Long Term Simulations of Space Charge and Beam Loss Observed in the CERN Proton Synchrotron

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Abstract. Long term storage of high intensity beams with small loss is required in the FAIR [1] project at GSI as well as for JPARC [2]. In this paper we discuss that an important contribution to the loss in bunched beams can be explained it terms of particles trapped into lattice and space charge driven islands. Dedicated experiments at the CERN-Proton Synchrotron to confirm the theoretical model have shown the existence of an emittance growth dominated regime for working points sufficiently far from the lattice resonance, and of a beam loss dominated regime for tunes very close to the resonance. While the emittance growth dominated regime has been investigated in previous studies [3], we focus here on the beam loss dominated regime and compare simulation results with measurements made in the CERN-Proton Synchrotron ring.

INTRODUCTION

The trapping of particles into resonance induced phase space islands has first been considered for single passage through a resonance. The efficiency of the process has been investigated in [4, 5] and mechanisms for detrapping have been studied as well [6]. Applications of the trapping of particles have been proposed as a method to clean the beam from halo particles [7] and recently for multi-turn extraction [8, 9]. Contrary to a "useful" controlled trapping/detrapping of particles, in the beam loss issue for high intensity we are dealing with uncontrolled trapping/detrapping phenomena. In the SIS100 [10], for example, storage for 1 second of a high intensity bunched beam (with 10^{12} U^{28+} and $\Delta Q_x \approx 0.2$) in a nonlinear lattice with a loss level not exceeding 1% is requested. The proposed mechanism is as follows: when the ring tune Q_{x0} is set close above a single nonlinear resonance, stable islands appears at a certain amplitude corresponding to the space charge detuning. Contrary to the single passage through a resonance, in which the island control was achieved by changing properly the machine tune, in the high intensity bunch *space charge* is now controlling the island position along the longitudinal axes. Islands have to be further out in the phase space in the center of the bunch ($z = 0$) to compensate the stronger space charge detuning, whereas in the head/tail ($z = \pm z_{max}$) the island will be further in due to the weaker space charge. Hence some particles will be periodically crossed by the lattice induced/space charge controlled islands. These particles then perform a periodic resonance crossing at the synchrotron frequency with the associated trapping-detrapping phenomena. The complexity of the single particle dynamics in the self-consistent field is considerable and so is computer simulation. A study of trapping conditions has been presented at this workshop [11]. A first simplified study was based on a model with a frozen coasting beam with a Gaussian distribution and a parametrically modulated intensity [12]. Unfortunately fully self-consistent simulations are affected by the artificial noise created by the PIC solvers [13]: long term simulations (thousands of machine turns) may transfer the usually acceptable PIC noise into unphysical emittance growth or excessive halo formation. Therefore, we have proposed a long-term simulation using an analytical expression for the electric field for a frozen charge distribution presented in detail in [3] and applied to the SIS100 in [14]. In order to check the validity of these simulations, a campaign of experiments was started in 2002 at the CERN-PS in a CERN-GSI collaboration. The aim of these measurements was two-fold. One goal was to investigate the new mechanism of beam loss/emittance increase and provide experimental data for code comparison. The second was the investigation of the Montague resonance with a purpose of a code benchmarking [15, 16]. The first code comparison presented in [3] employed a constant focusing lattice and was only able to confirm the experimentally observed emittance growth, but not the measured beam loss. In the present paper we extend the simulation to the full AG focusing lattice of the CERN-PS in order to get a better match of experimental and simulation conditions. In this comparison we are referring to recent experimental data obtained in 2003.

MEASUREMENTS

The measurements were all performed keeping the kinetic energy at the injection value of 1.4 GeV. Bunch profiles were first measured at 180 ms after injection (before powering the octupoles). Profiles were found to be Gaussian in both vertical and horizontal. The chromaticity was close to the natural one, and the effect of the momentum spread of 3.3×10^{-3} (at 2σ) was found to be 27% of the maximum space charge induced tune-spread $\Delta Q_x = 0.075$ for a particle with small amplitude. At 180 ms after injection two calibrated octupoles each with integrated strength of $K_3 = 0.6075 \times I$ m^{-3}, with I octupole current, were powered to -20 A in order to excite the resonance $4Q_{x0} = 25$. The measurement window was set to 1.2 s (4.4×10^5 turns) during which the bunch intensity was monitored with a current transformer. At selected times we measured transverse beam profiles with the flying wire (20 m/s), fitted them with a Gaussian profile, and determined the corresponding rms emittances. In most cases profiles were actually found quite close to Gaussian. The vertical machine tune was set to $Q_{y0} = 6.12$, and the horizontal tune was varied in the interval $6.24 < Q_{x0} < 6.32$ so as to link the octupole resonance crossing with different positions of the space charge tune-spread. The parameters of the experiment are summarized in Table 1. In Fig. 1 we plot, as a function of Q_{x0}, the emittances and beam intensity 1.2 s after injection. In the interval $6.28 < Q_{x0} < 6.32$ we find the typical emittance growth regime, which is characterized by the maximum emittance growth of 42% at the tune $Q_{x0} = 6.28$. The beam loss regime, instead, is located in the interval $6.25 < Q_{x0} < 6.28$, and the maximum beam loss of $\sim 32\%$ is found at $Q_{x0} = 6.265$. We also studied the time dependence of the longitudinal bunch profile. These results shown here in a standard waterfall plot (Fig. 2a) suggest that the lost particles make the longitudinal distribution shrink in amplitude and size. In order to quantify this visual pattern we computed the time evolution of the bunch length by performing a Gaussian fit for each of the profiles of Fig. 2a. These results are plotted in Fig. 2b. In the same picture we plot also the integrated intensity of each longitudinal bunch profile for each time measurement. The two curves in Fig. 2b show that there is a direct relation between beam loss and bunch shortening. Unless an unexpected transverse-longitudinal correlation takes place, Figs. 2 a,b suggest that lost particles are not only those with large transverse amplitude, but also with large synchrotron amplitude. This experimental evidence is consistent with the condition of periodic resonance crossing. In fact particles with small longitudinal amplitude and small transverse amplitude will always have an effective tune Q_x sufficiently below 6.25, because their motion is confined in the denser region of the bunch, hence they will never be extracted. Particles with large longitudinal amplitude, instead, can periodically cross the resonance and therefore may get trapped and eventually get lost.

TABLE 1. Summary of experimental settings

Parameter	Value	Units
Kinetic energy	1.4	GeV
Particles per bunch	1×10^{12}	
Bunch length (4σ)	180	ns
Emittances (normalized at 2σ)*	25.5/10	mm mrad
Momentum spread (2σ)	3.3×10^{-3}	
Derived maximum tuneshifts*	0.075/0.12	
PS circumference	628	m
Beam pipe axes*	14/7	cm
PS superperiodicity	10	
Octupole current	-20	A

* horizontal/vertical

SIMULATION

3D simulations so far have been performed by using a simplified axisymmetric frozen bunch model with density profile $\rho \propto (1-t)^2$ [17]. Here $t = (r/R)^2 + (z/Z)^2$, with $r = \sqrt{x^2+y^2}$, R the transverse beam edge radius and Z the longitudinal edge. This analytic frozen bunch model has the advantage that noise-free space charge forces are obtained. In the simplified axisymmetric model the horizontal and vertical axes were artificially equal, therefore we have progressed to modeling ellipsoidal bunches [18]. For the simulations reported here we used an ellipsoidal Gaussian distribution $\rho \propto \exp(-t/2)$, with $t = (x/\sigma_x)^2 + (y/\sigma_y)^2 + (z/\sigma_z)^2$, and $\sigma_x, \sigma_y, \sigma_z$ arbitrary rms bunch sizes. The tracking is performed by computing the average beam size in horizontal/vertical axes, then the space charge algorithm is initialized keeping the 3 bunch axes frozen. This modeling is neglecting the local transverse modulation of the bunch envelopes which we expect to have only a minor influence on the island position due to its rapid oscillation. A further improvement in simulations has been reached by using the AG focusing PS lattice as provided

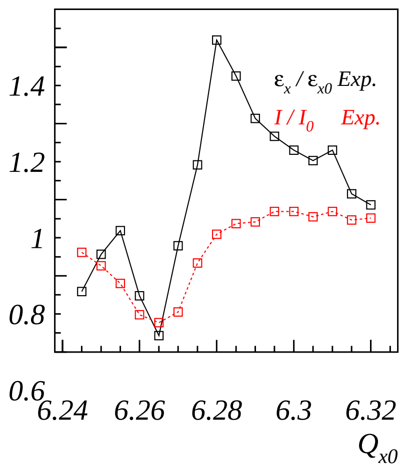

FIGURE 1. Experimental findings: normalized final emittance and beam intensity as function of the working point.

in [19], including the measured lattice nonlinearities as discussed in [20]. With this modeling the beam is now experiencing the correct nonlinear forces from the octupole.

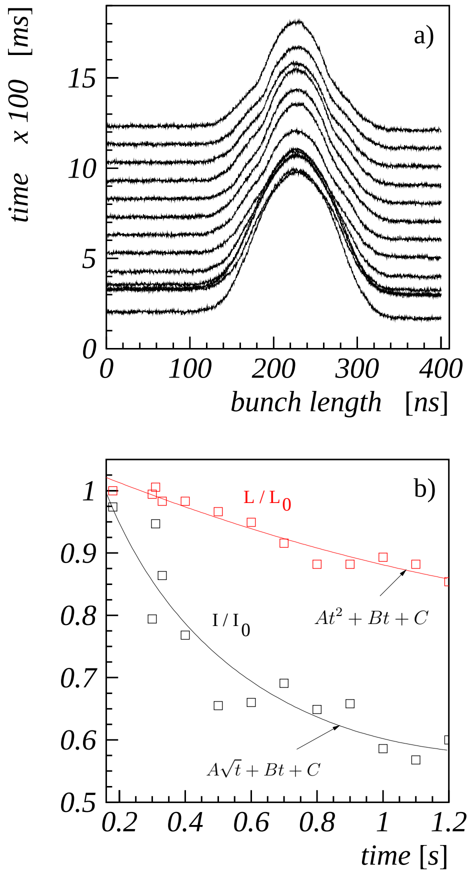

FIGURE 2. Measurements at $Q_{x0} = 6.265, Q_{y0} = 6.12$: a) Waterfall plot of the longitudinal profile as function of time; b) Beam intensity and bunch length as function of storage time. The solid lines are fitted with data by using the functional dependance reported.

Dynamic aperture and space charge

In previous work [3] the PS lattice was simulated in a constant focusing approximation. This simplified the particle dynamics, but it also introduced enhanced horizontal and reduced vertical octupolar nonlinear forces at the location of the external octupoles as in this case $\beta_x = \beta_y = 16$ m. In contrast, by using the real PS lattice, the beta functions are $\beta_x = 11.7$ m, $\beta_y = 22.2$ m at the location of the octupoles. It becomes then necessary firstly to investigate if beam loss may be attributed to an excessive shrinking of the dynamic aperture. To explore roughly the dynamic aperture of the PS we have carried out a numerical test by searching the maximum stable radius of test particles placed into 20 different directions in the upper half of the $x-y$ plane. In these calculations space charge is not included. The stability condition was searched within 10^3 turns (short term dynamic aperture). We firstly computed the dynamic aperture without external octupole ($I = 0$ A), and found that it is at 9σ, for all the tune range, where σ is the horizontal rms beam size of the injected beam. This value of the dynamic aperture sets the inner most part of the stable region practically on the beam pipe, i.e. the mechanical acceptance is all inside the stable region. By activating the octupoles with $I = -20$ A, the short term dynamic aperture is lowered to 8σ for $6.27 < Q_{x0} < 6.32$, but near $Q_{x0} = 6.25$ it shrinks to 3.5σ. This value is small enough to intercept the tail of a Gaussian distribution. However, a 2D multiparticle simulation (without space charge) on the same time scale has shown no particle loss. An upper bound to the beam loss can be obtained by cutting the 2D Gaussian distribution in energy at 3.5σ, i.e. keeping only particles such that $\varepsilon_{x,s}/\tilde{\varepsilon}_x + \varepsilon_{y,s}/\tilde{\varepsilon}_y < 3.5$, with $\varepsilon_{x/y,s}$ the single particle emittances, and $\tilde{\varepsilon}_{x/y}$ beam rms emittances. This estimate gives only 0.5% beam loss.

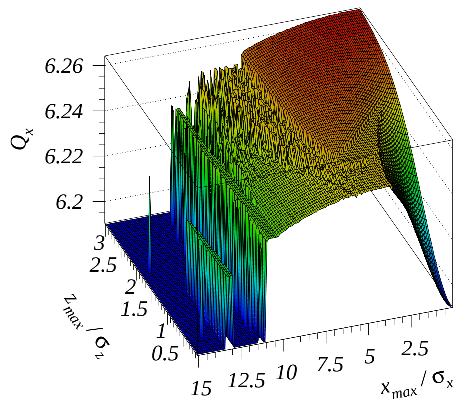

FIGURE 3. Nonlinear tune Q_x as function of the maximum initial amplitude (x_{max}, z_{max}) for $Q_{x0} = 6.265$. The longitudinal motion is frozen.

A further inspection at 10^5 turns shows that close to $Q_{x0} = 6.25$ the dynamic aperture without space charge shrinks from 3.5σ to 3σ. A 2D simulation for this timescale has shown beam loss of 0.1%. Again these results suggest that the dynamic aperture alone as modeled here and ignoring space charge cannot explain beam loss. We also note the significant difference between the study of the single particle stability, where the minimum dynamic aperture is located at $Q_{x0} = 6.25$, from the experimental finding of Fig. 1, where the maximum loss occurs at $Q_{x0} = 6.265$. In order to understand the origin of this difference we studied a simplified constant focusing 2D model of the PS with the nonlinearities of one magnet. In this model the longitudinal motion is kept frozen. The aim of this model is to study the nonlinear tune of a test particle as function of its initial coordinates $x = x_{max}, z = z_{max}, x' = z' = 0$. The tune was computed with an FFT in 4096 turns. In Fig. 3 we computed this "frequency map" [21]for the bare tune of $Q_{x0} = 6.265$. We see that for $x \sim z \sim 0$ the nonlinear tune Q_x has the lower value $Q_x = Q_{x0} - \Delta Q_x$ with $\Delta Q_x = 0.075$. At the longitudinal position of $z = 3\sigma_z$ the space charge is quite reduced and only the detuning due to the octupole is present. Note that detuning of octupole and space charge act in the same direction (they reduce the nominal tune) but in a different way: the space charge detuning is stronger on axis whereas the octupole detuning is stronger off axis as it can be seen at $z = 3\sigma_z$. The flat-top in the picture represents all the initial conditions of particles whose tune is *locked* on the 4th order resonance. It is approximately given where the space charge detuning matches the octupole induced resonance condition. It is interesting to note that this flat-top does not reach the plane $z = 0$, but stops at $z = 1$ as there a chaotic region is met. We also note that the flat-top reaches the maximum extension of 5σ far below the dynamic aperture of 10σ. However the chaotic region extends from 5σ to the dynamic aperture. This picture suggests that trapped particles follow the flat-top, and when they reach the chaotic region they can be brought to the dynamic aperture and get lost. This case occurs only at the tune $Q_{x0} = 6.265$. For comparison we have calculated the same "frequency map" for $Q_{x0} = 6.2685$. In this case the flattop crosses the plane $z = 0$ at 5σ. This becomes consequently the maximum extension of trapped particles which cannot be lost. For this tune only an emittance growth may occur.

3D simulations with synchrotron motion

Using the AG structure of the PS ring increases considerably the CPU-time required for simulations. For this reason we have restricted the number of turns to 1.5×10^5. Following the procedure of the experiment we have performed simulations for the relevant working points used in the measurements. The initial bunch distribution was Gaussian with 2000 macroparticles with transverse emittances $\varepsilon_x = 25.5$ mm-mrad, $\varepsilon_y = 10$ mm-mrad normalized at 2σ. Chromatic effects have been neglected here. In the simulations the PS beam pipe (14/7 cm) has been assumed constant throughout the ring, which is a reasonable assumption. In Fig. 4 we show the result of the simulations at 1.5×10^5 turns and plot for convenience also the final experimental data. The comparison shows a similar pattern: an emittance growth regime for larger tunes next to a loss dominated regime closer to $Q_x = 6.25$. In the emittance growth regime the agreement between measured and simulated data, taken at the same number of turns, is reasonably good. This is shown

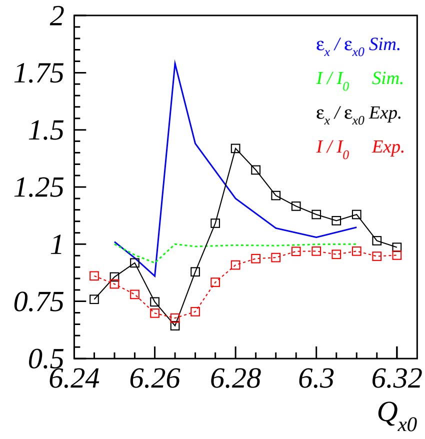

FIGURE 4. Final beam emittance (solid) and intensities (dashed) versus working point in simulations (no markers) up to 1.5×10^5 turns and experiment (markers) up to 4.4×10^5.

in Fig. 5 for the working point corresponding to the maximum emittance growth in the experiment. The discrepancy is larger in the loss dominated regime, where the simulation gives a maximum 8% for $Q_{x0} = 6.26$, while the maximum measured beam loss is found to be $\approx 25\%$ at $Q_{x0} = 6.265$ both after 1.5×10^5 turns. This suggests that we should re-examine all approximations made in simulations, but also the accuracy to which the nonlinear lattice used in the simulations represents the real machine nonlinearities at the specific working points considered here. In fact, we have also measured the loss at $Q_{x0} = 6.265$ after 4.4×10^5 turns with the octupole off and found $\approx 20\%$, compared with $\approx 30\%$ for the octupole on. The no-octupole measurement showed, however, no emittance growth regime, which suggests that the natural lattice octupoles are too weak to cause efficient trapping/detrapping. Hence, the loss difference be-

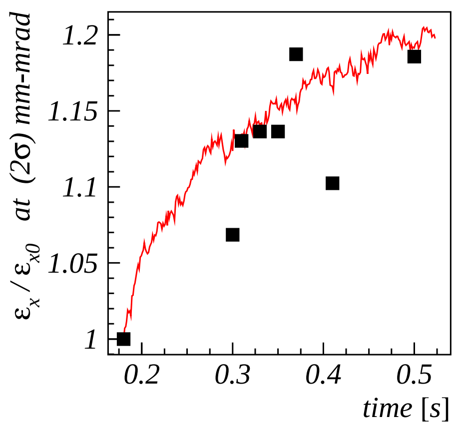

FIGURE 5. Comparison of emittance time evolution in measurements (markers) and simulation at $Q_{x0} = 6.28$.

tween octupole on/off, about 10%, should be mainly due to trapping/detrapping by the additional octupole. This loss difference is in much better agreement with the simulation result of maximum 8% loss with the octupole on. In future work the effect of chromaticity should also be added as it enters directly into the resonance condition and therefore into the trapping/detrapping probability as well as into the maximum halo radius. It should also be noted that experimental beam loss of the order of tens of percent leads this study into a realm, where self-consistent simulation would be desirable.

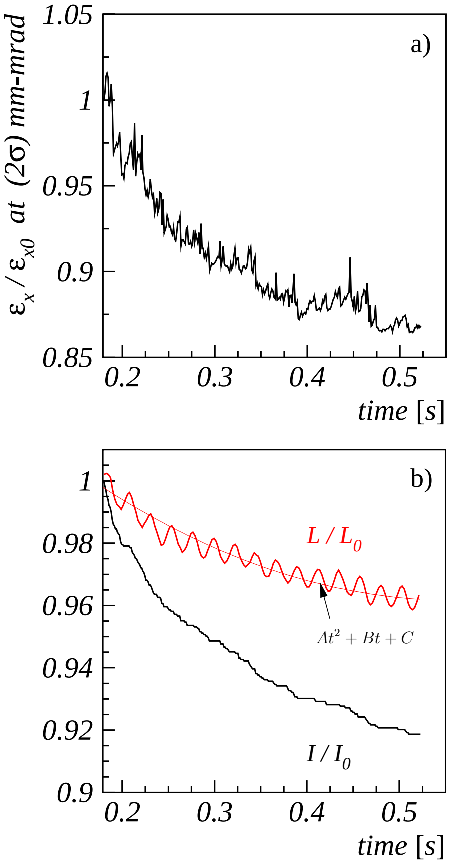

FIGURE 6. Simulation for $Q_{x0} = 6.26$: a) rms horizontal emittance versus time; b) beam intensity and bunch length with fitted curve as function of storage time.

We expect that with self-consistency the loss doesn't saturate, since with weakened space charge - by the loss - additional particles may cross the resonance. The small loss in the simulation of Fig. 6 makes it unlikely that self-consistency would make a noticeable change. To further support the interpretation of our findings we have studied the correlation between beam loss and bunch shortening, which is shown in Fig. 6. The bunch shortening is comparable with the experimental results of Fig. 2b at 1.5×10^5 turns, even if for a slightly different bare tune (in fact the experimental beam loss data are relatively flat between $Q_{x0} = 6.26$ and $Q_{x0} = 6.265$). Note that although the measured and simulated emittance and bunch length follow a similar evolution in the first 0.5 seconds, the beam loss is quite different.

CONCLUSION

The experimental study in the CERN-Proton Synchrotron and comparison with simulations has reached a sufficiently good agreement in the emittance growth regime. With respect to previous studies we now also succeeded to confirm the beam loss regime. There, the agreement with the measurement is also reasonably good if we take the difference in measurements with octupole on and off, which eliminates the pure dynamic aperture effect. Further steps will be needed to get a better quantitative code modelling of the machine dynamic aperture exactly at the working points used here. In order to confirm that the mechanism proposed here is indeed accelerator independent we are planning to carry out a similar measurement campaign in the SIS18 at GSI.

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