Simulation aspects of the code benchmarking based on the CERN-PS “Montague-resonance” experiment

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Abstract. Measurements of emittance exchange due to the Montague resonance in the CERN Proton Synchrotron in 2003 have provided detailed data, which are suitable for benchmarking of different simulation codes for high-intensity accelerators. We present here some characteristic features of the Montague resonance by using first simulations obtained with MICROMAP and IMPACT under simplifying conditions. The challenges for the planned code benchmarking are discussed.

Keywords: <Missing keywords>
PACS: <Missing classification>

INTRODUCTION

The request for benchmarking simulation codes for high-intensity synchrotrons and storage rings is obvious in view of the need to predict beam loss and degradation for the new projects, or explain and possibly improve the performance of running machines. Besides comparisons between codes developed in different laboratories, it is desirable to benchmark the codes with experiments and develop simulation quality criteria. Developing the required confidence in these codes is, of course, a complex undertaking evidenced by experiments at the level of some 10^3 turns in foil injection schemes [1, 2]. In synchrotrons, where beams need to be tracked for a larger number of turns - from 10^4 up to 10^6 the challenges increase, if the fully nonlinear lattice matters. Recently, relevant data were acquired from extended storage between 10^4 (“Montague resonance”) and 5×10^5 (“octupole driven resonance”) turns in the CERN Proton Synchrotron as reported at this conference [3, 4].

The intrinsic space charge driven fourth order resonance near the diagonal given by $2Q_x - 2Q_y = 0$ has first been studied on a single-particle basis by Montague [5] who pointed out that it can be avoided by sufficient splitting of tunes. The emittance exchange connected with this resonance was observed in several accelerators with multi-turn injection and in general avoided rather than studied in detail. A first comparison between data and simulation in the relatively dynamic situation of progressive foil stacking was carried out in the KEK synchrotron [6]. With the measurements at the CERN PS an in-depth analysis has become possible mainly due to the fact that they were taken under “static” conditions, by varying the horizontal tune from shot to shot, with constant current, energy and bunch length. This enabled the measurement of a full stop-band of emittance exchange with data that appear sufficiently accurate to serve as a first step of an inter-laboratory bench-marking of self-consistent particle-in-cell (PIC) codes.

In this paper we present some characteristics of the resonant emittance exchange by using simulation results obtained under simplifying conditions and summarizing characteristic features. The procedure proposed for the inter-laboratory bench-marking will be to compare codes at various levels of complexity up to the full scenario, which would take into account the fully non-linear lattice and 3D aspects. At a later time it is planned to also use the octupole driven resonance data for very long-term code comparison.

THE MONTAGUE RESONANCE STOP-BAND

Beyond the original single-particle analysis by Montague[5], this topic was not given much further attendance, until it was realized more recently that the performance of linear accelerators (see Ref. [7]) and of synchrotrons with high intensity merit more detailed consideration. Several theoretical papers were dedicated to this topic with the goal of exploring collective aspects of the resonance and the dependence of stop-bands on the distribution function and on beam parameters [8, 9].

The fast nature of the emittance exchange - typically 20 turns for present parameters - suggests that synchrotron motion is a weak effect only. We therefore consider as a first step the 2D coasting beam approximation in a constant focusing lattice. We employ a transverse Gaussian distribution ignoring the finite bunch length and synchrotron motion. As in the experiment, the proton energy is 1.4 GeV, and the initial normalized emittances are $\varepsilon_x = 30\times10^{-6}$, $\varepsilon_y = 10\times10^{-6}$ (normalized, defined at 2σ). The current is set to yield the maximum vertical tune shift of $\Delta Q_y = 0.106$ of the experiment (in the center of the Gaussian bunch), which corresponds to a maximum horizontal tune shift of $\Delta Q_x = 0.054$. Results are obtained with the MICROMAP-code employing 50.000 particles and a 128x128 grid with conducting boundary conditions on a square box of width 6 times the horizontal rms size of the beam.

The time behavior of three typical cases is shown in Fig. 1, where it is also seen that final emittances undergo small oscillations, which are weakly damped. No damping is found for the complete emittance exchange effect at $Q_{0,x} = 6.21$, which can be explained as a result of the spontaneous instability of a skewing (linear coupling) mode as described in Ref. [9]. The linear coupling here is only due to the space charge force from an initially infinitesimal beam rotation, which gets exponentially self-amplified. The time average of this periodic exchange is the arithmetic mean of the initial emittances.

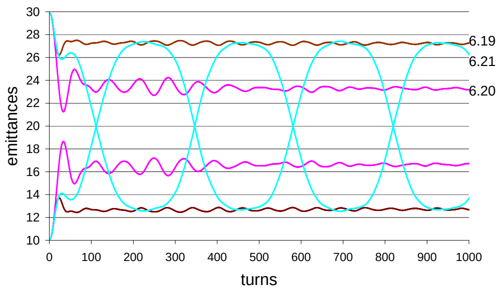

FIGURE 1. Time evolution of emittances (mm mrad) for $Q_{0,x} = 6.19, 6.20$ and 6.21 (2D PIC with MICROMAP).

In Fig. 2 we show the stop-band obtained for variable horizontal bare machine tunes and fixed vertical bare tune $Q_{0,y} = 6.21$. The plotted final emittances are determined by averaging the oscillating values between turn 500 and 1000. As far as the use of constant focusing we have found that there is no difference in the results, if compared with (linear) periodic focusing. This is due to the fact that what matters is the phase advance per turn as was already pointed out in Ref. [9].

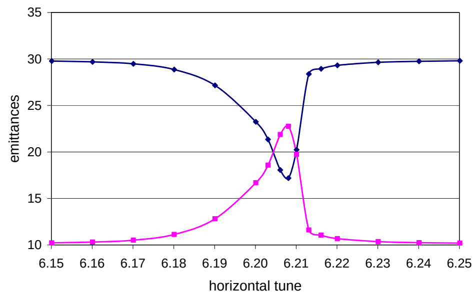

FIGURE 2. Simulated final emittances (mm mrad) for fixed $Q_{0,y} = 6.21$ as function of $Q_{0,x}$ (2D PIC with MICROMAP).

EXPERIMENT VS. SIMULATION

The measured data of the final emittance exchange are shown in Fig. 3. Due to the fast nature of the Montague emittance exchange the experimental data are not completely independent of the injection process, where a dispersion mismatch was present. Measurements were carried out 30 ms after injection corresponding to 11.000 turns. Each data point in Fig. 3 is an average over five subsequent shots, which improved the statistics compared with measurements in 2002. The measurements at $Q_{0,x} = 6.245$ have been used to infer the "initial" emittances – as quoted above – by assuming exchange is absent for this tune.

We compared the measurement with the simulation results obtained with the fully 3D particle-in-cell code IMPACT [10] employing a grid of 128x128x512 in x, y, z and 10^6 simulation particles, using a constant focusing lattice. These simulations have been extended over 2000 turns, but results are practically stationary after the first few hundred turns. The difference from the above 2D simulations is minor, which is not surprising, if one keeps in mind that the exchange takes place during about 5% of a synchrotron period. Note that the synchrotron period in simulation and experiment corresponds to about 600 turns.

In the following we draw some conclusions and outline open questions for future "benchmarking efforts":

- The agreement between measurement and simulation is reasonably good as far as width and asymmetry of the stop-band are concerned.
- The measurement shows a band between tune 6.19 and 6.21, where the – time averaged – emittances are equalized. Such a band is absent in the 2D and

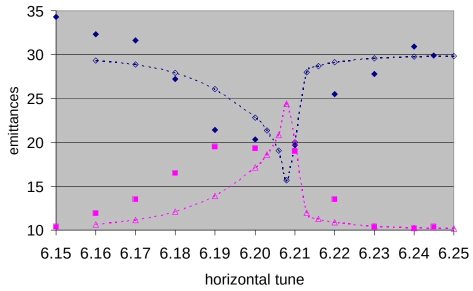

FIGURE 3. Final measured (full markers) and simulated (open markers) emittances for fixed $Q_{0,y} = 6.21$ as function of Q_x.

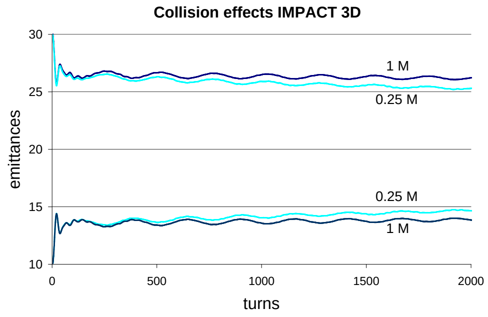

FIGURE 4. Emittances (mm mrad) for 1 M and 0.25 M simulation particles and $Q_{0,x} = 6.19$ in IMPACT 3D.

3D simulation, where, instead, overshoot is found between 6.205 and 6.21. Very close to 6.21 the simulation yields spontaneous linear coupling also averaging to equal emittances.

- This discrepancy needs to be checked in simulations using the full (nonlinear and coupled) lattice of the PS over as many as 11.000 turns, which is the next challenge to the benchmarking effort.
- The damping of emittance oscillations found in the simulation needs to be checked with better resolution.
- The effect of numerical collisions requires special attention, especially in the 3D simulations, as it may lead to additional emittance exchange. A comparison with IMPACT 3D simulation (at tune 6.19) using only 2.5×10^5 particles showed a rate of increase of $\varepsilon_y \approx 0.6 \times 10^{-6}$ π m-rad per 1000 turns (see Fig. 4), with an evidence of saturation above 2×10^6 particles at the rate of $\varepsilon_y \approx 0.2 \times 10^{-6}$ per 1000 turns; in ε_x an equally large symmetric decrease was found. Whether or not this “saturation” is associated with a particle number independent collisionality of the code, or with the synchrotron oscillation, or both, needs to be explored.

In summary, reproducing the experimental results to higher accuracy for the static tunes than achieved here with the linearized lattice (constant or periodic) is the next challenge of the planned benchmarking; thereafter, comparing simulation with experimental data on dynamic crossing over 35.000 turns (100 ms, see Ref. [3]) will add additional complexity to the comparison due to the interplay of space charge, lattice nonlinearity and synchrotron motion.

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