Space-Charge Experiments at the CERN Proton Synchrotron

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Abstract. Benchmarking of the simulation codes used for the design of the next generation of high beam power accelerators is of paramount importance due to the very demanding requirements on the level of beam losses. This is usually accomplished by comparing simulation results against available theories, and more importantly, against experimental observations. To this aim, a number of well-defined test cases, obtained by accurate measurements made in existing machines, are of great interest. Such measurements have been made in the CERN Proton Synchrotron to probe three space-charge effects: (i) transverse emittance blow-up due to space-charge induced crossing of the integer or half-integer stop-band, (ii) space-charge and octupole driven resonance trapping, and (iii) intensity-dependent emittance transfer between the two transverse planes. The last mechanism is discussed in detail in this paper and compared to simulation predictions.

Keywords: Space charge, Emittance exchange, Montague resonance.
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INTRODUCTION

An intensive study has been undertaken since the year 2002 to understand better the various high-intensity bottlenecks of the CERN Proton Synchrotron (PS) machine. Space-charge tune shifts can drive the beam onto linear and/or nonlinear resonances generating transverse emittance blow-up and sometimes subsequent beam loss. Depending on how the space-charge tune spread overlaps the resonance, i.e. whether the centre or tail particles are in the stop-band, the beam behaviour will be completely different. The first case leads to a loss-free (provided the mechanical aperture is sufficiently large) core-emittance blow-up, where the distribution is almost conserved. Measurements and comparisons with simulations were published for this case in Refs. [1,2]. The second case leads to a diffusion of the tail particles into a halo of increasing size, which may lead to particle losses due to the reduced dynamic aperture close to the resonance, extracting the halo particles. This case is discussed in detail in another paper of this workshop [3].

Another space-charge limitation, which is the subject of this paper, comes from the so-called Montague resonance [4]. High-intensity proton synchrotrons, having larger horizontal than vertical emittance, may suffer from this intrinsic space-charge driven fourth-order resonance $2Q_x - 2Q_y = 0$. In particular, such resonance may lead to emittance sharing and, possibly, beam loss due to vertical acceptance limitation. In circular accelerators with the same integer part for the transverse tunes and elliptical vacuum chambers, as is the case of the CERN PS, avoidance of this resonance may be desirable to achieve maximum high-intensity performance. Montague pointed out that this mechanism of emittance transfer can be avoided by a sufficient splitting of the transverse tunes.

Experimental observations made in the CERN PS from 2002 to 2004 on the Montague resonance have already been presented and partially compared to 3D Particle-In-Cell (PIC) simulations [5]. The emittance exchange related to this resonance was also observed few years ago during foil injection into the KEK synchrotron [6].

Both theoretical and simulation results are reviewed in the next section, while measurements are presented in the following one. The comparison between measurements and simulations is discussed in the last section.

THEORY AND SIMULATIONS

Montague-Baconnier First (Single-Particle) Approach

Montague showed in 1968 that the space-charge potential could excite the fourth-order coupling resonance $2Q_x - 2Q_y = 0$, known since as the Montague resonance [4]. This mechanism leads to an amplitude beating between the horizontal and vertical planes for the single-particle motion, resulting in an apparent increase in emittance in the plane of smaller emittance. This phenomenon is very fast as growths in few (~ 1-5) turns for a synchrotron at the space-charge limit (small-amplitude space-charge tune shift of about -0.3) are expected. Montague said that this effect should be taken into account in the choice of parameters for future high-intensity synchrotrons. Baconnier recognised in 1987 that the Montague stop-band was certainly one of the most effective mechanisms for losing particles at injection in the CERN PS [7].

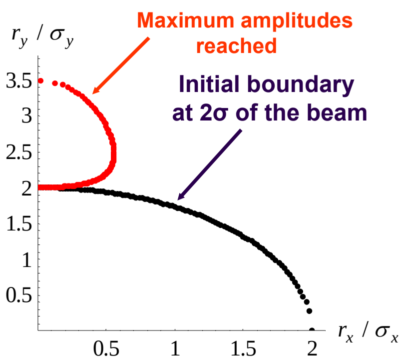


FIGURE 1. Initial and maximum transverse amplitudes in the "Montague stop-band" in an example case with PS parameters close to the experimental conditions (see measurements section).

In Ref. [7], Baconnier applied Montague's theory using Keil's computation [8] for the incoherent space-charge tune shift (instead of deducing it from the potential limited to 4$^{\text{th}}$ order) and developed a computation procedure, which applies only if the stop-band is approached from above (i.e. $Q_y > Q_x$) when the beam is larger in the horizontal than vertical plane. An example is given in Fig. 1, with the following numerical values: the transverse coherent tunes and small-amplitude space-charge tune shifts (assuming a transversally Gaussian bunch and considering a longitudinal parabolic line density) are $Q_x = 6.18$, $Q_y = 6.21$, $\Delta Q_{inc,x0} \approx -0.058$, $\Delta Q_{inc,y0} \approx -0.105$, and the transverse emittances are given by $\varepsilon_{x,2\sigma}^{norm} = 25\ \mu\text{m}$ and $\varepsilon_{y,2\sigma}^{norm} = 10\ \mu\text{m}$.

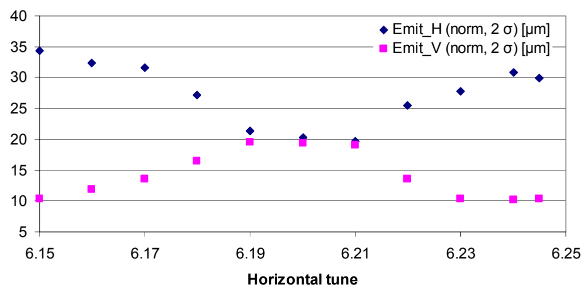
Self-Consistent Simulations

The original analysis by Montague was based on a single-particle approach using a "frozen-in" space-charge potential defined by the initial Gaussian distribution, which neglects the effect of the induced time-varying collective space-charge force. Self-consistent simulations have been explored in detail "in static conditions" (i.e. with constant tunes) in Refs. [9,10], and reveal that the Montague mechanism is in fact a combination of instability and resonance: it develops as a pure instability for a Kapchinskij-Vladimirskij (KV) distribution, which has no initial driving nonlinearity. The initial growth for the KV-case is found to be exponential, starting from noise level. An initial water-bag distribution, instead, has all features of a space-charge driven resonance due to the presence of a finite nonlinearity from the beginning. The instability is, however, still superimposed on the resonance phenomenon.

MEASUREMENTS IN THE CERN PS

Several measurements have been performed in the CERN PS machine from 2002 to 2004. The outcome results of almost all the measurements were recently published [5]. In the present paper, only two measurements, one in the static and one in the dynamic (i.e. slowly crossing the coupling resonance) case, are discussed. These measurements were performed on a single-bunch beam generated by the CERN Proton Synchrotron Booster (PSB), fast injected into the PS machine at 1.4 GeV kinetic energy, on harmonic $h = 8$. The number of protons per bunch was $N_b \approx 10^{12}$ p/b, the vertical tune was fixed at $Q_y = 6.21$, while the horizontal tune Q_x was varied between 6.15 and 6.25, the total bunch length (4 σ) was $\tau_b \approx 180$ ns, the normalized rms momentum spread was $\sigma_p / p \approx 1.2 \times 10^{-3}$, the synchrotron period was $T_s \approx 1.5\,\text{ms}$, the initial (un-coupled) horizontal and vertical emittances were $\varepsilon_{x,2\sigma}^{norm} \approx 35\,\mu\text{m}$ and $\varepsilon_{y,2\sigma}^{norm} \approx 10\,\mu\text{m}$ respectively. The transverse average betatron functions are given by $\beta_x \approx R/Q_x$ and $\beta_y \approx R/Q_y$, where the average machine radius is $R = 100\text{ m}$, the horizontal average dispersion function is $D_x \approx R/Q_x^2$, and the beam sizes ($\sqrt{2}\;\sigma$) are $a_0 = 0.012\text{ m}$, and $b_0 = 0.006\text{ m}$. This yields $\Delta Q_{inc,x0} \approx -0.054$ and $\Delta Q_{inc,y0} \approx -0.109$. The numerical values have been chosen after careful analysis of the tune diagram in order to avoid the non-compensated resonances in the PS machine.

In the first experiment, the horizontal tune was scanned from 6.25 to 6.15 in the static mode, i.e. constant value from injection to the measurement point. The measurements were performed on different cycles (one measurement per cycle), at the same time for each cycle, and the data were averaged over several measurements (see the results in Fig. 2(upper)).



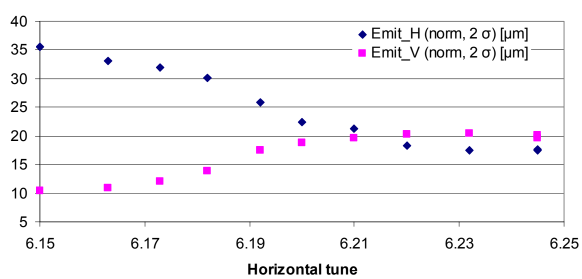


FIGURE 2. Measured intensity-dependent emittance transfer in 2003 for $Q_y = 6.21$ (upper) in the static case (constant horizontal tune from injection to the measurement point), (lower) in the dynamic case (the horizontal tune was changed linearly from 6.15 to 6.25 in 100 ms).

In the second experiment, based on what was done in the PS to study the mechanism of emittance exchange between the transverse planes due to linear betatron coupling [11,12], the Montague resonance was studied in dynamic conditions, contrary to what was done during the last 30 years. Measurement results were published in Ref. [5] (see Fig. 2(lower)), and predictions from numerical simulations were reported in Ref. [13].

COMPARISON BETWEEN MEASUREMENTS AND SIMULATIONS

Static Case

Both experimental results and simulation predictions are shown in Fig. 3. Due to the assumption of fixed tunes and the fast nature of the Montague resonance, the experimental data are not completely independent of the injection process. A dispersion mismatch at PS injection cannot be disentangled from the Montague resonance effect.

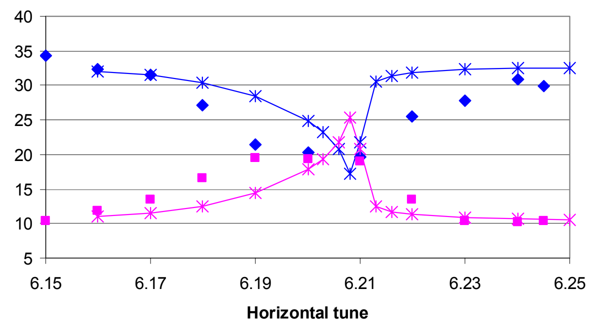


FIGURE 3. Measured (dots, see Fig. 2) and simulated (full line) intensity-dependent emittance transfer in the static case.

The simulations were performed using the fully 3D PIC code IMPACT [14] with a grid of 65×65×257 points in *x*, *y*, *z* and 10^6 particles in a constant focusing lattice. Initial distributions are assumed to be Gaussian in all phase planes. The simulations have been run over 1000 turns, which is found sufficient to determine the equilibrium emittances.

The pronounced asymmetry of the simulation response curve was observed already in 2D simulations for coasting beams and explained as a result of the collective response of the charge distribution [9]. It is worthwhile noting that for tunes just below 6.21 the emittances feature a shoot over, i.e. the vertical emittance is finally larger than the horizontal one. The time evolution for the exchange process in the simulation is by far too fast (~100 turns) to be resolved by the wire scan measurements limited to millisecond resolution for the working point

$Q_x = 6.208$ in the "overshoot" region. After about 100 turns the emittances have become equalized or "equipartitioned", thereafter a rapid overshoot occurs such that the final vertical emittance is 50% larger than the horizontal one. This leads to the spike in Fig. 3 for tune values just below 6.21. This might be related to the spontaneous instability of a skewing (linear coupling) mode as described in Ref. [10]. Obviously, this overshoot and related spike are not confirmed by the experiment. However, Fig. 3 reflects a reasonably good agreement between experiment and simulation. In particular the stop-band also extends mainly to the left of $Q_x = 6.21$, which would be inverted for the inverse emittance ratio.

Dynamic Case

When the coupling resonance is slowly crossed, a complete exchange of the emittances is predicted provided the crossing speed is much slower than the time it takes for the mechanism to develop, but much faster than the synchrotron period [13] (see Fig. 4). A good fit, inspired from Refs. [11] and [15], is given by

$$\varepsilon_x = \varepsilon_{x0} - \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{|C|^2 / 2}{\Delta^2 + |C|^2 + \Delta\sqrt{\Delta^2 + |C|^2}}, \quad (1)$$

$$\varepsilon_y = \varepsilon_{y0} + \left(\varepsilon_{x0} - \varepsilon_{y0}\right) \frac{|C|^2 / 2}{\Delta^2 + |C|^2 + \Delta\sqrt{\Delta^2 + |C|^2}}, \quad (2)$$

where $\varepsilon_{x0,y0}$ are the equilibrium (un-coupled) transverse emittances far from the resonance $Q_x = Q_y$, $\Delta = 2Q_y - 2Q_x$ and $|C|$ is the space-charge coupling strength. In Fig. 4, the fit is obtained using $\varepsilon_{x0} = 35\ \mu\text{m}$, $\varepsilon_{y0} = 10.5\ \mu\text{m}$, and $|C| = 0.037$.

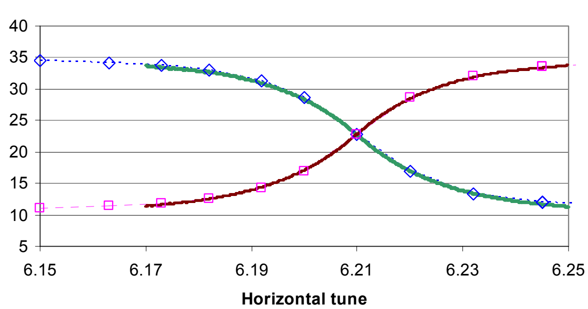


FIGURE 4. 3D simulation results (IMPACT code) in the case where the synchrotron period is much larger than the crossing time (full line), and fit (dotted line), using the numerical values of the measured case (see Fig. 2).

In the real measured case, when the crossing speed is not much faster than the synchrotron period, a mixing effect due to the longitudinal motion is predicted by simulations after the resonance crossing [13]. As seen in Fig. 5, the effect of the longitudinal motion goes in the good direction to explain the difference between measurements and simulations, but it seems not sufficient to entirely explain the observations.

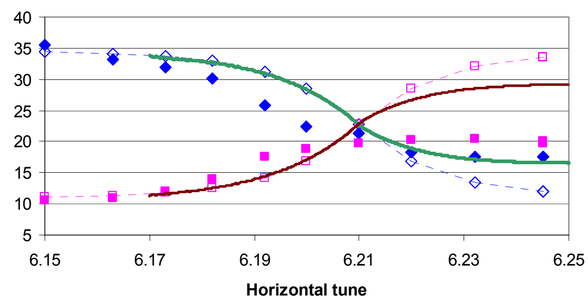


FIGURE 5. Measurements (dots, see Fig. 2), 3D simulation results (IMPACT code) in the real measured case where the synchrotron period is not much larger than the crossing time (full line), and fit of the 3D simulation results in the case where the synchrotron period is much larger than the crossing time (dotted line).

CONCLUSION

The phenomenon of emittance exchange has been found and studied in a series of measurements performed in 2002, 2003, and 2004 in the CERN PS machine. The main features of the mechanism are confirmed by 3D particle-in-cell simulations.

The new interesting feature in this field is the study of the Montague resonance in dynamic, i.e. slowly crossing the coupling resonance, as it was done few years ago in the PS machine to study the mechanism of emittance exchange between the transverse planes due to linear betatron coupling. In this case a complete exchange of the emittances is predicted provided the crossing speed is much slower than the time it takes for the mechanism to develop, but much faster than the synchrotron period [13] (see Fig. 4).

When the crossing speed is not much faster than the synchrotron period (usual case in practice), a mixing effect due to the longitudinal motion is predicted by simulations after the resonance crossing [13]. Such a mixing effect was observed in the measurements (see Fig. 5), but it is not sufficient to explain the whole process. Another mechanism is necessary to account for the remaining difference between measurements and simulations. Intra-Beam Scattering (IBS) is suspected to play a significant role after the resonance crossing, and will need to be investigated in detail.

The possible future developments of this work could now be to go in two directions. In the first, more complete, detailed, quantitative predictions are required. This will lead eventually to self-consistent 3D simulations with IBS, and perhaps other mechanisms, included. In the second direction, one could try to understand why, in the case where the crossing speed is much faster than the synchrotron period, the evolution of the transverse emittances near the coupling resonance seems to be very well described using only two parameters, the coupling strength and the tune distance from the resonance (see Fig. 4, Eqs. (1) and (2)).

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