overlap - nuclear overlap calculation

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1 Introduction

The nuclear overlap model, as described by Eskola in Nucl. Phys. B323(1989)37, expresses a nucleus-nucleus collision in terms of binary collisions between nucleons. The mass distributions within the two colliding nuclei is used to calculate the number of participating nucleons and the number of nucleon-nucleon (NN) collisions as a function of the impact parameter, as well as the total geometric cross section.

This note describes the program `overlap`. The source code is `overlap.f`. The input, the calculation, and the output are described below. The lengths are expressed in fm and cross sections in barn, mb, or fm^2. Please note that

$$\begin{aligned} 1\text{ barn} &= 1000\text{ mb} \\ 1\text{ barn} &= 100\text{ fm}^2 \\ 1\text{ fm}^2 &= 10\text{ mb} \\ 1\text{ fm}^{-2} &= 0.1\text{ mb}^{-1} \end{aligned}$$

2 Input

A	–	mass number of the projectile nucleus
B	–	mass number of the target nucleus
denflag	–	density profile selection, 1 for sharp sphere or 2 for Woods-Saxon
sigma_NN	–	inelastic NN cross section in mb, recommended values are 30 for 10-200 GeV LAB, 37, 41, 42, 60 for $\sqrt{s}$=56, 130, 200, 5500 GeV, respectively
statistics	–	number of trials per integral, 1000 is good for quick tests

Please note that the description of the density becomes unrealistic for low A. The systematic error related to this can be estimated by running the code with B=1, and comparing the obtained TAB(0) value with TA(0), the latter being the relevant quantity for pA. The result is shown below. The discrepancy is largest for Woods-Saxon profile with low A.

profile	A	B	TAB(0)/TA(0)
Woods-Saxon	236	1	0.9
Woods-Saxon	12	1	0.6
sharp sphere	236	1	1.0
sharp sphere	12	1	0.9

3 Calculation

3.1 Density

Density is the number of nucleons per unit volume. The user has a choice between two density profiles, sharp sphere

$$n_A(r) = \begin{cases} n_0, & r \leq R \\ 0, & r > R \end{cases} \tag{1}$$

with

$$\begin{aligned} n_0 &= 0.17 \text{ fm}^{-3}, \\ R &= \left(\frac{3A}{4\pi \cdot n_0} \right)^{1/3} \end{aligned} \tag{2}$$

and Woods-Saxon

$$n_A(r) = \frac{n_0}{1 + \exp(\frac{r-R}{d})} \ . \tag{3}$$

with

$$\begin{aligned} n_0 &= 0.17 \text{ fm}^{-3}, \\ R &= (1.12 \ A^{1/3} - 0.86 \ A^{-1/3}) \text{ fm}, \\ d &= 0.54 \text{ fm}. \end{aligned} \tag{4}$$

Both densities are normalized to the number of nucleons:

$$\int d^3r \ n_A(r) = 4\pi \int_0^\infty r^2 \ dr \ n_A(r) = A. \tag{5}$$

3.2 Thickness function

Thickness function is the density integrated along the beam axis z:

$$T_A(b) = \int_{-\infty}^{\infty} dz \ n_A(\sqrt{b^2 + z^2}). \tag{6}$$

The integral of the thickness function is

$$\int d^2b \ T_A(b) = A. \tag{7}$$

In the sharp sphere case the thickness function is the product of the density and the path length:

$$\begin{aligned} T_A(b) &= 2n_0\sqrt{R^2 - b^2} && \xrightarrow{b\to 0} \quad 2n_0 R \\ T_A(b) &= 3/2 \ \tfrac{A}{\pi R^3} \ \sqrt{R^2 - b^2} && \longrightarrow \quad \tfrac{3A}{2\pi R^2} \\ T_A(b) &= 2 \ n_0 \sqrt{\left(\tfrac{3A}{4\pi n_0}\right)^{2/3} - b^2} && \longrightarrow \quad \left(\tfrac{6An_0^2}{\pi}\right)^{1/3} \quad \approx \quad 0.38 \ A^{1/3}. \end{aligned} \tag{8}$$

Physically $T_A(b) \ \sigma_{\rm NN}$ can be interpreted as the number of NN collisions encountered by a nucleon passing through a nucleus at an impact parameter b. The parameter $\sigma_{\rm NN}$ is the total inelastic nucleon-nucleon cross section.

Hard processes like D production scale with the number of NN collisions. If we denote the D production cross section in nucleon-nucleon collision by $\sigma_{\rm NN}^D$, then the D meson multiplicity in a pA collision at impact parameter b is given by $T_A(b) \ \sigma_{\rm NN}^D$. The inclusive D meson production cross section in pA, which can be obtained by integrating the D multiplicity over b, is then equal to $A \ \sigma_{\rm NN}^D$.

3.3 Overlap function

For a given impact parameter $\vec{b} := \langle \vec{r}_B \rangle - \langle \vec{r}_A \rangle$, the overlap function is defined as the product of the thickness functions of the colliding nuclei A and B, integrated over the two transverse dimensions:

$$T_{AB}(b) = \int d^2s\; T_A(\vec{s})\; T_B(\vec{s} - \vec{b}) \tag{9}$$

with $\vec{b}$ and $\vec{s}$ being perpendicular to the beam direction z. The overlap function can also be calculated directly from densities by a four-dimensional integration:

$$T_{AB}(b) = \int d^2s\; dz_A\; dz_B\; n_A(\vec{s}, z_A)\; n_B(\vec{s} - \vec{b}, z_B). \tag{10}$$

The integral of the overlap function is

$$\int d^2b\; T_{AB}(b) = AB. \tag{11}$$

With the sharp-sphere density description, the overlap function for the central A+A collision is

$$\begin{aligned} T_{AB}(0) &= 2\pi R^4 n_0^2 \\ T_{AB}(0) &= \tfrac{9A^2}{8\pi R^2} \\ T_{AB}(0) &= \left(\tfrac{81A^4 n_0^2}{32\pi}\right)^{1/3} \approx 0.29A^{4/3}. \end{aligned} \tag{12}$$

Physically, $T_{AB}(b)\; \sigma_{\rm NN}$ can be interpreted as the number of NN collisions in a A+B collision at impact parameter b. The parameter $\sigma_{\rm NN}$ is the total inelastic nucleon-nucleon cross section.

Hard processes like D production scale with the number of NN collisions. If we denote the D production cross section in nucleon-nucleon collision by $\sigma^D_{\rm NN}$, then the D meson multiplicity in a A+B collision at impact parameter b is given by $T_{AB}(b)\; \sigma^D_{\rm NN}$. The inclusive D meson production cross section in A+B, which can be obtained by integrating the D multiplicity over b, is then equal to $AB\; \sigma^D_{\rm NN}$.

3.4 Number of participants

Participants are nucleons that have encountered at least one NN collision. The mean number of participants in pA and A+B collisions at an impact parameter b can be calculated via

$$N\text{part}_{pA}(b) = \left\{1 - \left[1 - \frac{\sigma_{\rm NN}\; T_A(b)}{A}\right]^A\right\} + T_A(b)\; \sigma_{\rm NN} \tag{13}$$

$$N\text{part}_{AB}(b) = \int d^2s\; T_A(\vec{s}) \left\{1 - \left[1 - \frac{\sigma_{\rm NN}\; T_B(\vec{s} - \vec{b})}{B}\right]^B\right\} \tag{14}$$

$$+ \int d^2s\; T_B(\vec{s}) \left\{1 - \left[1 - \frac{\sigma_{\rm NN}\; T_A(\vec{s} + \vec{b})}{A}\right]^A\right\}. \tag{15}$$

The two terms represents the contributions from the two colliding objects. The factor repeatedly present in these formulas represents the probability for a nucleon to pass through the nucleus without any collision:

$$p(ncol = 0) = \left[1 - \frac{\sigma_{\rm NN}\; T_A(b)}{A}\right]^A \tag{16}$$

This probability is obtained using the binomial distribution of number of NN collisions. A numerically similar result can be obtained using the simpler Poissonian

$$p(ncol = 0) = e^{-\sigma_{\rm NN}\ T_A(b)}. \tag{17}$$

The probability of becoming participant is $p(ncol > 0) = 1 - p(0) = 1 - e^{-\sigma_{\rm NN}\ T_A(b)}$, and the number of participants is then

$$N{\rm part}_{pA}(b) = \left\{1 - e^{-\sigma_{\rm NN}\ T_A(b)}\right\} + T_A(b)\ \sigma_{\rm NN} \tag{18}$$

$$N{\rm part}_{AB}(b) = \int d^2s\ T_A(\vec{s}) \left\{1 - e^{-\sigma_{\rm NN}\ T_B(\vec{s}-\vec{b})}\right\} + \int d^2s\ T_B(\vec{s}) \left\{1 - e^{-\sigma_{\rm NN}\ T_B(\vec{s}+\vec{b})}\right\}. \tag{19}$$

3.5 Total geometric cross section

The total geometric cross section is the cross section for such A+B collisions in which at least one NN collision occurs:

$$\sigma_G = \int d^2b\ \left[1 - e^{-T_{AB}(b)\ \sigma_{\rm NN}}\right]. \tag{20}$$

For the sharp sphere density profile and in the limit of an infinite nucleon-nucleon cross section $\sigma_{\rm NN}$ the total geometric cross section should approach

$$\sigma_G = \pi(R_A + R_B)^2 = \left(\frac{9\ \pi}{16\ n_0^2}\right)^{1/3} \left[A^{1/3} + B^{1/3}\right]^2 \approx 3.94 \left[A^{1/3} + B^{1/3}\right]^2. \tag{21}$$

4 Output

4.1 Output format

The code prints the values of the arguments, then the calculated total cross section. The rest of output consists of several columns of numbers. The first column is the abscissa and represents the radius r or the impact parameter b in fm. Following are the densities of the colliding nuclei $n_A(r)$ and $n_B(r)$, their thickness functions $T_A(b)$ and $T_B(b)$, the overlap function $T_{AB}(b)$, then again the overlap function but calculated using a different method (a disagreement between the two is a sign that the used statistics was too low), numbers of participants from A and from B (A_{part},B_{part}), the contribution of the given impact parameter slice to the total cross section ($sigma$), and finally the centrality, i.e. the cross section for collisions with an impact parameter between 0 and b divided by the total cross section. An example of the output is shown below.

```
******************************************************************************************
 Nuclear overlap calculation
 Enter A,B
 Select density profile: 1 for sharp sphere or 2 for Woods-Saxon
 Enter sigma_NN in mb
 Enter number of shots per integral (e.g. 10000)
******************************************************************************************
 A ................  208.
 B ................  197.
 sigma_NN .........  38. mb
 statistics ........  30000
 density profile ...  Woods-Saxon
******************************************************************************************
 Calculating TA for A and B
 Calculating TAB for A+B collision
 Calculating Npart and TAB for A+B collision
 Calculating cross section for A+B collision
 cross section ..... 7.092 barn
```

```
*********************************************************************************************
 b ................ impact parameter or radius (fm)
 nA ............... density of nucleus A (1/fm^3)
 nB ............... density of nucleus B (1/fm^3)
 TA ............... thickness function of A (1/fm^2)
 TB ............... thickness function of B (1/fm^2)
 TAB .............. overlap function of A+B (1/fm^2)
 TABcheck.......... TAB calculated via TA and TB
 Apart ............ number of participants from A
 Bpart ............ number of participants from B
 sigma ............ cross section ds/db*0.1 fm (fm^2)
 cent ............. fraction of total cross section
*********************************************************************************************
```

b	nA	nB	TA	TB	TAB	TABcheck	Apart	Bpart	sigma	cent
0.05	0.170	0.170	2.215	2.174	300.524	306.128	198.495	190.450	0.031	0.000
0.15	0.170	0.170	2.202	2.153	304.809	305.482	198.380	190.315	0.094	0.000
0.25	0.170	0.170	2.212	2.167	299.831	302.380	196.504	188.693	0.157	0.000
0.35	0.170	0.170	2.215	2.160	305.086	309.382	200.418	192.350	0.220	0.001
0.45	0.170	0.170	2.189	2.162	301.327	300.823	196.282	188.342	0.283	0.001
0.55	0.170	0.170	2.191	2.147	288.262	311.906	201.823	193.817	0.346	0.002
0.65	0.170	0.170	2.194	2.169	300.122	303.332	197.622	189.716	0.408	0.002
0.75	0.170	0.170	2.185	2.144	300.392	302.418	196.487	188.337	0.471	0.003
0.85	0.170	0.170	2.187	2.155	304.341	299.117	195.480	187.891	0.534	0.004
0.95	0.170	0.170	2.189	2.138	292.538	297.281	194.179	186.329	0.597	0.004
1.05	0.170	0.170	2.171	2.143	289.995	297.381	193.628	186.793	0.660	0.005
1.15	0.170	0.170	2.170	2.121	301.532	292.714	192.400	185.654	0.723	0.006
1.25	0.170	0.170	2.169	2.137	291.566	294.301	192.815	185.939	0.785	0.007
1.35	0.170	0.170	2.150	2.129	300.017	289.067	190.803	183.807	0.848	0.009
1.45	0.170	0.170	2.157	2.102	294.802	294.474	192.743	186.157	0.911	0.010
1.55	0.170	0.170	2.132	2.096	292.712	280.834	185.871	178.748	0.974	0.011
1.65	0.170	0.170	2.122	2.098	288.827	284.375	187.610	181.018	1.037	0.013
1.75	0.170	0.170	2.126	2.069	280.473	283.571	187.595	180.799	1.100	0.014
1.85	0.170	0.170	2.119	2.076	280.946	283.873	187.305	181.206	1.162	0.016
1.95	0.170	0.170	2.107	2.059	277.115	280.157	187.221	178.951	1.225	0.018
2.05	0.170	0.170	2.089	2.058	281.614	276.426	183.524	177.734	1.288	0.020
2.15	0.170	0.170	2.079	2.037	273.103	267.322	180.559	173.641	1.351	0.021
2.25	0.170	0.170	2.061	2.016	266.790	270.640	180.946	176.012	1.414	0.023
2.35	0.170	0.170	2.056	2.021	272.931	261.966	176.429	171.052	1.477	0.026
2.45	0.170	0.170	2.027	1.980	265.807	273.173	182.624	177.808	1.539	0.028
2.55	0.170	0.170	2.017	1.981	256.821	269.147	180.497	175.334	1.602	0.030
2.65	0.170	0.170	2.011	1.956	256.049	258.155	174.493	169.258	1.665	0.032
2.75	0.170	0.170	1.988	1.950	250.209	253.325	170.716	166.772	1.728	0.035
2.85	0.170	0.170	1.972	1.940	250.668	251.617	172.166	166.258	1.791	0.037
2.95	0.170	0.170	1.948	1.907	244.754	250.879	171.457	166.992	1.854	0.040
3.05	0.170	0.170	1.943	1.893	252.517	245.776	168.228	163.369	1.916	0.043
3.15	0.170	0.170	1.905	1.876	245.027	242.584	166.632	162.837	1.979	0.045
3.25	0.170	0.169	1.893	1.866	236.961	242.516	166.361	163.004	2.042	0.048
3.35	0.169	0.169	1.874	1.830	237.807	234.087	162.111	157.320	2.105	0.051
3.45	0.169	0.169	1.856	1.816	238.464	236.683	163.793	159.612	2.168	0.054
3.55	0.169	0.169	1.823	1.781	230.838	234.208	162.615	158.587	2.231	0.057
3.65	0.169	0.169	1.810	1.766	231.862	223.043	156.247	152.728	2.293	0.061
3.75	0.169	0.169	1.786	1.734	227.528	225.927	157.622	154.567	2.356	0.064
3.85	0.169	0.168	1.747	1.696	210.380	217.560	153.424	150.190	2.419	0.067
3.95	0.168	0.168	1.717	1.682	223.550	214.991	151.775	148.671	2.482	0.071
4.05	0.168	0.168	1.702	1.650	209.885	217.344	152.942	149.982	2.545	0.074
4.15	0.168	0.167	1.676	1.611	201.937	203.763	145.635	142.618	2.608	0.078
4.25	0.167	0.167	1.632	1.573	206.610	203.748	146.803	143.782	2.670	0.082
4.35	0.167	0.166	1.615	1.541	202.463	200.713	144.125	141.442	2.733	0.086
4.45	0.166	0.165	1.568	1.504	197.503	197.960	144.203	140.753	2.796	0.090
4.55	0.165	0.164	1.535	1.474	194.053	188.864	137.449	135.095	2.859	0.094
4.65	0.165	0.163	1.491	1.429	195.888	190.419	137.499	135.400	2.922	0.098
4.75	0.163	0.162	1.446	1.380	185.580	183.938	134.705	131.160	2.985	0.102
4.85	0.162	0.160	1.414	1.342	181.589	181.672	134.770	131.709	3.047	0.106
4.95	0.161	0.159	1.366	1.284	178.346	175.311	130.461	126.446	3.110	0.111
5.05	0.159	0.156	1.318	1.246	173.757	177.727	130.948	128.992	3.173	0.115
5.15	0.157	0.154	1.282	1.190	170.356	163.767	124.367	121.069	3.236	0.120
5.25	0.154	0.151	1.225	1.146	164.250	163.723	123.847	121.984	3.299	0.124
5.35	0.152	0.148	1.171	1.102	158.491	163.844	123.802	120.046	3.362	0.129
5.45	0.148	0.144	1.122	1.044	160.115	162.438	122.609	120.586	3.424	0.134
5.55	0.145	0.139	1.059	0.968	149.566	162.292	123.342	120.132	3.487	0.139
5.65	0.140	0.134	1.006	0.918	156.551	154.345	118.167	116.525	3.550	0.144
5.75	0.136	0.129	0.959	0.873	147.020	146.136	113.127	111.721	3.613	0.149
5.85	0.130	0.123	0.886	0.797	145.015	144.633	113.254	110.865	3.676	0.154
5.95	0.124	0.116	0.822	0.747	144.700	138.271	109.195	107.907	3.739	0.159
6.05	0.118	0.109	0.765	0.677	134.790	139.807	110.156	108.725	3.801	0.165
6.15	0.111	0.102	0.705	0.620	133.980	134.977	106.914	106.877	3.864	0.170
6.25	0.104	0.094	0.644	0.568	133.752	131.886	106.737	103.210	3.927	0.176
6.35	0.096	0.087	0.579	0.503	134.261	126.877	102.774	100.185	3.990	0.181
6.45	0.088	0.079	0.521	0.454	126.816	118.877	97.455	96.310	4.053	0.187
6.55	0.080	0.071	0.471	0.407	119.377	117.256	96.236	94.573	4.115	0.193
6.65	0.073	0.063	0.420	0.356	115.103	118.588	96.476	95.884	4.178	0.199

6.75	0.065	0.056	0.367	0.313	110.514	114.716	95.800	93.229	4.241	0.205
6.85	0.058	0.049	0.319	0.271	108.120	111.950	92.493	92.660	4.304	0.211
6.95	0.051	0.043	0.283	0.234	103.300	103.864	87.453	86.736	4.367	0.217
7.05	0.045	0.038	0.247	0.206	101.995	102.951	88.986	85.378	4.430	0.223
7.15	0.039	0.032	0.212	0.174	99.222	100.927	86.590	85.215	4.492	0.230
7.25	0.033	0.028	0.181	0.149	97.376	95.427	83.418	80.672	4.555	0.236
7.35	0.029	0.024	0.158	0.128	92.586	93.376	81.550	79.385	4.618	0.243
7.45	0.025	0.020	0.133	0.107	87.821	90.211	78.730	78.633	4.681	0.249
7.55	0.021	0.017	0.114	0.092	86.584	86.684	76.759	76.658	4.744	0.256
7.65	0.018	0.015	0.097	0.078	86.116	84.392	75.303	74.954	4.807	0.263
7.75	0.015	0.012	0.082	0.065	80.289	81.637	73.533	72.348	4.869	0.269
7.85	0.013	0.010	0.069	0.055	78.867	77.348	71.054	70.602	4.932	0.276
7.95	0.011	0.009	0.058	0.047	77.872	74.950	68.849	68.639	4.995	0.283
8.05	0.009	0.007	0.049	0.039	73.964	71.445	66.565	66.340	5.058	0.291
8.15	0.008	0.006	0.041	0.033	74.792	69.825	65.473	64.956	5.121	0.298
8.25	0.006	0.005	0.034	0.028	68.475	66.144	63.270	62.186	5.184	0.305
8.35	0.005	0.004	0.029	0.023	67.783	64.049	61.589	61.201	5.246	0.313
8.45	0.004	0.004	0.024	0.020	59.608	61.346	59.548	59.409	5.309	0.320
8.55	0.004	0.003	0.020	0.016	59.443	59.525	58.754	57.368	5.372	0.328
8.65	0.003	0.002	0.017	0.014	56.385	56.964	56.867	55.962	5.435	0.335
8.75	0.003	0.002	0.014	0.011	56.644	56.001	55.929	54.912	5.498	0.343
8.85	0.002	0.002	0.012	0.010	50.866	52.092	53.357	52.169	5.561	0.351
8.95	0.002	0.001	0.010	0.008	49.712	48.628	50.306	49.657	5.623	0.359
9.05	0.001	0.001	0.008	0.007	46.466	47.980	49.994	49.368	5.686	0.367
9.15	0.001	0.001	0.007	0.006	47.500	45.004	47.884	47.096	5.749	0.375
9.25	0.001	0.001	0.006	0.005	44.230	43.505	47.087	45.985	5.812	0.383
9.35	0.001	0.001	0.005	0.004	40.375	40.607	44.537	43.753	5.875	0.391
9.45	0.001	0.001	0.004	0.003	39.624	37.742	41.843	41.417	5.938	0.400
9.55	0.001	0.000	0.003	0.003	37.242	37.117	41.074	40.752	6.000	0.408
9.65	0.000	0.000	0.003	0.002	32.536	35.251	40.270	39.836	6.063	0.417
9.75	0.000	0.000	0.002	0.002	34.081	33.107	38.013	37.575	6.126	0.425
9.85	0.000	0.000	0.002	0.002	30.728	31.131	36.022	35.861	6.189	0.434
9.95	0.000	0.000	0.002	0.001	29.514	27.940	33.093	33.559	6.252	0.443
10.05	0.000	0.000	0.001	0.001	27.184	27.773	33.722	32.964	6.315	0.452
10.15	0.000	0.000	0.001	0.001	25.260	27.471	33.210	33.191	6.377	0.461
10.25	0.000	0.000	0.001	0.001	23.936	24.835	30.750	30.422	6.440	0.470
10.35	0.000	0.000	0.001	0.001	22.186	22.725	29.079	28.242	6.503	0.479
10.45	0.000	0.000	0.001	0.001	21.891	21.744	28.073	27.969	6.566	0.488
10.55	0.000	0.000	0.001	0.000	19.365	20.731	26.774	26.820	6.629	0.498
10.65	0.000	0.000	0.000	0.000	16.989	18.089	24.326	24.108	6.692	0.507
10.75	0.000	0.000	0.000	0.000	17.007	17.499	23.211	23.792	6.754	0.517
10.85	0.000	0.000	0.000	0.000	16.930	16.248	22.510	22.344	6.817	0.526
10.95	0.000	0.000	0.000	0.000	17.012	15.406	21.600	21.534	6.880	0.536
11.05	0.000	0.000	0.000	0.000	14.102	14.787	21.128	20.943	6.943	0.546
11.15	0.000	0.000	0.000	0.000	12.022	12.562	18.447	18.437	7.006	0.556
11.25	0.000	0.000	0.000	0.000	11.616	12.392	18.466	18.144	7.069	0.566
11.35	0.000	0.000	0.000	0.000	12.367	11.386	17.507	17.017	7.131	0.576
11.45	0.000	0.000	0.000	0.000	10.363	10.594	16.276	16.433	7.194	0.586
11.55	0.000	0.000	0.000	0.000	9.684	9.522	15.205	14.927	7.257	0.596
11.65	0.000	0.000	0.000	0.000	8.406	8.459	13.844	13.391	7.320	0.606
11.75	0.000	0.000	0.000	0.000	8.091	8.114	13.237	13.321	7.383	0.617
11.85	0.000	0.000	0.000	0.000	7.169	7.315	12.359	12.205	7.446	0.627
11.95	0.000	0.000	0.000	0.000	7.137	6.484	11.047	11.238	7.508	0.638
12.05	0.000	0.000	0.000	0.000	6.115	5.970	10.460	10.393	7.571	0.649
12.15	0.000	0.000	0.000	0.000	5.642	5.370	9.627	9.649	7.634	0.659
12.25	0.000	0.000	0.000	0.000	4.871	5.084	9.409	9.136	7.697	0.670
12.35	0.000	0.000	0.000	0.000	4.350	4.455	8.354	8.320	7.760	0.681
12.45	0.000	0.000	0.000	0.000	4.152	4.043	7.709	7.762	7.823	0.692
12.55	0.000	0.000	0.000	0.000	3.681	3.534	6.811	6.938	7.885	0.703
12.65	0.000	0.000	0.000	0.000	3.138	3.193	6.461	6.278	7.948	0.714
12.75	0.000	0.000	0.000	0.000	2.829	2.836	5.770	5.732	8.011	0.726
12.85	0.000	0.000	0.000	0.000	2.427	2.620	5.412	5.455	8.073	0.737
12.95	0.000	0.000	0.000	0.000	2.299	2.385	5.030	5.051	8.135	0.749
13.05	0.000	0.000	0.000	0.000	2.231	2.068	4.493	4.399	8.198	0.760
13.15	0.000	0.000	0.000	0.000	1.942	1.885	4.165	4.072	8.257	0.772
13.25	0.000	0.000	0.000	0.000	1.497	1.623	3.644	3.596	8.297	0.783
13.35	0.000	0.000	0.000	0.000	1.552	1.476	3.409	3.296	8.365	0.795
13.45	0.000	0.000	0.000	0.000	1.289	1.294	3.000	2.984	8.388	0.807
13.55	0.000	0.000	0.000	0.000	1.164	1.130	2.724	2.610	8.411	0.819
13.65	0.000	0.000	0.000	0.000	0.967	1.040	2.506	2.462	8.359	0.831
13.75	0.000	0.000	0.000	0.000	0.863	0.892	2.194	2.150	8.314	0.842
13.85	0.000	0.000	0.000	0.000	0.803	0.778	1.958	1.894	8.290	0.854
13.95	0.000	0.000	0.000	0.000	0.658	0.677	1.733	1.673	8.045	0.865
14.05	0.000	0.000	0.000	0.000	0.580	0.606	1.565	1.531	7.855	0.877
14.15	0.000	0.000	0.000	0.000	0.490	0.517	1.368	1.312	7.510	0.887
14.25	0.000	0.000	0.000	0.000	0.461	0.464	1.247	1.185	7.402	0.898
14.35	0.000	0.000	0.000	0.000	0.444	0.418	1.143	1.075	7.349	0.908
14.45	0.000	0.000	0.000	0.000	0.330	0.348	0.952	0.921	6.486	0.917
14.55	0.000	0.000	0.000	0.000	0.276	0.298	0.828	0.793	5.936	0.925
14.65	0.000	0.000	0.000	0.000	0.254	0.263	0.736	0.718	5.701	0.934
14.75	0.000	0.000	0.000	0.000	0.214	0.215	0.615	0.585	5.156	0.941

14.85	0.000	0.000	0.000	0.000	0.207	0.196	0.571	0.535	5.085	0.948
14.95	0.000	0.000	0.000	0.000	0.166	0.171	0.497	0.477	4.393	0.954
15.05	0.000	0.000	0.000	0.000	0.141	0.139	0.411	0.393	3.932	0.960
15.15	0.000	0.000	0.000	0.000	0.120	0.123	0.368	0.349	3.482	0.965
15.25	0.000	0.000	0.000	0.000	0.109	0.108	0.329	0.312	3.258	0.969
15.35	0.000	0.000	0.000	0.000	0.087	0.086	0.260	0.249	2.724	0.973
15.45	0.000	0.000	0.000	0.000	0.087	0.077	0.239	0.224	2.736	0.977
15.55	0.000	0.000	0.000	0.000	0.069	0.066	0.209	0.193	2.267	0.980
15.65	0.000	0.000	0.000	0.000	0.060	0.057	0.181	0.168	1.998	0.983
15.75	0.000	0.000	0.000	0.000	0.049	0.049	0.159	0.145	1.679	0.985
15.85	0.000	0.000	0.000	0.000	0.043	0.040	0.131	0.118	1.512	0.987
15.95	0.000	0.000	0.000	0.000	0.037	0.036	0.119	0.107	1.304	0.989
16.05	0.000	0.000	0.000	0.000	0.028	0.030	0.098	0.088	1.011	0.991
16.15	0.000	0.000	0.000	0.000	0.026	0.026	0.088	0.078	0.947	0.992
16.25	0.000	0.000	0.000	0.000	0.021	0.021	0.072	0.064	0.794	0.993
16.35	0.000	0.000	0.000	0.000	0.020	0.018	0.064	0.055	0.762	0.994
16.45	0.000	0.000	0.000	0.000	0.017	0.016	0.055	0.047	0.637	0.995
16.55	0.000	0.000	0.000	0.000	0.013	0.013	0.045	0.039	0.514	0.996
16.65	0.000	0.000	0.000	0.000	0.012	0.011	0.039	0.033	0.476	0.996
16.75	0.000	0.000	0.000	0.000	0.010	0.010	0.034	0.029	0.404	0.997
16.85	0.000	0.000	0.000	0.000	0.008	0.008	0.028	0.024	0.322	0.998
16.95	0.000	0.000	0.000	0.000	0.008	0.006	0.023	0.020	0.300	0.998
17.05	0.000	0.000	0.000	0.000	0.005	0.006	0.020	0.017	0.210	0.998
17.15	0.000	0.000	0.000	0.000	0.005	0.004	0.016	0.014	0.188	0.998
17.25	0.000	0.000	0.000	0.000	0.004	0.004	0.014	0.012	0.170	0.999
17.35	0.000	0.000	0.000	0.000	0.004	0.003	0.012	0.010	0.147	0.999
17.45	0.000	0.000	0.000	0.000	0.003	0.003	0.010	0.009	0.126	0.999
17.55	0.000	0.000	0.000	0.000	0.002	0.002	0.008	0.007	0.103	0.999
17.65	0.000	0.000	0.000	0.000	0.002	0.002	0.007	0.006	0.086	0.999
17.75	0.000	0.000	0.000	0.000	0.002	0.002	0.006	0.005	0.064	0.999
17.85	0.000	0.000	0.000	0.000	0.001	0.001	0.005	0.004	0.063	1.000
17.95	0.000	0.000	0.000	0.000	0.001	0.001	0.004	0.004	0.046	1.000
18.05	0.000	0.000	0.000	0.000	0.001	0.001	0.004	0.003	0.045	1.000
18.15	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.003	0.036	1.000
18.25	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.030	1.000
18.35	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.026	1.000
18.45	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.020	1.000
18.55	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.018	1.000
18.65	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.014	1.000
18.75	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.014	1.000
18.85	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.010	1.000
18.95	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.009	1.000
19.05	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.008	1.000
19.15	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.006	1.000
19.25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	1.000
19.35	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	1.000
19.45	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	1.000
19.55	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	1.000
19.65	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.75	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.85	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.95	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000

4.2 Do the obtained numbers make sense?

The calculated numbers can be compared to some popular expectations. The agreement should be exact for hard sphere and reasonable for Woods-Saxon.

1. Compare the radius at which the density drops to 50% with 1.2 fm $A^{1/3}$:

$$R_A \stackrel{?}{=} 1.2 \text{ fm } A^{1/3} \ , \ n(R_A) = n_0/2. \tag{22}$$

2. Compare the peak density function with the product of nuclear density and nuclear diameter:

$$T_A(0) \stackrel{?}{=} 2 \ R \ n_0 \approx 0.38 \ A^{1/3}. \tag{23}$$

3. Compare the integral of the thickness function with the number of nucleons:

$$A \stackrel{?}{=} \int d^2b \ T_A(b) \approx \sum_{i=1}^{200} 2\pi b \cdot 0.1 \text{ fm} \cdot T_A(b). \tag{24}$$

4. Compare the peak overlap function with the analytic formula:

$$T_{AB}(0) \stackrel{?}{=} 2\pi R^4 n_0^2 \approx 0.29 A^{4/3}. \tag{25}$$

5. Compare the integral of the overlap function with AB:

$$AB \stackrel{?}{=} \int d^2b\; T_{AB}(b) \approx \sum_{i=1}^{200} 2\pi b \cdot 0.1\text{ fm} \cdot T_{AB}(b). \tag{26}$$

6. Compare the total cross section with $\pi {b_{\max}}^2$:

$$\sigma_G \stackrel{?}{=} \pi (R_A + R_B)^2 \approx 3.94\text{ fm}^2 \left[A^{1/3} + B^{1/3}\right]^2. \tag{27}$$

7. Compare the mean number of NN collisions in a minimum bias A+B collision with the value expected from the normalization conditions of T_A and T_{AB} (Eqs. 7,11):

$$\langle N_{\text{coll}} \rangle_{\text{minb}} \stackrel{?}{=} AB \frac{\sigma_{\text{NN}}}{\sigma_G} \tag{28}$$