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Transverse Radial Expansion and Directed Flow.

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ABSTRACT

I believe that the high statistics E877 '94 data permits an interesting study of an interplay of directed flow and isotropic transverse expansion of the thermalized hadron source. In this note in a simple model I discuss how the radial expansion affects the directed flow signal.

Transverse directed flow is (by definition) a result of a movement of an effective source. In our note with Wen-Chen Chang [1] it is shown that if particles are produced by one thermal source moving in the transverse direction with velocity β_a, the directed flow signal (the value of the first Fourier coefficient in the decomposition of the particle azimuthal distribution) depends on this velocity in a following way:

$$v_1 = \frac{\beta_a p_t}{2T_{eff}}, \tag{1}$$

where T_{eff} is an effective (Boltzmann) temperature parameter of the transverse momentum distribution of particles with a given rapidity.

Below I discuss the case when the thermal source undergoes an isotropic expansion. I study the expansion in a simple model which permits to perform all calculations analytically to have clear the qualitative features of the effect. Directed flow of protons is the most pronounced and less affected by shadowing, so I consider in this note only proton flow. I treat protons as non-relativistic particles. (A relativistic approach is discussed in brief at the end of the note.) This is certainly justified in the source rest frame, where the proton effective energy is of the order of temperature $T_{eff} \ll m$, where m is the proton mass. I always work in the frame moving longitudinally with the effective source rapidity, hence the longitudinal proton momenta are always relatively small. To have possibility to treat protons non-relativistically in the transverse direction I have to assume that $m_t - m << m$, what restricts the consideration to the region $p_t \leq 0.5\ GeV$. Under these assumptions the transverse momentum distribution of protons from the thermal source can be written as:

$$\frac{d^2n}{d\mathbf{p_t}} = \frac{1}{2\pi(2mT_{eff})} \exp\left(-\frac{\mathbf{p_t}^2}{2mT_{eff}}\right). \tag{2}$$

The transverse isotropic expansion of the source can be described as a superposition of different sources moving radially with an expansion velocity β_0.

$$\frac{d^2n}{d\mathbf{p_t}} = \frac{1}{(2\pi)^2\,(2mT_{eff})} \int d\psi \exp\left(-\frac{(p_x - p_0\cos(\psi))^2 + (p_y - p_0\sin(\psi))^2}{2mT_{eff}}\right), \tag{3}$$

where $p_0 = m\beta_0$. Such an expansion results in the distribution:

$$\frac{d^2n}{d\mathbf{p_t}} = \frac{1}{2\pi(2mT_{eff})} \exp\left(-\frac{p_t^2 + p_0^2}{2mT_{eff}}\right) I_0(\xi), \tag{4}$$

where I_0 is the modified Bessel function, and $\xi = \beta_0 p_t / T_{eff}$. On top of the isotropic expansion we have an overall movement in the flow direction (below we assume that this movement is along the "x" axis). The velocity of a such movement we denote as β_a. It results in a shift $p_x \to p_x - \Delta p_x = p_x - m\beta_a$ and, consequently, in an anisotropy of the final distribution. In order to calculate the first Fourier coefficient of the azimuthal distribution I perform an expansion of the distribution in Δp_x. I use

$$\frac{d}{dp_x} \exp\left(-\frac{p_t^2}{2mT_{eff}}\right) = -\exp\left(-\frac{p_t^2}{2mT_{eff}}\right) \frac{p_x}{mT_{eff}}, \tag{5}$$

and

$$\frac{d}{dp_x} I_0(\xi) = I_1(\xi) \frac{\beta_0 p_x}{p_t T_{eff}}. \tag{6}$$

The expansion of the exponent is valid if $p_t \beta_a / T_{eff} \ll 1$. The expansion of the Bessel function is valid if $\beta_0 \beta_a m / T_{eff} \ll 1$.

Using the above formulae I get an expression for v_1:

$$v_1(p_t) = \frac{p_t \beta_a}{2T_{eff}} (1 - \frac{m\beta_0}{p_t} \frac{I_1(\xi)}{I_0(\xi)}). \tag{7}$$

At relatively small values of the isotropic expansion velocity β_0 and/or relatively large p_t this expression coincides with (1). Remarkably, at some parameter values, namely such that $m\beta_0^2/2 > T_{eff}$, $v_1(p_t)$ has a region (at small p_t), where it is negative (see Fig. 1). Physically it corresponds to the case, when particle production with such a value of p_t is more probable from the part of the source which moves to the opposite direction than the flow direction (when directed flow and "expansion" flow compensate each other).

Below I derive more general expression for v_1, assuming that the expression for the invariant distribution in the source rest frame is known

$$\frac{Ed^3n}{d^3p} \equiv J(\mathbf{p_t}, y) = J(p_t, y). \tag{8}$$

Source movement in the transverse direction will result in $J \to J' = J(\mathbf{p_t}', y)$, where $\mathbf{p_t}'$ is defined by the Lorentz shift with velocity β_a along the "x" axis. Assuming that the shift is small ($p_x' = p_x - \beta_a E$), and taking into account that $\partial p_t / \partial p_x = \cos(\phi)$, one gets

$$v_1 = \frac{\int d\phi J(\mathbf{p_t}', y) \cos(\phi)}{\int d\phi J(\mathbf{p_t}', y)} = -\frac{\beta_a E}{2} \frac{d \ln J(p_t, y)}{dp_t} \tag{9}$$

For radially expanding thermal source the invariant distribution can be written in the form:

$$
\begin{aligned}
J &\propto \int d\psi E^* e^{-E^*/T} \\
&= \int d\psi (E\cosh(y_t) - p_t\cos(\psi)\sinh(y_t))\exp(-(E\cosh(y_t) - p_t\cos(\psi)\sinh(y_t))/T) \\
&= Te^{-\chi}(\chi I_0(\xi) - \xi I_1(\xi)),
\end{aligned} \tag{10}
$$

where $\chi = E\cosh(y_t)/T$, $\xi = p_t\sinh(y_t)/T$; the transverse rapidity $y_t = 0.5\ln((1+\beta_0)/(1-\beta_0))$; I_0 and I_1 are modified Bessel functions.

The application of the formula (9) to the distribution (10) yields:

$$
v_1 = \frac{\beta_a p_t \cosh(y_t)}{2T}\left\{1 - \frac{I_1(\xi)E^2\sinh(y_t)\cosh(y_t) + I_0(\xi)(p_t T\cosh(y_t) - ET\sinh(y_t)^2)}{p_t T\cosh(y_t)(\chi I_0(\xi) - \xi I_1(\xi))}\right\}. \tag{11}
$$

(The application of the formula (9) to the (non-relativistic) distribution (4) yields the dependence (7).)

For illustration purposes in Fig. 2 I present a compilation (made by M. Pollack) of preliminary results on proton $v_1(pt)$ for different centralities and rapidity regions (not corrected for the reaction plane resolution; to correct one should divide the values by 0.6 – 0.7). The behaviour of $v_1(p_t)$ for the rapidity region $2.4 < y < 2.8$ is very similar to the one discussed in the text. For lower rapidities directed flow looks rather weak. For higher rapidities (close to the beam rapidity) the isotropic expansion could be weak. Besides that the contribution of "cold" spectators could also smear out the picture.

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[1] W.-C. Chang and S. Voloshin, "Proton and Pion Production Relative to the Reaction Plane in Au+Au Collisions at 10.8 GeV", E877 Note # SUNYSB-96-003, September 1996.

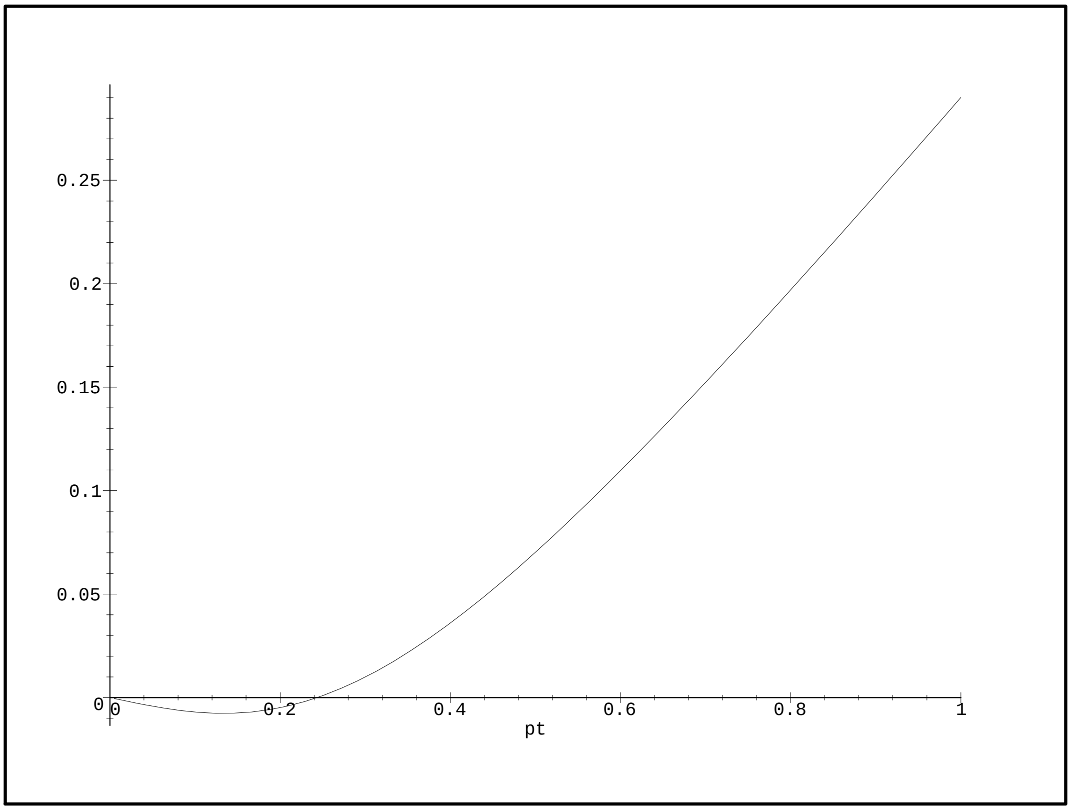

FIG. 1. The dependence of v_1 on the transverse momentum. $T_{eff} = 0.1\ GeV$, $\beta_0 = 0.5$, and $\beta_a = 0.1$.

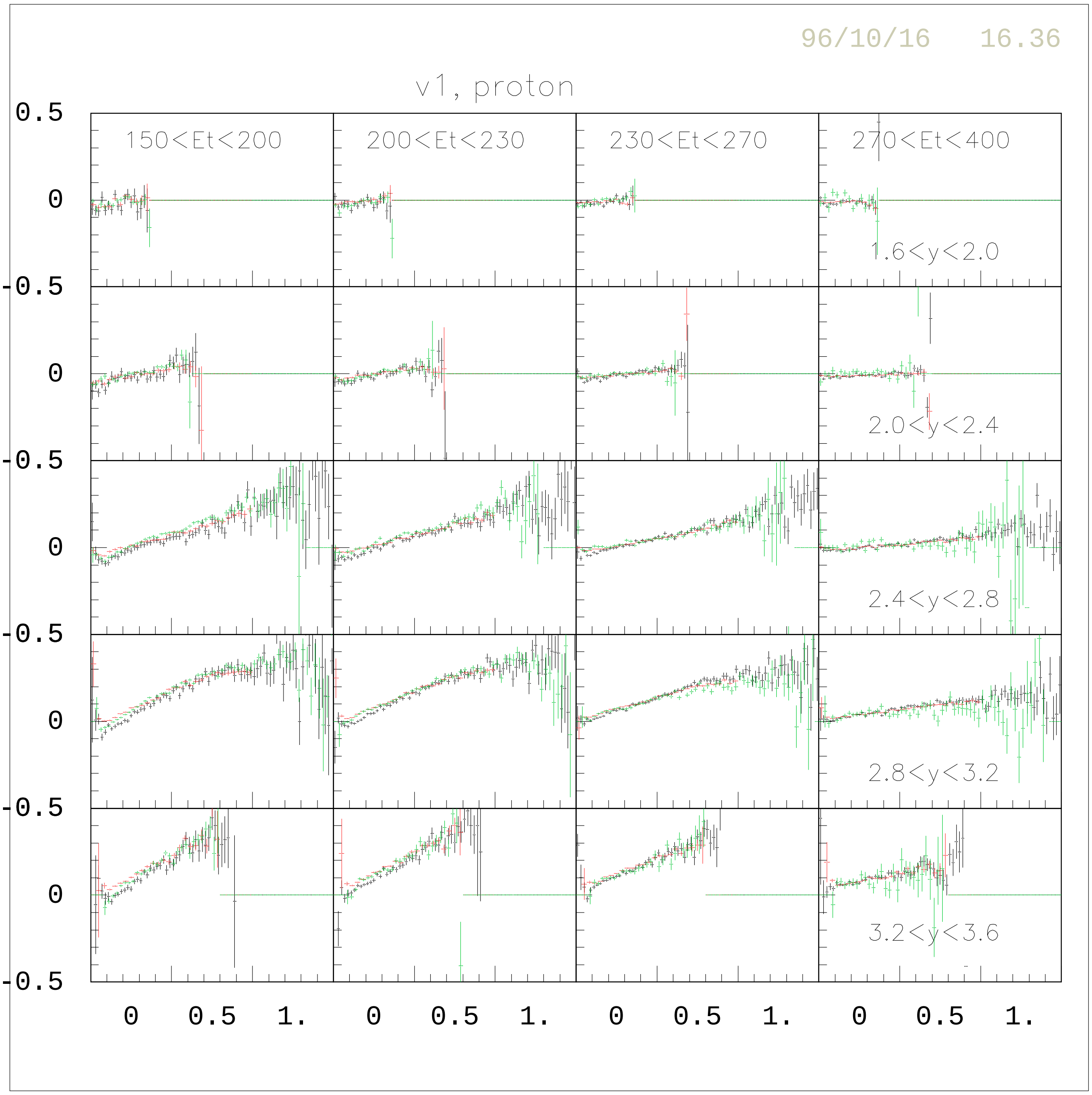

FIG. 2. $v_1(p_t)$. Protons. Red – '94 (M. Pollack), Black – '94 (S. V.), Green – '93 (W.-C. Chang).