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REACTION PLANE RESOLUTION FROM FLOW MEASUREMENTS IN TWO RAPIDITY WINDOWS

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ABSTRACT

We discuss the application of the maximum likelihood technique to the problem of event by event resolution of the reaction plane angle using the information on directed flow in two different (pseudo)rapidity windows (detectors).

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In this note we address the problem of determination of the reaction plane in the case where measurements of energy flow from more than one detector (or from more than one rapidity "window" in the same detector) is available. The notation used is generally that of References 1 and 2.

We start from the distribution in magnitudes of directed flow and flow angles in two (independent) (pseudo)rapidity windows. The distribution in each of the windows (here window "a") is [1]:

$$\frac{d^2w}{v_{1a}\, dv_{1a}\, d(\psi_{1a}-\psi_r)} = \frac{1}{2\pi\sigma_a^2}\exp(-\frac{\tilde{v}_{1a}^2 + v_{1a}^2 - 2\tilde{v}_{1a}v_{1a}\cos(\psi_{1a}-\psi_r)}{2\sigma_a^2}), \qquad (1)$$

where the parameter $\tilde{v}_{1a}$ describes the average magnitude of flow in this rapidity region, the parameter σ_a takes into account the finite multiplicity fluctuations and detector resolution; v_1 and ψ_{1a} are flow parameters measured in each event. The distribution corresponding to the window "b" has the same form. It is convenient to introduce the normalized variables $\xi_1 = v_1/\sigma$. The distribution in this case reads:

$$\frac{d^2w}{\xi_{1a}\, d\xi_{1a}\, d(\psi_{1a}-\psi_r)} = \frac{1}{2\pi}\exp(-\frac{\tilde{\xi}_{1a}^2 + \xi_{1a}^2 - 2\tilde{\xi}_{1a}\xi_{1a}\cos(\psi_{1a}-\psi_r)}{2}), \qquad (2)$$

The combined distribution in $\xi_{1a}, \psi_{1a}, \xi_{1b}, \psi_{1b}$

$$\frac{d^4w}{d\xi_{1a}^2 d\psi_{1a} d\xi_{1b}^2 d\psi_{1b}} = F(\xi_{1a}, \psi_{1a}, \xi_{1b}, \psi_{1b}; \psi_r) \qquad (3)$$

is the product of two distributions (2) for windows "a" and "b".

From the point of view of the estimate of ψ_r, the reaction plane angle resolution the function $F(\xi_{1a}, \psi_{1a}, \xi_{1b}, \psi_{1b}; \psi)$ can be considered as a likelihood function, which can be maximized to yield the most probable value of ψ_r for a given set of experimental values of $\xi_{1a}, \psi_{1a}, \xi_{1b}, \psi_{1b}$. The solution of the corresponding equation for an extremum gives for an estimator of ψ_r the expression

$$\psi = \psi_{1a} + \arctan(\frac{\xi_{1b}\tilde{\xi}_{1b}\sin(\psi_{1b}-\psi_{1a})}{\xi_{1a}\tilde{\xi}_{1a} + \xi_{1b}\tilde{\xi}_{1b}\sin(\psi_{1b}-\psi_{1a})}) \qquad (4)$$

Due to the periodicity of tangent function one should be careful in the evaluation of ψ and check that the extremum corresponds to a maximum in the likelihood function at a given value of ψ.

We have performed a simple Monte Carlo calculation, constructing the distribution in ψ using Eq. (4) for a few values of parameters $\tilde{\xi}_{a,b}$. The results are presented in Figure 1. It was assumed in the simulation that the angle $\psi_r = 0$.

For comparison we also show the distribution which arises from the "weighted angles" method, when the flow angles measured in different windows are weighted with weights inversely proportional to the angular resolution

$$\psi_{wa} = \frac{\psi_{1a}\langle(\psi_{1b} - \psi r)^2\rangle + \psi_{1b}\langle(\psi_{1a} - \psi r)^2\rangle)}{\langle(\psi_{1b} - \psi r)^2\rangle + \langle(\psi_{1a} - \psi r)^2\rangle} \tag{5}$$

One of the reasons why the maximum likelihood method gives better results is that it takes into account the fluctuations in the magnitude of flow in each window: when the magnitude of flow is small the resolution in flow angle is poor. Having this in mind it is also possible to improve the weighting method introducing weights which include the magnitude of flow seen in each rapidity window:

$$\psi_{iwa} = \frac{\psi_{1a}\tilde{\xi}_{1a}\xi_{1a} + \psi_{1b}\tilde{\xi}_{1b}\xi_{1b}}{\tilde{\xi}_{1a}\xi_{1a} + \tilde{\xi}_{1b}\xi_{1b}} \tag{6}$$

In Table 1 we present the values of $\langle cos(\psi)\rangle$ for different parameters of flow in two windows. The values of mean $cos(\psi)$ indicate the resolution in the different methods. $\tilde{\xi}_{1a}$ and $\tilde{\xi}_{1b}$ are the input values for Monte Carlo simulation to produce a Gaussian distribution according to Eq. 1 and 2. The mean values of $\cos(\psi_{1a})$ and $\cos(\psi_{1a})$ are presented in columns "W1" and "W2". The three methods of determining the reaction plane angle are compared in the last 3 columns of the table. The notation "LF", "WA", and "IWA" are used to denote the likelihood function method (Eq. 4) and the two weighted angles methods (Eq. 5 and 6, respectively). In each case the likelihood function shows somewhat better resolution than the two methods of weighted angles. Qualitatively the same conclusion can be drawn from the distributions of ψ shown in Fig. 1.

Table 1: Mean cosines of different flow angles for different parameters of flow

flow parameters		mean cosines				
$\tilde{\xi}_{1a}$	$\tilde{\xi}_{1b}$	W1	W2	LF	WA	IWA
0.5	0.5	0.303	0.305	0.418	0.379	0.410
0.5	1.0	0.303	0.559	0.608	0.565	0.599
1.0	1.0	0.559	0.555	0.710	0.663	0.703
0.5	1.5	0.304	0.735	0.758	0.743	0.750

References

[1] S. Voloshin and Y. Zhang, hep-ph/9407282.

[2] J. Barrette *et al.*, Phys. Rev. Lett. **73**, 2532 (1994).

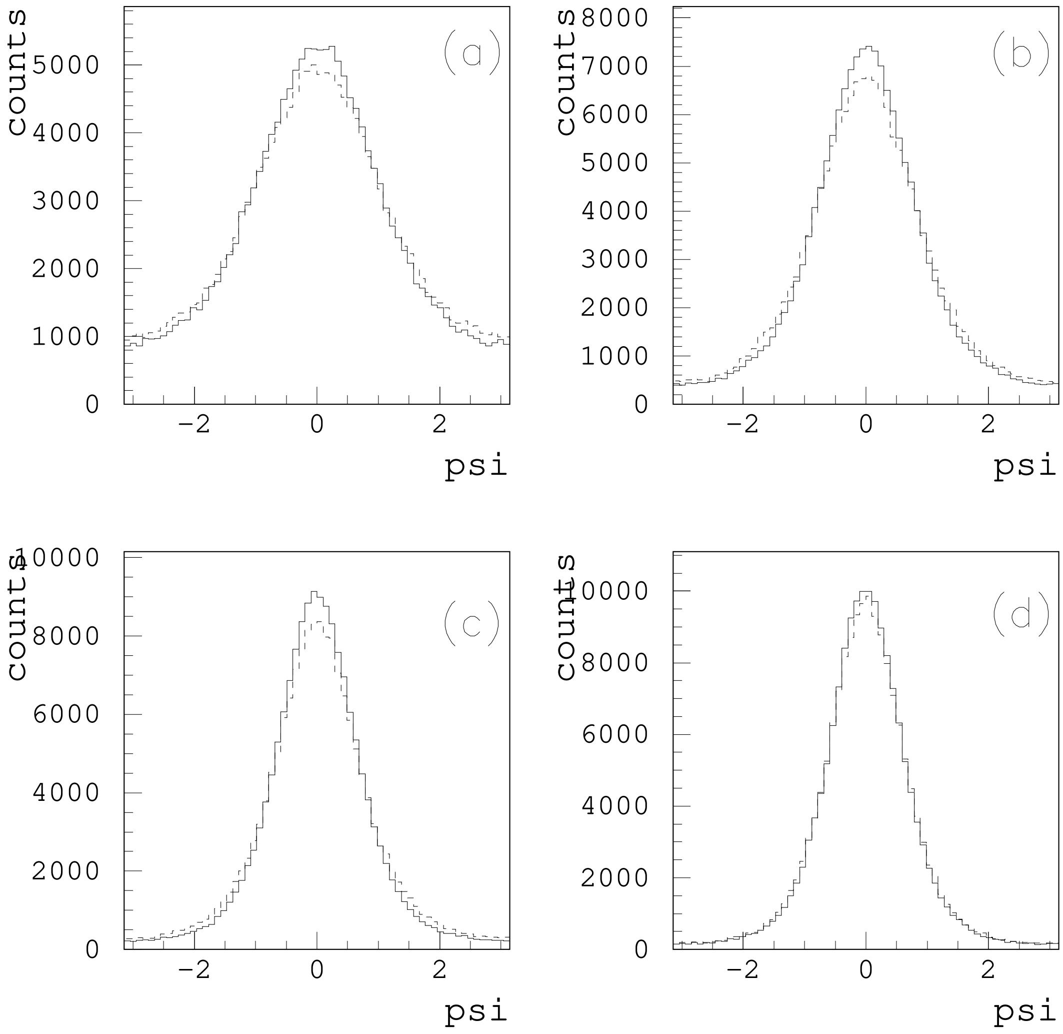

Figure 1: The reaction plane resolution by the maximum likelihood method (solid histograms) compared with the "weighted angles" method (dashed histograms for different parameters of flow. (a) $\tilde{\xi}_{1a} = 0.5$, $\tilde{\xi}_{1b} = 0.5$ (b) $\tilde{\xi}_{1a} = 0.5$, $\tilde{\xi}_{1b} = 1.0$ (c) $\tilde{\xi}_{1a} = 1.0$, $\tilde{\xi}_{1b} = 1.0$ (d) $\tilde{\xi}_{1a} = 0.5$, $\tilde{\xi}_{1b} = 1.5$