Plan of lectures

- 1 15.04.2015 Preliminary Discussion / Introduction

- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (bound-state solutions of Dirac equation, quantum numbers)
- 6 20.05.2015 Theory (bound-state Dirac wavefunctions, QED corrections)

- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)
- 8 03.06.2015 Theory (single and multiple scattering, energy loss mechanisms, channeling regime)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ….)

- 10 17.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 11 24.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)

- 12 01.07.2015 Theory (atomic-physics tests of the Standard Model, search for a new physics)
- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)

Bound-state solutions of Dirac equation and QED effects

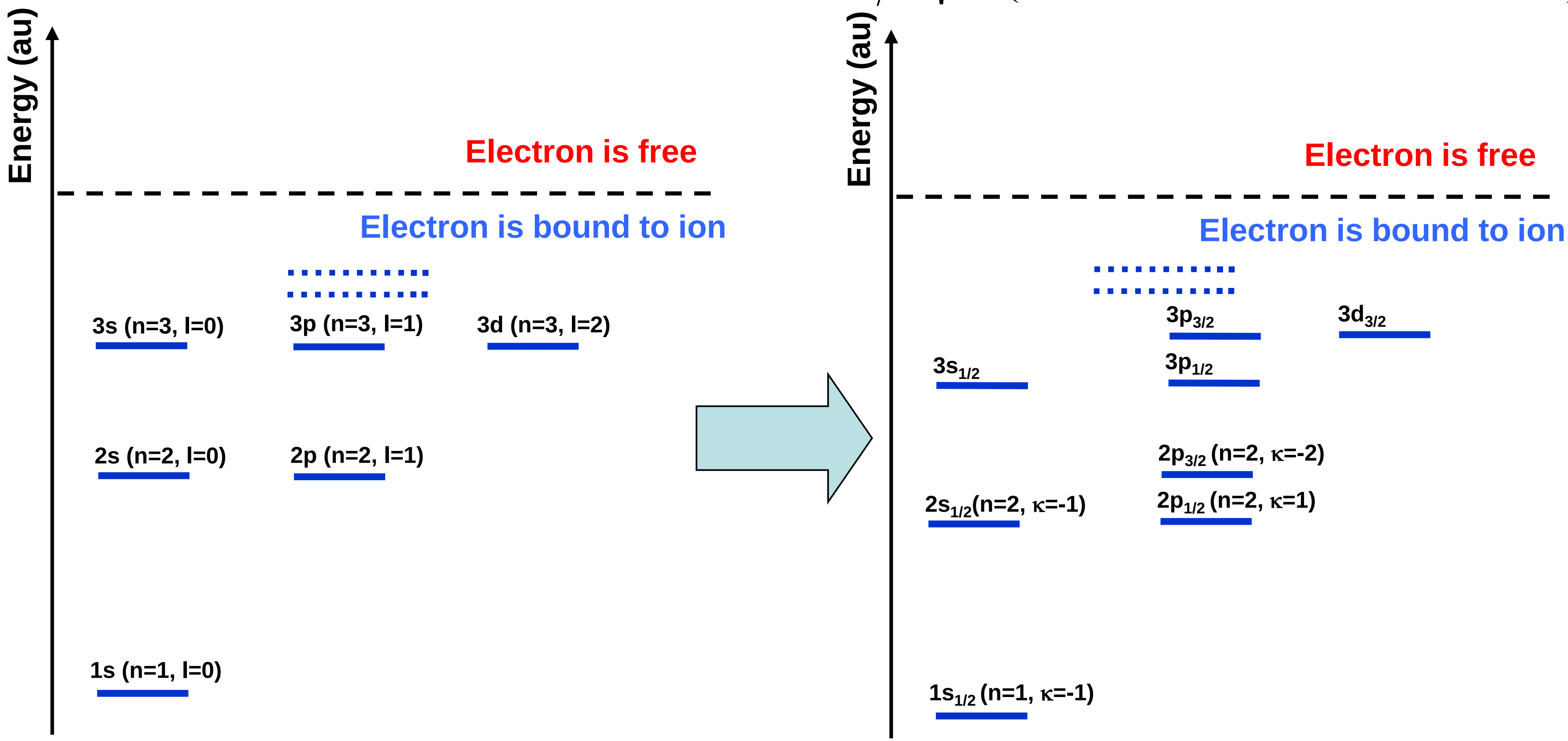
Plan of lecture

- Reminder from the last lecture: Bound-state solutions of Dirac equation
- **Higher-order corrections to Dirac energies:**
  - **Radiative corrections (QED effects)**
  - **Hyperfine interaction**

Energy levels of hydrogen ion

$$E_n = -Z^2\varepsilon_0 \big/ 2n^2$$

$$E_{nj} = mc^2 \Big/ \sqrt{1+\left(\frac{Z\alpha}{n-|j+1/2|+\sqrt{(j+1/2)^2-(Z\alpha)^2}}\right)^2}$$



All state with the same n are degenerated!

All state with the same n and j are degenerated!

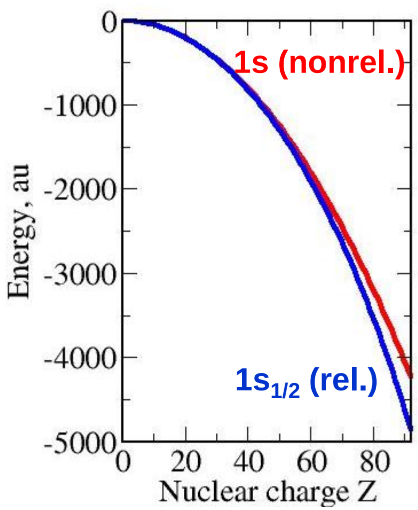
Energy levels of hydrogen ion

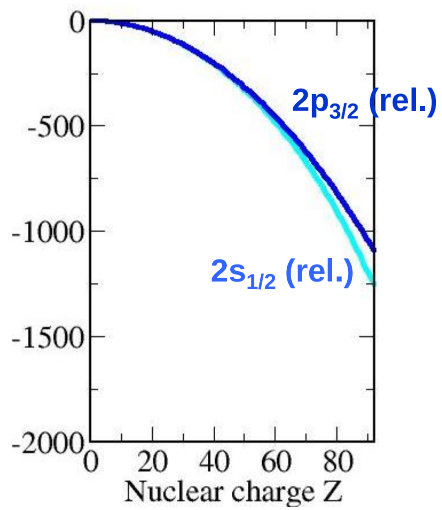
$$E_{nj} = mc^2 \Bigg/ \sqrt{1 + \left(\frac{Z\alpha}{n - |j + 1/2| + \sqrt{(j+1/2)^2 - (Z\alpha)^2}} \right)^2}$$

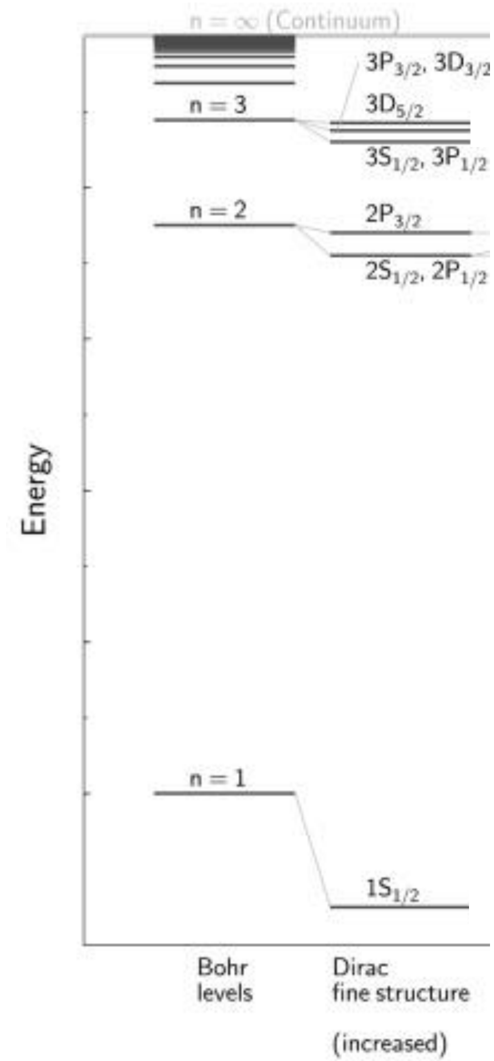
$$\approx mc^2 \left(1 - \frac{1}{2}\frac{(\alpha Z)^2}{n^2} - \frac{1}{2}\frac{(\alpha Z)^4}{n^3}\left(\frac{1}{j+1/2} - \frac{3}{4n} \right) - \ldots \right)$$

Rest mass term **Nonrelativistic energy** **First relativistic correction**

- **Relativistic effects results both in shifting and splitting of energy levels.**







Structure of Dirac wavefunctions

- **Stationary Dirac equation for particle in Coulomb field reads:**

$$\left(-i\hbar c\boldsymbol{\alpha}\cdot\nabla - \frac{Ze^2}{r} + m_e c^2\alpha_0\right)\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r})$$

- **The four-spinor** $\psi(\boldsymbol{r}) = \begin{pmatrix} \varphi_1(\boldsymbol{r}) \\ \varphi_2(\boldsymbol{r}) \\ \varphi_3(\boldsymbol{r}) \\ \varphi_4(\boldsymbol{r}) \end{pmatrix}$ **is more convenient to write as:** $\psi(\boldsymbol{r}) = \begin{pmatrix} g(\boldsymbol{r}) \\ f(\boldsymbol{r}) \end{pmatrix}$

What are the (large and small) components of wavefunction?

- **Please, remind yourself our (wrong) guess:**

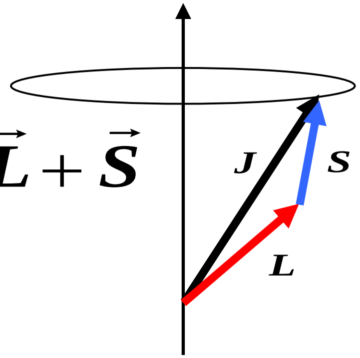
$$\psi_{nlm_l sm_s}(\boldsymbol{r}) = R_{nl}(r) Y_{lm_l}(\theta,\varphi)\,\chi_{sm_s}(\sigma)$$

What is wrong here? We already learned that *l* and *s* should be coupled together to form total angular momentum *j*.

Building Dirac spinor

- **We shall “couple” together angular momentum and spin to obtain total angular momentum:**

$$\vec{J} = \vec{L} + \vec{S}$$



$$Y_{lm_l}(\theta,\varphi)\,\chi_{sm_s}(\sigma)$$

$$\Omega_{ljm_j}(\hat{\boldsymbol{r}}) = \sum_{m_l m_s} \left(l m_l \; s m_s \middle| j m_j \right) Y_{lm_l}(\theta,\varphi)\,\chi_{sm_s}(\sigma)$$

Clebsch-Gordan coefficients
(more detailed discuss comes later)

- **Dirac spinors are the eigenfunctions of operators J^2 and J_z:**

$$\hat{\boldsymbol{J}}^2 \Omega_{jm_j} = j(j+1)\hbar^2 \Omega_{jm_j} \qquad \hat{J}_z \Omega_{jm_j} = m_j \hbar \Omega_{jm_j}$$

Structure of Dirac wavefunctions

- **Stationary Dirac equation for particle in Coulomb field reads:**

$$\left(-i\hbar c\boldsymbol{\alpha}\cdot\nabla - \frac{Ze^2}{r} + m_e c^2 \alpha_0\right)\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r})$$

- **Wavefunctions can be written now as:** $\psi_{nljm_j}(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} g_{nj}(r)\Omega_{ljm_j}(\hat{\boldsymbol{r}}) \\ i\, f_{nj}(r)\Omega_{l'jm_j}(\hat{\boldsymbol{r}}) \end{pmatrix}$

- **Where the angular and spin dependence is in Dirac spinors:**

$$\Omega_{ljm_j}(\hat{\boldsymbol{r}}) = \sum_{m_l m_s} \left(lm_l\ sm_s \middle| jm_j\right) Y_{lm_l}(\theta,\varphi)\,\chi_{sm_s}(\sigma)$$

- **And *g(r)* and *f(r)* are the large and small radial components of the Dirac wavefunction.**

◆ **How to find these radial components?**

Coupled radial equations

- **By substituting wavefunction** $$\psi_{nljm_j}(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} g_{n\kappa}(r)\Omega_{ljm_j}(\hat{\mathbf{r}}) \\ i\, f_{n\kappa}(r)\Omega_{l'jm_j}(\hat{\mathbf{r}}) \end{pmatrix}$$
- **into Dirac's equation** $$\left(-i\hbar c\boldsymbol{\alpha}\cdot\nabla - \frac{Ze^2}{r} + m_e c^2 \alpha_0\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$$
- **we obtain the coupled radial equations:**

$$\left(\frac{d\, f_{n\kappa}(r)}{dr} - \frac{\kappa}{r} f_{n\kappa}(r)\right) = -\left(E - V(r) - m_e c^2\right) g_{n\kappa}(r)$$

$$\left(\frac{dg_{n\kappa}(r)}{dr} + \frac{\kappa}{r} g_{n\kappa}(r)\right) = \left(E - V(r) + m_e c^2\right) f_{n\kappa}(r)$$

- **which can be solved and ...**

Dirac‘s radial components

- **We finally may derive analytic expressions for the radial components of the Dirac’s equation (for point-like nucleus!):**

$$f_{n\kappa}(r) = N_{n\kappa}\sqrt{1+W_{n\kappa}}\,r(2qr)^{s-1}\mathrm{e}^{-qr} \times \left[-n'F(-n'+1,2s+1;2qr) - \left(\kappa - \frac{\alpha Z}{q\lambda_c}\right)F(-n',2s+1;2qr)\right],$$

$$g_{n\kappa}(r) = -N_{n\kappa}\sqrt{1-W_{n\kappa}}\,r(2qr)^{s-1}e^{-qr} \times \left[n'F(-n'+1,2s+1;2qr) - \left(\kappa - \frac{\alpha Z}{q\lambda_c}\right)F(-n',2s+1;2qr)\right],$$

where $n' = n - |\kappa| = 0, 1, 2, \ldots$ denotes the number of nodes of the radial components, $\lambda_c = \hbar/m_e c$ the Compton length of the electron, and

$$s = \sqrt{\kappa^2 - (\alpha Z)^2},$$

$$q = \frac{Z}{\sqrt{(\alpha Z)^2 + (n'+s)^2}}.$$

the so-called hypergeometric function

Moreover, the normalization factor

$$N_{n\kappa} = \frac{\sqrt{2}q^{5/2}\lambda_c}{\Gamma(2s+1)}\left[\frac{\Gamma(2s+n'+1)}{n'!(\alpha Z)(\alpha Z - \kappa q \lambda_c)}\right]^{1/2}$$

- **Radial components of the Dirac’s equation are implemented in many computer codes so there is usually no need to re-program these relations again.**

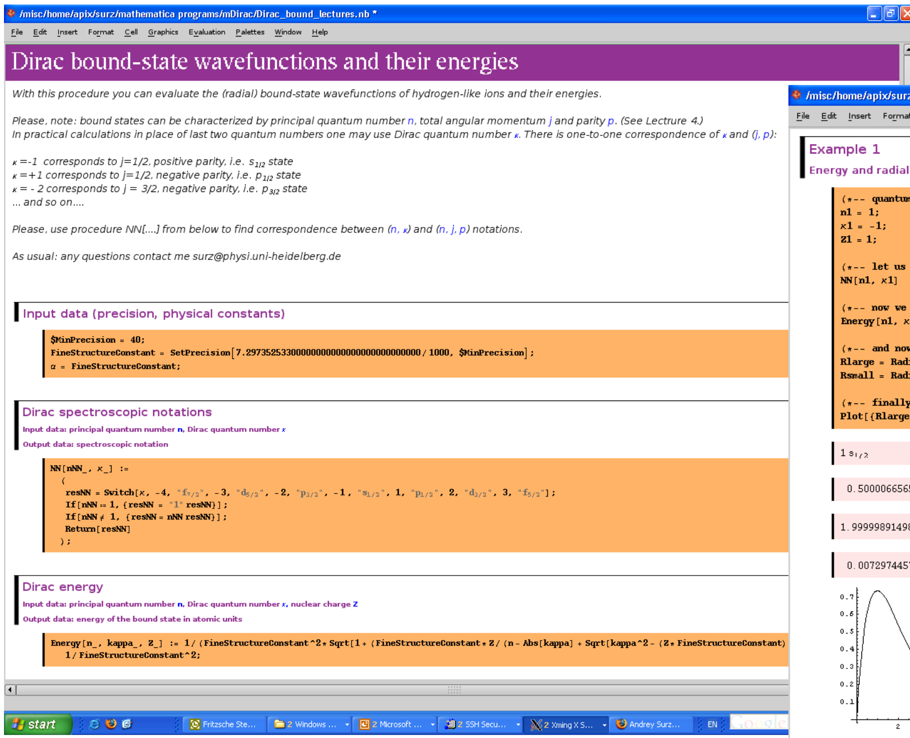
Dirac's radial components

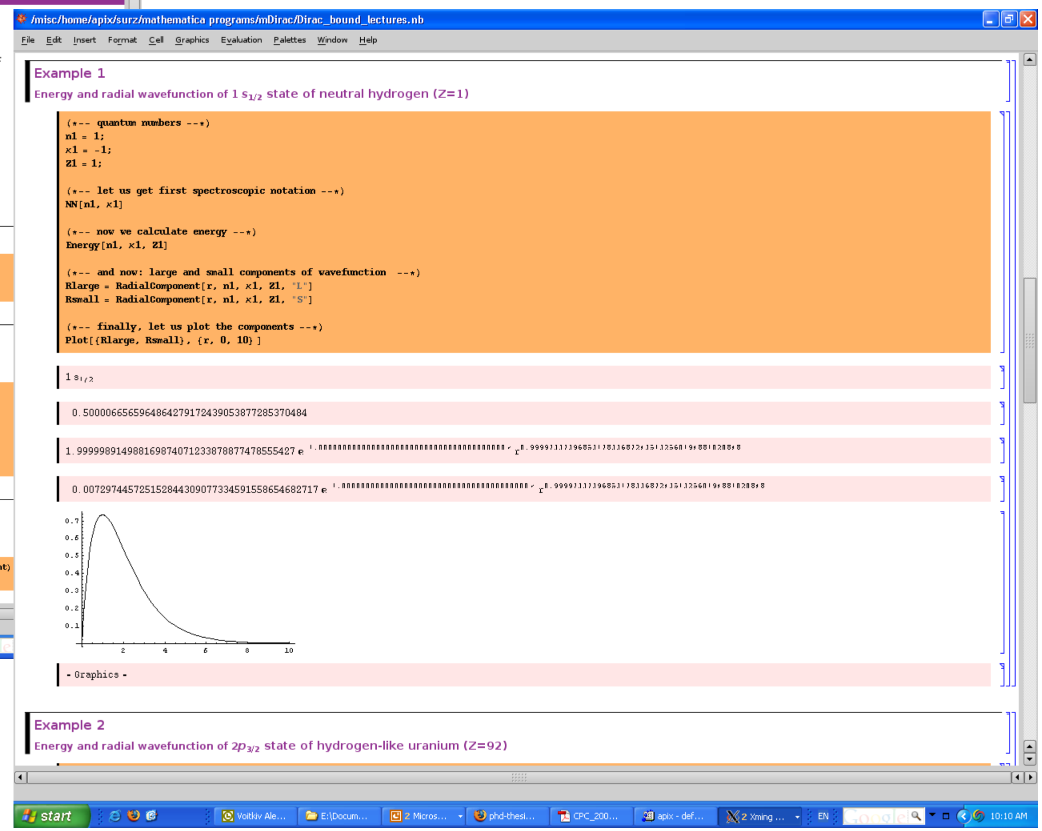
(Mathematica package)

Please, find zipped .nb files with the Mathematic notebooks at:

http://web-docs.gsi.de/~stoe_exp/lectures/SS2015/lectures.php

(password: dirac2015)



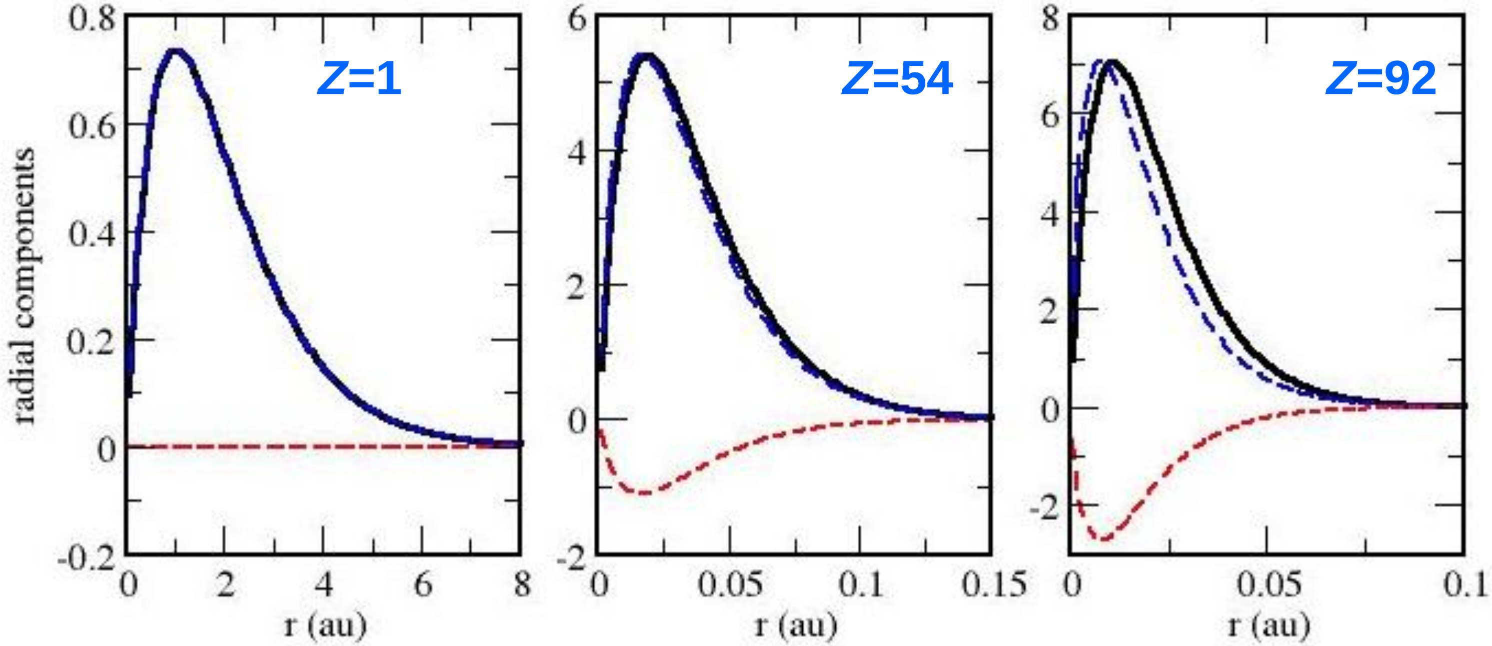


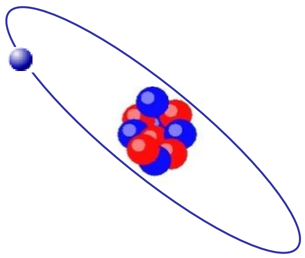
Dirac's radial components

(...behaviour)

- **Let us consider radial components of the wavefunction** $\psi_{nljm_j}(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} g_{nj}(r)\Omega_{ljm_j}(\hat{\boldsymbol{r}}) \\ i\, f_{nj}(r)\Omega_{l'jm_j}(\hat{\boldsymbol{r}}) \end{pmatrix}$

for particular case of $1s_{1/2}$ ground state.





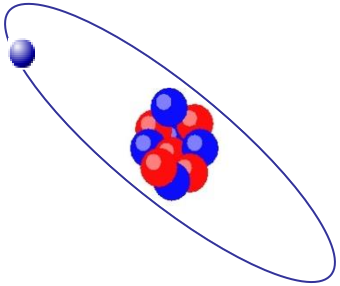
- **For low-Z regime: Dirac and Schrödinger wavefunctions basically coincides.**
- **For high-Z regime: small component becomes significant and ...**

Relativistic contraction of atomic orbitals

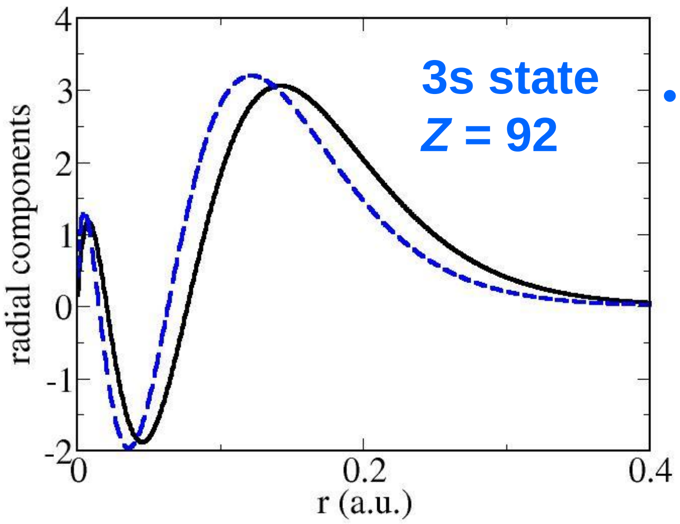
- **From the simple model one can "estimate" the electron "velocity" in the ground state:**

$$v = (\alpha Z)\, c$$

Speed of light



- **For hydrogen-like Uranium (Z=92):** $\alpha Z \approx 0.67$



- **due to STR:** $m_{el} = \frac{m_{el}^{0}}{\sqrt{1-(v/c)^2}}$ **(electron becomes heavier)**
- **due to simple Bohr's model:** $r_n \propto \frac{n^2}{Z\, m_{el}}$

As the electron's mass increases, the radius of an orbit with constant angular momentum shrinks proportionately.

Plan of lecture

- **Reminder from the last lecture: Bound-state solutions of Dirac equation**
- **Higher-order corrections to Dirac energies:**
  - **Radiative corrections (QED effects)**
  - **Hyperfine interaction**

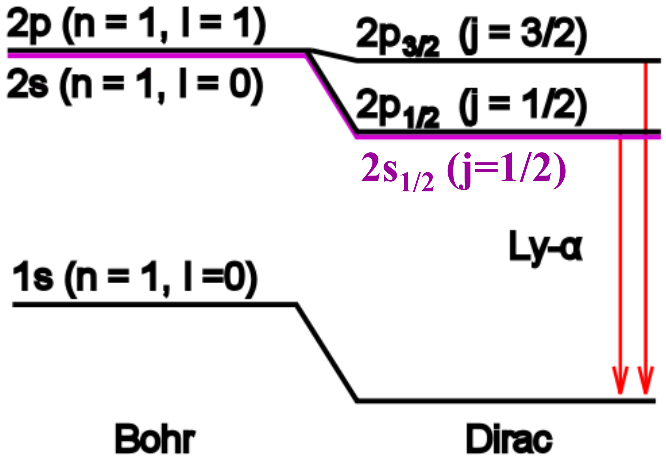
Bound-state solutions of Dirac equation

(reminder from the last lecture)

- **Main question we have to answer today is:**

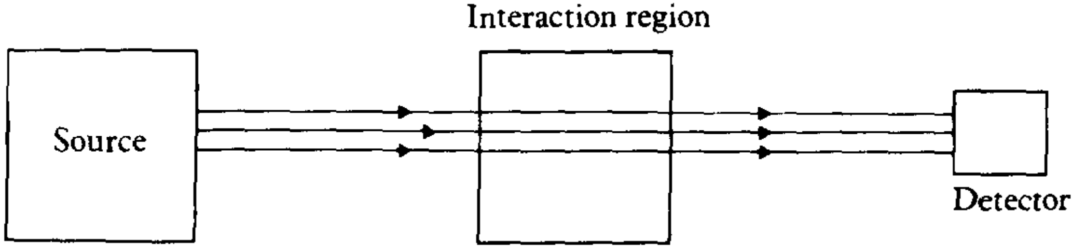
Are any further corrections to Dirac energies?

$2s_{1/2}$ – $2p_{1/2}$ energy splitting



- **From the middle of 30's several measurements have been reported which <u>probably</u> indicated that $2s_{1/2}$ and $2p_{1/2}$ levels do not coincide.**
- **The problem of these (first) experiments was their technique: optical spectroscopy of Ly-α lines.**

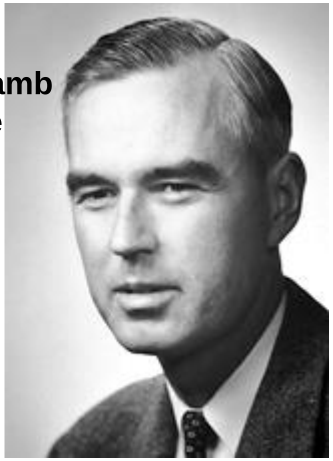
- **Another approach has been used in brilliant experiment by Lamb and Retherford (1947) who used *microwave* techniques to stimulate a direct transition between $2s_{1/2}$ and $2p_{1/2}$ levels.**



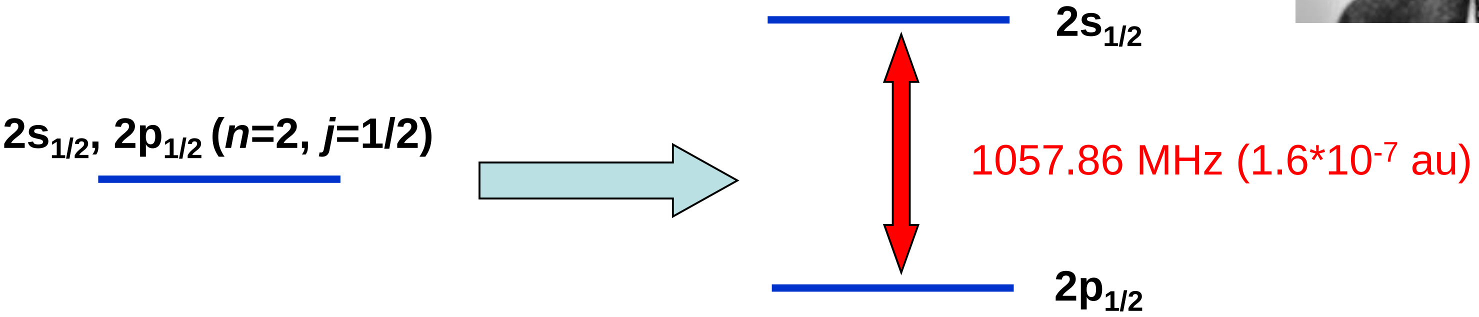
5.16 Schematic diagram of the Lamb–Retherford experiment. The source produces an atomic beam of hydrogen containing a small fraction of atoms in the $2s_{1/2}$ level. The beam is passed through a region of a radio-frequency electric field and a variable magnetic field and is detected by an apparatus which records only atoms in the $n = 2$ level.

Lamb shift

- **According to Dirac theory, levels $2s_{1/2}$ and $2p_{1/2}$ should be degenerated (since the have the same *j*).**
- **However, in 1947 Willis Lamb and Robert Retherford have a small difference in energy between these two levels!**



Willis Eugene Lamb
1955 Nobel prize

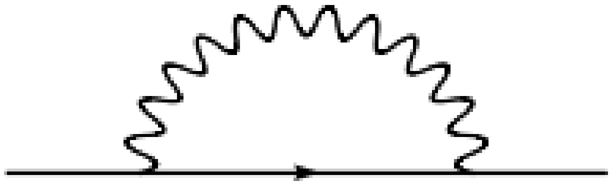


- **To compare: energy of $2s_{1/2}$ and $2p_{1/2}$ levels is -0.125 au.**

Obviously: some effects which beyond the Dirac theory have to be taken into account!
By which???

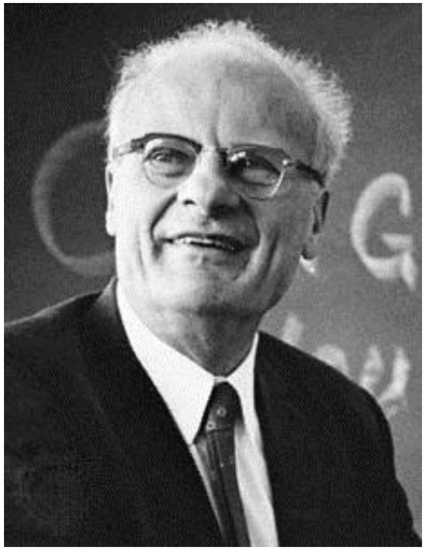
Lamb shift: Idea of radiative corrections

- **From the end of 1930th : interaction of electron with radiation filed. But which field?**



- **Ideas: electron may interact with its own field. The Coulomb potential is therefore perturbed by a small amount and the degeneracy of the two energy levels is removed.**

- **But: problems with divergence of results!**



Hans Bethe

- **In 1947 Hans Bethe has shown how to identify the divergent terms and to substract them from the theoretical expression.**

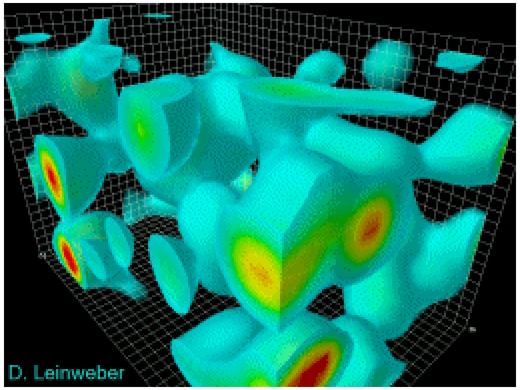
Development of
Quantum Electrodynamics (QED)
and Quantum Field Theory (QFT)

Lamb shift: Idea of radiative corrections

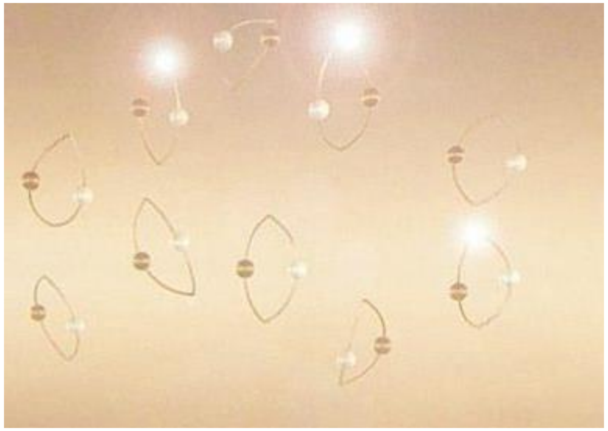
- **What is vacuum?**

- **With “classical” vacuum it is clear: it is a volume of space that is essentially empty of “everything”.**

“An Experiment on a Bird in the Air Pump”
by Joseph Wright of Derby



QCD vacuum fluctuations

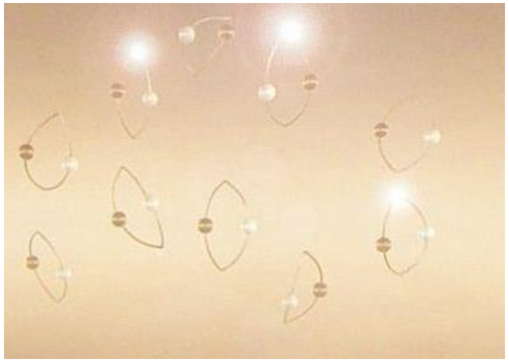


- **According to present-day understanding of what is called quantum vacuum, it is "by no means a simple empty space”.**

- **The quantum vacuum is not truly empty but instead contains fleeting electromagnetic waves and particles that pop into and out of existence.**

Virtual particles

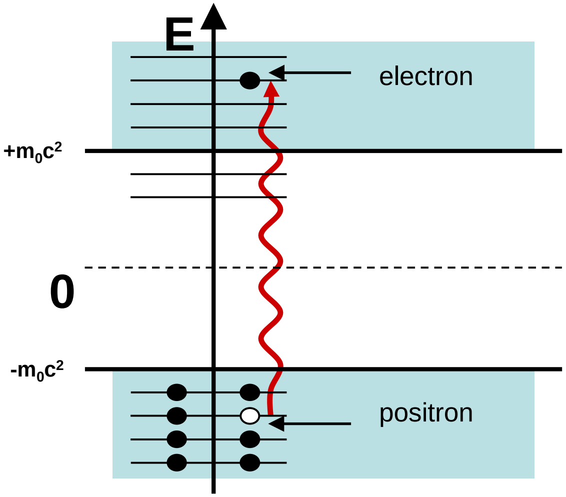
- **The uncertainty principle allows virtual particles (each corresponding to a quantum field) continually materialize out of the vacuum, propagate for a short time and then vanish.**



- **Roughly speaking, we "borrow" energy for a short time.**

- **For example, let us borrow at least $2mc^2$ and born for a short time (with temporary violation of conservation of energy) electron-positron pair.**

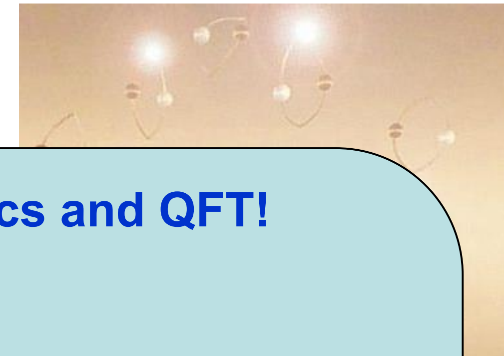
$$\Delta E\ \Delta t \geq \hbar / 2$$



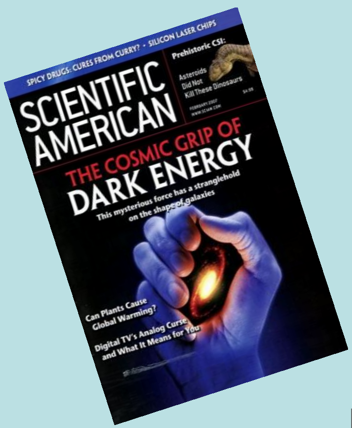
From Heisenberg principle it is clear: virtual particles can not exists infinitely long and can not "escape to infinity".

Virtual particles

- The uncertainty principle allows virtual particles (each corresponding to a quantum field) continually materialize out of the vacuum, propagate for a short time and then va

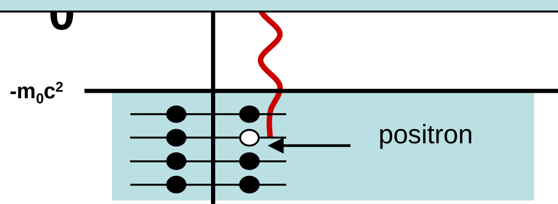


“Philosophical aspects” of quantum mechanics and QFT!



“Are virtual particles really constantly popping in and out of existence? Or are they merely a mathematical bookkeeping device for quantum mechanics?”

http://www.sciam.com/article.cfm?id=are-virtual-particles-rea&topicID=13

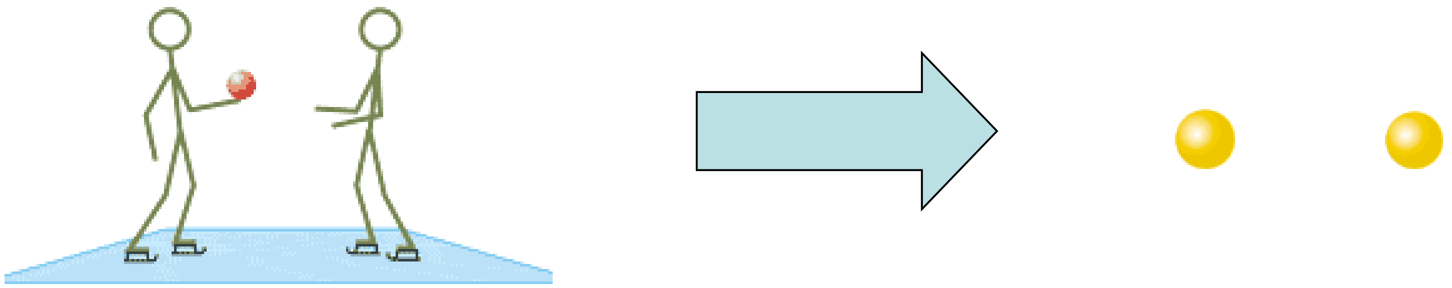


not exists infinitely long and can not “escape to infinity”.

Virtual particles and interactions

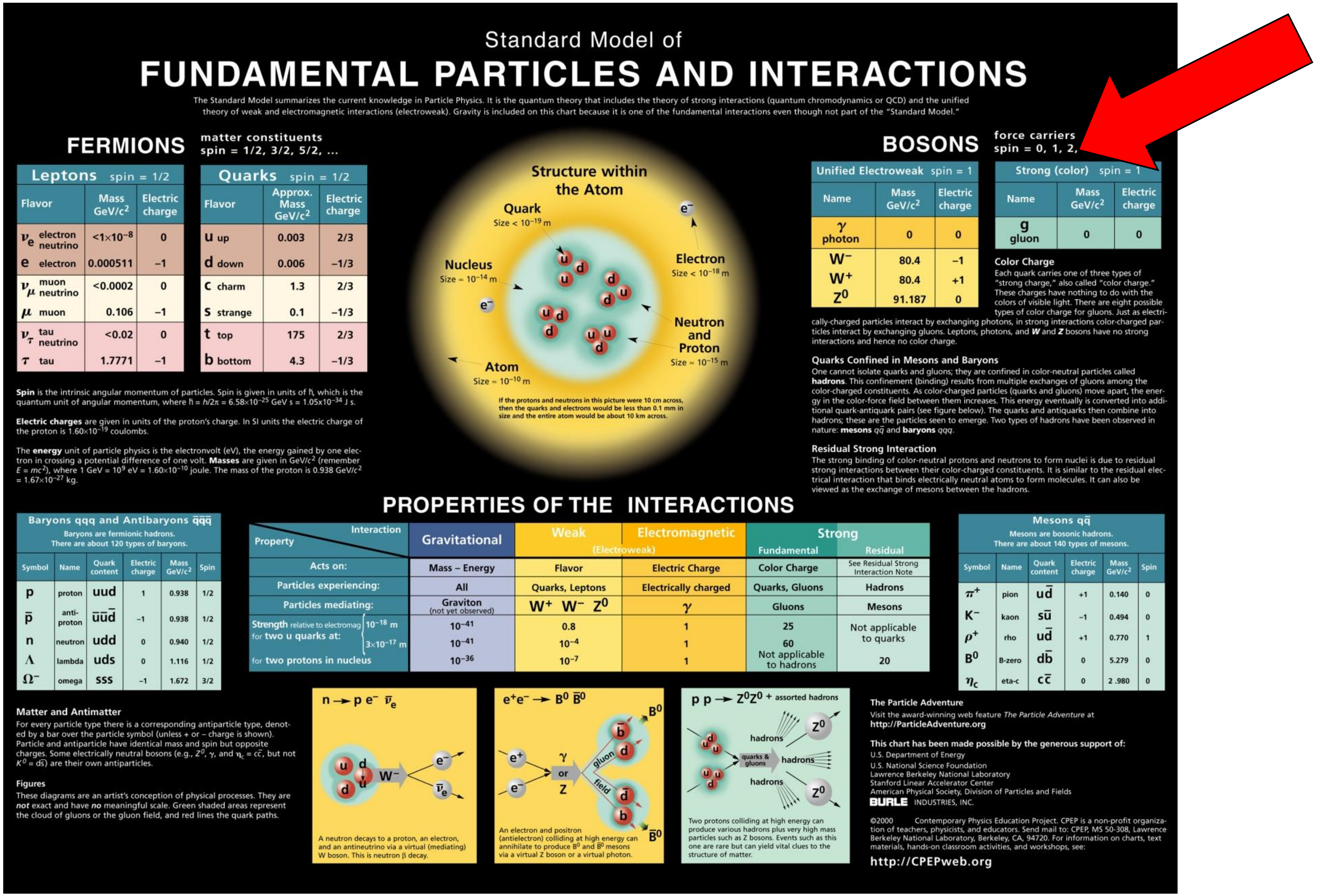
- **The concept of virtual particles necessarily arises in the (perturbation theory of) quantum field theory, where interactions (essentially, forces) between real particles are described in terms of exchanges of virtual particles.**

- **Idea of exchange forces!!!**



From: http://resources.schoolscience.co.uk/PPARC/16plus/partich6pg2.html

Virtual particles and interactions



Feynman diagrams

- **Feynman diagrams is a nice tool to represent exchange forces and, hence, to perform calculations of scattering processes.**

Richard Feynman

electron emits photon

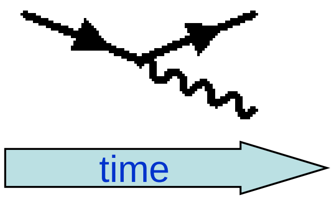
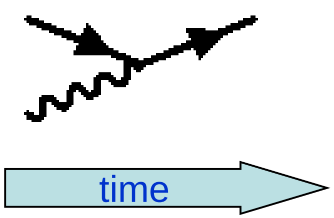
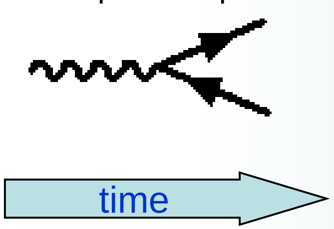


Image	Description	Particle Represented
	straight line, arrow to the right	**electron**
	straight line, arrow to the left	**positron**
	wavy line	**photon**

electron absorbs photon



photon produces electron-positron pair



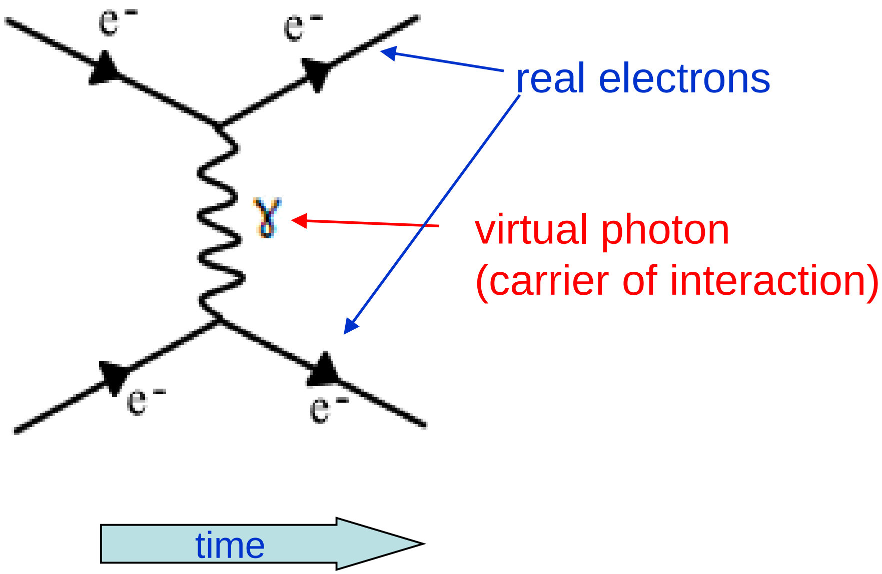
These are "building blocks" to construct interactions/scattering processes.

Feynman diagrams

- **Feynman diagrams is a nice tool to represent exchange forces and, hence, to perform calculations of scattering processes.**

Richard Feynman

Electron-electron scattering



Feynman diagrams

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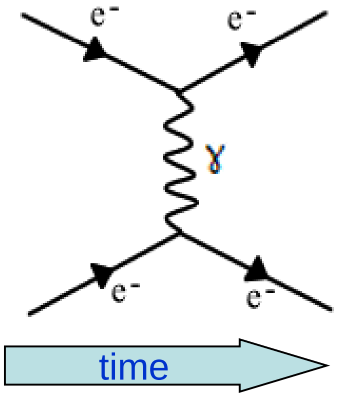
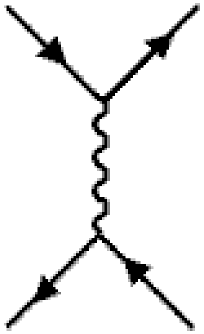
Richard Feynman

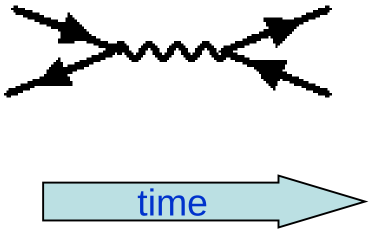
Electron-electron repulsion



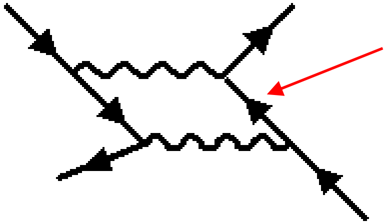
Electron-positron scattering



Electron-positron annihilation and creation



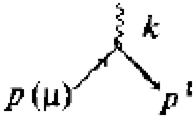
… and many many other processes like:



virtual electron

Feynman technique

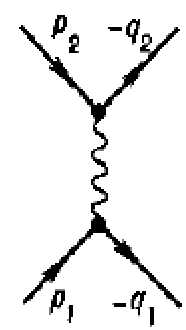
- **Feynman diagrams is a nice tool to perform calculations of scattering processes.**
- **Each Feynman diagram an amplitude for some process!**
- **In order to "translate" the Feynman diagram to language of formulas, one has to use set of rules.**

vertex		
virtual photon		$\frac{1}{(2\pi)^4 i}\frac{\varepsilon_{\mu,\nu}}{-k^2}$
virtual electron (positron)		$\frac{1}{(2\pi)^4 i}\frac{m+\hat{p}}{m^2-p^2}\hat{p} = \gamma^\mu \rho_\mu$
photon		$\frac{(e^\alpha(k))_\mu}{(2\pi)(3/2)\sqrt{2k_0}}$
outgoing electron		$(2\pi)^{-3/2}\vec{v}_\sigma(\rho)$
incoming electron		$(2\pi)^{-3/2}\vec{v}_\rho(\rho)$

Feynman technique

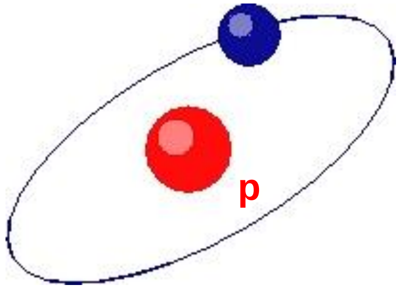
vertex		$(2\pi)^4\, ie\gamma^\mu\, \delta^{(4)}(p + k - p')$
virtual photon		$\frac{1}{(2\pi)^4 i}\frac{\varepsilon_{\mu,\nu}}{-k^2}$
virtual electron (positron)		$\frac{1}{(2\pi)^4 i}\frac{m+\hat{p}}{m^2-p^2}\hat{p} = \gamma^\mu \rho_\mu$
photon		$\frac{(e^\alpha(k))_\mu}{(2\pi)^{(3/2)}\sqrt{2k_0}}$
outgoing electron		$(2\pi)^{-3/2}\vec{v_\sigma}(\rho)$
incoming electron		$(2\pi)^{-3/2}\vec{v_\rho}(\rho)$

- **Amplitude for one of the simplest processes (electron-electron or Moller scattering)**

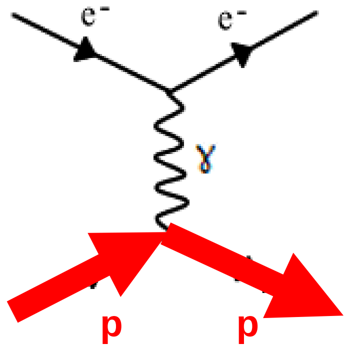


$$M(p_1,p_2,-q_1,-q_2) = \frac{e^2}{i(2\pi)^2}\delta(p_1+p_2+q_1+q_2)\frac{g_{\mu,\nu}}{(p_1+q_1)^2}\vec{v_\sigma}(-q_1)\gamma^\mu v_\rho(p_1)\vec{v_\kappa}(-q_2)\gamma^\nu v_\lambda(p_2)$$

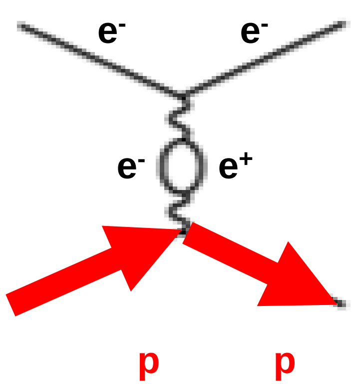
Back to Lamb shift



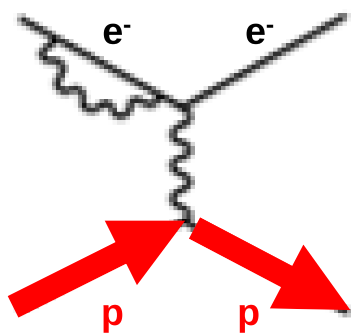
- **Interaction of electron with atomic/ionic nucleus in zero approximation:**



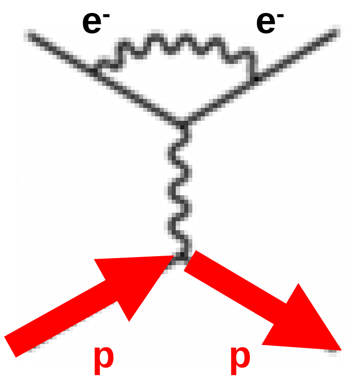
- **But we can now consider the next-order corrections:**



vacuum polarization



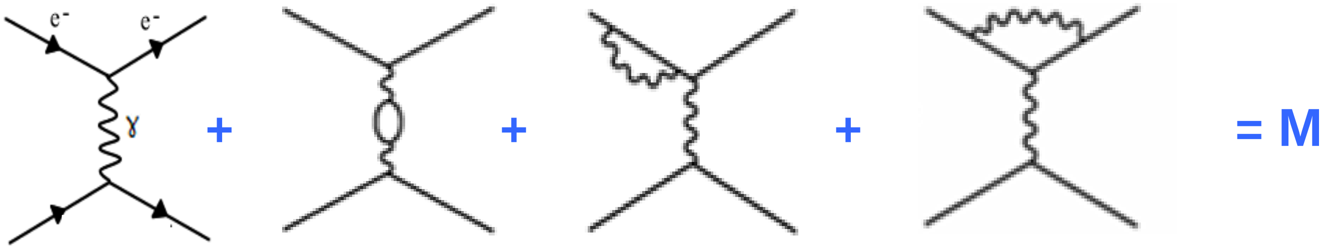
self energy



vertex correction
(anomalous magnetic moment)

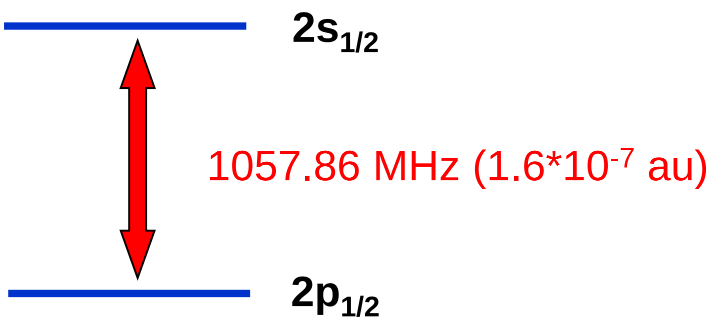
Lamb shift in neutral hydrogen

- **Remembering that every Feynman diagram can be attributed to some amplitude, one may evaluate the numerical value of the Lamb shift.**



- **For the neutral hydrogen we find:**

Effect	Energy contribution
Vacuum polarization	-27 MHz
Electron mass renormalization	+1017 MHz
Anomalous magnetic moment	+68 MHz
Total	+1058 MHz

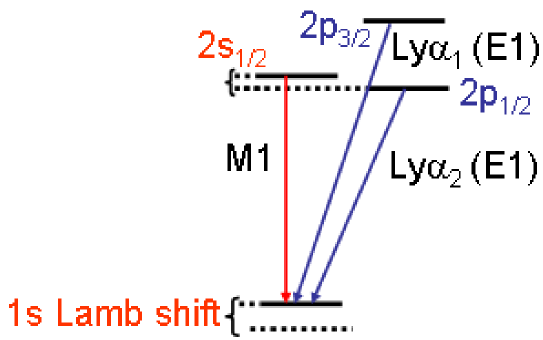


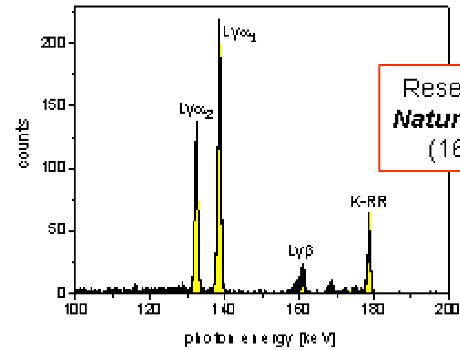
- **Another important result: Lamb shift increases with increasing of nuclear charge Z!**

→ **Experiments with high-Z, hydrogen-like ions are performed!**

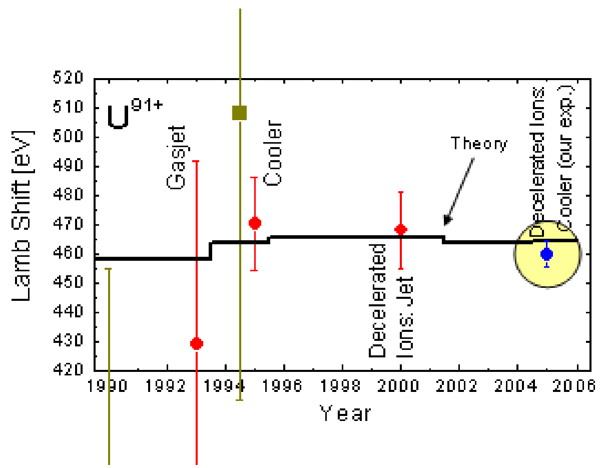
Lamb shift for high-Z ions

- **At the GSI facility in Darmstadt experiments are performed to measure Lamb shift of the ground $1s_{1/2}$ level of hydrohen-like uranium.**



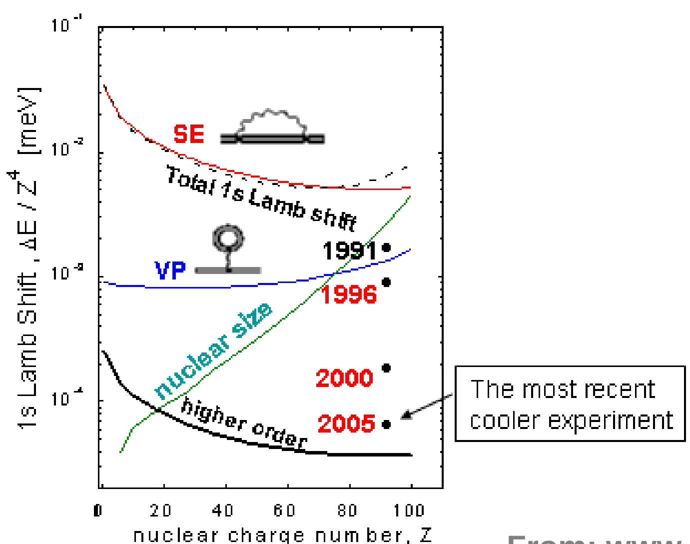


Research Highlights
***Nature* 435**, 858-859
(16 June 2005)



The most recent result from the electron cooler exp.

- **1% sensitivity to the 1s Lamb shift**
- **Most stringent test of bound-state QED for one-electron high-Z systems**

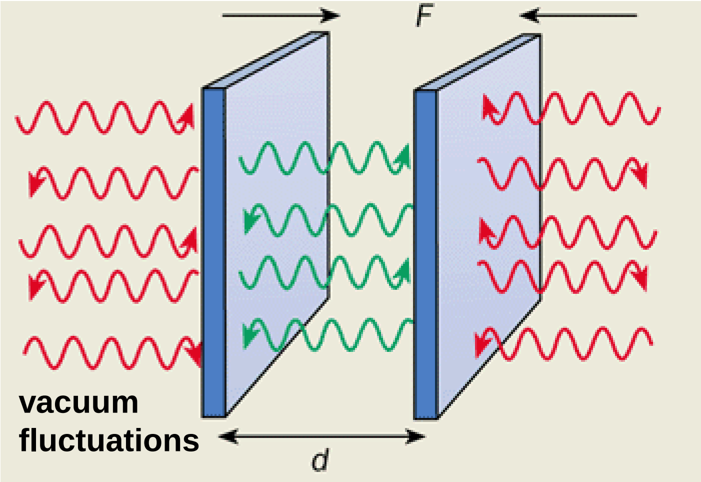


From: www.gsi.de

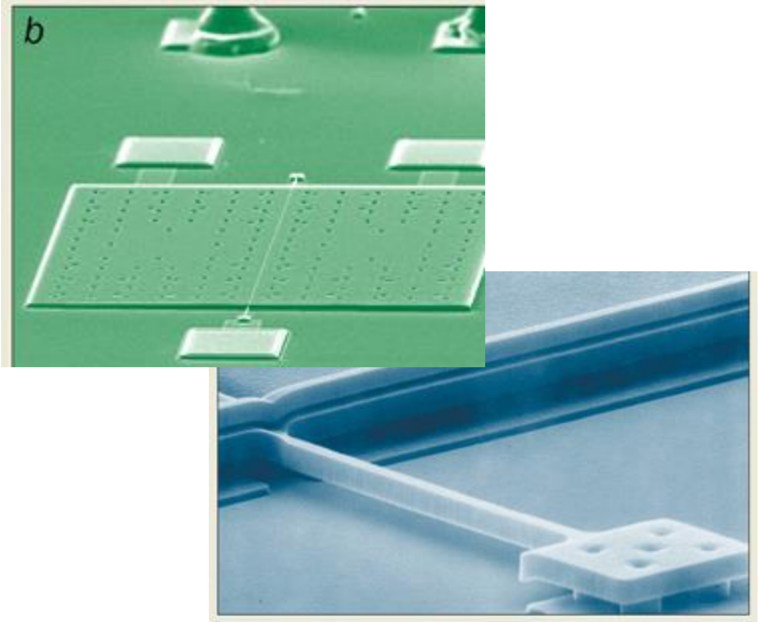
Problem 6.1: Compare the 1s-Lamb shift in hydrogen-like Uranium (see the previous slide) with the relativistic shift of the 1s energy (i.e. $\Delta E_{1s} = E_{1s}^{non-rel} - E_{1s}^{rel}$)

Casimir effect

- **Another, very interesting manifestation of the (properties of) physical vacuum is the so-called Casimir effect.**
- **The Casimir effect is a small attractive force which acts between two close parallel *uncharged* conducting plates (in vacuum).**
  - **On average the external "radiation pressure" (red arrows) is greater than the internal pressure (green arrows).**



From: http://physicsworld.com/

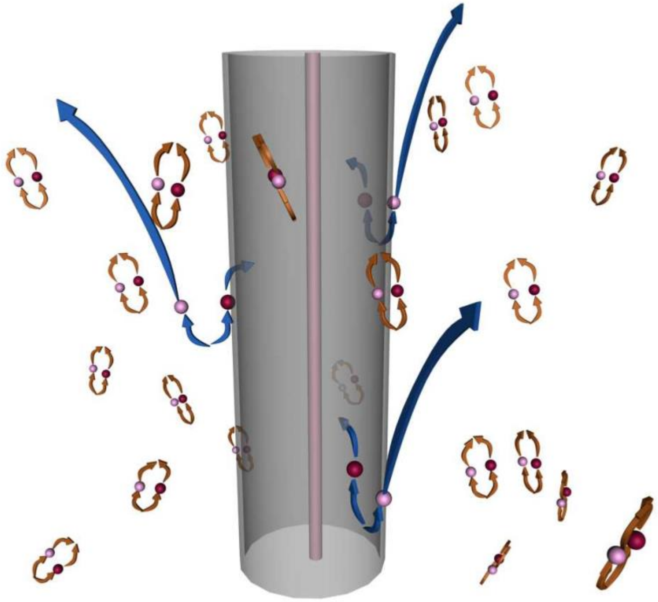


- **Nowadays: Casimir effect studies have a significant impact for the development of nanotechnologies.**

Hawking radiation

- **Vacuum fluctuations cause a particle-antiparticle pair to appear close to the event horizon of a black hole. One of the pair falls into the black hole whilst the other escapes.**
- **To an outside observer, it would appear that the black hole has just emitted a particle.**

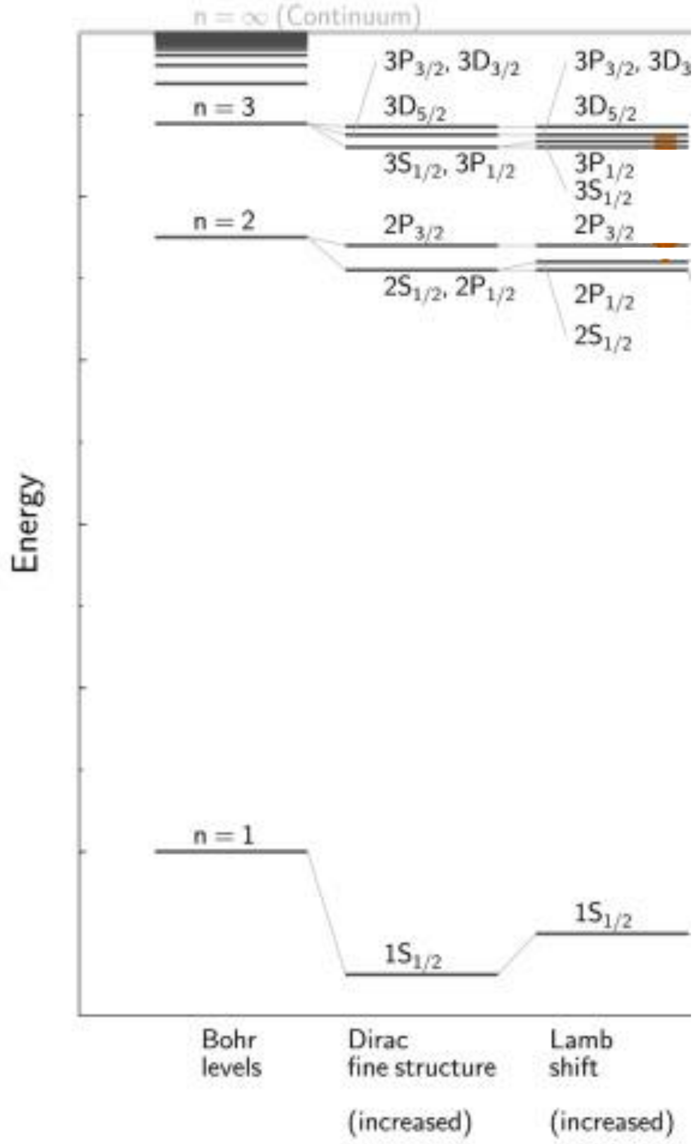
Stephen Hawking



- **Until now Hawking radiation has not been observed experimentally.**

Energy levels of hydrogen-like ions

From Schrödinger to Dirac to QED



- **Do we expect some more effects which can influence the bound-state structure of hydrogen-like ions?**

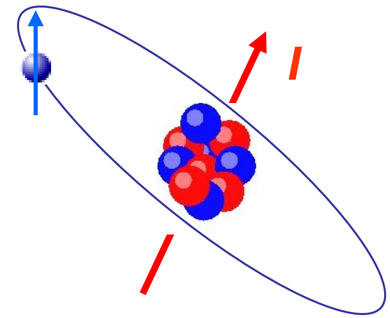
Actually, yes!

Plan of lecture

- **Reminder from the last lecture: Bound-state solutions of Dirac equation**
- **Higher-order corrections to Dirac energies:**
  - **Radiative corrections (QED effects)**
  - **Hyperfine interaction**

Nuclear spin and magnetic moment

- **Until now we have assumed in our analysis that nucleus has *zero* nuclear spin.**
- **However, there are many isotopes having non-zero (integer or half-integer) nuclear spin *I*.**



Nucleus	*Spin* I	*Landé factor* g_I	*Magnetic moment* $\mathcal{M}_N$ (*in nuclear magnetons*)
proton p	1/2	5.5883	2.79278
neutron n	1/2	−3.8263	−1.91315
deuteron ^2_1D	1	0.85742	0.85742
^3_2He	1/2	−4.255	−2.1276
^4_2He	0	—	0
$^{12}_6C$	0	—	0
$^{16}_8O$	0	—	0
$^{39}_{19}K$	3/2	0.2609	0.3914
$^{67}_{30}Zn$	5/2	0.35028	0.8757
$^{85}_{37}Rb$	5/2	0.54108	1.3527
$^{129}_{54}Xe$	1/2	−1.5536	−0.7768
$^{133}_{55}Cs$	7/2	0.7369	2.579
$^{199}_{80}Hg$	1/2	1.0054	0.5027
$^{201}_{80}Hg$	3/2	−0.37113	−0.5567

- **Associated with each nuclear spin there is a magnetic moment:**

$$\boldsymbol{M}_N = g_N \, \mu_N \boldsymbol{I} / \hbar$$

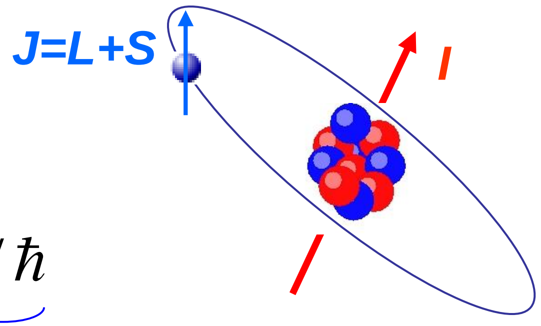
nuclear magneton

nuclear g factor

How non-zero nuclear spin may affect energy levels of hydrogen-like ions?

Hyperfine interaction

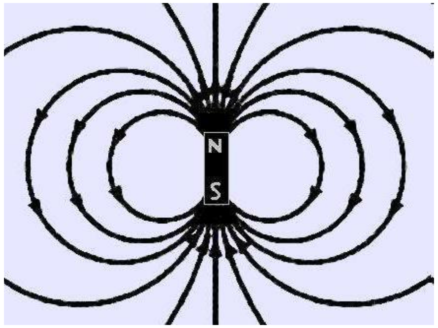
- The magnetic field due to the magnetic dipole moment of the nucleus will interact with the electron dipole momentum:



$$\underbrace{\hat{\boldsymbol{M}}_N = g_N\, \mu_N \hat{\mathbf{I}} / \hbar}_{\text{nuclear magnetic dipole moment}} \qquad \underbrace{\hat{\boldsymbol{\mu}} = -\mu_0 (\hat{\boldsymbol{L}} + g\hat{\boldsymbol{S}}) / \hbar}_{\text{electron moment}}$$

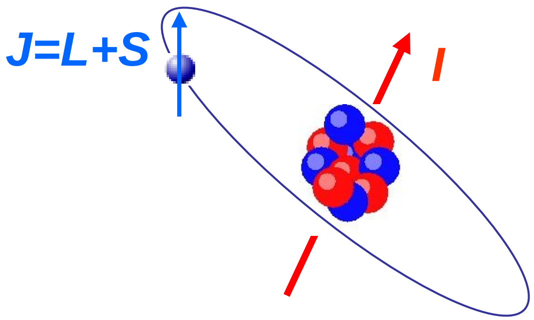
- i.e. it will interact both with electron orbital momentum and spin.

$$\hat{H}_1 \propto \mu_N \mu_0\, \boldsymbol{L} \cdot \boldsymbol{I} \qquad \hat{H}_2 \propto \mu_0\, \boldsymbol{S} \cdot \boldsymbol{B}$$



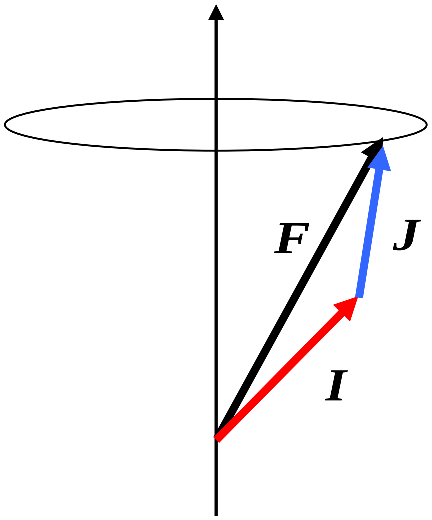
Again, we have to re-consider set of quantum numbers to describe our quantum states! (Please, remind yourself the case of spin-orbit interaction).

Total angular momentum *F* of an ion



- **We shall introduce total angular momentum F of the system "electron+ion"**

$$\vec{F} = \vec{J} + \vec{I}$$



- **Again, any other angular momentum it satisfies:**

$$\hat{F}^2 \Psi_{FM_F} = F(F+1)\hbar^2 \Psi_{FM_F}$$

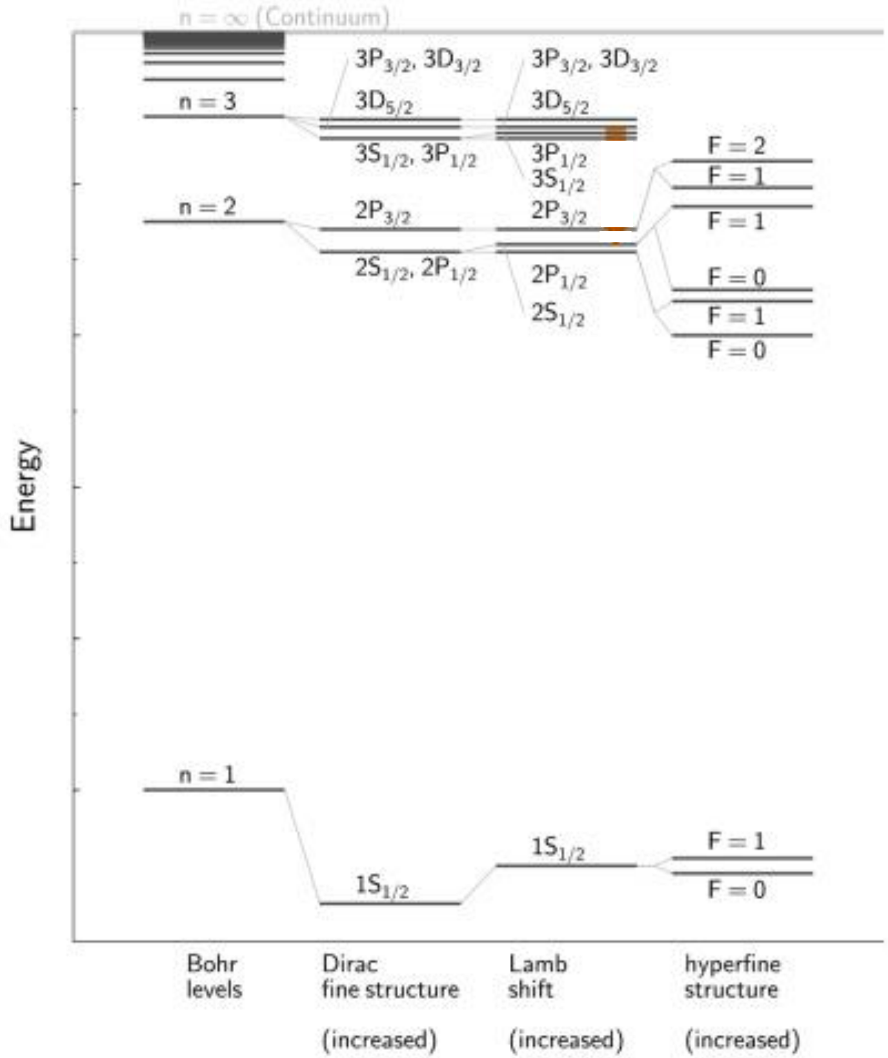
$$\hat{F}_z \Psi_{FM_F} = \hbar M_F \Psi_{FM_F}$$

- **Levels with the same n and j appear to be split one more time!**

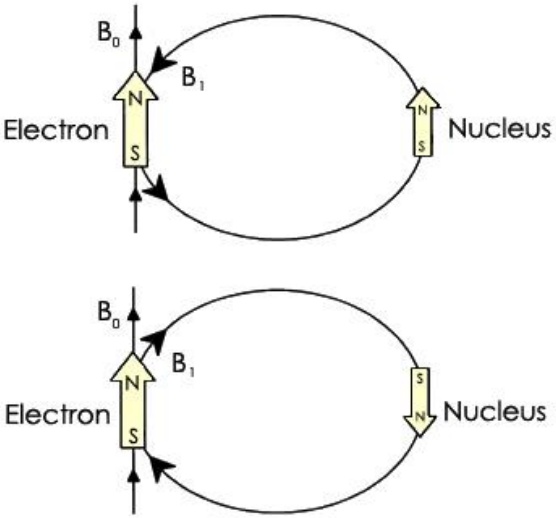
$$\Delta E_{HF} \approx a\left(F(F+1) - I(I+1) - J(J+1)\right)$$

Energy levels of hydrogen-like ions

From Schrödinger to Dirac to QED th HF interaction

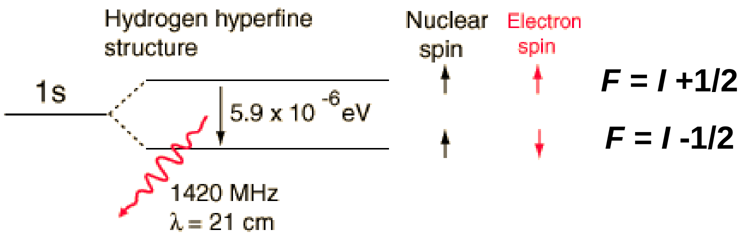


- **Note: even levels with *L*=0 (s states) appear to be split because of "spin-spin" interaction.**

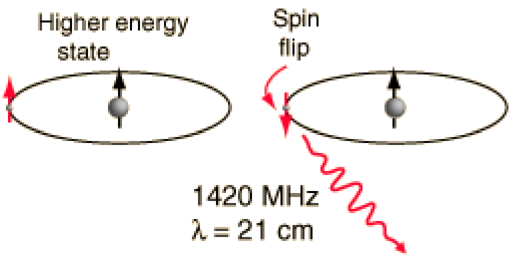


Hyperfine splitting in astrophysics

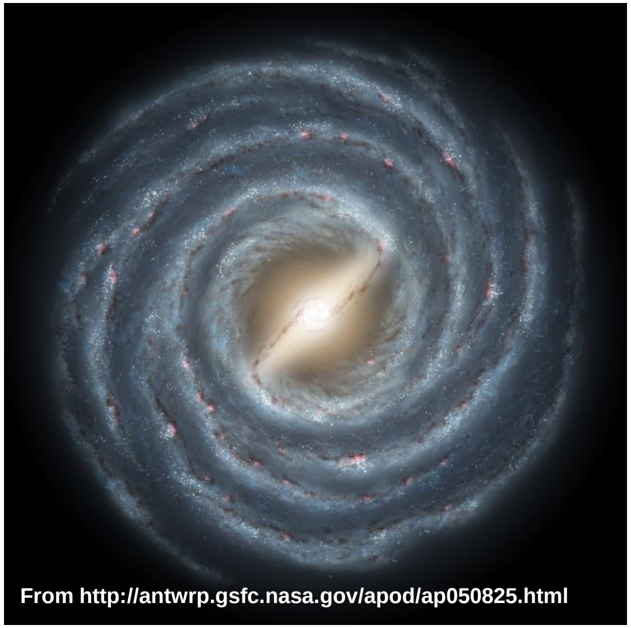
- **For the case of ground $1s_{1/2}$ state (j=1/2) of hydrogen, HF interaction results in splitting of energy level into two levels.**



- **One can observe transition between two HF levels: famous 21 cm line in astrophysics!**



From: http://http://hyperphysics.phy-astr.gsu.edu



From http://antwrp.gsfc.nasa.gov/apod/ap050825.html

- **21 cm radiation is used, for example, to measure radial velocities of spiral arms of Milky Way.**
- **Analysis of the properties of galaxies.**

Plan of lectures

- 1 15.04.2015 Preliminary Discussion / Introduction

- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (bound-state solutions of Dirac equation, quantum numbers)
- 6 20.05.2015 Theory (bound-state Dirac wavefunctions, QED corrections)

- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)
- 8 03.06.2015 Theory (single and multiple scattering, energy loss mechanisms, channeling regime)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ….)

- 10 17.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 11 24.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)

- 12 01.07.2015 Theory (atomic-physics tests of the Standard Model, search for a new physics)
- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)