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The Perturbative Effect of Higher Fourier Moments on the Shape of Azimuthal Distributions

Wen-Chen Chang

State University of New York, Stony Brook, NY 11794-3800

ABSTRACT

This brief note is to provide a mathematical and graphical demonstration of how the shape of a circular distribution varies with the perturbation of higher Fourier moments.

The starting point is that the zeroth moment of Fourier analysis of an azimuthal distribution represents a circular component in polar space. This could be expressed in term of a radius function $r(\phi)$ or the Cartesian coordinate x_0, y_0

$$\frac{dN}{d\phi} = r(\phi) = r_0 = \sqrt{x_0^2 + y_0^2}$$

If there appears only the perturbation of the first moment $\varepsilon(\ll 1)$, then,up to the order of ε^2 accuracy,

$$\begin{aligned}
\frac{dN}{d\phi} &= r_0(1 + \varepsilon \cos \phi) && (1)\\
&\simeq r_0(1 + 2\varepsilon \cos \phi)^{1/2} && (2)\\
&\simeq (r_0^2 + 2\varepsilon r_0^2 \cos \phi)^{1/2} && (3)\\
&\simeq (x_0^2 + y_0^2 + 2\varepsilon' x_0)^{1/2} && (4)\\
&\simeq ((x_0 + \varepsilon')^2 + y_0^2)^{1/2} && (5)
\end{aligned}$$

which could be viewed as the approximation of the shift of the center along x axis. $\varepsilon' = \varepsilon r_0$ should be much less than x_0 to validate the approximation and, however, this means that the approximation is never valid around the region of $\phi = \pi/2, 3\pi/2$.

The case of second moment can be done in the similar way,

$$\begin{aligned}
\frac{dN}{d\phi} &= r_0(1 + \varepsilon \cos 2\phi) && (6)\\
&\simeq r_0(1 + 2\varepsilon \cos 2\phi)^{1/2} && (7)\\
&\simeq (r_0^2 + 2\varepsilon r_0^2(\cos \phi^2 - \sin \phi^2))^{1/2} && (8)\\
&\simeq (x_0^2 + y_0^2 + 2\varepsilon x_0^2 - 2\varepsilon y_0^2)^{1/2} && (9)\\
&\simeq ([(1 + \varepsilon)x_0]^2 + [(1 - \varepsilon)y_0]^2)^{1/2} && (10)
\end{aligned}$$

which is the simultaneous constant scaling of the x, y axis and makes the shape elliptical. Here only $\varepsilon \ll 1$ is the only requirement.

As for the third moment,

$$
\begin{aligned}
\frac{dN}{d\phi} &= r_0(1+\varepsilon\cos 3\phi) && (11)\\
&\simeq r_0(1+2\varepsilon\cos 3\phi)^{1/2} && (12)\\
&\simeq (r_0^2+2\varepsilon r_0^2(\cos\phi^3-3\sin\phi^2\cos\phi))^{1/2} && (13)\\
&\simeq (x_0^2+y_0^2+\varepsilon' x_0^3-3\varepsilon' y_0^2 x_0)^{1/2} && (14)\\
&\simeq ([(1+\varepsilon'' x_0)x_0]^2+[(1-3\varepsilon'' x_0)y_0]^2)^{1/2} && (15)
\end{aligned}
$$

which is still the scaling of coordinate but the scaling factors depends on x_0. The assumption that $\varepsilon'' x_0 = \varepsilon x_0/r_0 \ll 1$ satisfied everywhere.

In principle, this interpretation can be extended to higher moments. We show graphical results in the figures up to the $4th$ moments. Observation of these figures shows that the $1st$ moment effectively shifts the center of distribution and changes the circular shape simultaneously. The other higher moments only deform the overall shape symmetrically.

I want to thank Yingchao for all the initiatives for thinking this problem.

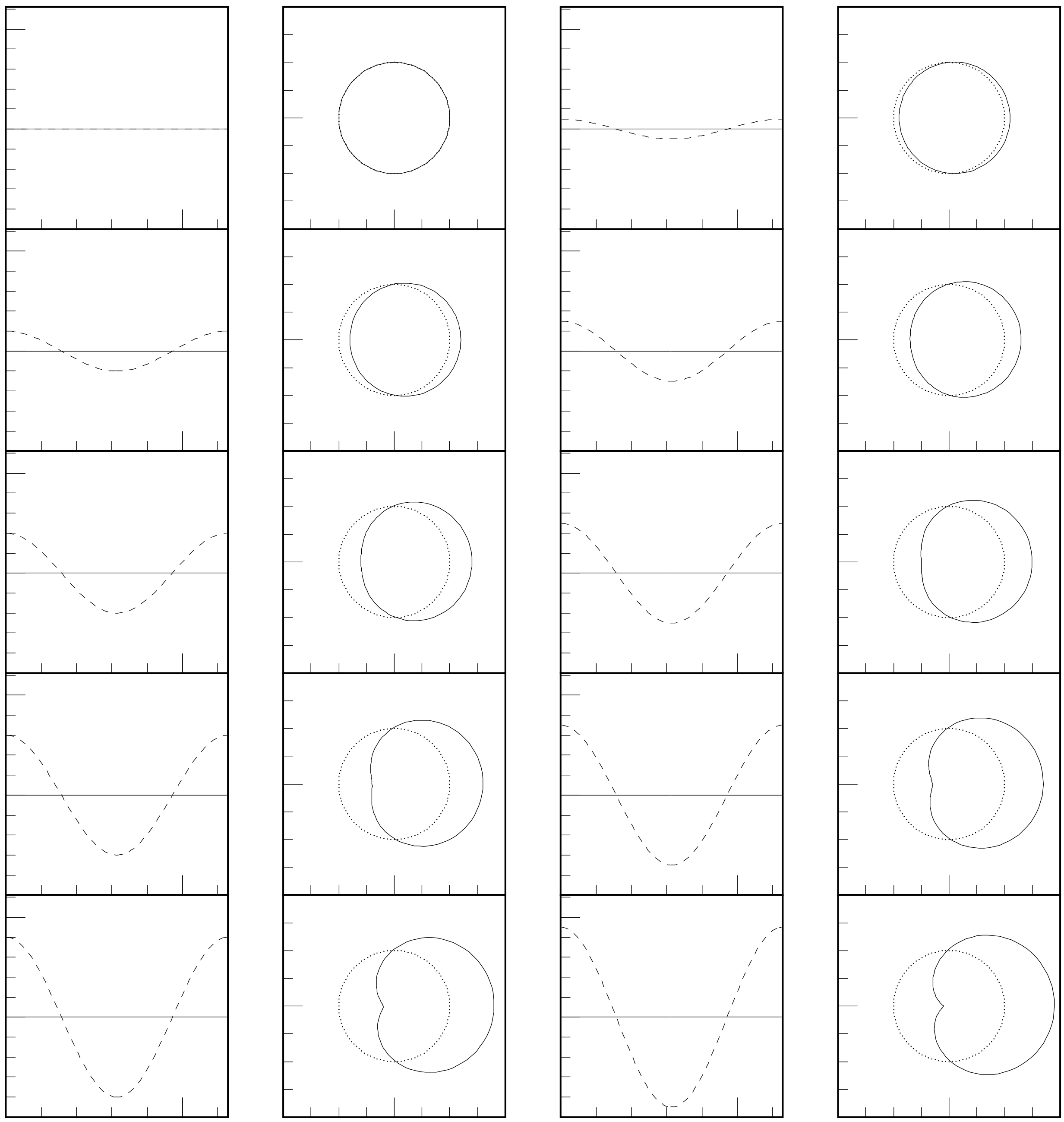

Figure 1: Azimuthal distribution and representation in polar coordinate of $\frac{dN}{d\phi} = (1 + \varepsilon \cos\phi)$, with ε varies from 0 to 0.9 in steps of 0.1

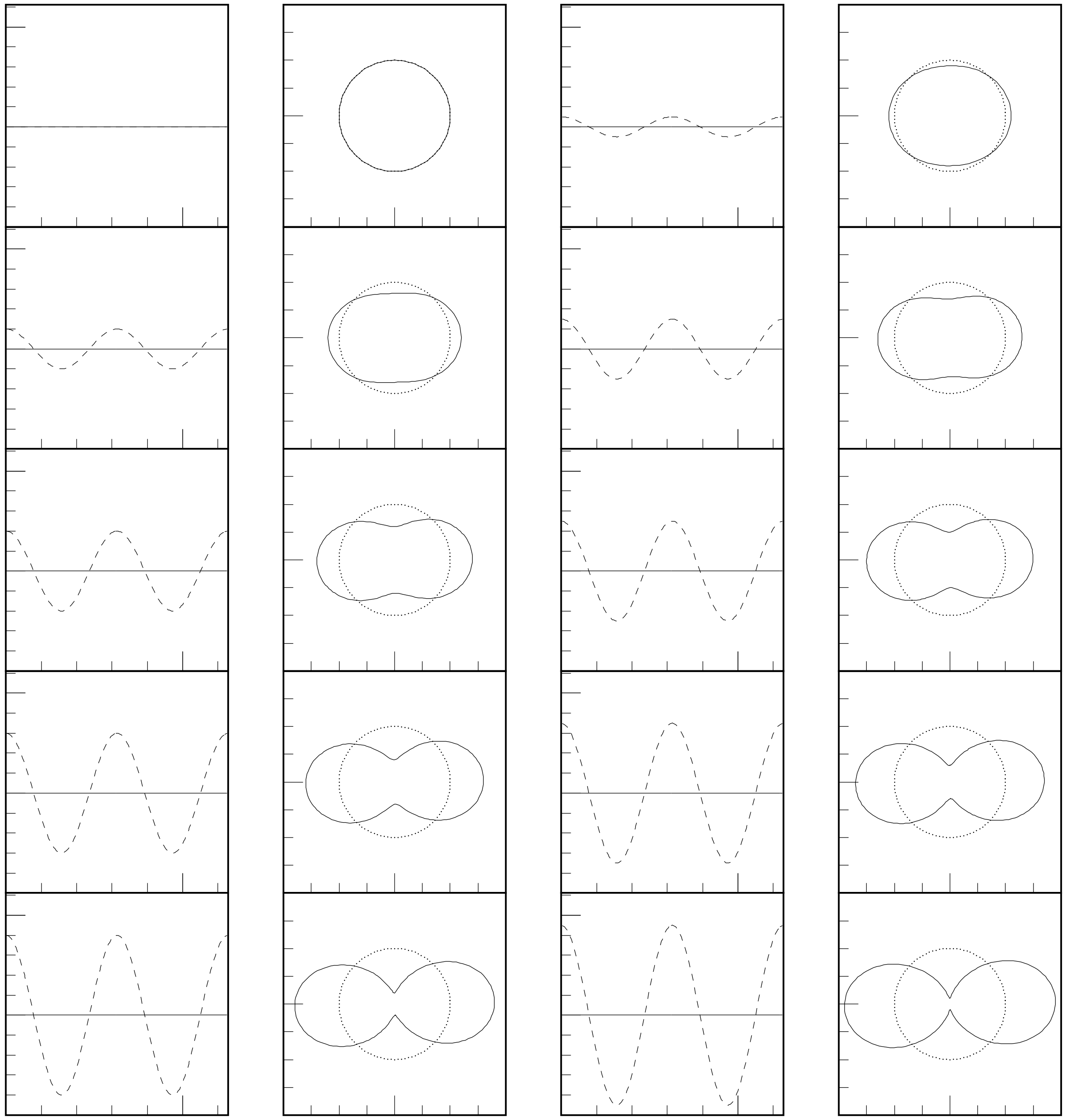

Figure 2: Azimuthal distribution and representation in polar coordinate of $\frac{dN}{d\phi} = (1 + \varepsilon \cos 2\phi)$, with ε varies from 0 to 0.9 in steps of 0.1

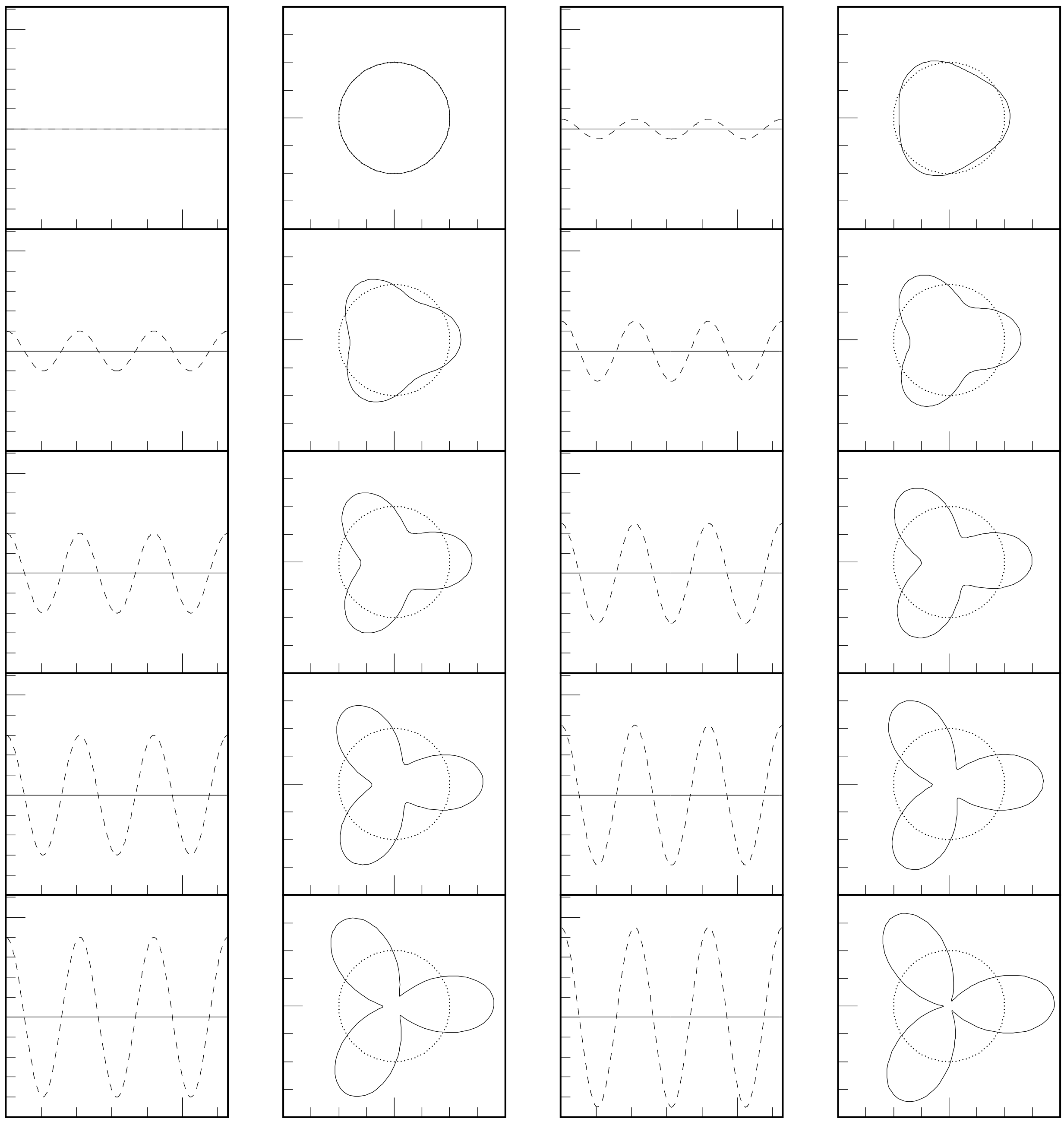

Figure 3: Azimuthal distribution and representation in polar coordinate of $\frac{dN}{d\phi} = (1 + \varepsilon \cos 3\phi)$, with ε varies from 0 to 0.9 in steps of 0.1

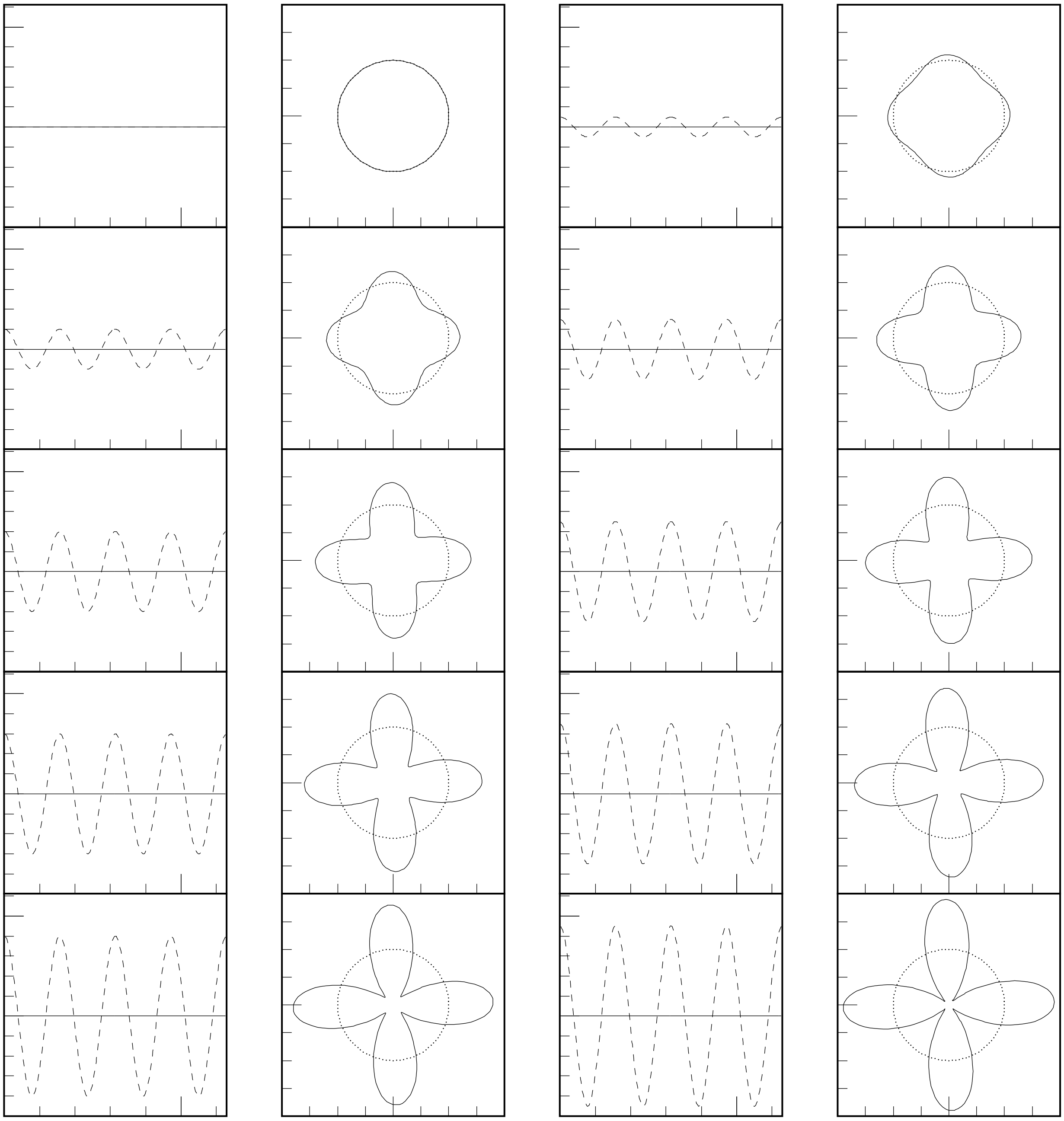

Figure 4: Azimuthal distribution and representation in polar coordinate of $\frac{dN}{d\phi} = (1 + \varepsilon \cos 4\phi)$, with ε varies from 0 to 0.9 in steps of 0.1