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Calculation of the Coulomb Effect in Bunch Compression near Transition

I. Hofmann and G. Franchetti
GSI Darmstadt

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Abstract

It is shown that Coulomb repulsion during pulse compression near transition energy in a proton driver can be evaluated by means of a single dimensionless Coulomb parameter Σ ($\propto 1/\eta$, with η the slip factor), without explicitly solving the longitudinal envelope equation. Σ directly gives the (undesirable) extra coherent momentum spread during compression as well as the enhanced rf voltage to hold a bunch and to compress it. We find that for practical design Σ should not exceed unity, which determines the minimum η to avoid a "Coulomb explosion" (i.e. excessive coherent momentum spread).

1 Introduction

Bunch compression near transition is an attractive scheme for proton drivers taking advantage of the reduction of the required rf voltage for small η. Coulomb repulsion is shown here to set a lower limit to the smallness of $\mid \eta \mid$. For given intensity there is a critical $\mid \eta \mid$ below which a noticeable coherent momentum spread appears during compression (disappearing again at the time focus), and there is only minor saving in rf voltage. As an example we consider 10^{13} protons at 1.5 GeV and assume for the compressed bunch $\Delta p/p = 0.0115$ and a half-length of 1.05 m. The exact solution of the envelope equation (see below) for $\eta = -0.001$ and compression starting from 6 m half-length is shown in Fig. 1. This "Coulomb explosion" can be avoided (as shown below) by choosing larger $\mid \eta \mid$.

To determine the practically useful η quantitatively it is entirely sufficient, as shown here, to calculate for the parameters of the compressed bunch (at the time focus, denoted by subscript f) the dimensionless Coulomb parameter Σ (generalizing an earlier derivation with $\mid \eta \mid \approx 1$, see Refs.[1, 2]):

$$\Sigma_f \equiv \frac{3}{2} \frac{gNr_p}{\beta^2\gamma^3 \mid \eta \mid (\Delta p/p)_f^2 \, z_f} \tag{1}$$

with $g = 0.5 + 2ln(R_p/R_b)$ (R_p and R_b the pipe and beam radii), N the bunch intensity and $r_p = 1.53 \ 10^{-18} \ m$. Physically speaking, Σ_f is a measure of the extra energy required to compress against space charge.

2 Solution of Envelope Equation

We use the longitudinal envelope equation to describe the bunch length behavior during bunch compression:

$$\frac{d^2z}{ds^2} + k_{z0}^2 z - \frac{K}{z^2} - \frac{\epsilon^2}{z^3} = 0 \tag{2}$$

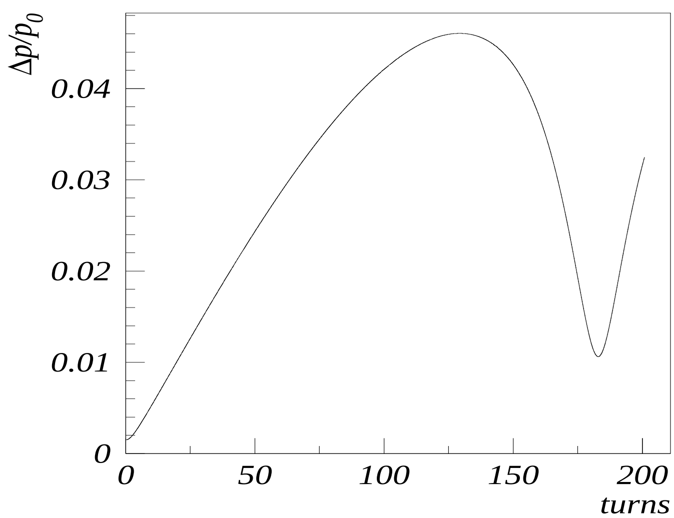

Figure 1: Momentum spread during compression against large space charge and $\eta = 0.001$, showing a coherent spread 4 times the final incoherent one.

with $K \equiv -\frac{3}{2} g N r_p \eta / (\beta^2 \gamma^3)$, and $k_{z0}^2 = eVh \mid \eta \mid / (2\pi R^2 \gamma \beta^2 mc^2)$ describing the linear rf focusing force for a voltage V at harmonic h, and the longitudinal emittance (evaluated at a waist with bunch half length z_0), $\epsilon = \mid \eta \mid \left(\frac{\Delta p}{p}\right)_0 z_0$. Note that the Σ introduced above is the ratio of the space charge term over the emittance term in Eq. 2.

2.1 Drift Approximation

For an estimation of the space charge effect we integrate backwards from the desired time focus to an initial length z_i by ignoring the rf term, k_{z0}, and multiplying Eq. 2 by dz/ds. We thus obtain for the difference between initial and final states ($dz/ds = 0$ at time focus):

$$\left(\frac{dz}{ds}\right)_i^2 = \epsilon^2 \left(z_f^{-2} - z_i^{-2}\right) + 2K \left(z_f^{-1} - z_i^{-1}\right) \tag{3}$$

The envelope gradient allows us to calculate the coherent momentum spread of a tilted longitudinal phase space ellipse by using the relationship

$$\left(\frac{\Delta p}{p}\right)_{coh}^2 = \left(\frac{dz/ds}{\eta}\right)^2 + \left(\frac{\epsilon}{\eta z}\right)^2 . \tag{4}$$

We thus obtain the coherent momentum spread (i.e. maximum momentum deviation) needed to compress a bunch against its own space charge in the drift approximation, which applies to the case where the rf force acts instantaneously (as a kick) and the bunch ends drift towards each other only exposed to the space charge force:

$$\left(\frac{\Delta p}{p}\right)_{coh,drift}^2 = \left(\frac{\Delta p}{p}\right)_f^2 \left(1 + 2\Sigma_f \left(1 - \chi\right)\right) . \tag{5}$$

Here we have introduced the compression factor χ

$$\chi \equiv z_f/z_i \tag{6}$$

for compression from an initial length z_i to a final length z_f. The factor to the r.h.s. of Eq. 5 expresses the additional energy required to compress against space charge.

2.2 Correction with RF Force Term

For our case of a continuously present rf force Eq. 5 somewhat over-estimates the required coherent spread. We have found that for strong compression the continuous rf force case approaches asymptotically the result of Eq. 5. The general case can be written as:

$$\left(\frac{\Delta p}{p}\right)^2_{coh} \approx \left(\frac{\Delta p}{p}\right)^2_f \left(1 + 2\lambda\Sigma_f\,(1-\chi)\right) \tag{7}$$

Comparing results for a variety of parameters we have found that λ depends only on the compression factor χ for all cases of practical interest (where the space charge effect is non-negligible). This makes Eq. 7 useful for design considerations, without having to really solve the envelope equation. The numerical result for λ is shown in Fig. 2.

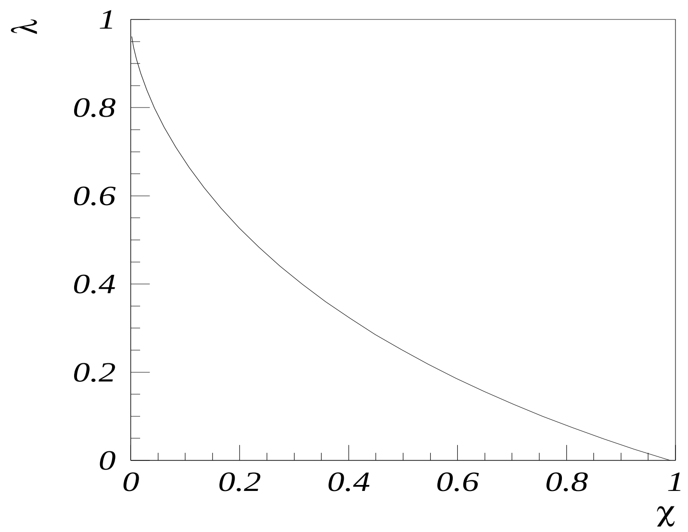

Figure 2: Correction factor λ as function of compression ratio χ.

3 Space Charge Modification of the RF Potential

The voltage to hold a bunch stationary prior to compression is also modified by space charge. It follows directly from Eq. 2 that for a stationary bunch the rf force gradient (hence rf amplitude) needs an enhancement compared with the case of no space charge ($k_{sc=0}$) given by:

$$k_{0z}^2 = k_{sc=0}^2 \left(1 + \Sigma_i\right) \tag{8}$$

Moreover, the rf voltage for compression against space charge is found similar to Eq. 5 as (for $\chi << 1$)

$$k_{0z}^2 = k_{sc=0}^2 \left(1 + \frac{2}{1+\chi} \Sigma_f \right), \tag{9}$$

which agrees with Eq. 8 in the limit $\chi = 1$. Here Σ_f is again evaluated at the time focus. It is noted that $\Sigma_f / \Sigma_i = \chi$, hence the relative significance of space charge is enhanced for the uncompressed bunch. The holding voltage for the uncompressed bunch is therefore even more controlled by space charge than the compression voltage.

4 Discussion of Results

In the example of Fig. 1 we readily see from Eq. 7 that $\Sigma_f = 18$ (assuming $g = 1.6$). Since $\chi = 1/6$, we have taken the reduction factor $\delta \approx 0.55$ according to Fig. 2. From this we calculate the unacceptable large coherent momentum blow up by a factor 4, which is fully in agreement with the explicit solution of the envelope equation shown in Fig. 1. Σ_f can be reduced by choosing a larger $\mid \eta \mid$. We suggest as conservative upper bound

$$\Sigma_f \approx 1, \tag{10}$$

which implies that the intermediate coherent momentum spread is only 40% higher (for a compression ratio 1:6) than the incoherent spread at final compression. For the above example this requires $\eta = -0.018$. The rf voltage required for compression in this case is then only 54% larger than with $\eta = -0.001$. This expresses the fact that the benefit of small η vanishes with large space charge. In other words, the independence from η of the term $k_{sc=0}^2 \Sigma_f$ in Eq. 9 makes it impractical to reduce $\mid \eta \mid$ beyond the threshold $\Sigma_f \approx 1$; the enhanced coherent momentum spread would not be justified by the relatively small saving in rf voltage.

The present study is based on parabolic bunch models; we assume that the main conclusions will remain unchanged for more general bunch forms. The possibility of tails in the momentum distribution at the time focus (with a width related to the intermediate coherent spread) is one of the characteristic features of non-parabolic bunch models as shown in Ref. [2]. Also, it should be mentioned that higher order terms in the phase shear (for very small η) have been neglected here.

Acknowledgment

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References

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[2] I. Hofmann, Proc. Symposium on Accelerator Aspects of Heavy Ion Fusion, Darmstadt, March 29 - April 2, 1982, p. 181