Effect of Nonlinearity Errors on HIDIF Storage Ring

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We have analyzed the particle dynamics in the 4D phase space via the computer code Plato [1] with regard to effects of non-systematic sextupolar field errors. At a fixed ring position, the 4D phase space coordinates are given by the vector $\hat{\mathbf{x}} = (\hat{x}, \hat{p}_x, \hat{y}, \hat{p}_y)$. Here $\hat{x}, \hat{p}_x, \hat{y}, \hat{p}_y$ are the Courant-Snyder coordinates. A particle has the initial coordinates $\hat{\mathbf{x}}_0$, the coordinates $\hat{\mathbf{x}}_1$ after the first turn, etc.. . The set $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, ...$ is here called the "transverse orbit". We define a "single particle emittance" $E_x = \hat{x}^2 + \hat{p}_x^2$ and $E_y = \hat{y}^2 + \hat{p}_y^2$.

Nonlinear magnetic fields lead to emittance growth and beam losses, if the tunes q_x and q_y satisfy the resonance condition $mq_x + nq_y = p$, ($m, n \in Z$ and $p \in N$). In our computer simulations we have considered the HIDIF storage ring. The magnetic nonlinearities are restricted to a sextupolar component in the bending magnets simulated by a thin lens approximation (one kick). The sextupolar errors are characterized by means of the 27 integrated gradients $k_1, k_2, ..., k_{27}$ in the 27 HIDIF bending magnets. Such a single error distribution is called a "seed". Our non-systematic errors are characterized by the property that the average of all gradients is zero.

Fig. 1b shows a single particle simulation using the working points $q_x = 8.65$ and $q_y = 8.78$ with $E_{x0} = E_{y0} = 50$ $mm\ mrad$ (fig. 1a). The transverse orbit is deformed and $E_{x,y}$ oscillate with a constant amplitude (fig. 1c). Because the orbit depends on the particles' initial coordinates, the simulation have been carried out with 200 particles. The initial coordinates of the particles represent a random distribution ($E_x + E_y \leq 50$ $mm\ mrad$). After each turn one observes particles with a $E_{x,y} > 50$ $mm\ mrad$. We have studied the evolution of the beam emittance due to sextupolar errors by logging the relative number of particles ρ that have $E_{x,y} < E_1$ after 1000 turns, (here $E_1 = 55$ $mm\ mrad$). For the seed of fig. 1a we find $\rho = 97.5\%$.

An important result is that the parameter ρ strongly depends on the property of the seed used. Therefore we have repeated the same simulation for a set of 10 different seeds, which were randomly generated. Each seed of the set has the average strength zero and the same standard deviation σ_k of its gradients. For the $i-th$ seed of the set we calculate the corresponding parameter ρ_i. We then compute the average R and the standard deviation σ_R for the 10 parameter ρ_i. For the set $\sigma_k = 0.1$ we find $R = 93.8 \pm 9.8\%$. The large σ_R reflects that the betatron motion of a particle is significantly affected by how the errors are distributed around the machine.

By space charge effects the incoherent tunes of the particles will move along the working line $(q_x, q_y) \in [(8.65, 8.78), ..., (8.45, 8.58)]$. We have analyzed the change of R and σ_R along this line. The result is shown in fig. 1d. Investigating the results for different parameter σ_k (fig. 1e) one observes three resonances, where the emittance grows with the strength σ_k of the errors.

The 3rd order resonance $q_x + 2q_y = 26$ explains the strongest emittance growth in fig. 1d and fig. 1e. The two weaker effect are explained by the 4rd order resonances $2q_x = 17$ and $4q_y = 35$. Our simulation result show that higher order resonances are driven by non-systematic sextupolar field errors. We have explored the tune diagram around the working line. A contour plot (fig. 1f) for $\sigma_k = 0.1$ shows the resonance lines and their width as they appear via the "emittance growth parameter" R.

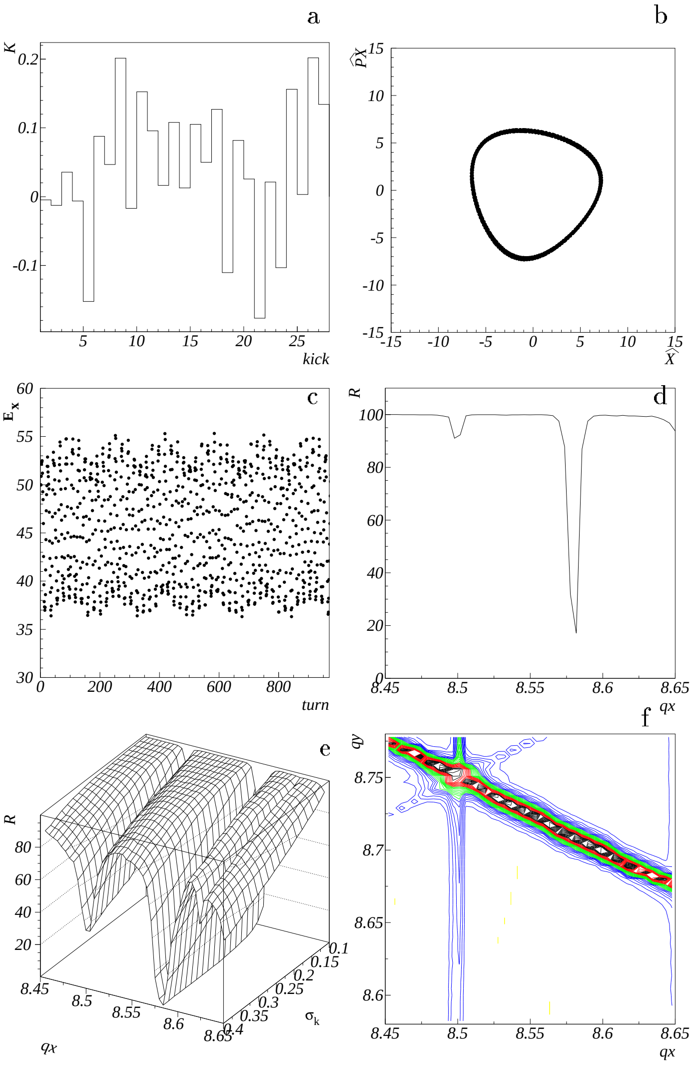

Figure 1: a) strength of 27 sext. field gradient is called a "seed"; b) and c) show only the horizontal plane; d) parameter R along the working line; e) R along the working line for different σ_k; f) landscape diagram.

Refereces

[1] M. Giovannozzi and E.Todesco, A.Bazzani, R.Bartolini, "Plato: a Program Library for Analysis of nonlinear betatronic motion", CERN-PS/96-12 (PA)