# The Early Universe

## The first few microseconds

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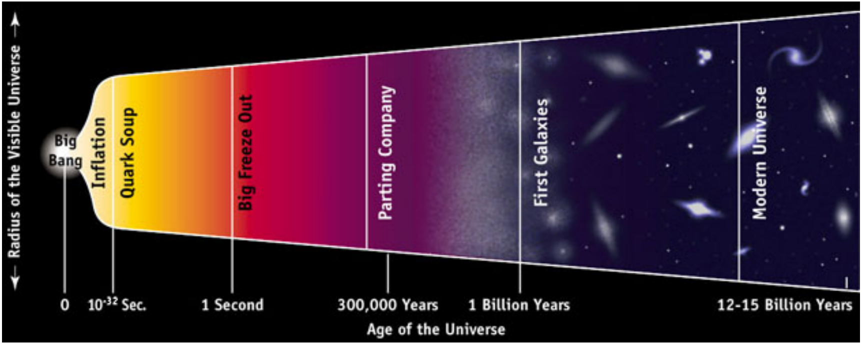
# Outline

# The Expanding Universe



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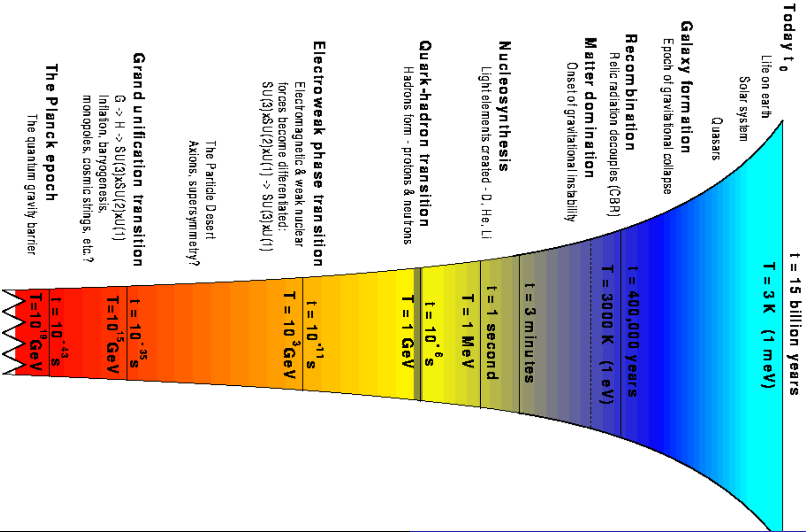
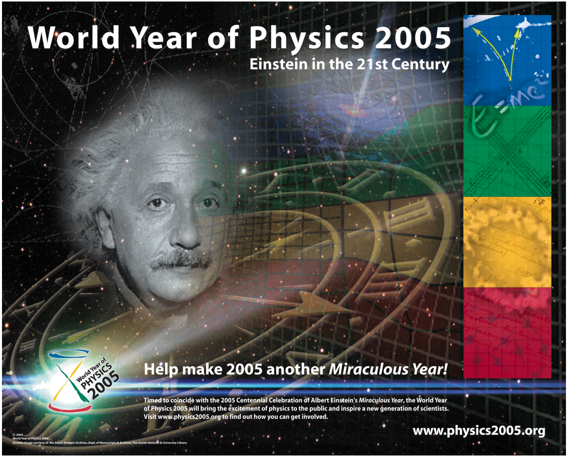


# Einstein Year




## Papers in 1905

- Photoeffect
- Brownian motion
- Special relativity
- $E = mc^2$

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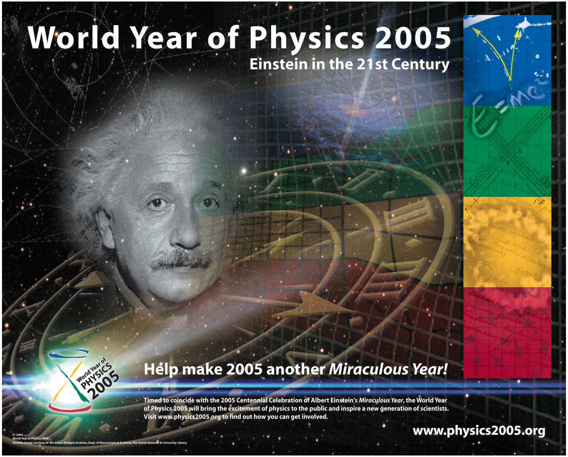


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# General Theory of Relativity (GTR)

## Special Relativity (1905)

- Unify Mechanics and Electrodynamics
- Equations of motion the same in inertial frames
- The principle of relativity

## General Theory of Relativity (1915)

- Unify Gravity and Relativity
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- Inspired by Mach's Principle (Rotation relative to distant stars)
- Einstein's equivalence principle
  („eliminate" gravitation in local inertial frame, laws of special relativity)

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# Where do we need GTR?

- Strong gravitational fields
  - Compact stars, black holes
  - GTR $\xrightarrow{\textit{weak fields}}$ Newtonian gravity
- Large distances
  - Cosmology
  - Expansion of the Universe
  - Cosmological redshift
- High precision in weak fields
  - Gravitational redshift
  - Deflection of light in gravitaional fields
  - Precession of planetary orbits
  - Syncronization of clocks in gravitaional fields (GPS)
- Gravitational waves
  - Slowing down of binary pulsars

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# Space-time metrics
## flat space-time

### Minkowski metric

- $d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2 = \sum_{\mu\nu} \eta_{\mu\nu} dx^\mu dx^\nu$
  $\mu, \nu \in \{0, 1, 2, 3\}$
- $\eta_{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$
- $x^\mu = (x^0, x^1, x^2, x^3) \equiv (t, x, y, z)$
- $x_\mu = \eta_{\mu\nu} x^\nu \equiv \sum_\nu \eta_{\mu\nu} x^\nu$ (Summation convention)
- Scalar product $A_\mu B^\mu = \eta_{\mu\nu} A^\mu B^\nu$ Lorentz scalar
- $A_\mu$ covariant, $A^\mu$ contravariant vectors

# Space-time metrics
## curved space-time

### GTR: Gravity $\Longleftrightarrow$ curvature of space-time

- $d\tau^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$
- Cannot bring $g_{\mu\nu}(x)$ in Minkowski form by coord. trafo
- $g^{\mu\nu} = g^{\nu\mu}$, $g^{\mu\nu} g_{\nu\lambda} = \delta^\mu{}_\lambda$
- Equation of motion

$$\frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

- Connection coefficient (Cristoffel symbol)

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left\{ \frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right\}$$

# Curved space-time
## Useful relations

### Four-velocity

- $u^\mu = dx^\mu/d\tau \qquad u_\mu = dx_\mu/d\tau$
- Equations of motion valid in an arbitrary frame

$$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0$$

$$\frac{du_\mu}{d\tau} - \frac{1}{2}\frac{\partial g_{\nu\lambda}}{\partial x^\mu} u^\nu u^\lambda = 0$$

- $\partial g_{\nu\lambda}/\partial x^\mu = 0 \;\Rightarrow\; u^\mu = \text{constant}$

### Weak field limit

- $g_{00} = 1 + 2\phi \qquad \phi = -GM/r$ (Newtonian grav. potential)

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# Covariant derivatives

- In curved space-time the differential $dA^\mu$ is not a vector
- $DA^\mu = A^\mu{}_{;\nu} dx^\nu$ transforms like a vector

$$A^\mu{}_{;\nu} = \frac{\partial A^\mu}{\partial x^\nu} + \Gamma^\mu_{\lambda\nu} A^\lambda \qquad \text{Covariant derivative}$$

$$A_{\mu;\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \Gamma^\lambda_{\mu\nu} A_\lambda$$

- Scalar product

$$\frac{\partial A_\mu B^\mu}{\partial x^\nu} = \frac{\partial A_\mu}{\partial x^\nu} B^\mu + A_\mu \frac{\partial B^\mu}{\partial x^\nu}$$

- Metric tensor: $DA_\mu = g_{\mu\nu} DA^\nu \Rightarrow Dg_{\mu\nu} = 0$

# From Riemann to Einstein

## Tensors

- Second covariant derivative $A_{\mu;\nu;\lambda} - A_{\mu;\lambda;\nu} = R^{\sigma}_{\mu\nu\lambda} A_{\sigma}$

$$R^{\sigma}_{\mu\nu\lambda} = \frac{\partial \Gamma^{\sigma}_{\mu\lambda}}{\partial x^{\nu}} - \frac{\partial \Gamma^{\sigma}_{\mu\nu}}{\partial x^{\lambda}} + \Gamma^{\sigma}_{\alpha\nu}\Gamma^{\alpha}_{\mu\lambda} - \Gamma^{\sigma}_{\alpha\lambda}\Gamma^{\alpha}_{\mu\nu}$$

- Riemann tensor vanishes if and only if space-time is flat
- Ricci tensor $R_{\mu\nu} = R^{\nu}_{\mu\nu\lambda}$
- Scalar curvature $R = g^{\mu\nu} R_{\mu\nu}$
- Einstein tensor $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$
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# Source of curvature

## Energy-momentum tensor

- Free gas ($f(\vec{k}, x)$ phasespace density):

$$T^{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3} \frac{k^\mu k^\nu}{k^0} f(\vec{k}, x)$$

- In local restframe for system in local equilibrium
$T^{\mu\nu}(x) = \text{diag}(\varepsilon, p, p, p)$
$p =$ pressure, $\varepsilon =$ energy density, $E = \int T^{00} d^3x$
- In a general frame (Minkowski metric)
$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - \eta^{\mu\nu} p$
- Conservation of energy and momentum: $\partial_\nu T^{\mu\nu} = 0$
- In curved space-time $T^{\mu\nu}{}_{;\nu} = 0$

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# Energy-momentum tensor

## A gas of photons

- In rest frame of gas ($\beta = 1/T$)

$$\varepsilon = 2\int \frac{d^3k}{(2\pi)^3}|\vec{k}|\frac{1}{e^{\beta|\vec{k}|}-1} = \frac{\pi^2}{15}T^4$$

$$p = \frac{\pi^2}{45}T^4 = \frac{1}{3}\varepsilon$$

- General property of gas of massless particles:

$$p = \varepsilon/3 \;\; \Rightarrow \;\; T^{\mu}{}_{\mu} = \varepsilon - 3p = 0$$

- $\varepsilon + 3p > 0$

# Energy-momentum tensor II

## Massive particles

- For $T << m$

$$\varepsilon = \sum_{spin} \int \frac{d^3k}{(2\pi)^3} k^0 f(\vec{k}) \simeq m \sum_{spin} \int \frac{d^3k}{(2\pi)^3} f(\vec{k}) \simeq m\, n$$

$$p = \sum_{spin} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{k^0} f(\vec{k}) \simeq T\, n << \varepsilon$$

- $T^{\mu\nu} \simeq \mathrm{diag}(\varepsilon, 0, 0, 0), \qquad T^{\mu}{}_{\mu} \simeq \varepsilon$
- $\varepsilon + 3\,p > 0$

# Energy-momentum tensor III

## Vacuum

- Vacuum should be Lorentz invariant

$$T^{\mu\nu} = \varepsilon_{vac}\,\eta^{\mu\nu} = \text{diag}(\varepsilon_{vac}, -\varepsilon_{vac}, -\varepsilon_{vac}, -\varepsilon_{vac})$$

- $p_{vac} = -\varepsilon_{vac}$
- „Normal" vacuum: $p_{vac} = \varepsilon_{vac} = 0$
- Unstable vacuum: $\varepsilon_{vac} > 0$, $p_{vac} < 0$,

$$\varepsilon_{vac} + p_{vac} = 0$$

- $\varepsilon + 3\,p < 0$

# Einstein's field equation

## Motivation

- Generalize Newton's law of gravity

$$\vec{\nabla}^2\phi(\vec{x}) = 4\pi G\,\rho(\vec{x}) \qquad \rho(\vec{x}) = mn(\vec{x})$$

- Connect curvature of space with energy momentum content. Ansatz: $G^{\mu\nu} = \kappa T^{\mu\nu}$ $\qquad (G^{\mu\nu}{}_{;\nu} = 0)$
- Determine $\kappa$. Newtonian limit: $G^{00} = \vec{\nabla}^2 g^{00} = \kappa\varepsilon \simeq \kappa mn$
$g^{00} \simeq 1 + 2\phi \Rightarrow \kappa = 8\pi G$
- Einstein's field equation:
$G^{\mu\nu} = 8\pi G\,T^{\mu\nu}$
- Alternative form: $R^{\mu\nu} = 8\pi G\,(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T)$ $\quad (T = T^{\mu}{}_{\mu})$

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# Einstein's field equation II

## $G^{\mu\nu} = 8\pi G\, T^{\mu\nu}$

- The field equation determines „everything".
  - Space-time geometry $G^{\mu\nu}$ as function of sources $T^{\mu\nu}$
  - Dynamics of sources in the curved space-time determined by $T^{\mu\nu}{}_{;\nu} = 0$ and equation-of-state
- $G^{\mu\nu}{}_{;\nu} = 0$ automatically satisfied $\Leftrightarrow$ freedom of choice of coordinate system.
- In vacuum $R^{\mu\nu} = 0$ ($T^{\mu\nu}$ does not include grav. field)
- Cosmological constant *Lambda* (vacuum energy)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - g^{\mu\nu}\Lambda = 8\pi G\, T^{\mu\nu}$$

$T^{\mu\nu}_{vac} = \frac{\Lambda}{8\pi G} g^{\mu\nu}$

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# Cosmology

Cosmology deals with the evolution of the universe on the largest scales of space and time. Cosmology is one of the most important applications of GTR.

## Basic observational facts

- The Universe consist of visible matter (stars in galaxies), radiation, dark matter and dark energy (vacuum energy)
- The Universe is expanding
- On large scales the Universe is isotropic and homogeneous (Cosmological principle)

## Notation

- all massless particles = radiation
- all massive particles = matter

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# The composition of the universe

- Most visible matter is in galaxies.
- Average density of visible matter ($\rho \equiv \varepsilon$)

$$\rho_{visible}(t_0) \sim 10^{-31} g\, cm^{-3} \sim 1 proton/m^3$$

- Cosmic Microwave Background $T_{CMB}(t_0) = 2.726K$

$$\rho_{CMB}(t_0) \sim 10^{-34} g\, cm^{-3}$$

- CMB one of the strongest pieces of evidence for Big Bang

## Approximate fractions

| | | |
|---|---|---|
| Dark energy | .7 | |
| Matter | .3 | (visible matter .04) |
| Radiation | $5 \times 10^{-5}$ | |

# The composition of the universe

- Most visible matter is in galaxies.
- Average density of visible matter ($\rho \equiv \varepsilon$)

$$\rho_{visible}(t_0) \sim 10^{-31} g\, cm^{-3} \sim 1 proton/m^3$$

- Cosmic Microwave Background $T_{CMB}(t_0) = 2.726K$

$$\rho_{CMB}(t_0) \sim 10^{-34} g\, cm^{-3}$$

- CMB one of the strongest pieces of evidence for Big Bang

**Approximate fractions**

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| Dark energy | .7 | | |
| Matter | .3 | (visible matter | .04) |
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# The universe expands

- The spectra from distant stars are redshifted
- If interpreted in terms of Dopplershift

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = Z \quad (\frac{v}{c} << 1)$$

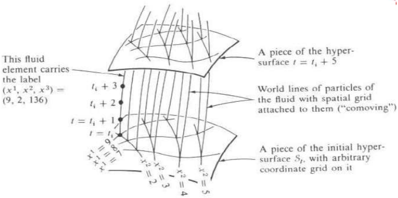
- Empirically $v = H_0 d$ (Hubble's law)
  Hubble constant $H_0 = 72 \pm 7 \frac{km}{s} \frac{1}{Mpc}$ Hubble parsec
- Hubble's law $\Rightarrow$ homologous expansion (no distortions)

# Friedman-Robertson-Walker metric (FRW)

- Simplest homogeneous, isotropic metric (flat space)

$$d\tau^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$$

  - $a(t)$ scale factor, $x, y, z$ comoving, synchronous coordinates
  - At fixed time $ds^2 = a(t)^2(dx^2 + dy^2 + dz^2)$ physical distances given by $X = \int a(t)dx = a(t)x$ etc.
  - $g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$



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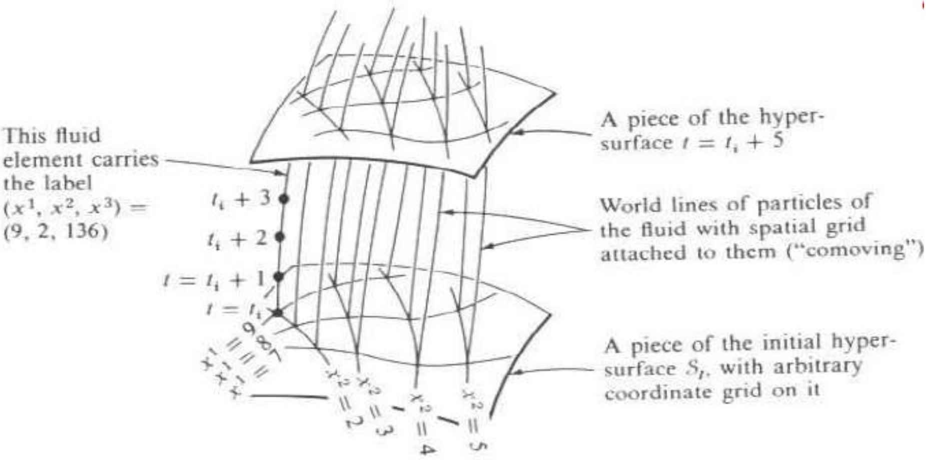

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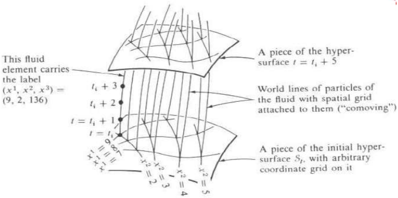

# FRW II

- More general coordinates

$$d\tau^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1 - k\,r^2} + r^2 d\Omega^2\right) \qquad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

- $k = 1$ closed, $k = 0$ flat, $k = -1$ open universe
- Since our universe almost flat, consider only $k = 0$
- Consider volume $\Delta V$
- Universe homogeneous $\Rightarrow$ no heat flow

$$d(\Delta E) = -p\, d(\Delta V) \qquad \text{(energy conservation)}$$

- $\Delta E = \rho\,\Delta V \qquad \Delta V_{coord} = \Delta x\,\Delta y\,\Delta z$
  $\Delta V = a(t)^3 \Delta V_{coord}$

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# Scaling of $\rho$

- Energy conservation

$$\frac{d}{dt}(\rho a(t)^3) = -p\,\frac{d}{dt}(a(t)^3)$$

- Matter dominated universe $\frac{d}{dt}(\rho a^3) = 0$

$$\rho(t) = \rho(t_0)\left(\frac{a(t_0)}{a(t)}\right)^3 \qquad \text{isoergic expansion}$$

- Radiation dominated universe $p = \frac{1}{3}\rho$

$$\rho(t) = \rho(t_0)\left(\frac{a(t_0)}{a(t)}\right)^4 \qquad \text{isentropic expansion}$$

- $\rho \sim T^4 \Rightarrow T(t) = T(t_0)\frac{a(t_0)}{a(t)}$ Redshift due to $p\,dV$ work

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# Scaling of $\rho$ II

- Vacuum energy dominated universe

$$T^{\mu\nu} = \rho_{vac}\, g^{\mu\nu} \qquad p_{vac} = -\rho_{vac}$$

- $\frac{d}{dt}(\rho_{vac} a(t)^3) = \rho_{vac} \frac{d}{dt}(a(t)^3) = -p_{vac} \frac{d}{dt}(a(t)^3)$
- Energy grows proportional to volume
  $p\, dV$ work negative, energy conserved
- What drives expansion?

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- What drives expansion?

# Dynamics of FRW

- $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$
- Compute $G_{\mu\nu}$ for

$$d\tau^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right)$$

- Use orthonormal basis, where $T_{\mu\nu} = \text{diag}(\rho, p, p, p)$

**FRW field equations:**

$$\frac{3}{a^2}(k + \dot{a}^2) = 8\pi\, G\, \rho$$

$$-\frac{1}{a^2}(k + \dot{a}^2 + 2\, a\, \ddot{a}) = 8\pi\, G\, p$$

**Combining eqn's:**

$$(\dot{a})^2 = -k + \frac{8\,\pi\, G}{3}\,\rho\, a^2$$

$$\frac{\ddot{a}}{a} = -\frac{8\,\pi\, G}{6}(\rho + 3\, p)$$

# Critical density

- Start with $k = 0$
  $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\,\rho$$
- $H_0^2 = \frac{8\,\pi\,G}{3}\,\rho_0$
- Present energy density in flat universe = critical density
  $$\rho_{crit} = \frac{3\,H_0^2}{8\,\pi\,G} = 1.88 \times 10^{-29} h^2\, g\, cm^{-3}$$
  $$h = H_0/100 \frac{km}{s} \frac{1}{Mpc}$$
- $\Omega_m = \frac{\rho_m(t_0)}{\rho_{crit}}$, $\Omega_r = \frac{\rho_r(t_0)}{\rho_{crit}}$, $\Omega_v = \frac{\rho_v(t_0)}{\rho_{crit}}$
  $$\Omega = \Omega_m + \Omega_r + \Omega_v = \frac{\rho(t_0)}{\rho crit}$$

# Energy density

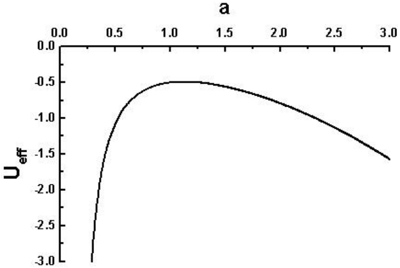
- In a flat universe $\Omega = 1$
  Our universe: $\Omega_m \simeq 0.3$, $\Omega_r \simeq 5 \times 10^{-5}$, $\Omega_v \simeq 0.7$
- Choose $a(t_0) = 1$, then

$$\rho(a) = \rho_{crit}(\Omega_v + \Omega_m/a^3 + \Omega_r/a^4)$$

- Dynamical equation:

$$\frac{1}{2H_0^2}\dot{a}^2 + U_{eff}(a) = 0$$

$$U_{eff}(a) = -\frac{1}{2}\left(\Omega_v a^2 + \frac{\Omega_m}{a} + \frac{\Omega_r}{a^2}\right)$$



# Simple flat universes

## Matter dominated universe

- $\Omega_m = 1, \Omega_r = \Omega_v = 0$
- $\frac{da}{dt} = H_0 \frac{1}{\sqrt{a}} \Rightarrow a(t) = (\frac{3}{2} H_0\, t)^{2/3} \sim t^{2/3}$
- $a(t_0) = 1 \Rightarrow t_0 = \frac{2}{3\, H_0} \simeq 9 \times 10^9 years$

## Radiation dominated universe

- $\Omega_r = 1, \Omega_m = \Omega_v = 0$
- $\frac{da}{dt} = H_0 \frac{1}{a} \Rightarrow a(t) = (2H_0\, t)^{1/2} \sim t^{1/2}$
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# Simple flat universes II

## Vacuum dominated universe

- $\Omega_v = 1, \Omega_m = \Omega_r = 0$
- $\frac{da}{dt} = H_0\, a \Rightarrow a(t) = a(t_0)\, e^{H_0(t-t_0)} \sim e^{H_0\, t}$
- Cosmological constant $\Lambda = 8\,\pi\, G\, \rho_v$
- Positive vacuum energy $\Rightarrow$ repulsive gravitation $\Rightarrow$ exponential growth of the universe!
  $\frac{\ddot{a}}{a} = -\frac{8\,\pi\, G}{6}(\rho + 3\, p)\; > \; 0$

- First solutions with an expanding universe by Friedman (1922) and by Lemaitre (1927). Lemaitre suggested that the universe evolved from a singularity. The name big bang coined by Fred Hoyle.

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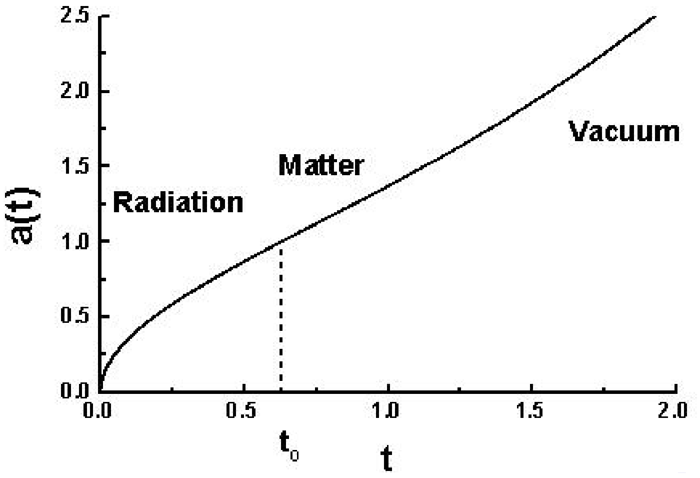
# Solution for $\Omega_r = \Omega_m = \Omega_v = \frac{1}{3}$



# Schematic model

- Abundance of a particle species depends only on $m_i/T$.
- Schematic model:

$$\rho_i = 0 \text{ for } m_i/T > 1$$

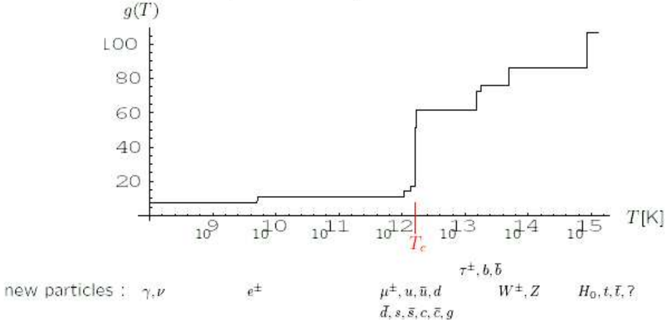
$$\rho_i = g_i \frac{\pi^2}{30} S_i T^4 \text{ for } m_i/T < 1$$

$$S_i = 1 \text{ for bosons, } S_i = \frac{7}{8} \text{ for fermions}$$

# Particle content of the early universe

**Content of the early universe**

$$\rho = \left( \sum_b g_b + \frac{7}{8} \sum_f g_f \right) \frac{\pi^2}{30} T^4 \equiv \frac{\pi^2}{30} g(T) T^4$$



# Photon decoupling

- At $T > 10^3 - 10^4$ K, $t_{dec} \simeq 10^5$ years atoms are ionized
  $\Rightarrow$ the universe is opaque to photons
- At smaller $T$, $e, p, n$ form neutral atoms
  $\Rightarrow$ the universe is transparent to photons
- From $t_{dec}$ the photons propagate freely, except for gravitation
- Expansion of the universe, or equivalently, the interaction with the gravitational field

  $\Rightarrow$ Cosmological redshift $\lambda_0 = \lambda_{dec} \frac{a(t0)}{a(t_{dec})} \simeq 1000 \lambda_{dec}$
- $T_{CMB} = 2.75$ K $T_{dec} \simeq 1000 T_{CMB} \simeq 3000$ K

# Phase transitions

## Timeline of the very early universe

- The Planck epoch $t \simeq 10^{-43}$ sec

  before this time need quantum gravity - not yet understood!
- Grand unification $t \simeq 10^{-33}$ sec

  electromagnetism, weak and strong interaction of the same strength
- Cosmic inflation and reheating

  The temperature at which inflation occurs is not known
- Baryogenesis

  Supposed to explain why there are slightly more baryons than anti-baryons in the universe - not yet fully understood

# Phase transitions II

## Timeline of the very early universe cont'd

- The electroweak epoch $t \simeq 10^{-12}$ sec

  Electromagnetic and weak interactions separate
- Confinement transition $t \simeq 10^{-5}$ sec

  Quarks and gluons are confined into hadrons
- Nucleosynthesis $t \simeq 1$ sec

  Light nuclei are formed

# Why do we need inflation?

## Horizon problem

- CMB uniform to one part in $10^5$ wmap
- Difficult to understand, unless all parts of the visible universe were in thermal contact at the time of decoupling
- The horizon at decoupling (causally connected) $\Leftrightarrow$ an angle of 2 degrees today

## Monopole problem

- Particle physics theories predict a variety of relics:
  - Magnetic monopoles
  - Domain walls
  - Supersymmetric particles ...
- Not seen. Needs to be diluted!

# Why do we need inflation? II

## Flatness problem

- Einstein's Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G}{3}\rho - \frac{k}{a^2}$$

$$|\Omega - 1| = \frac{|k|}{(\dot{a})^2}$$

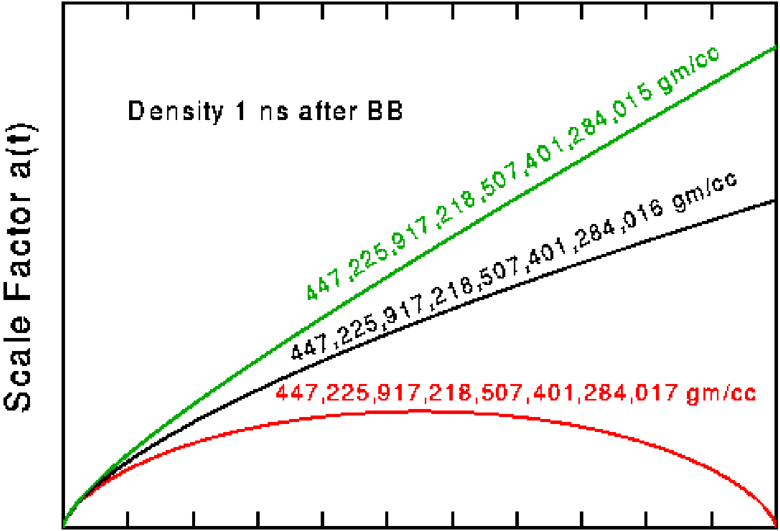
- Standard expansion $a(t) \sim t^\alpha \qquad \alpha = 1/2, 2/3$

$$|\Omega - 1| \sim t^{2(1-\alpha)} \to \infty$$

- $\Omega = 1$ unstable.

  Finetuning problem to achieve $|\Omega - 1| < 1$ today.

# Flatness problem



# Solution: Inflation

- Need an epoch where $|\Omega - 1| = |k|/(\dot{a})^2 \to 0$,

  i.e., where the finetuning is automatic
- Inflation $\Leftrightarrow \ddot{a} > 0$

$$\Leftrightarrow \frac{d}{dt}\frac{1}{(\dot{a})^2} < 0, \text{ i.e. } \Omega \to 1$$

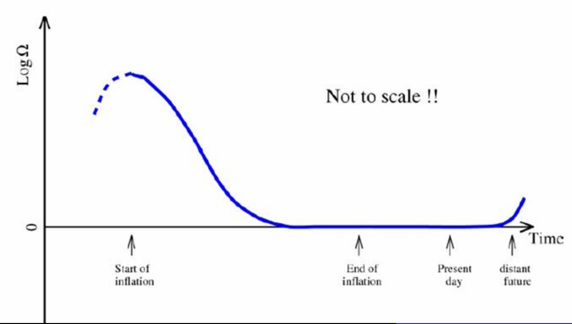
$$\Leftrightarrow \rho + 3p < 0$$

## Standard inflation

- $p = -\rho$, $a(t) \sim e^{H\,t}$
- During inflation the horizon remains $\simeq$ constant in comoving coordinates
- Typically one needs $H\,\Delta t > 70$
  $\Rightarrow a(t)$ grows by factor $> 10^{30}$ during inflation!

# Inflation II

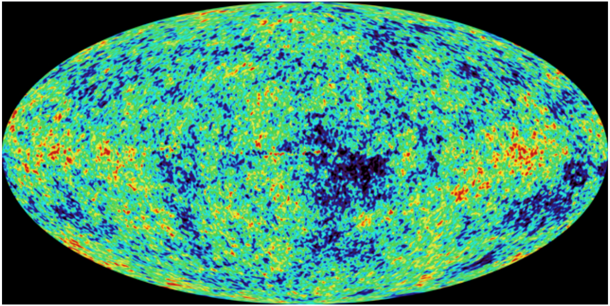
- Inflation solves horizon problem by blowing up the causally connected region by $\sim 10^{30}$
- Monopole problem solved by dilution
- Solves the flatness problem by $\Omega \to 1,\ |\Omega - 1| \sim 10^{-60}$



# Inflation III

- Inflation solves many problems, but cause of inflation is not understood! Present models: inflation caused by some unspecified scalar field.
- Not very satisfactory, but inflation is also consistent with fluctuations in CMB temperature:
  - Prior to inflation, quantum fluctuations in scalar field on microscopic scale.
  - During inflation scale of fluctuations grow ($\sim 10^{30}$) and freeze out (no causal connection)
  - Fluctuations in $\rho \Rightarrow$ fluctuations in grav. field, which in turn attract matter.
  - Lumps of matter act as seeds for galaxy formation.
  - Fluctuations in temperature arise because photons from deeper grav. potential are red shifted.

# WMAP

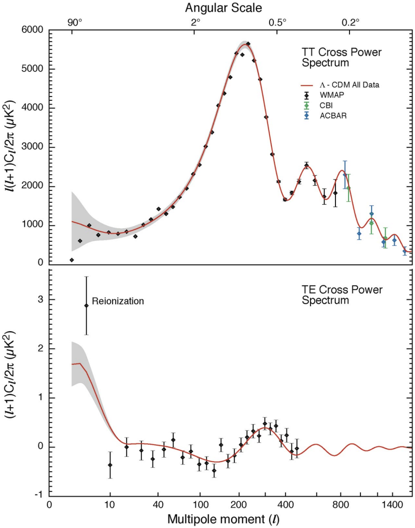


- Correlation function

$$C(\theta) = \langle T(\vec{n}_1) T(\vec{n}_2) \rangle \qquad \vec{n}_1 \cdot \vec{n}_2 = \cos(\theta)$$

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos(\theta))$$

# WMAP II



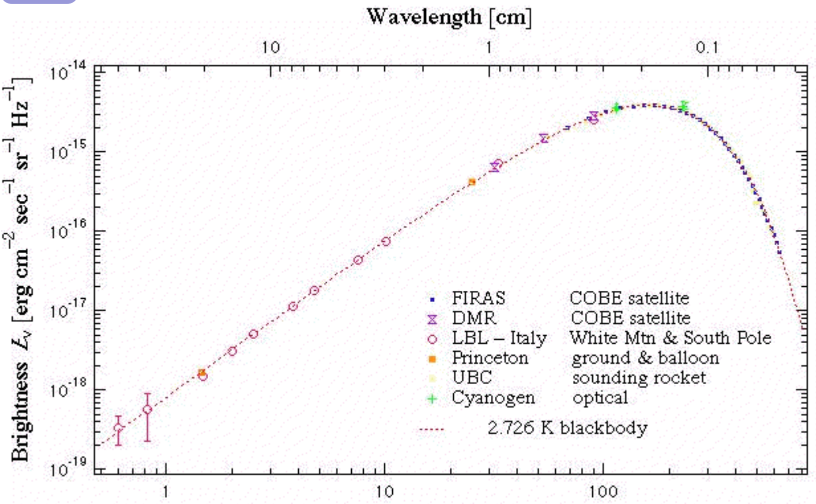

- Cosmological parameters ($\Lambda$-CDM):
  $\Omega = 1.02 \pm 0.02$
  $\Omega_v = 0.73 \pm 0.04$
  $\Omega_b = 0.044 \pm 0.004$
  $\Omega_m = 0.27 \pm 0.04$
  $Z_{dec} = 1089 \pm 1$
  $h = 0.71 \pm 0.04$
  $t_0 = 13.7 \pm 0.2) \times 10^9$ years

# Summary

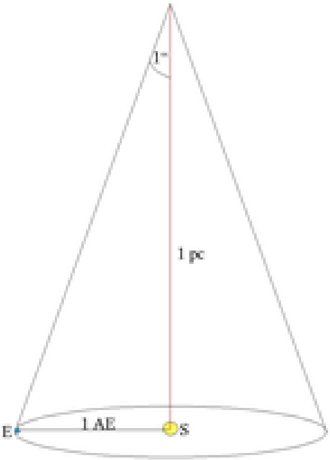
- Cosmology is an important application of GTR
- The universe expands, vacuum energy, dark matter play an important role
- Strong phenomenological support for inflation, but cause not understood
- Several phase transitions in early universe
- Explore matter at $T \sim 10^2$ MeV ($\Leftrightarrow 10^{-5}$ sec) in ultra-relativistic heavy-ion collisions

# CMB spectrum

return



# parsec




1 arcsecond

$$1pc = 3.09 \times 10^{13}\text{km}$$
$$= 3.26 \text{ light years}$$

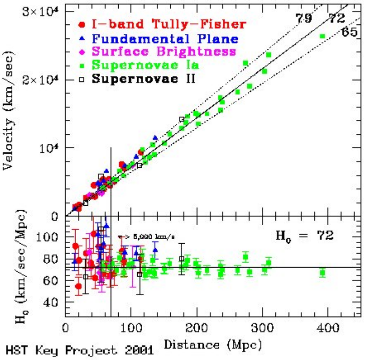
Alpha Centauri $\sim 1.2pc$

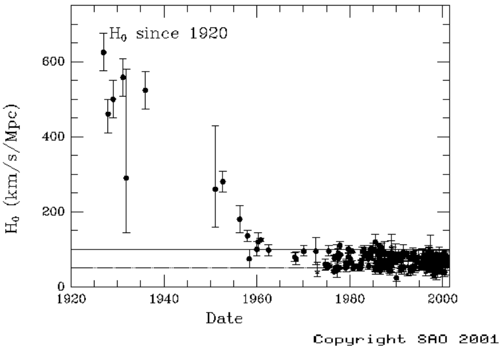
Galactic center $\sim 1kpc$

Virgo (nearest large cluster) $\sim 20Mpc$

return

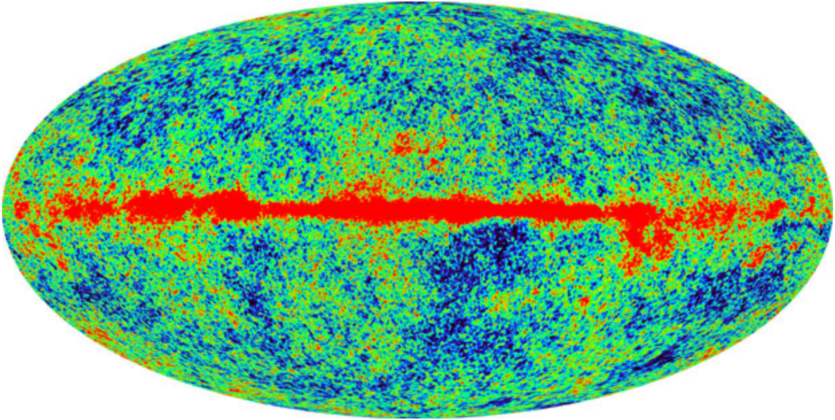
# Hubble constant




return

# CMB temperature (WMAP)



◂ return