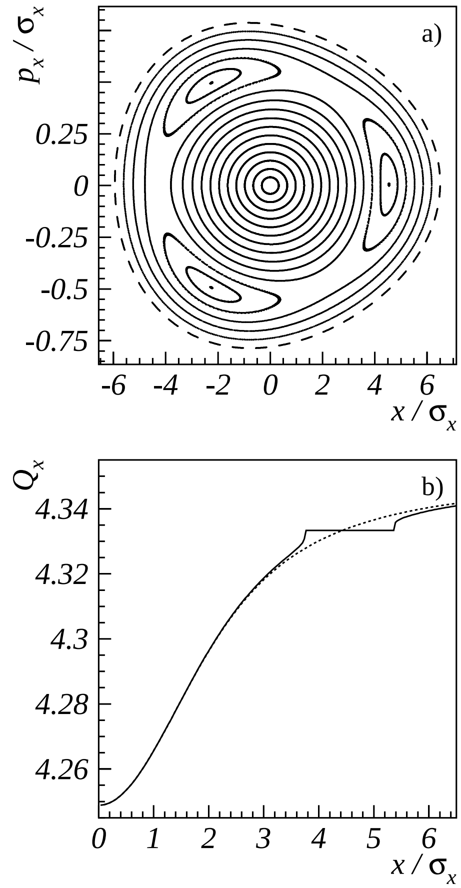

FIGURE 1. a) phase space orbits at $z = 0$; b) correspondent tunes.

$x = y = 0$; instead for $z > z_t$ no islands are present in the phase space. If $Q_{x0} - Q_{x,res} > \Delta Q_x$ then the resonance condition never occurs.

For the parameters used here $z_t = 1.89\sigma_z$. In Fig. 3 we plot the projection (cut at $p_x = 0$, $x > 0$) of the fixed point and edge of the island as function of z. The island has the outer position and maximum size at $z = 0$, then for larger z the fixed point approaches the closed orbit. Note that the islands merge into the origin at $z = 1.9\sigma_z$ in good agreement with the prediction of Eq. 6. We also computed the fractional tune Q_{xf} on the island fixed point (Fig. 4). The tune Q_{xf} is maximum in $z = 0$ and approaches zero at z_t.

TRAPPING INTO THE RESONANCE

During the synchrotron motion a particle sees at each instant, i.e. at each z, an instantaneous phase space topology as described by the analysis of the frozen system. The dynamical picture is that of an island which oscillates in the phase space.

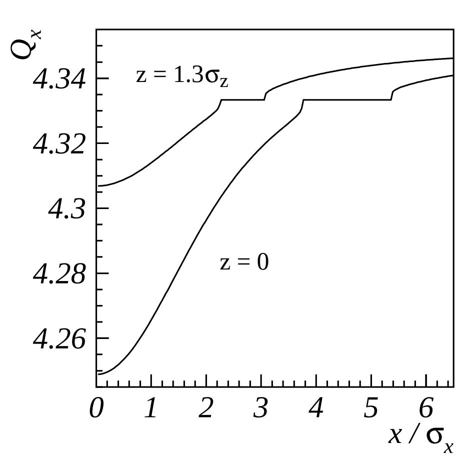

FIGURE 2. Island position as function of z.

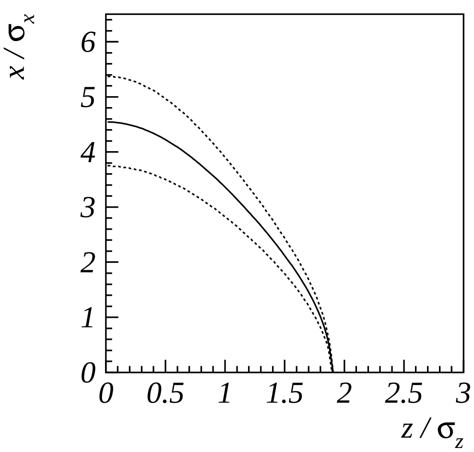

FIGURE 3. Island position: fixed point and edge vs. z.

The results of the previous section can be maintained in terms of the single particle transverse ε_x and longitudinal ε_z emittances. In fact we can define the threshold longitudinal emittance as $\varepsilon_{zt} = z_t^2 Q_{z0}/R$. Then for a particle with $\varepsilon_x \sim 0$, if $\varepsilon_z < \varepsilon_{zt}$, there is no longitudinal position allowed by the synchrotron motion in which the island intercepts the orbit of the test particle. Viceversa, for $\varepsilon_z > \varepsilon_{zt}$, there are two longitudinal positions where the island overlaps with the particle orbit. The resulting dynamics is that the oscillation of the island crosses repeatedly the particle trajectory.

When the island crossing is "adiabatic", a particle can be trapped into the island if the island area S is such that $S' > 0$ during the passage [8]. According to this theory the particle remains trapped until the island

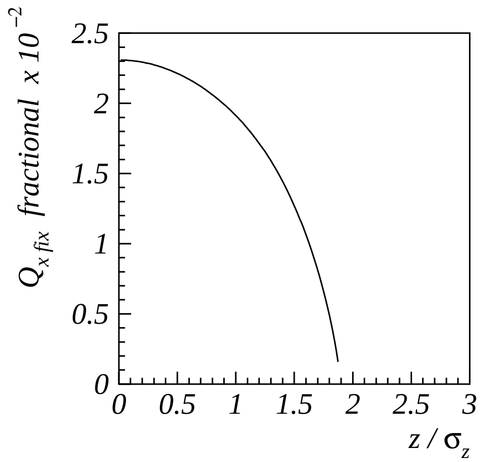

FIGURE 4. Tune of the fixed point vs. z.

area returns back to the same value, where the original trapping occurred [9]. The condition of adiabaticity is determined by:

- The speed of the island, which we approximate as the speed of the fixed point $\partial x_f(n)/\partial n$. Here $x_f(n)$ is the position of the fixed point;
- The size of the island $\Delta x(n)$;
- The tune of the fixed point $Q_{xf}(n)$.

The variable n is the number of turns. If during one revolution around the fixed point the island moves more than its size, the particle may not remain trapped. This condition can be formulated in terms of an adiabaticity parameter

$$T \equiv \frac{\partial x_f(n)}{\partial n} \frac{1}{Q_{xf}(n)\Delta x(n)}. \tag{7}$$

The condition of adiabaticity is expressed by $T \ll 1$. The parameter T is equivalent to the local adiabaticity parameter ε used in the analysis of the pendulum with time-varying amplitude (see [10]) where $\Delta x(n)/(\partial x_f(n)/\partial n)$ gives the characterisitc time scale of the perturbation (locally) and $1/Q_{xf}$ the time scale of the revolution around the fixed point. Typically $Q_{xf}(n)$ and $\Delta x(n)$ are depending on the space charge tune shift ΔQ_x and on the strength of the resonance A_2, whereas $\partial x_f(n)/\partial n$ depends also on Q_{z0}: the smaller Q_{z0} the slower the crossing and the more the adiabaticity is fulfilled. In Fig. 5 are shown two examples of the dependence of T from z for different longitudinal tunes. The adiabaticity is more critical at large synchrotron amplitudes.

In Fig. 6 is shown the motion of a particle for $Q_{x0} = 4.35$, $Q_{z0} = 5 \times 10^{-5}$ and initial coordinates $x = 1.5\sigma_x$,

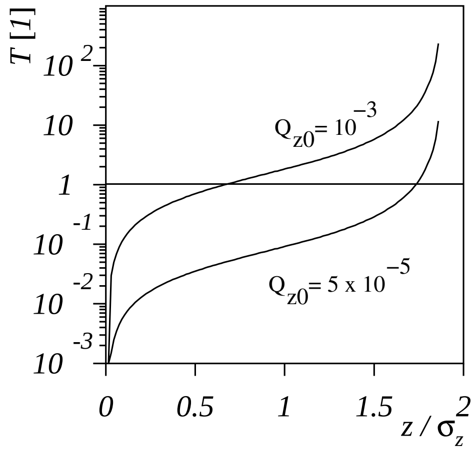

FIGURE 5. Adiabaticity parameter T as function of z for two longitudinal tunes.

$z = 3\sigma_z$; all other coordinates are zero. As Q_{z0} is small, the trapping in one synchrotron oscillation into the third order island is allowed. In a) is plotted the single particle invariant $\varepsilon_x/\varepsilon_{x0}$ during 1 synchrotron oscillation. Note that the trapping occurs after 0.15 synchrotron oscillations which corresponds to $z = 1.8\sigma_z$ consistently with Fig. 5; the detrapping occurs at 0.35 synchrotron oscillations, the symmetric point in the second half of the bunch. The second trapping shown in Fig. 6a occurs during the second half of the synchrotron oscillation. The differences between values of $\varepsilon_x/\varepsilon_{x0}$ before and after trapping are due to quasi random jumps of the adiabatic invariant at the separatrix [8, 9]. In Fig. 6b is shown that the outer position of the trapped particle at $5\sigma_x$ is given by the farthest position of the islands of the frozen system.

If we consider synchrotron tunes closer to those used in standard operation, for example $Q_{z0} = 10^{-3}$, the dynamics is completely different. In this case, as shown in Fig. 5, only particles with $|z/\sigma_z| < 0.7$ will be crossed by an island in an adiabatic regime. The resulting dynamics is shown in Fig. 7: the repeated resonance crossing induces a "scattering" of the single particle invariant. Over many synchrotron oscillations the repeated scattering produces a stochastic diffusion which brings the particle to large transverse amplitudes. When the particle transverse amplitude is large enough the crossing of islands through the particle orbit occurs at small $|z/\sigma_z|$ ensuring adiabatic trapping. In fact in Fig. 8 trapping occurs when $\varepsilon_x/\varepsilon_{x0} \simeq 5$ that is for an amplitude of $x = 3.3\sigma_x$. From Fig. 3 the island can cross this particle amplitude only at $z = 0.7\sigma_z$, but there $T \sim 1$ (see Fig. 5) and the adiabatic trapping can occur.

The diffusive process is nonlinear as the scattering is

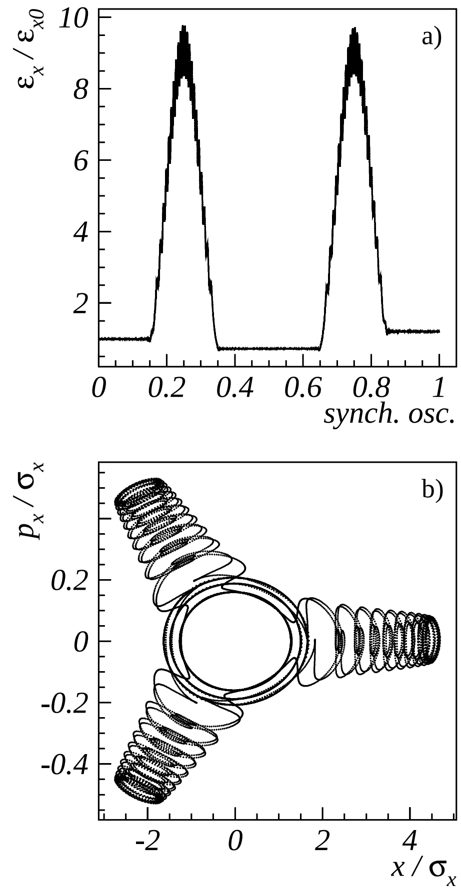

FIGURE 6. Trapping of a particle during one synchrotron oscillation: single particle invariant a); phase space b).

depending on the size of the island and its speed of crossing the particle orbit, which are depending on the transverse position. The number of turns needed to reach an amplitude large enough to ensure particle trapping into islands is therefore sensitive to the initial condition. For better representing the diffusive stochastic behaviour we track 10000 particles with different initial conditions in the range $0 < x < 4.5\sigma_x$, $z = 3\sigma_z$ (all other coordinates zero). In Fig. 9 we plot in synch. osc. units the average time needed in order that the test particle reaches $x = 4.5\sigma_x$. The average is obtained by taking the time of a group of 50 consecutive initial conditions. The correlation is clearly shown: the smaller the initial amplitude, the larger the number of crossings to reach $x = 4.5\sigma_x$.

HALO FORMATION

As the outer position of the island is roughly given by the interception of $Q_{x,res}$ with the effective depressed tune Q_x, the outer position of a particle is a function of the distance from a resonance $Q_{x0} - Q_{x,res}$, the space charge

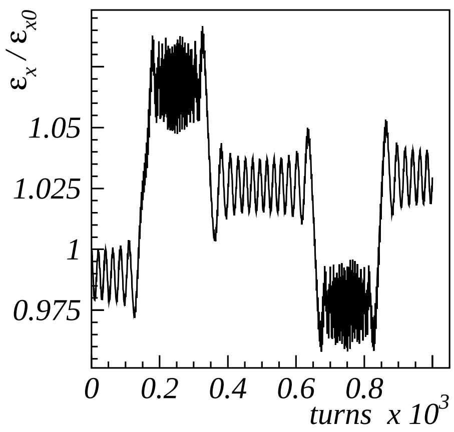

FIGURE 7. Scattering of ε_x during 1 synchrotron oscillation.

tune shift ΔQ_x, and the resonance strength. As guideline the above discussed mechanism allows to infer that for tunes approaching the resonance $Q_{x0} \to Q_{x,res}$, the fixed points, virtually, are shifted to infinity. Eventually particles reach the dynamic aperture and get lost. In Fig. 10 we plot the outer position of a particle for different tunes. The test particle has initial condition $x = 1.5\sigma_x, z = 3\sigma_z$, all the other coordinates are zero. The outer position is taken over 2×10^6 turns.

As previously discussed, all particles with $\varepsilon_x \sim 0$ and $\varepsilon_z > \varepsilon_{zt}$ are periodically crossed by the islands. In the longitudinal phase space the fraction of particles $\Delta N/N$, which satisfies this condition, is given by the area determined by the condition $\varepsilon_z > \varepsilon_{zt}$. For a Gaussian distribution we find

$$\frac{\Delta N}{N} = e^{-\varepsilon_{zt}/(2E_z)}, \tag{8}$$

which together with Eq. 6 yields

$$\frac{\Delta N}{N} = \frac{Q_{x0} - Q_{x,res}}{\Delta Q_x}. \tag{9}$$

If the maximum amplitude of the fixed points is beyond $3\sigma_x$ then Eq. 9 underestimates the fraction of particles which will be extracted from the bunch and brought to large amplitude populating the halo. If the external part of the halo intercepts the beam pipe or a dynamic aperture chaotic region, then more than $\Delta N/N$ particles are lost during long term storage.

EFFECT OF THE CHROMATICITY

Until now the analysis has been carried out for zero momentum spread. When this effect is included the transverse tunes depend on the particle off momentum $\delta p/p$

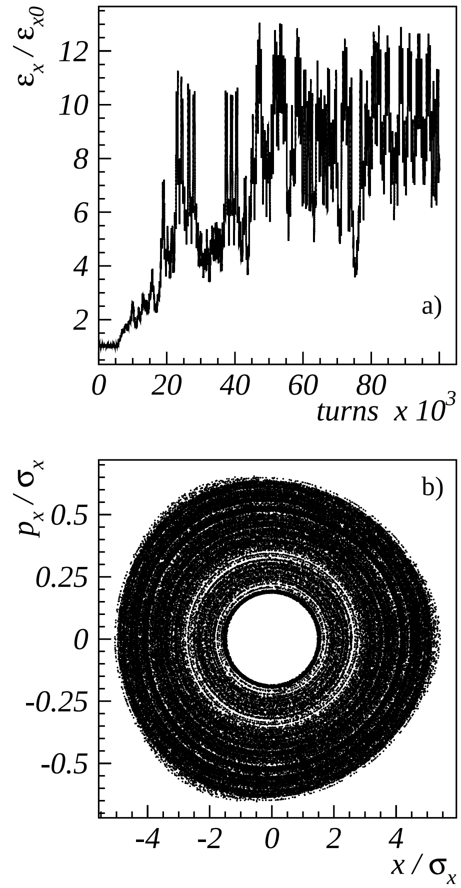

FIGURE 8. Scattering and trapping of a particle during 100 synchrotron oscillations. The longitudinal tune is $Q_{z0} = 10^{-3}$.

via chromaticity. For simplicity we consider here the natural relative chromaticity $\xi_x = \xi_y = -1$ which yields

$$\tilde{Q}_{x0} = Q_{x0}\left(1 - \frac{\delta p}{p}\right), \qquad \tilde{Q}_{y0} = Q_{y0}\left(1 - \frac{\delta p}{p}\right), \tag{10}$$

where now we denote with $\tilde{}$ the tunes of the off-momentum particle in the absence of space charge. When Eqs. 10 are added to Eqs. 4 the dynamics becomes more complex. A first consequence is that the tune Q_x of a test particle can distinguish if the particle is gaining or losing longitudinal momentum. The contribution of the chromaticity to the detuning is $\Delta Q_{xc} = \pm Q_{x0}\sqrt{\varepsilon_z/\beta_z - z^2/\beta_z^2}/|\eta|$ with η the slip factor. The sign +/- is used for loss/gain of longitudinal particle momentum. If the test particle has small transverse amplitude the total transverse detuning becomes

$$\Delta Q(z) = \Delta Q_x e^{-z^2/(2\sigma_z^2)} \pm \frac{\Delta Q_{xc0}}{z_{max}}\sqrt{z_m^2 - z^2}, \tag{11}$$

where $\Delta Q_{xc0} = Q_{x0}\sqrt{\varepsilon_{z_{max}}/\beta_z}/|\eta|$ is the maximum chromatic detuning for a particle with amplitude $z_{max} =$

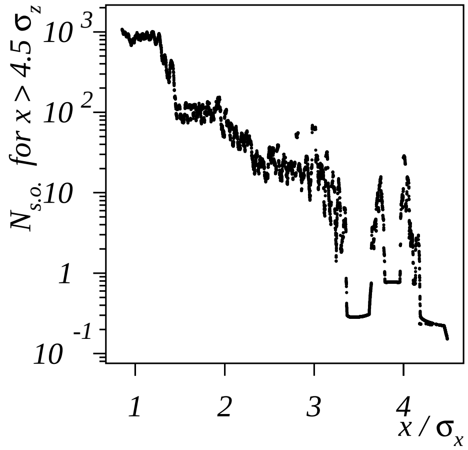

FIGURE 9. Turns, in synch. osc. units, necessary to bring the test particle to maximum amplitude.

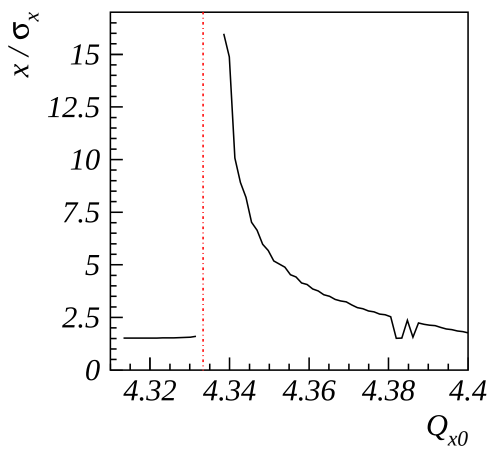

FIGURE 10. Halo radius vs tune Q_{x0}. The dot-dashed line represents the position of the resonance. The halo radius missing is at infinity (particle loss).

$\sqrt{\beta_z \varepsilon_{z_{max}}}$, the bucket is therefore defined by $z_{max}, \Delta Q_{xc0}$. In Eq. 11 z_m is the maximum amplitude of the test particle.

In Fig.11a is shown the tune Q_x along the bunch for a particle with $z_m = 3\sigma_z$ in a bucket defined by $z_{max} = 3\sigma_z$, $\Delta Q_{xc0} = 0.01$. The space charge tune shift is taken as $\Delta Q_x = 0.1$. Note that, due to chromaticity, different effective maximum detuning $\Delta Q_f / \Delta Q_b$ are obtained when the particle moves forward/backward in the bunch with respect to its average velocity. In Fig. 11b is shown the evolution of the single particle invariant in one synchrotron oscillation for $Q_{z0} = 5 \times 10^{-5}$. The test particle has initial coordinates $x = 1.5095\sigma_x$, $z = 3\sigma_z$, all other coordinates are zero. The difference in the maximum value of the invariant depends on the sign of $\delta p/p$. In fact the farthest position of the island is approximately given by the interception of the tune curve at $z = 0$ with the $Q_{x,res} = 4 + 1/3$, and the presence of the chromaticity shifts the full tune curve of $\pm\Delta Q_{xc} = 0.01$ according to whether the particle is losing or gaining momentum. The shift is affecting the position of interception with $Q_{x,res}$, which is occurring at smaller transverse amplitudes when the particle loses momentum, and at larger amplitudes during the other half of the synchrotron oscillation. This is shown in Fig. 12 where the transverse tune was modelled as

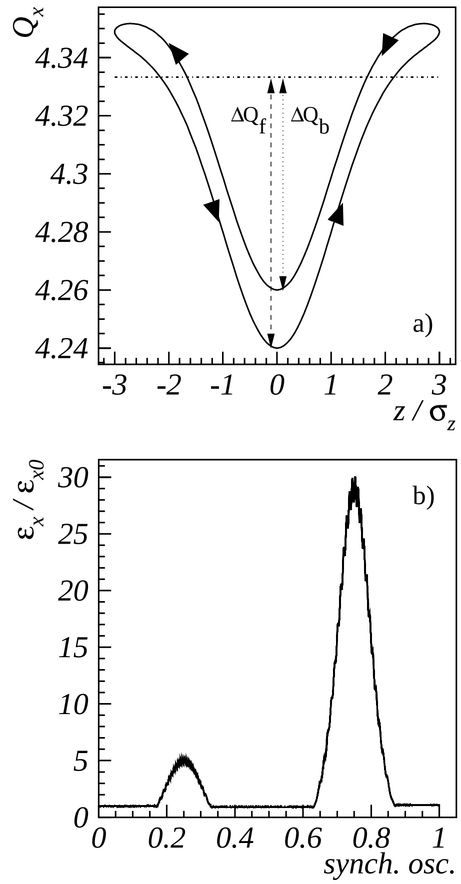

FIGURE 11. Single particle tune Q_x a) and invariant b) of a particle during one synchrotron oscillation in presence of chromaticity.

$$\Delta Q(x,z) = \frac{\Delta Q_x \, e^{-z^2/(2\sigma_z^2)}}{1 + [x/(2\sigma_x)]^2} \pm \frac{\Delta Q_{xc0}}{z_{max}} \sqrt{z_m^2 - z^2}. \quad (12)$$

This approximation for amplitudes $x > 3\sigma_x$ has a relative error with respect to the numerical values less than 2×10^{-4}. By imposing in Eq. 12 the condition

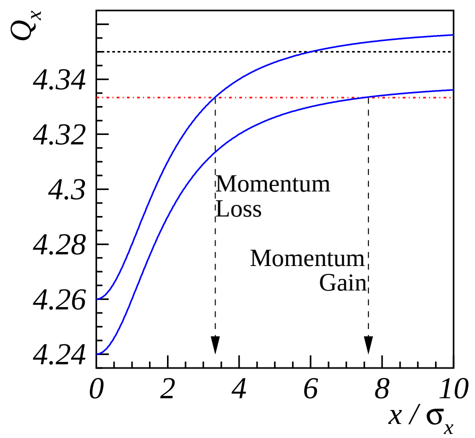

FIGURE 12. Tune dependence on amplitude at $z = 0$ for the maximum and minimum $\delta p/p$. The arrows show the position of the islands.

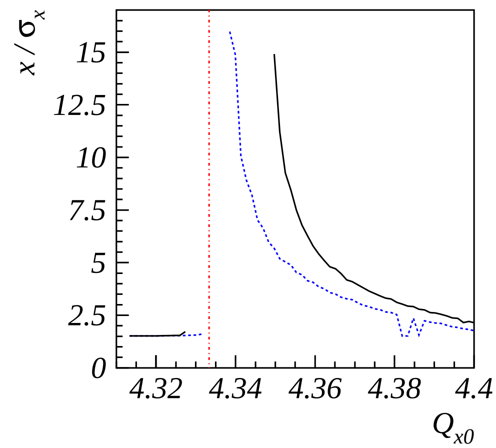

FIGURE 13. Halo radius vs. Q_{x0} in presence of chromaticity. The dashed (blue) line is the halo radius with $\Delta Q_{xc0} = 0$.

$Q_{x0} - Q_{x,res} = \Delta Q_x(\tilde{x}, z)$ we find approximately the position of the island $\tilde{x}$ and of the value of the invariants $\varepsilon_x/\varepsilon_{x0} = (\tilde{x}/1.5)^2$, from which we retrieve then Fig. 6a, and Fig. 11b.

CHROMATICITY INDUCED STOP-BAND

When the bare tune Q_{x0} is set in the band $Q_{x,res} < Q_{x0} < Q_{x,res} + \Delta Q_{xc0}\, z_m/z_{max}$ the limit tune $Q_x(x \to \infty)$ for weak resonances always crosses twice the resonance during one synchrotron oscillation. That means that the islands are always (virtually) brought to infinity. All the particles with maximum amplitude z_m such that

$$z_m \geq z_{mt} = \frac{Q_{x0} - Q_{x,res}}{\Delta Q_{xc0}} z_{max} \tag{13}$$

and whose transverse amplitude is crossed by the resonance will be eventually lost. The time scale in which the particles are lost is dependent on the scattering and trapping process (see Fig. 9). Note that according to Eq. 13 when $Q_{x0} \to Q_{x,res}$ then $z_{mt} \to 0$, but that does not mean that all particles of the bunch are lost. In fact in this case the islands are pushed to very large amplitudes, but not for all particles the islands will reach $x = 0$. This limits then the fraction of unstable particles. An estimate of the fraction of particles which are unstable via space charge and chromaticity will be the subject of future studies.

In Fig 13 we show the position of the outer most particle during 2×10^6 turns as function of the tune Q_{x0}. For better showing the effect of the chromaticity we plot in dashed blue the halo radius curve for $\Delta Q_{xc0} = 0$. Note that the tunes, where the halo is brought to infinity, is shifted by 0.01, which is exactly the maximum chromaticity. At $Q_{x0} = 4.35$ the halo extension is $5.7\sigma_x$ for the curve without chromaticity, and $15\sigma_x$ for the curve with chromaticity. These two halo radii correspond to $\varepsilon_x/\varepsilon_{x0}$ equal to 14.5 and 100 respectively. The discrepancy with Figs. 6a, and 11b stems from the fact that in those two examples the particles trapped remained always close to the fixed points ($5\sigma_x$ and $8\sigma_x$ respectively).

EFFECT OF CHROMATICITY ON THE HALO

Chromaticity has an interesting consequence on the halo formation. For example in Fig. 12 in the synchrotron oscillation half, in which the particle loses momentum, the halo radius h_2 is located at $3\sigma_x$, whereas in the other half the halo h_1 is located at $7.5\sigma_x$. This leads to the simultaneous presence of two halos that may share the same particles. However these two halos have been computed for a test particle with maximum amplitude $z_m = 3\sigma_z$. Inside the bunch each particle has its own maximum amplitude z_m and consequently its own halo radii h_1, h_2. In Fig. 14 we show the extension of the halos for particles with different z_m. The two halos merge at $z_m = 0$ because a particle of zero longitudinal amplitude has $\delta p/p = 0$. Fig. 14 suggests that the effect of the chromaticity is to make the global halo non-uniform.

As for on-momentum beams, each particle has a characteristic $\tilde{z}_t$ where the islands merge into $x = 0$. This quantity is a function of the maximum $\delta p/p$ of the test particle considered, and $\tilde{z}_t$ can be computed with Eq. 11. By using Eq. 6 we find $z_t = 1.89\sigma$, and from Eq. 11 we find that for the halo h_1 the interception points $\tilde{z}_t$ of the islands with the closed orbit satisfy $z_m > \tilde{z}_t > z_t$. This means that the fraction of particles to be brought to the halo h_1 is again larger than $\Delta N/N = (Q_{x0} - Q_{x,res})/\Delta Q_x$. For the smaller halo h_2 we find $\tilde{z}_t < z_t$ and therefore the number of particles, which populate it is larger than in h_1.

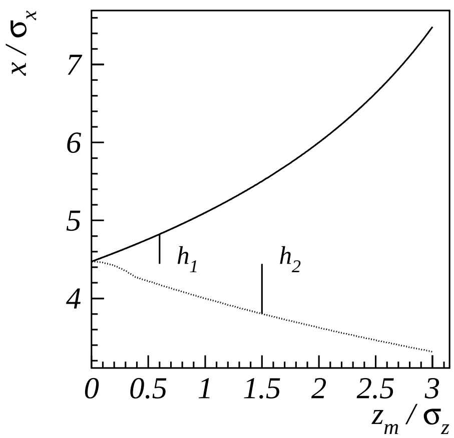

FIGURE 14. Halo radius for particles with different maximum longitudinal amplitude z_m.

CONCLUSION

The problem of the long term storage of a high intensity bunch has been described in terms of the single particle dynamics in a simplified model. We have investigated the effect of the chromaticity on the previously studied trapping mechanisms [2]. We found that the chromaticity in addition to space charge and synchrotron motion creates a stop band as large as the maximum chromatic detuning. The beam loss in the stop band is then a function of the vicinity of the bare tune to the resonance. This effect of chromaticity may help to explain the loss regime observed in a previous experiment [3], but detailed simulations are needed to quantify the effect. On the emittance growth regimes we find that the presence of a momentum spread in a bunch does not destroy the process of trapping/detrapping but rather enhances the halo radius. In this study we found evidences that the simultaneous presence of two halos does not change substantially the time (turns) needed to reach full trapping and transport to maximum halo amplitude. The effect of the simultaneous presence of two halos and of two crossing regimes on beam loss merits further studies.

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