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Analysis of Azimuthal Distributions With Respect to the Reaction Plane

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ABSTRACT

We discuss the procedure of determination of flow parameters by Fourier analysis of the azimuthal distribution with respect to the measured reaction plane. Due to the finite angular resolution, the necessary correction of each moment is shown to depend on $\langle\cos(n\Delta\Psi_1)\rangle$, where $\Delta\Psi_1$ is the difference between measured and true reaction planes. We derive the analytical expression of calculating $\langle\cos(n\Delta\Psi_1)\rangle$as a function of flow parameters including finite multiplicity fluctuations. The value of $\langle\cos(\Delta\Psi_1)\rangle$ is a convenient measurement of the reaction plane resolution. We show how it can also be evaluated by the correlation of flow angles in different windows. The graphs amd tables of the interrelation of different parameters used for the reaction plane resolution are presented.

1 Introduction

In the previous approach of flow analysis[1], signals are extracted by statistically fitting the centroid of distributions of trigonometric moments calculated event by event under the same centrality. There is severe limitation on the bin size of pseudo-rapidity window because of enough statistics is required. The alternative way is to reconstruct $d^2E/d\phi d\theta$ of one specific detector(or some components of it) with respect to the reaction plane estimated exclusively by the other detector(or the other components of it if there is no information sharing)[2]. It has the advantages of both avoidance of autocorrelation and much narrower pseudo-rapidity windows, even down to the intrinsic detector granularity. However, this much simply method heavily depends on the precision of the determination of reaction plane, because of the difference between measured reaction plane(Ψ_1) and true reaction plane(Ψ_r) due to the finite particle multiplicity. Fourier coefficients obtained by analyzing $dE/d\phi$ have to be corrected back to the real ones by dividing some factor which could be computed in the event-averaged sense. The derivation of the formulism and the by-product of check up of some equations in [1] are shown in Sect 2. Then we turn to the topic of evaluation of reaction plane resolution. This can in principle be done in general $\mathbf{v}_n$ space. Constructing the angular distribution of $\bar{\Psi_n}(\bar{\Psi_n}\Psi_n - n\Psi_r)$, we calculate some useful quantities like $\langle\bar{\Psi_n}^2\rangle$,$\langle\cos n(\Psi_n - \Psi_r)\rangle$,and $\langle\cos(\Psi_n - \Psi_r)\rangle$in term of $\frac{\tilde{v_n}}{\sigma}$. The commonly used difinitions of resolution like $sqrt(\langle\bar{\Psi_n}^2\rangle)$, R,$\langle\cos\bar{\Psi_n}\rangle$ are plotted as functions of $\frac{\tilde{v_1}}{\sigma}$and their correspondence are expressed explictly in the table.

2 The Recovering Factor

At each pseudo-rapidity window, we can construct $dE_T/d\phi'$, and perform Fourier analysis,

$$\begin{aligned}
\frac{dE_T}{d\phi'} &= \frac{\langle E_T \rangle}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}(a_n \cos(n\phi') + b_n \sin(n\phi')) \\
\phi' &= \phi_{Lab} - \Psi_1 \\
\mid \tilde{\mathbf{v}}_n{}' \mid &= \frac{\sqrt{a_n^2 + b_n^2}}{\langle E_T \rangle} \\
sign(\tilde{\mathbf{v}}_n{}') &= sign(a_n)
\end{aligned}$$

where ϕ_{Lab} is the azimuthal angle of tower in the lab system, Ψ_1 is the reaction plane estimated by the other detector in each event, and ψ' is equivalent to the particle emission angle in the analysis. Because of the mirror symmetry of reaction plane, b_n should be equal to zero. The nonzeroness of b_n could be understood the systematic error of a_n.[3]

The next step is to recover the signals back to those in the true reaction plane.

Here,

$$\begin{aligned}
\phi' &= \phi_{Lab} - \Psi_1 \\
&= \phi_{Lab} - (\Psi_r + \Delta\Psi_1) \\
&= (\phi_{Lab} - \Psi_r) - \Delta\Psi_1 \\
&= \phi_r - \Delta\Psi_1
\end{aligned}$$

$\Delta\Psi_1$: angular resolution

ϕ_r : particle emission angle with respect to true reaction plane

, and

$$\begin{aligned}
\tilde{\mathbf{v}}_n{}' &= \frac{\langle E_T \cos(n\phi')\rangle}{\langle E_T\rangle} \\
&= \frac{\langle E_T \cos(n\phi_r)\rangle\langle\cos(n\Delta\Psi_1)\rangle}{\langle E_T\rangle} - \frac{\langle E_T \sin(n\phi_r)\rangle\langle\sin(n\Delta\Psi_1)\rangle}{\langle E_T\rangle} \\
&= \tilde{\mathbf{v}}_n\langle\cos(n\Delta\Psi_1)\rangle
\end{aligned}$$

$\langle\sin n\Delta\Psi_1\rangle = 0$ because of the symmetric distribution around $\Psi_r = 0$.

3 The Derivation of $\langle\cos(n\Delta\Psi_1)\rangle$

In order to derive the analytical expression of $\langle\cos(n\Delta\Psi)\rangle$, we need some formula in hand.

$$\begin{aligned}
I_n(x) &= \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)\cos(n\theta)d\theta \\
\int_0^\infty x^{a-1}\exp(-bx^2)I_n(cx)dx &= 2^{-n-1}b^{\frac{-a-n}{2}}c^n\frac{\Gamma(\frac{a+n}{2})}{\Gamma(1+n)}\,{}_1F_1(\frac{a+n}{2}, 1+n, \frac{c^2}{4b}) \\
{}_1F_1(a,a,x) &= \exp(x) \\
\Gamma(x+1) &= x\Gamma(x) \\
\Gamma(1/2) &= \sqrt{\pi}
\end{aligned}$$

First, we derive eq(17) in [1] as an exercise. The way of interpretation of the result is pretty straightforward . After specifying the direction of the reaction plane to be the X axis, the centroid of the two-dimensional Gaussian distribution is also in the same direction by symmetry, $\langle p_x\rangle N = \langle v_1\cos\Psi_1\rangle = \langle v_1\cos\Delta\Psi_1\rangle$ is now the mean of the projection of v_1 on the X axis and that is just the magnitude of the centroid $\tilde{v_1}$. The complete derivation can be done following.

$$
\begin{aligned}
\langle p_x \rangle N &= \langle v_1 \cos \Psi_1 \rangle \\
&= \langle v_1 \cos \Delta\Psi_1 \rangle \\
&= \frac{1}{2\pi\sigma^2} \int d^2\mathbf{v}_1 v_1 \cos \Psi_1 \exp(-\frac{(\mathbf{v}_1 - \tilde{\mathbf{v}}_1)^2}{2\sigma^2}) \\
&= \frac{1}{2\pi\sigma^2} \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \int_0^\infty v_1^2 dv_1 \exp(-\frac{v_1^2}{2\sigma^2}) * \\
&\quad \int_0^{2\pi} d\Delta\Psi_1 \cos \Delta\Psi_1 \exp(\frac{v_1 \tilde{v}_1}{\sigma^2} \cos \Delta\Psi_1) \\
&= \frac{1}{\sigma^2} \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \int_0^\infty v_1^2 \exp(-\frac{v_1^2}{2\sigma^2}) I_1(\frac{v_1 \tilde{v}_1}{\sigma}) dv_1 \\
&= \frac{1}{\sigma^2} \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) 2^{-2} (2\sigma^2)^2 (\frac{\tilde{v}_1}{\sigma^2}) \frac{\Gamma(2)}{\Gamma(2)} \, {}_1F_1(2, 2, \frac{\tilde{v_1}^2}{2\sigma^2}) \\
&= \tilde{v_1}
\end{aligned}
$$

Then, we derive $\langle \cos(n\Delta\Psi_1) \rangle$. Eq(18) in [1] is the special case of $n = 1$.

$$
\begin{aligned}
\langle \cos(n\Delta\Psi_1) \rangle &= \frac{1}{2\pi\sigma^2} \int d^2\mathbf{v}_1 \cos(n\Delta\Psi_1) \exp(-\frac{(\mathbf{v}_1 - \tilde{\mathbf{v}}_1)^2}{2\sigma^2}) \\
&= \frac{1}{\sigma^2} \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \int_0^\infty v_1 \exp(-\frac{v_1^2}{2\sigma^2}) I_n(\frac{v_1 \tilde{v}_1}{\sigma}) dv_1 \\
&= 2^{-n/2-1} \frac{\Gamma(n/2)}{\Gamma(n)} (\frac{\tilde{v_1}}{\sigma})^n \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \, {}_1F_1(1 + \frac{n}{2}, 1 + n, \frac{\tilde{v_1}^2}{2\sigma^2})
\end{aligned}
$$

When $n = 1$,

$$
\begin{aligned}
\langle \cos(n\Delta\Psi_1) \rangle &= 2^{-3/2} \frac{\Gamma(1/2)}{\Gamma(1)} (\frac{\tilde{v_1}}{\sigma}) \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \, {}_1F_1(\frac{3}{2}, 2, \frac{\tilde{v_1}^2}{2\sigma^2}) \\
&= \frac{\sqrt{\pi}}{2\sqrt{2}} (\frac{\tilde{v_1}}{\sigma}) \exp(-\frac{\tilde{v}_1^2}{2\sigma^2}) \, {}_1F_1(\frac{3}{2}, 2, \frac{\tilde{v_1}^2}{2\sigma^2})
\end{aligned}
$$

The validity of this equation can be checked by its asymptotic behavior. When $(\frac{\tilde{v_1}}{\sigma})$ comes down to zero, which means no flow signal or infinitely bad resolution, ${}_1F_1 \to 1, \langle\cos(n\Delta\Psi_1)\rangle \to 0$. On the other hand, when $(\frac{\tilde{v_1}}{\sigma})$ approaches infinity, which means very large flow signal and(or), larger multiplicity(i.e. $\Delta\Psi_1 \to 0$),

$$ {}_1F_1(a,b,x) = \frac{\Gamma(b)}{\Gamma(a)}\exp(x)x^{a-b}[1+O(|x|)^{-1}], \text{for large } |x| $$

,$\langle\cos(n\Delta\Psi_1)\rangle \to 1$ as expected.

We can also evaluate the dependence of the correction factor $\langle\cos(n\Delta\Psi_1)\rangle$ with respect to $(\frac{\tilde{v_1}}{\sigma})$ for different n. The Fig.4 in [1] shows this in $n = 1$. Here $\langle\cos(n\Delta\Psi_1)\rangle$ is plotted as a function of $(\frac{\tilde{v_1}}{\sigma})$ for $n = 1, 2, 3, 4$ in Fig. 1[3]. Just like what is expected, $\langle\cos(n\Delta\Psi_1)\rangle$ of the same shape parameter $(\frac{\tilde{v_1}}{\sigma})$ is smaller for larger n , and this means that higher-order flow signals suffer more seriously from angular resolution and need more significant correction. This also indicates that enough statistics to make the error less than the magnitude of $\tilde{\mathbf{v}}'$ is necessary to validate these correction procedures.

4 Resolution of Reaction Plane

Resolution of reaction plane can be defined as the RMS of the correlation distribution of Ψ_1 and Ψ_r in the $\mathbf{v}_1$ space and this can be also generalized to higher moments[4]. In the general $\mathbf{v}_n$ space, we express each vector in polar coordinate.

$$ \begin{aligned} \mathbf{v}_n &: (v_n, n\Psi_n) \\ \tilde{\mathbf{v}_n} &: (\tilde{v_n}, n\tilde{\Psi}_n) = (\tilde{v_n}, n\tilde{\Psi}_r) \end{aligned} $$

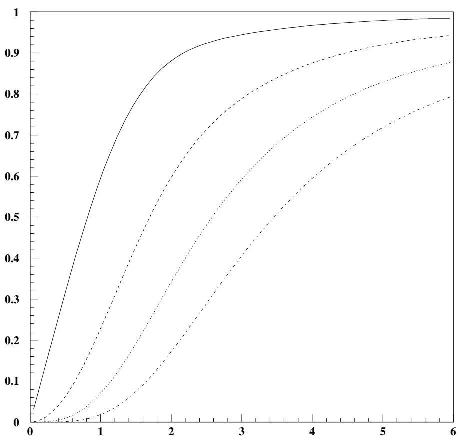

Figure 1: $\langle\cos(n\Delta\Psi_1)\rangle$as a function of $\frac{\tilde{v_1}}{\sigma}$for different moments, $n=1$(solid), 2(dash), 3(dot), 4(dash-dot)

So the angle between $\mathbf{v}_n$(charactering one event) and $\tilde{\mathbf{v}_n}$(average of events) in this space is defined as $\bar{\Psi}_n \equiv n\Psi_n - n\Psi_r$. The distribution of it can be expressed analytically as a function of $\frac{\tilde{v_n}}{\sigma}$[1].

$$\begin{aligned}
\frac{dN}{d\bar{\Psi}_n} &= \frac{1}{2\pi\sigma^2}\int v_n \exp(-\frac{\tilde{v_n}^2 + v_n^2 - 2\tilde{v_n}v_n\cos\Psi}{2\sigma^2})dv_n \\
&= \frac{1}{2\pi}[\,\exp(-\frac{1}{2}(\frac{\tilde{v_n}}{\sigma})^2) + \sqrt{\frac{\pi}{2}}\cos\Psi(\frac{\tilde{v_n}}{\sigma})\exp(-\frac{1}{2}\sin^2\Psi\,(\frac{\tilde{v_n}}{\sigma})^2)(1+erf(\frac{1}{2}\cos\Psi\,(\frac{\tilde{v_n}}{\sigma})))]
\end{aligned}$$

Fig. 2 is just the 2-d plot of the event distribution of $\bar{\Psi}_n$ and $(\frac{\tilde{v_n}}{\sigma})$. The width of distribution of $\bar{\Psi}_n$ become narrower with increasing larger flow signal detected.

With this distribution in hand, we can calculate several quantities which are convenient in the estimation of resolution.

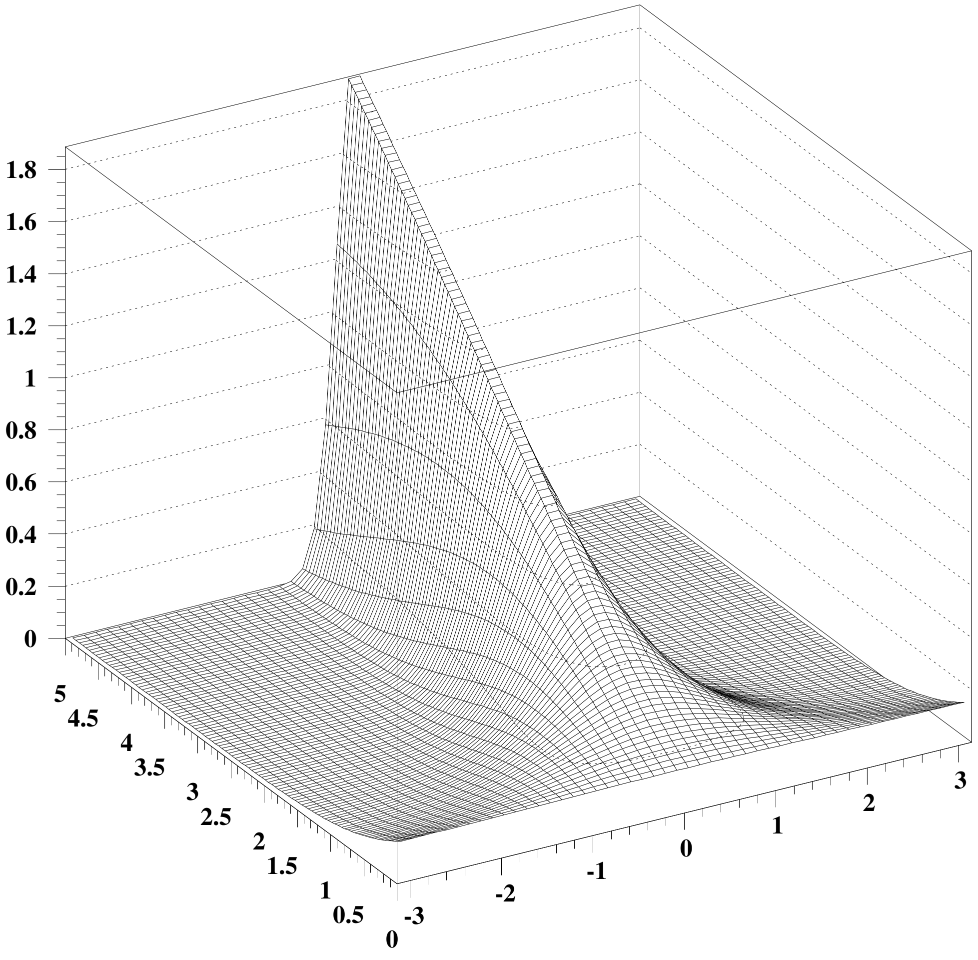

Figure 2: Distribution of events as function of $\bar{\Psi}_n$(X axis)and $\frac{\tilde{v_n}}{\sigma}$($Y$ axis)

- $\sqrt{\langle \bar{\Psi_n}^2 \rangle}$ (Plotted as function of $(\frac{\tilde{v_n}}{\sigma})$ in Fig. 3.)

- $\langle \cos(\Psi_n - \Psi_r) \rangle$ (Plotted as function of $(\frac{\tilde{v_n}}{\sigma})$ in Fig. 4.)

$$
\begin{aligned}
\langle \cos(\Psi_n - \Psi_r) \rangle &= \langle \cos(\frac{1}{n}\bar{\Psi}_n) \rangle \\
&= 2^{-1/2n-1}\frac{\Gamma(1/2n)}{\Gamma(1/n)}(\frac{\tilde{v_n}}{\sigma})^{1/n}\exp(-\frac{\tilde{v}_n^2}{2\sigma^2})\,{}_1F_1(1+\frac{1}{2n}, 1+\frac{1}{n}, \frac{\tilde{v_n}^2}{2\sigma^2})
\end{aligned}
$$

- $\langle \cos n(\Psi_n - \Psi_r) \rangle$(Plotted as a function of $(\frac{\tilde{v_n}}{\sigma})$ in Fig. 3 and as a function of $\sqrt{\langle \bar{\Psi_n}^2 \rangle}$ in Fig. 5.)

$$
\begin{aligned}
\langle \cos n(\Psi_n - \Psi_r) \rangle &= \langle \cos \bar{\Psi}_n \rangle \\
&= \frac{\sqrt{\pi}}{2\sqrt{2}}(\frac{\tilde{v_n}}{\sigma})\exp(-\frac{\tilde{v}_n^2}{2\sigma^2})\,{}_1F_1(\frac{3}{2}, 2, \frac{\tilde{v_n}^2}{2\sigma^2})
\end{aligned}
$$

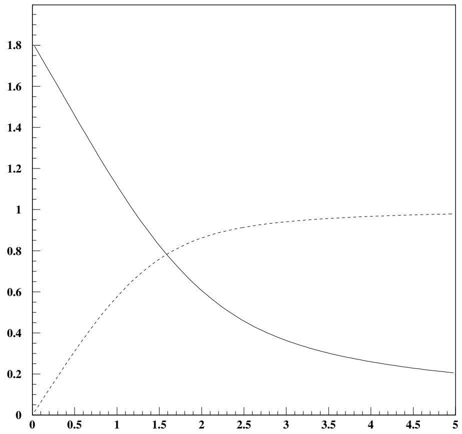

Figure 3: sqrt($\langle \bar{\Psi_n}^2 \rangle$)(solid) and $\langle \cos n(\Psi_n - \Psi_r) \rangle$(dash) as a function of $\frac{\tilde{v_n}}{\sigma}$

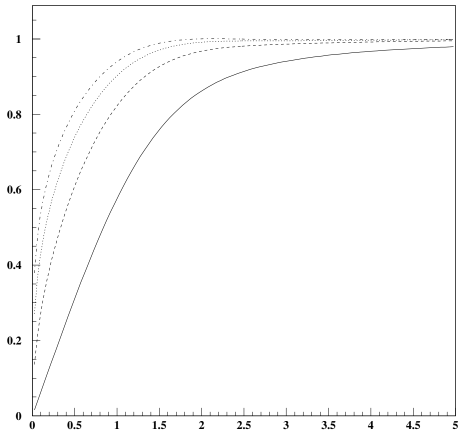

Figure 4: $\langle \cos(\Psi_n - \Psi_r) \rangle$as a function of $\frac{\tilde{v_n}}{\sigma}$for $n = 1$(solid),2(dash),3(dot),4(dash-dot)

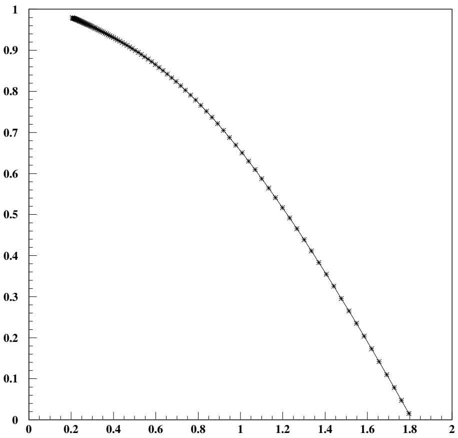

Figure 5: $\langle \cos n(\Psi_n - \Psi_r)\rangle$as a function of sqrt($\langle \bar{\Psi_n}^2 \rangle$)

References

[1] S.Voloshin, Y.Zhang, *hep-ph/9407282* (July 1994)

[2] S.Voloshin, *E877 Note#PITT-95-001*

[3] Y.Zhang, private communication

[4] S.Voloshin, private communication