Off-Momentum Particle Dynamics for the HIDIF Telescoping Scenario

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In the HIDIF scenario the accumulation of the required total number of heavy ions is an important issue [1]. The linac beam is injected into storage rings by using a multiturn injection scheme suitable to avoid that the recirculating beam hits the septum [2]. In the HIDIF telescoping issue one option (not included in the standard scenario) for reducing the number of storage rings was the accumulation of ions with different longitudinal momentum in the same storage ring.

After injecting ions at the design momentum, new ions with $\delta p = \Delta p/p_0 = 0.01$ are injected. The space charge due to the bunched beam causes a tune shift in the off-momentum particle that in one turn could hit the septum. Here we present a preliminary investigation on the effects that the bunched beam has on the off-momentum particle. Since the off-momentum particle is moving fast with respect to the lattice, it feels in average a space charge force as if the bunch envelope weren't changed by the betatronic motion. Since the longitudinal dynamics is not considered we approximate the bunch shape to a frozen ellipsoid uniformly filled of ions, we neglect also the effect of the image charges. Under these guesses the three semi axes of the ellipsoid are kept constant. Horizontal and vertical size have the same value a and the longitudinal length c is assumed throughout to be $c > a$. The electric field E_x, E_y is found out by integrating the analytical expression [4] of the electric field generated by an ellipsoidal bunch. Since bunch reference frame is supposed orthogonal, in a bending magnet an error in E_x, E_y due to the geometric approximation is introduced. The analytic expression of the integrated transverse electric field is $E_x = M(x,y,z)x$ and $E_y = M(x,y,z)y$, where x,y and z are the transverse and longitudinal coordinates and

$$M(x,y,z) = \frac{qn}{2\epsilon_0}\frac{a^2}{c^2}\left[\frac{\beta}{\alpha(1-\alpha\beta^2)} - \frac{1}{2\alpha^{3/2}}\ln\left(\frac{1+\sqrt{\alpha}\beta}{1-\sqrt{\alpha}\beta}\right)\right]$$

$\alpha = 1-(a/c)^2$, $\beta = 1/\sqrt{1+s_0/c^2}$, n is the ions density, q the charge state and $s_0 = s_1\Theta(s_1)$ ($\Theta(s)$ is the Heavside function) and s_1 is the solution of the equation $(x^2+y^2)/(a^2+s_1) + z^2/(c^2+s_1) = 1$. We simulate the transverse electric field generated by the bunched beam on the off momentum particle with the field E_x and E_y generated by its nearest bunch. This approximation introduce a discontinuity in $\partial E_x/\partial z$, $\partial E_y/\partial z$ when the particle crosses the border of a bucket. We describe the transverse single particle dynamics in a magnetic lattice by using the micromap technique [3]. The final form of the transverse micromap is

$$\mathbf{x}(s+\triangle s) = L_{s,s+\triangle s}\mathbf{K}(\mathbf{x}(s)) + \mathbf{D}_{s,s+\triangle s}\delta p$$

where $\mathbf{K}(\mathbf{x}(s)) = (x, x' + \triangle s\tilde{f}_x, y, y' + \triangle s\tilde{f}_y)$, $\mathbf{x}(s) = (x,x',y,y')_s$, $\tilde{f}_\zeta = f_\zeta + qE_\zeta/(v_0p_0)$, $\zeta = x,y$ and v_0 is the design longitudinal speed, p_0 is the design momentum. $\mathbf{D}_{s,s+\triangle s} = (D_{s,s+\triangle s}, D'_{s,s+\triangle s})$ is a particular solution of the horizontal transverse equation of motion correspondent to the initial condition $\mathbf{x}(s) = 0$ and $\delta p = 1$. Assuming the single particle dynamics linear within one turn, we use each of the periods of the HIDIF ring to evaluate the tune. In one period the initial coordinates $\mathbf{x}_0$ are changed in $\mathbf{x}_1$ and in the next period they become $\mathbf{x}_2$. Since Courant-Snyder theory shows that the trace of the transfer matrix is $2\cos[\psi_x(l/3)]$ where $\psi_x(l/3)$ is the phase advance in one period, one finds $\cos(2\pi q_x/3) = 0.5(x_0x_2' - x_2x_0')/(x_0x_1' - x_1x_0')$ and then q_x. When $\delta p \neq 0$ the coordinates (x,x') are meant with respect to the new closed orbit. We considered the existing beam made of ellipsoidal bunches with transverse size of 2.5 cm and longitudinal length 17.36 cm filled with $1.32\cdot 10^{13}$ ions each with $\delta p = 0$. We choose the transverse size of 2.5 cm since considering a frozen ellipsoid the bunch feels an average beta function of $\beta_x = \beta_y \sim 11$ m. The ion dynamics has been computed with a micromap of length $\Delta s \sim 14.7$ cm. In order to check the code we have used first a constant focusing lattice of length $L = 442,8$ m and $q_x = 8.715$. A test particle with initial coordinates $x_0 = y_0 = 15$ mm and $p_{x,0} = p_{y,0} = 0$ mrad and $\delta p = 10^{-3}$ has been tracked for two overtaking evaluating the tune (Fig. 1a), we consider such an off-momentum as to show the physics involved in the process. The test particle moves from the border of the bucket and since it is far from the bunch it doesn't feel a strong space charge therefore the tuneshift is zero. Moving ahead the particle's motion begins to be effected by the bunch space charge that grows reaching the maximum when the particle enters inside the bunch. Since the electric field inside the bunch is linear the tuneshift $\Delta q_x = \Delta q_y = 0.0375$ is constant. Fig. 1b shows the tuneshift evolution for the test particle with $x_0 = y_0 = 50$ mm and $p_{x,0} = p_{y,0} = 0$ mrad and $\delta p = 10^{-3}$. Since the particle is outside the bunch the space charge is reduced and consequently maximum tuneshift ($\Delta q_x = \Delta q_y = 0.008$). The fluctuations at the values 0.2 and 0.65 ms are due to the nonlinearity of the space charge ($E_x \propto x/r$ where $r = \sqrt{x^2+y^2}$), they are reduced when the particle is far from the center of the bunch. Fig. 1c,d show the strong influence of the betatronic motion of the test particle on the evaluation of the tune for the HIDIF lattice. The tune behaves like Fig. 1b but now it depends on the initial transverse condition of the particle. The betatronic motion makes $\sqrt{\beta_x\epsilon_x}$ of the test particle oscillate each 11 m (β_x wavelength) while a, c are constant and the wavelentgh in space charge intensity is ~ 18.45 m. As consequence the tuneshift exhibits fast oscillation of amplitude proportional to the space charge and with wavelength $\propto \delta p$. The difference in the spread of the tunes in the two planes is due to the effect of the bending. The same case has been considered for a test particle with $\delta p = 0.01$ in 20 overtaking (Fig. 1e,f). The tune oscillation for a strong off-momentum stays limited, the parabolic-like shape of Fig 1c,d is lost and the tune footprint (Fig. 1f) shows the spread of the tunes. The change of the dynamics due to the space charge on the off-momentum particle can be analyzed looking at the orbits on the Poincare´ section correspondent to the septum. We consider the $x - x'$ plane. The test particle with $\delta p = 0.01$ and initial coordinates $p_{x,0} = p_{y,0} = 0$ mrad, $x_0 = y_0 = 2 \cdot n, n = -50, ..., 50$ mm has been tracked for 20 turns (correspondent to $\sim$ 20 overtake). In Fig. 1g the continuous line represents the final position of the particle without the interaction with the bunched beam. The dashed line shows what is obtained when the effect of the bunched beam is included. The new closed orbit is in $x_{co} = D \cdot \delta p = 10.6$ mm ($D_x = 1.06$ m). Composing the average linear force generated by the lattice $-\overline{k}x$ with the centrifugal force due to the off momentum $\delta p/\rho$ one finds the equilibrium point x_{co}. The effect of the bunched beam moves the closed orbit outward. Considering these two contributions to the dynamics, the average potential V felt by the test particle is a parabola-like that in the bottom has either three stationary points ($\partial V/\partial x = 0$) two stable with one unstable in between (strong space charge) or simply one stable point (weak space charge). When the particle is injected on the closed orbit ($x = 0$), since this is not the closed orbit, the transverse oscillation has a different tune form $q_{x,0}$. As the initial conditions are moved outward ($x > 0$) we approach the minimum of the potential and beyond the same physical situation is repeated until $V(x) = V(0)$. Moving further outward the particle feels completely the nonlinearity of the space charge, this effect is shown in the big deformation of the dashed line in Fig. 1g. Furtheron the effect of the nonlinearity in the potential is reduced with respect to the quadratic-like shape and the deformation of the dashed line is reduced. Fig 1h shows the horizontal and vertical shift $\Delta x, y$ with respect to the unperturbed case, we see that for positive value of x the particle is beyond the septum since $x + y > x_0 + y_0$.

These results give a first indication that the space charge effect of the on-momentum beam on the off-momentum ions is significant. Hence a new optimization is required to make the off-momentum multiturn injection (nearly) loss free. Possibly the space charge effect could be reduced by choosing a lattice with a significant large dispersion.

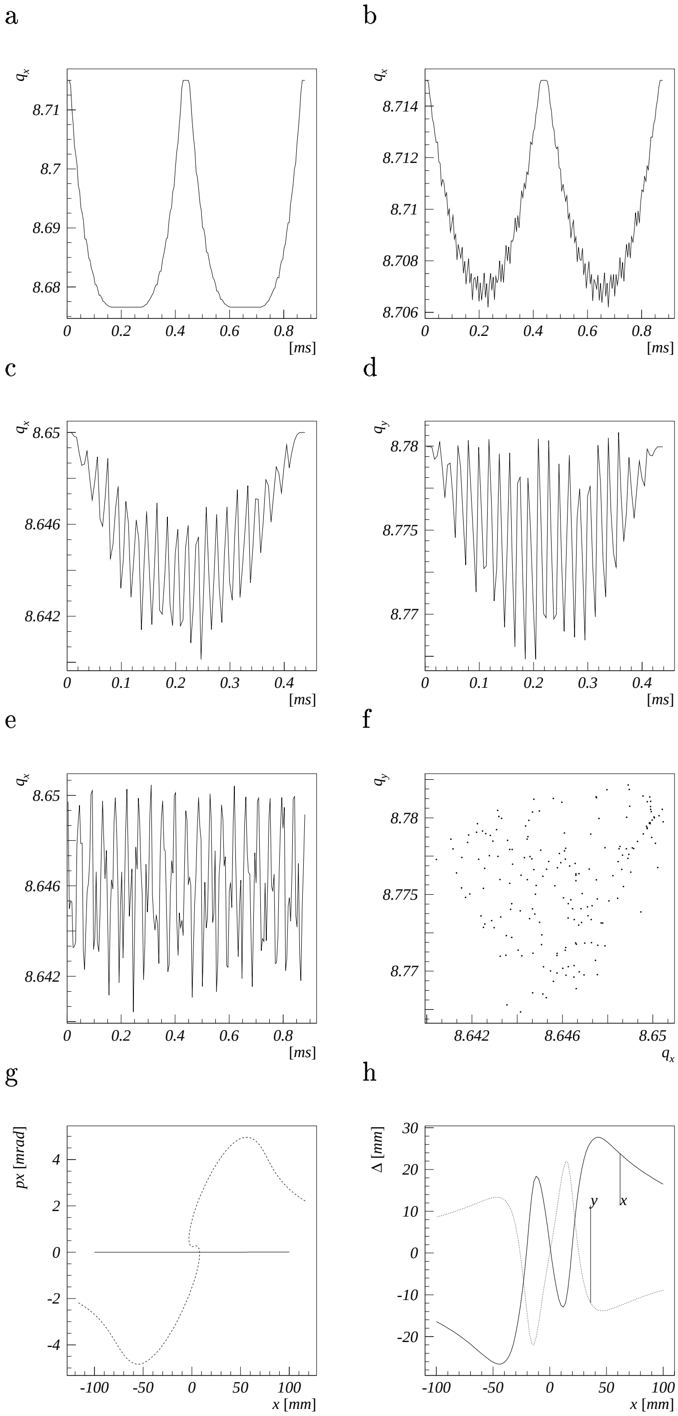

Figure 1: a) Tuneshift of a test particle crossing two bunches in a constant focusing lattice; b) Tuneshift of a test particle which is never inside any bunch; c) Tuneshift of a test particle in the HIDIF lattice; e) Evolution of the tuneshift in 20 turns; f) Tune footprint for the 20 turns case; g) Poincare´ section on the septum of a line of initial conditions for the test particle; h) Shift of the horizontal and vertical position with respect to the case without beam interaction (picture correspondent to the data of Fig. 1g).

References

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