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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Six-dimensional approach to the beam dynamics in HIDIF scenario

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Abstract

In the HIDIF scenario the accumulation of the required total number of heavy ions ($2–3\times 10^{15}$ per ring) is an important issue. However nonlinear magnetic fields can lead to emittance growth and beam losses if the tunes q_x and q_y satisfy usual the resonance condition, as a result of the space-charge tune depression. The transfer map technique is the usual approach suitable to include lattice nonlinearities in the transverse single-particle dynamics. An extension of the transfer map concept to six dimension is presented. Space-charge, lattice nonlinearities and longitudinal electric fields are included by a kick approximation technique. A first application to the HIDIF scenario in a debunching process is presented. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The development to high intensities and phase-space densities is a goal for heavy ion accelerators to drive inertial confinement fusion (HIDIF) [1] as well as for SIS (GSI). In order to study the bunch dynamics including space-charge effects, we use the transfer map technique.

In Section 1, a micro transfer map is introduced. In Section 2, a model of the space-charge calculation is presented. Section 3 shows an application to the case of a microbunch in the HIDIF storage ring.

2. Micromap theory

We will describe the single-particle dynamics in a magnetic lattice in the coordinate system where x, y are the horizontal and vertical axes perpendicular to the reference orbit and s is the curvilinear coordinate. Such a frame moves on the design orbit with constant velocity v_0 and the curvilinear

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coordinate of the origin at time t is $s = v_0 t$. In these coordinates the transverse equations of motion are

$$x'' - \left(k(s) + \frac{1}{\rho(s)^2}\right)x = f_x(x, y, s), \tag{1}$$

$$y'' + k(s)y = f_y(x, y, s), \tag{2}$$

where $' = \mathrm{d}/\mathrm{d}s$, $k(s)$ is the quadrupole gradient, $\rho(s)$ is the curvature radius of the reference orbit, $f_x(x, y, s)$ and $f_y(x, y, s)$ describe the lattice nonlinearities [2]. We call off momentum the quantity $\delta p = (p - p_0)/p_0$, denoting with $\tilde{p} = p - p_0$ the relative momentum of the particle in the local frame. p is the longitudinal momentum and p_0 is the longitudinal momentum of the design particle. For a linear lattice ($f_x = f_y = 0$) and a particle with off momentum $\delta p = 0$ the solution of the Eqs. (1) and (2) for the initial condition $\boldsymbol{x}(s) = (x, x', y, y')_s$ can be written as the transverse transfer map

$$\boldsymbol{x}(s + \Delta s) = L^{\mathrm{t}}_{s,s+\Delta s_0}\boldsymbol{x}(s) \tag{3}$$

where $L^{\mathrm{t}}_{s,s+\Delta s_0}$ is a diagonal blocks matrix [2] correspondent to the longitudinal step Δs_0. If $\delta p \neq 0$ the map needs to be corrected adding a dispersion term that causes a deviation of the closed orbit from the reference orbit. In order to preserve the simplicity of the 6D map we will keep the transverse map in the form (3). With this assumption for the maximum δp in the HIDIF scenario we have a maximum error on the transverse trajectory of $|\Delta x| \sim 1.36\,\mathrm{mm}$ and $|\Delta x'| \sim 0.34\,\mathrm{mrad}$.

If the lattice is not linear we can introduce the transverse forces f_x,f_y and preserve the symplectic character of the transfer map, by using the one kick approximation [2]. The effect of the nonlinearities on the particle moving along the arc $[s, s + \Delta s_0]$ on the lattice path is described by a transverse kick $\boldsymbol{k}^{\mathrm{t}}$ as follows

$$\boldsymbol{x}(s) \overset{\boldsymbol{k}^{\mathrm{t}}}{\mapsto} \boldsymbol{x}(s)' \overset{L^{\mathrm{t}}}{\mapsto} \boldsymbol{x}(s + \Delta s_0) \tag{4}$$

with $\boldsymbol{k}^{\mathrm{t}}(\boldsymbol{x}) = (x, x' + \Delta s_0 f_x, y, y' + \Delta s_0 f_y)_s$. Therefore, the transverse nonlinear map is

$$\boldsymbol{x}(s + \Delta s_0) = L^{\mathrm{t}}_{s,s+\Delta s_0}\boldsymbol{k}^{\mathrm{t}}(\boldsymbol{x}(s)). \tag{5}$$

In order to include the space-charge effects in the transverse map (5) using the one kick approximation we look at a particle moving in a drift. The equation of motion for the x coordinate is

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = F_{x,sc}. \tag{6}$$

It is straightforward to show that the relative error ΔA between $\mathrm{d}p_x/\mathrm{d}t$ and $m\gamma_0(\mathrm{d}v_x/\mathrm{d}t)$ is less then $(\gamma_0^2 - 1)(x'^2 + y'^2)$ where $\gamma_0 = (1 - \beta_0^2)^{-1/2}$. Using the HIDIF scenario parameters we find $\Delta A < 1.36 \times 10^{-5}$ therefore Eq. (6) can be written as $x'' = f_{x,sc}$ where $x'' = q^* E_{x,sc}/v_0 p_0$ is the x component of the space-charge force, q^* is the macroparticle electric charge, $E_{x,sc}$ the x component of the electric field. v_0 is the design longitudinal speed and p_0 is the design momentum. The same results hold for the y coordinate and the final form of the transverse kick is

$$\boldsymbol{K}^{\mathrm{t}}(\boldsymbol{x}) = \begin{pmatrix} x \\ x' + \Delta s_0\left(f_x + \dfrac{q^* E_{x,sc}(x, y, z)}{v_0 p_0}\right) \\ y \\ y' + \Delta s_0\left(f_y + \dfrac{q^* E_{y,sc}(x, y, z)}{v_0 p_0}\right) \end{pmatrix}_s .$$

Next, we will introduce the longitudinal map with respect to the z-axis tangential to the reference orbit in the point s (Fig. 1). The longitudinal conjugate coordinates are $\boldsymbol{z} = (z, \tilde{p})$ where $\tilde{p} = p - p_0$. In order to build the symplectic longitudinal map, we write the linear map for the constant off-momentum particle, then we add the longitudinal electric field E_z (due to space-charge and cavity). The longitudinal linear map represents in the local frame the drift

$$L^{l}_{t,\,t+\Delta t}\colon \begin{cases} z(t + \Delta t) = z(t) + \Delta s(\tilde{p}, t) - v_0 \Delta t \\ \tilde{p}(t + \Delta t) = \tilde{p}(t), \end{cases}$$

where $\Delta s(\tilde{p}, t)$ is the arc on the reference orbit corresponding to an arc $v(s)\Delta t$ on the orbit of the off momentum particle. As a consequence if Q, Q' are the positions of the particle at times $t, t + \Delta t$ its projections P,P' on the reference orbit have arc lengths $s + z$ and $s' + z'$ whose difference is $\Delta s(\tilde{p}, t)$.

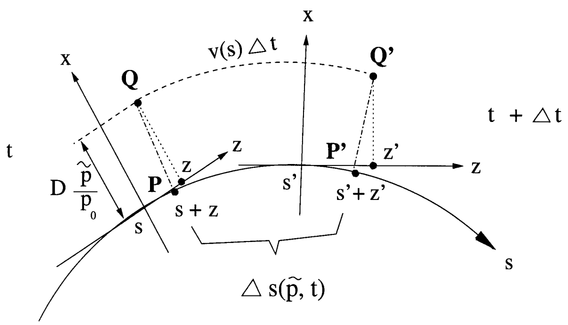

Fig. 1. Definition of the various quantities used for the derivation of the longitudinal map.

Since $s' - s = v_0\,\Delta t$ is the advance of the design particle with momentum p_0, sitting on the x,y plane, the longitunal map follows (we have choosen a coordinate frame for each particle with momentum p with the z tangential to the reference orbit whose origin moves with a velocity v_0). Assuming that the real trajectory of the particle is approximated by the real dispersion orbit we find that $\Delta s(\tilde{p},t)$ satisfies the equation

$$\Delta s(\tilde{p},t) + \int_s^{s+\Delta s(\tilde{p},t)} \frac{D(s')}{\rho(s')}\,\mathrm{d}s' \frac{\tilde{p}}{p_0} = v(s)\Delta t \tag{7}$$

where $D(s)$ is the closed periodic dispersion functions and $s = v_0 t$. The form of the longitudinal kick in the interval $[t, t+\Delta t]$ is $\boldsymbol{K}^l(z) = (z, \tilde{p} + q^* E_z \Delta t)$, then the longitudinal map is

$$z(t+\Delta t) = L^l_{t,t+\Delta t}\boldsymbol{K}^l(z(t)).$$

The final form of the symplectic 6D micromap is

$$\begin{pmatrix} \boldsymbol{x} \\ z \end{pmatrix}_{t+\Delta t} = \begin{pmatrix} L^{\mathrm{t}}_{s,s+\Delta s_0} & 0 \\ 0 & L^l_{t,t+\Delta t} \end{pmatrix} \begin{pmatrix} \boldsymbol{K}^{\mathrm{t}}(\boldsymbol{x}) \\ \boldsymbol{K}^l(z) \end{pmatrix}_t$$

Without space-charge and cavity, transverse and longitudinal plane are decoupled. The approximation, $\Delta s_0 = v_0 \Delta t$ instead of $\Delta s(\tilde{p},t)$, introduce an error in the single transverse dynamics, but a small effect on the particle distribution.

Applying this micromap to each particle of the beam we can compute the beam evolution for a time step Δt. The calculation of the electric fields E_x,E_y,E_z is described in the following section.

3. Space-charge calculation

In order to make this application for HIDIF more transparent we summarize some details of the method of Ref. [3]. We restrict our discussion to the case of a bunch which has ellipsoidal symmetry. In this case the particle density has the form

$$n(x,y,z) = n(\psi)$$

written as function of the isodensity parameter $\psi = (x/a_x)^2 + (y/a_y)^2 + (z/a_z)^2$ where a_x,a_y,a_z are the r.m.s. dimension of the bunch. The electric field is given by the formulas [3]

$$E_x = \frac{q^*}{\varepsilon_0} M_x x; \quad E_y = \frac{q^*}{\varepsilon_0} M_y y; \quad E_z = \frac{q^*}{\varepsilon_0} M_z z$$

where

$$M_u = \frac{a_x a_y a_z}{2} \int_0^\infty \frac{n(\lambda)\,\mathrm{d}s}{(a_u^2 + s)\sqrt{(a_x^2+s)(a_y^2+s)(a_z^2+s)}}, \tag{8}$$

$\lambda = x^2/(a_x^2+s) + y^2/(a_y^2+s) + z^2/(a_z^2+s)$ and $u = x,y,z$. In order to evaluate the integral in Eq. (8) when $a_z \gg a_x$,a_y the following change is made $s = \alpha^2(1/u^2 - 1)$ where $\alpha = (a_x a_y a_z)^{1/3}$ which maps the half line into a finite interval without the square root end point singularity.

In order to expand the particle density $n(\psi)$ as a Fourier series we assume the symmetry $n(\psi) = n(-\psi)$. If Ψ is the maximum value of ψ then

$$n(\psi) = \frac{c_0}{2} + \sum_{\ell=1}^{\infty} c_\ell \cos\left(\frac{\ell\pi\psi}{\Psi}\right)$$

where

$$c_\ell = \frac{2}{\Psi}\int_0^\Psi n(\psi)\cos\left(\frac{\ell\pi\psi}{\Psi}\right)\mathrm{d}\psi. \tag{9}$$

Next, we will rewrite the Eq. (9) in a suitable form to be used in numerical simulation. The integral (9) can be transformed as a sum dividing the interval $[0,\Psi]$ in small intervals of equal length $\Delta\psi = \Psi/M$

$$c_\ell = \frac{2}{\Psi}\sum_{i=1}^{M} n(\psi_i)\cos\left(\frac{\ell\pi\psi_i}{\Psi}\right)\Delta\psi$$

where $\psi_i = i\,\Delta\psi$. By doing the variable substitution $N = nV = n(4\pi/3)a_x a_y a_z \psi^{3/2}$ where V is the volume of the ellipsoid with parameter ψ, we get $\mathrm{d}\psi = \psi^{-1/2}/(n(\psi)2\pi a_x a_y a_z)\,\mathrm{d}N$

$$c_\ell = \frac{1}{\Psi \pi a_x a_y a_z} \sum_{i=1}^{M} \Delta N_i \psi_i^{-1/2} \cos\left(\frac{\ell \pi \psi_i}{\Psi}\right) \tag{10}$$

In Eq. (10) ΔN_i denotes the number of particles in the ellipsoidal shell defined by $[\psi_i, \psi_{i+1}]$, the empty shells giving zero contribution to the sum. The spectrum of the distribution $n(\psi)$ can be computed then as

$$c_\ell = \frac{1}{\Psi \pi a_x a_y a_z} \sum_{i=1}^{N} \psi_i^{-1/2} \cos\left(\frac{\ell \pi \psi_i}{\Psi}\right).$$

If we substitute the expansion of $n(\psi)$ in the integral (8) we can express the electric field, for instance E_x, as

$$E_x = \frac{c_0}{2} F_0^x(x, y, z) + \sum_{\ell=1}^{\infty} c_\ell F_\ell^x(x, y, z),$$

where $F_\ell^x(x, y, z) = x M_x q_x^* / \varepsilon_0$ and M_x is given by Eq. (8) where the density n is replaced with $\cos(\ell \pi \psi / \Psi)$.

This method describing the particle density as smooth distribution can represent an arbitrary density profile with ellipsoidal symmetry.

4. Example

In the HIDIF scenario the linac beam is injected into storage rings by using a multiturn injection scheme suitable to avoid that the re-circulating beam hitting the septum [4].

The linac beam has a current of 400 mA and is made of microbunches of emittances $\varepsilon_x = \varepsilon_y =$ 4 mm mrad, length of 4.25 ns and momentum spread $\Delta p / p_0 = 1.25 \times 10^{-4}$. When it is injected into a storage ring each microbunch is subject to a debunching process and therefore the incoherent tune shift is reduced, however the initial tune shift can affect the efficiency of the injection scheme.

As an application of the theoretical tools developed in Sections 1 and 2 we simulate the dynamics of a microbunch in the debunching process. We will investigate the space-charge effects on a single particle belonging to the microbunch in order to see how fast the tune shift is reduced. This is a first contribution to a more complete description of the actual multiturn injection scheme.

In the simulation we have used a constant focusing lattice with nominal tunes $q_x = q_y = 8.715$. The bunch has been taken with 200 macroparticles and the space-charge calculation was performed up to the 10th harmonic. The time step has been chosen $\Delta t = 38.40$ ns and we used in the transverse map Δs instead of Δs_0 since the short number of turns considered.

In the simulation we have taken one particle in the center of the bunch and we found out its tunes from its dynamics. By using the assumption that in a path of 10 m the single-particle dynamics is linear, evaluating three transverse position of the particle in two steps, we can calculate the trace of the transfer matrix and then the tunes.

At the beginning of the debunching the space-charge effect is the strongest and the tune depression maximum, $\Delta q_x = \Delta q_y = -0.038$. In the following debunching, the density and therefore the space charge is reduced, due to the repulsive Columb force. The test particle feels a weaker space charge and then its incoherent tune shift is reduced as shown in Fig. 2. The noise present in these pictures depends on the small number of macroparticles chosen. Numerical tests indicate that the tune fluctuation is reduced as the number of macroparticles is increased.

Fig. 3a shows the tune footprint for each determination of the tunes. The initial tunes in Fig. 3a lie on the left bottom of the picture. Next tune evaluation should be a new point in the diagram displaced toward the nominal tune and so on for all the tunes measured. Due to the numerical noise each tune jump has a component that shifts its direction from the expected 45° toward the nominal tune. This effect is maximum at the beginning of the debunching because the granularity of the bunch is emphasized by the strong space charge. During the debunching this effect is reduced and the tunes converge to a line as soon as the tune approaches the nominal one. Fig. 3a shows also that tunes are accumulating toward the nominal tune because of its asymptotic behavior (Fig. 2). During the debunching each particle of the microbunch gains longitudinal velocity and therefore the initial

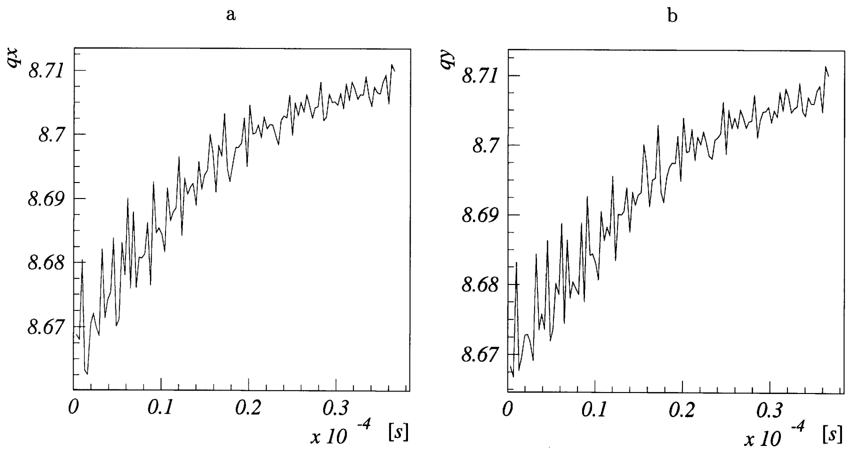

Fig. 2. Time evolution of the tune q_x and q_y of a macroparticle chosen in the center of the microbunch.

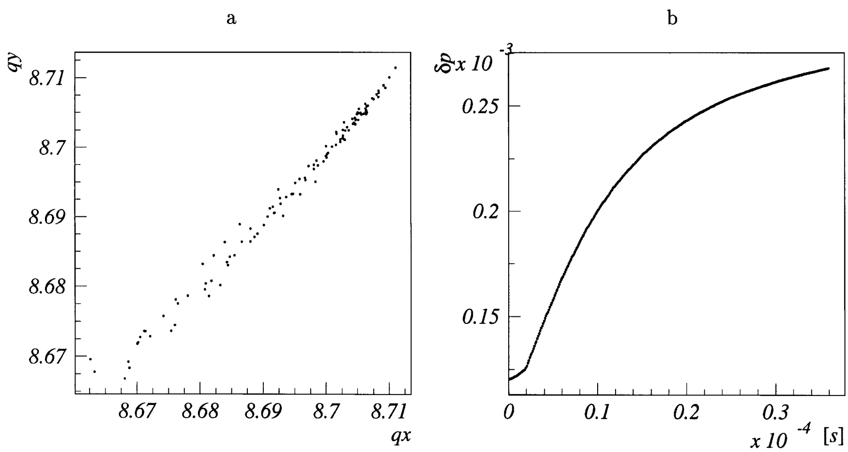

Fig. 3. (a) tune footprint; (b) time momentum spread evolution during debunching.

momentum spread is changed. Fig. 3b shows the evolution of the momentum spread. The momentum spread of the bunch increases due to the transformation of the initial electrostatic energy into kinetic energy as function of the initial spatial distribution. The asymptotic behavior of the momentum spread follows from energy conservation.

We conclude from this simulation that in ~ 4 turns the microbunch expansion reaches almost a constant velocity and the tune shift of the single bunch is reduced to ~ 0.015. Simulations on the stationary bunch dynamics are in progress.

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