Modeling of Space Charge Induced Resonance Loss

G. Franchetti and I. Hofmann

GSI, 64291 Darmstadt, Germany

Abstract. For high-current circular machines like the SIS100, which has been proposed as driver for a future radioactive beam facility at GSI, an exploratory study of space charge effects on resonances during long-term beam storage is indispensable. We discuss the effect of space charge on sextupole error resonances by a 4D model using an analytical expression for a stationary space charge. The effect of space charge induced resonance crossing in a bunch beam is approached by a simplified model.

INTRODUCTION

Beam loss control is an important issue for high-intensity heavy ion rings [1]. Long term bunched beam storage, typically 10^6 turns, and resolution of beam loss under the percent level are very demanding for PIC simulation. As pointed out in [2] numerical noise generated in PIC solvers is responsible for an artificial emittance growth. This effect may get controlled by increasing the number of macroparticles used in the simulations at expenses of larger CPU time. A study of this effect in 3D simulation was presented in [3]. Since small beam loss is of concern, a particle-core model provides a way to avoid numerical noise artifacts while retrieving important aspects of the dynamics during long term beam storage. If loss are found to be small and collective effects may be neglected a frozen core model may give reasonable predictions.

2D MODEL

The model is based on a coasting beam in a constant focusing lattice. We consider a 2D axisymmetric beam with frozen transverse Gaussian particle distribution of rms radius σ. The analytic space charge electric field [4] used in the particle equation of motion provides a particle-core model. In order to simplify the discussion we consider initially the particle dynamics only in the x-plane setting for any test particle $y = y' = 0$. The equation of motion becomes

$$x'' + \left(\frac{\nu_0}{R}\right)^2 x = K\frac{x}{r^2}(1 - e^{-\frac{r^2}{2\sigma^2}}) \qquad (1)$$

where $()' = d/ds$, ν_0 is the ring bare tune, R is the ring radius, K is the perveance, $r = \sqrt{x^2+y^2}$. In this case $r = x$. Calling $C = 2\pi R$, and by using the coordinate transformation $x = \tilde{x}\sigma$, $s = \tilde{s}C$ we rescale Eq. 1 to the dimensionless equation

$$\ddot{\tilde{x}} + (2\pi\nu_0)^2\tilde{x} = 2(2\pi)^2(2\nu_0\Delta\nu - \Delta\nu^2)\frac{1}{\tilde{x}}(1 - e^{-\frac{\tilde{x}^2}{2}}) \qquad (2)$$

where $\dot{()} = d/d\tilde{s}$. In Eq. 2 we use the tuneshift as a measure of the beam intensity, while the perveance is given by $K = 2(\sigma/C)^2(2\pi)^2(2\nu_0\Delta\nu - \Delta\nu^2)$. Equation 2 can be integrated through a micromap + nonlinear kick symplectic method. Dividing the ring in N integration steps the nonlinear transport map becomes

$$\begin{pmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{pmatrix} = \begin{pmatrix} \mathscr{C} & \frac{\mathscr{S}}{2\pi\nu_0} \\ -2\pi\nu_0\mathscr{S} & \mathscr{C} \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \dot{\tilde{x}} + f_n(\tilde{x}) \end{pmatrix} \qquad (3)$$

with $\mathscr{S} = \sin(2\pi\nu_0/N)$, and $\mathscr{C} = \cos(2\pi\nu_0/N)$. The nonlinear kick f_n is $f_n(\tilde{x}) = \frac{2}{N}(2\pi)^2(2\nu_0\Delta\nu - \Delta\nu^2)(1 - e^{-\tilde{x}^2/2})/\tilde{x}$. The lattice nonlinearities can be added to $f_n(\tilde{x})$ at any integration step through the kick $A_m\tilde{x}^m$. Here A_m is the integrated strength of a m-th order nonlinear thin element in the frame $\{\tilde{s}, \tilde{x}\}$. The relation with its integrated strength K_m in the laboratory frame is $A_m = C\sigma^{m-1}K_m$.

RESONANCES AND SPACE CHARGE

With this model we study the effect of space charge on lattice nonlinearity induced resonances. Here the space charge acts as a perturbation leaving unchanged the main mechanisms of single particle resonances. We consider a ring with $\nu_0 = 13.36$, a beam with maximum tuneshift $\Delta\nu = 0.2$, and use 1060 integration steps to resolve the dynamics with space charge. With this choice the resonances $3\nu_0 = 40$, and $4\nu_0 = 53$ can be met if the depressed tune scans over $\nu = 13.16, ..., 13.36$. We show in Fig. 1a the nonlinear tune ν for particles with initial coordinates $\dot{\tilde{x}} = 0$. The calculation of ν is obtained with a FFT interpolated method (see in [5]) by using the particle coordinates in a fixed ring section over 2048 turns. As expected, at the center of the coasting beam $\nu = 13.16$, which moves asymptotically towards $\nu_0 = 13.36$ if the particle is taken very far from the beam center. Note that third and fourth order resonances are crossed at $x = 5.5\sigma$, and $x = 2\sigma$ respectively. We excite these two resonances by using 20 equal sextupolar kicks with $A_2 = 0.01$, and

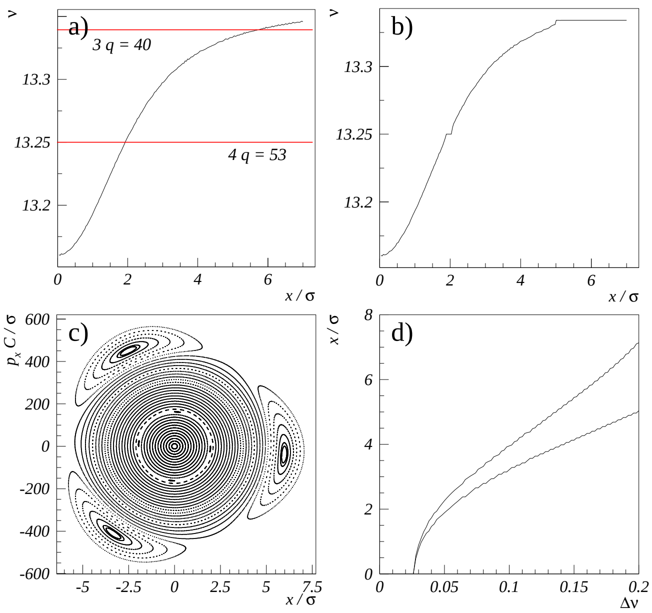

FIGURE 1. a) space charge induced nonlinear tunes, b) effect of resonances on nonlinear tunes, c) Poincare' section for $\Delta\nu = 0.2$, d) amplitude of 3rd order island as function of $\Delta\nu$.

53 equal octupolar kicks with $A_3 = 0.005$. In Figs. 1b,c we show the nonlinear tune and Poincaré section. The amplitude of the island may be measured by the size of the flat region in the nonlinear tune which crosses the resonance. The position of the fixed point is where $\nu_0 - \Delta\nu$ equals the harmonic of the nonlinearity driving term; the area of the islands seems to depend on the gradient of the nonlinear tune: the weaker the nonlinear tune gradient the bigger is the island. From the example in Fig. 1b it follows that large islands are obtained at 6σ. Setting properly the bare tune ν_0 and varying $\Delta\nu$ it is possible to create islands at any position. In Fig. 1d we plot the outer and inner separatrix of the third order resonance (at $\dot{\tilde{x}} = 0$) as function of the space charge intensity expressed in terms of $\Delta\nu$. The bare tune is still $\nu_0 = 13.36$. This picture shows consistently with the interpretation of Figs. 1a and b) that the third order islands move outward and increase their areas as the tuneshift increases. From this study we find that typically we may create big islands far beyond 3σ, whereas the islands within 3σ are much smaller.

Dynamics in a Bunch

The situation may be different for the single particle dynamics in a bunch. In fact synchrotron oscillations induce oscillations of the instantaneous current: when the particle is in the center of the bunch the local transverse current is maximum whereas on the bunch head or tail the current is minimum. The global effect is that a single particle experiences an oscillating instantaneous beam current with frequency much slower than the betatron oscillations. We consider first a linear current ramp which takes the tuneshift to $\Delta\nu = 0.2$. In first order the current ramp causes a migration of islands consistent with Fig. 1d. If the test particle has initial coordinates $x' = 0, x < 7.5\sigma$ the third order island eventually will cross the particle phase space orbit. The *speed* of the island at the crossing is crucial for the effect on a particle. In Fig 2 we explore this effect. In the left column we plot pictures of the single particle orbits. In the right column the time evolution of the $\tilde{x}$ coordinate. In the case a) (first two pictures at the top) a positive linear ramp brings the tuneshift to $\Delta\nu = 0.2$ in 2048 turns. The particle initial coordinates are $\tilde{x} = 4, \dot{\tilde{x}} = 0$. At the beginning the particle lies on an elliptic orbit, the frequency of the oscillations (picture on the right) decreases due to the nonlinear tune change for effect of the space charge ramp. The increasing space charge moves the third or-

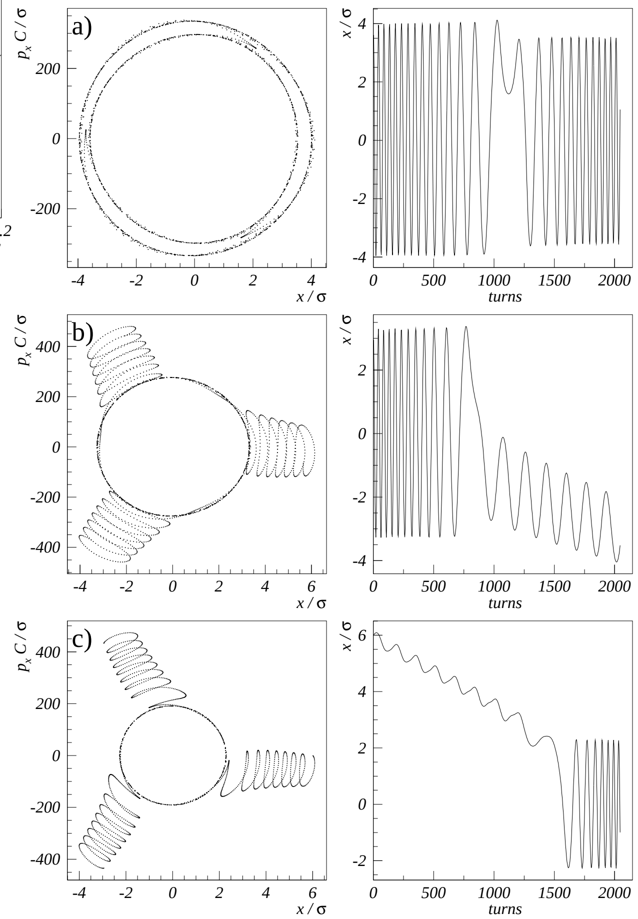

FIGURE 2. Phase space orbit (left) and time evolution of the particle amplitude plotted every third turn (right).

der islands outward. When the islands reach the particle orbit the third order resonance tries to capture the particle. This happen for half of an island revolution. Afterwards, the particle is closer the inner separatrix, and due to the outward motion of the island it crosses the separatrix again and gets unlocked from the resonance. The total effect is that the particle orbit gets kicked inward by an amplitude comparable with the island width, then its orbit amplitude remains unaffected. The orbit jump is visible after 1000 turns in the left picture. Simulations showed also the inverse effect for a decreasing current: in this case the orbit jump is outward. We conclude that the resonance crossing causes an orbit jump which has a direction opposite to the motion of the islands. The partial capture process depends strongly on the particle and island position in the phase space and on the speed of the island when approaching the particle. If a particle inside an island during half a revolution around the fixed point moves less than the island size, it may get trapped and follow the fixed point. In Figs. 2b,c we show two examples of particles trapped by the resonance. In b) the particle with $\tilde{x} = 3.3, \dot{\tilde{x}} = 0$ is trapped and moves outwards as the current increases linearly to reach $\Delta\nu = 0.2$. In c) a particle with $\tilde{x} = 6, \dot{\tilde{x}} = 0$ gets trapped and moves inward following the island. When the trapping condition gets broken, the orbit gets unlocked and follows the usual ellipse. From these simulations we conclude that a particle in a bunch may be subject to periodic resonance crossing: the islands oscillate back and forth periodically, kicking the particle orbit in and out. The single particle dynamics has then a nonlinear resonance induced stochastic regime. Over many synchrotron oscillations the particle may reach the trapping condition and get transported to the farest transverse position determined by the position of the island at the maximum intensity. Fig 3a shows the evolution of the $\tilde{x}$ coordinate of one particle with $\tilde{x} = 1.4, \dot{\tilde{x}} = 0$ over 10^6 turns. The space charge oscillations are simulated with $\Delta\nu = 0.2[1+\cos(2\pi\nu_s n)]/2$, where synchrotron tune is $\nu_s = 10^{-3}$. Note the diffusive-like process in the first $2 \cdot 10^5$ turns while above it spikes bring the particle to $\sim 7\sigma$. Due to the reversible nature of the resonance trapping process the particle does not remain long at the maximum distance but falls within 3σ and later it gets trapped again and is brought to $6-7\sigma$. Note that $\sim 6\sigma$ is the position of the fixed point for $\Delta\nu = 0.2$ as shown in Fig 1d.

4D Tracking and Outlook

We repeat the same simulation for the 4D system (the condition $\tilde{y} = \dot{\tilde{y}} = 0$ is removed). The bare tunes are $\nu_{x0} = 13.91, \nu_{y0} = 13.58$, and the maximum tuneshift is $\Delta\nu_x = \Delta\nu_y = 0.3$. We excite now the harmonics 41 by using equal sextupolar errors with $A_2 = 0.01$. In Fig. 3b

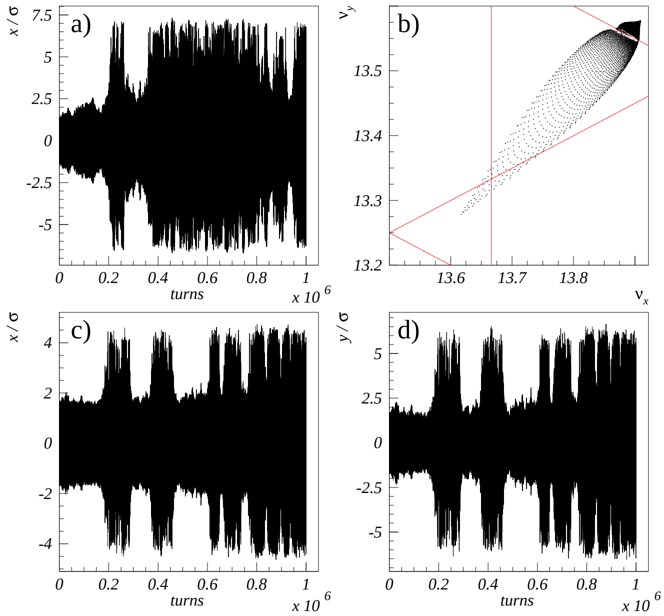

FIGURE 3. a) Particle evolution in a bunch (2D case), b) resonance lines and tune footprint for 4D case, c),d) particle evolution in the 4D case.

we plot the third order resonance lines and the tune footprint for test particles distributed over a larger area compared with the beam size. Note that the vicinity of the line $2\nu_y - \nu_x = 41$ is evacuated, which shows the existence of a stop band. The position of this resonance is chosen to be near the bare tunes. According to the 2D interpretation we expect again that the periodic resonance crossing will create a diffusion, and the resonance trapping would eventually bring the particle at the farest position. These effects are found in Figs. 3c,d for a particle with $\tilde{y} = \tilde{x} = 1.5, \dot{\tilde{x}} = \dot{\tilde{y}} = 0$. This study shows that in a bunched beam the most dangerous resonances appear to lie near the bare tune and not in the most densely populated part of the footprint corresponding to the bunch case. Consequently the single particle dynamics in a bunch is then very sensitive with respect to the particle synchrotron amplitude as it effects the tune oscillations. In order to quantify losses induced by the periodic resonance crossing a 3D self consistent space charge calculation along with a consistent modeling of synchrotron oscillation is needed.

REFERENCES

1. C. D. R. http://www.gsi.de/GSI Future/cdr/ .
2. J. Struckmeier, Phys. Rev. **E54**, 830 (1996).
3. J. Qiang and R. Ryne, these Proceedings.
4. M. B. Ottinger, T. Tajima, and K. Hiramoto, 1653 , 6th European Particle Accelerator Conference, Stockholm, Sweden, 22-26 Jun 1998.
5. R. Bartolini *et al.*, Part. Accel. **52**, 147 (1996).