Plan of lectures

- 1 15.04.2015 Preliminary Discussion / Introduction

- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (bound-state solutions of Dirac equation, quantum numbers)
- 6 20.05.2015 Theory (bound-state Dirac wavefunctions, QED corrections)

- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)
- 8 03.06.2015 Theory (description of the light-matter interaction)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ….)
- 10 17.06.2015 Theory (interaction of charged particles with matter)

- 11 24.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 12 01.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)

- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)

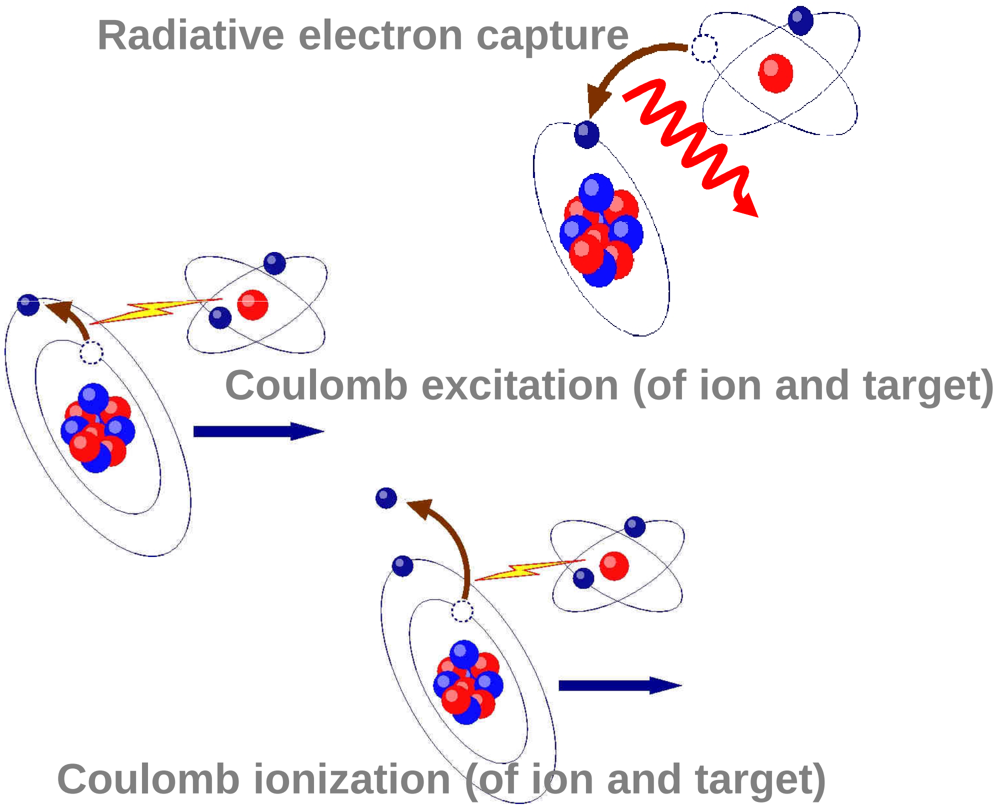
Interaction of charged particles with matter

Plan of lecture

- **Motivation: Why to study interaction of particles with matter?**
- **Structure of matter (very briefly)**
- **Interaction of positively charged particles with matter**
  - **Energy loss**
  - **Bethe-Bloch equation**
  - **Energy straggling**
- **Interaction of electrons with matter**
  - **Radiation and ionization losses**
  - **Track patterns**
- **Channeling regime and its applications**

Motivation for the lecture

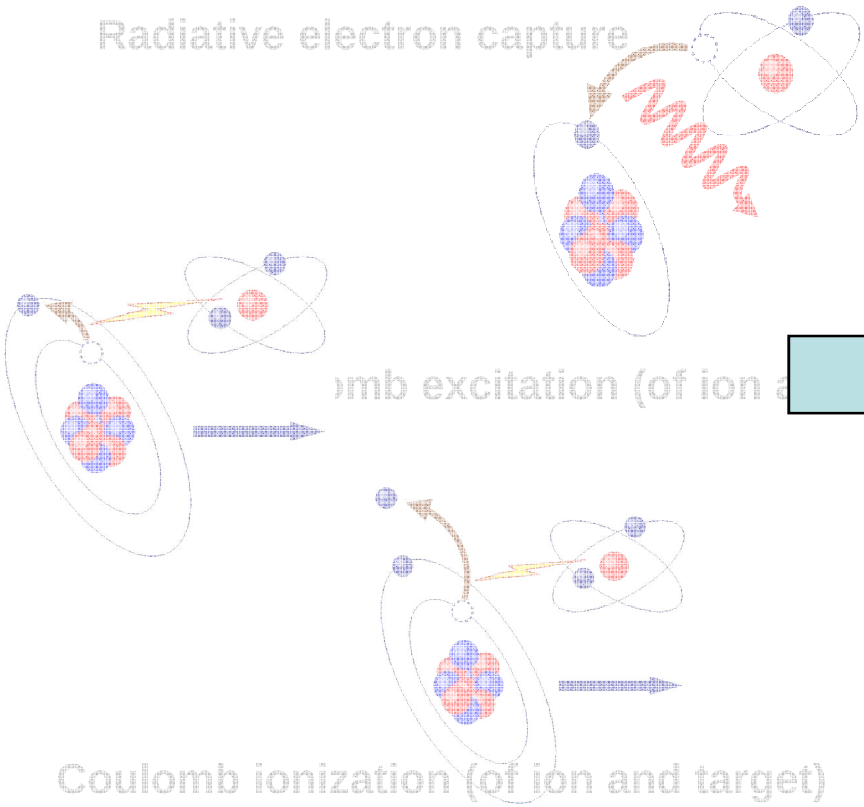
- **Usually one discusses in the lectures the single collisions/interactions.**



"one projectile – one target"

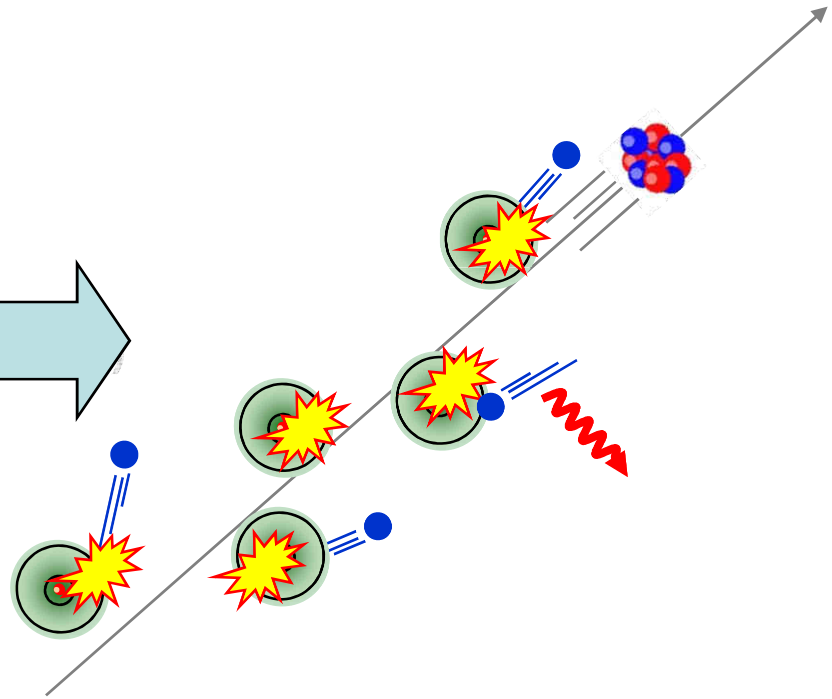
Motivation for the lecture

- Up to now (in our previous lectures) we have studied "single collision mode" for atomic/ionic collisions.



"one projectile – one target"

- **In reality, however this single collision regime is rather unusual.**

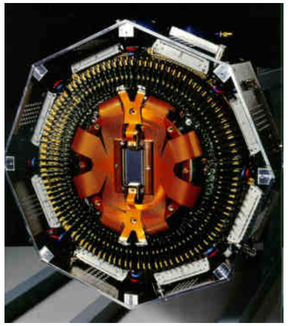


Many different processes happen!

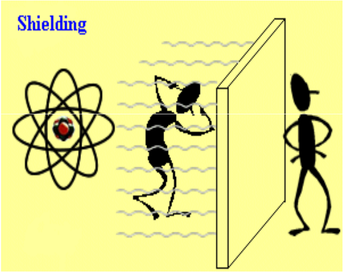
Motivation for the lecture

- **Why it is important to understand well the interactions of particles with matter?**

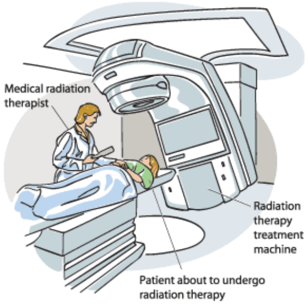
➡ **Development of novel detectors**



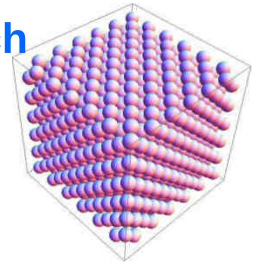
➡ **Design of radiation shielding**



➡ **Medical applications**



➡ **Fundamental research**



Plan of lecture

- Motivation: Why to study interaction of particles with matter?
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  - Bethe-Bloch equation
  - Energy straggling
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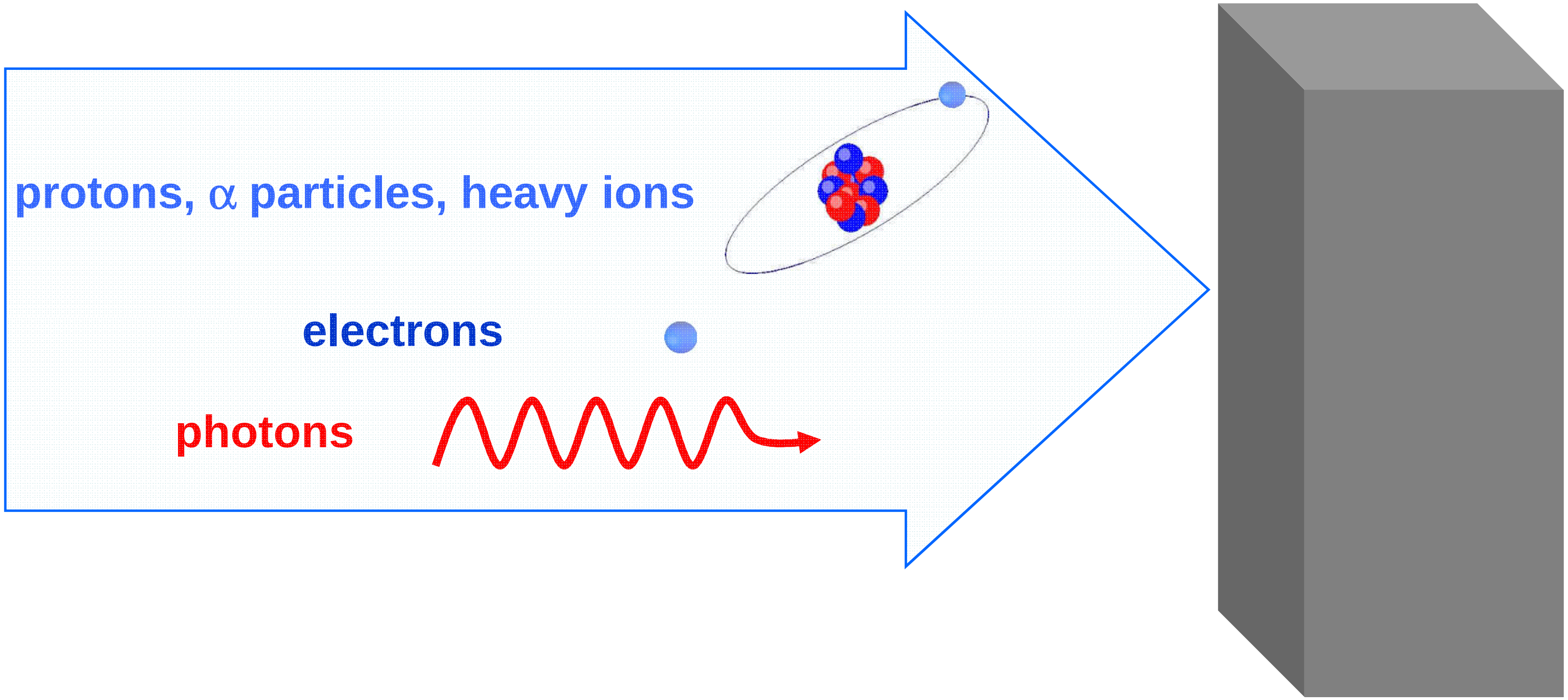
Structure of matter

(some characteristics)

- **First, let us discuss "what interacts with what".**



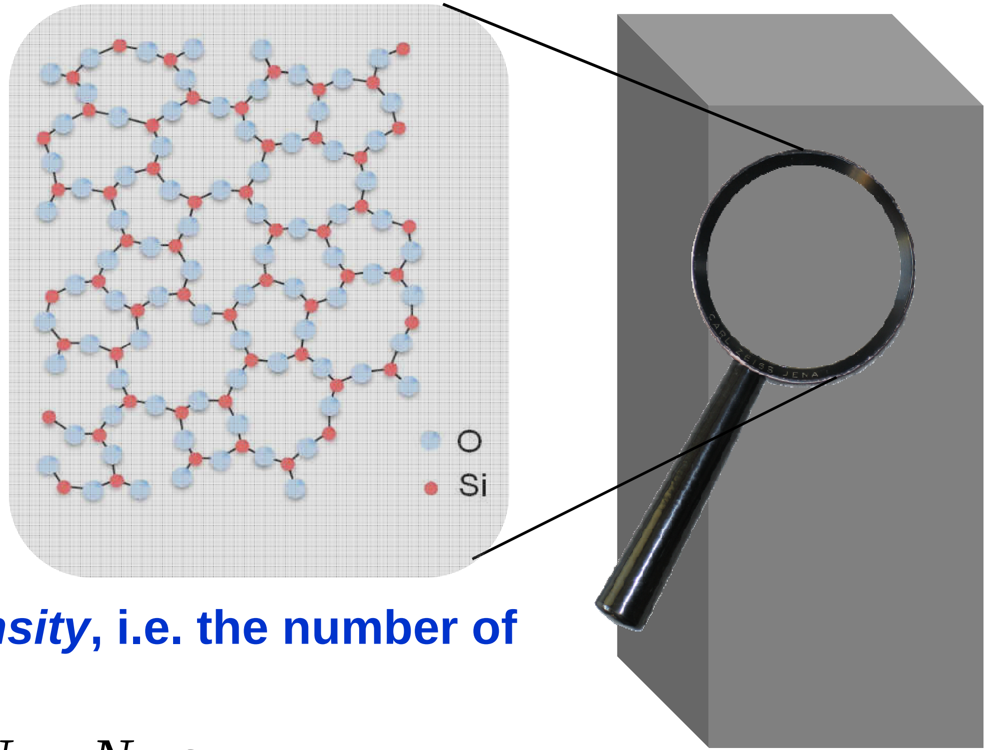
- **... and even more particles: neutrons, neutrino.....**

Structure of matter

(some characteristics)

- **For the moment we consider amorphous matter (there is no long-range order of the positions of the atoms).**

  - **Example: glassy silica (SiO_2)**
    From: www.wikipedia.org



- **Let us estimate *number density*, i.e. the number of atoms per unit volume:**

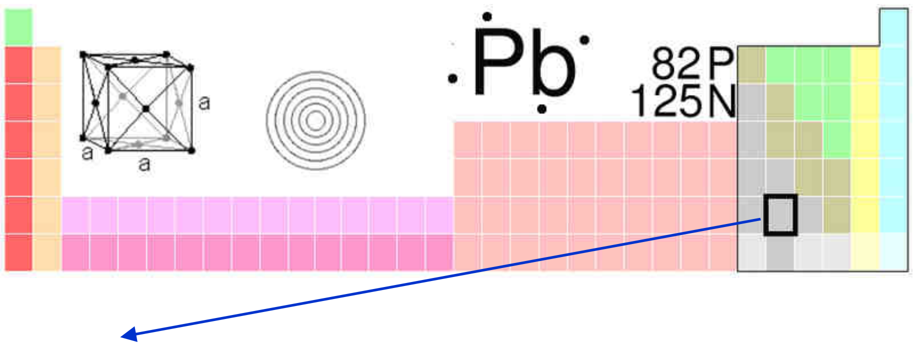
$$n = \frac{N}{V} = \frac{N_A \rho}{A}$$

N_A=6.02x10^{23} mol^{-1} is Avogardro's number, *A* is atomic weight and ρ is the density.

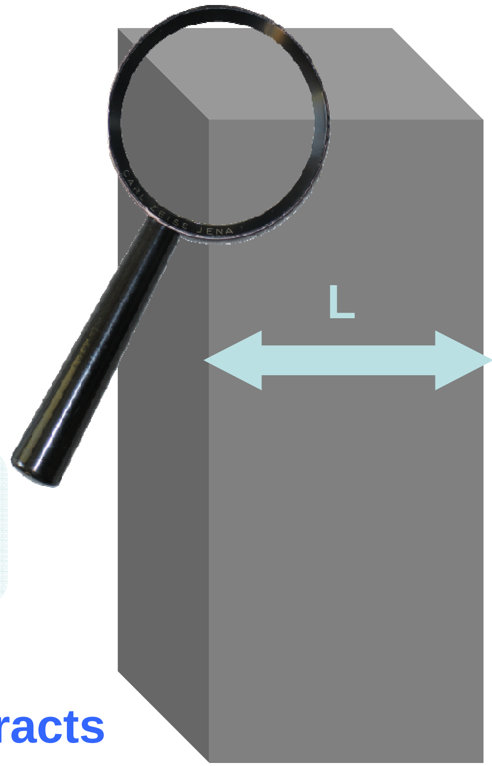
Structure of matter

(some characteristics)

- Lets us consider for example lead *Pb*.



***A* = 207.2 g mol^{-1} , ρ = 11.34 g cm^{-3}**



- ***Number density* of lead:** $$n = \frac{N_A \rho}{A} = 3.3 \times 10^{22} \, cm^{-3}$$

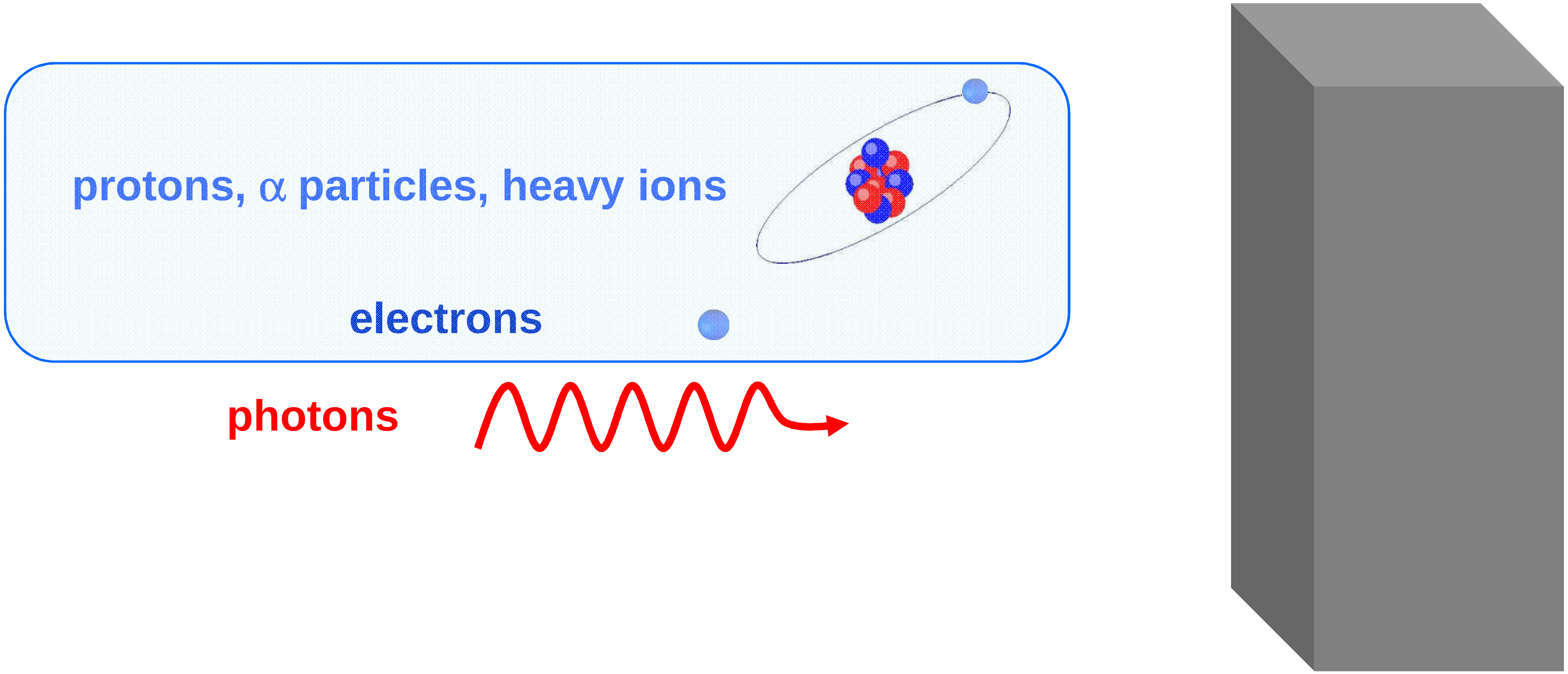
While penetrating solid target projectile particle interacts with huge amount of electrons/nuclei in the medium!

- **In order to understand interaction of particles with matter we shall again recall the “elementary” (single-event) processes.**

Structure of matter

(some characteristics)

- **First, let us discuss "what interacts with what".**



- **... and even more particles: neutrons, neutrino.....**

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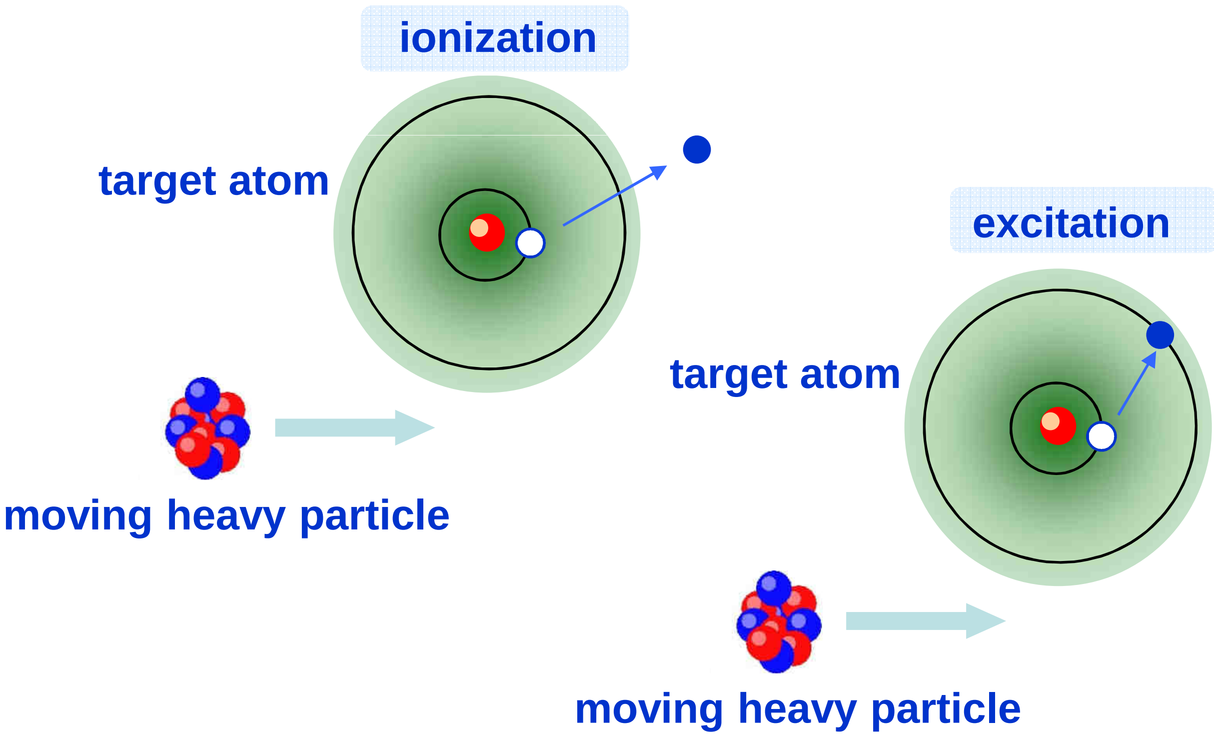
Heavy particles: Basic interaction processes

- **Let us restrict for the moment for the simplest case of proton or bare ion (i.e. ion without electrons).**
- **What can happen with such an ion during the matter penetration?**

In fact, there are many scenarios. We will choose the main!



Ion (proton) may ionize or excite target atoms.

Heavy particles: Basic interaction processes

- **Let us restrict for the moment for the simplest case of proton or bare ion (i.e. ion without electrons).**

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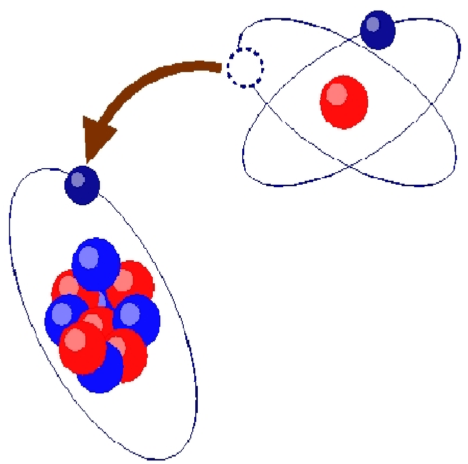
In fact, there are many scenarios. We will choose the main!



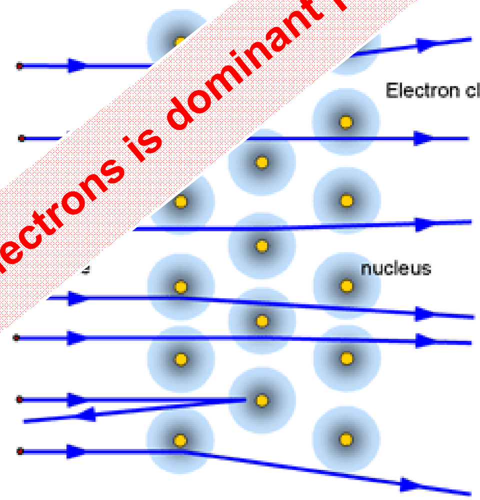
Ion may capture an electron from a target (i.e. to change its charge state).

But! It will be-very probable-ionized again since the ionization cross section is much larger than the capture one.

For the moment we will neglect this possibility!

Heavy particles: Basic interaction processes

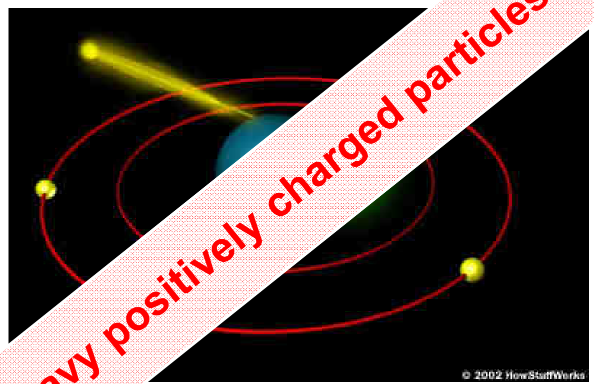
- **There is a possibility to interact with target nuclei.**
- **Probability is much smaller comparing to the probability for interaction with atomic electrons.**
  - **Size of the atom:** approx. 10^{-11} m
  - **Size of the nucleus:** approx. 10^{-15} m



For heavy positively charged particles (inelastic) collision with atomic electrons is dominant process!

Inelastic collisions with nuclei

- **Particle may radiate (bremsstra**



- **excite the nucleus.**

Elastic collisions with nuclei

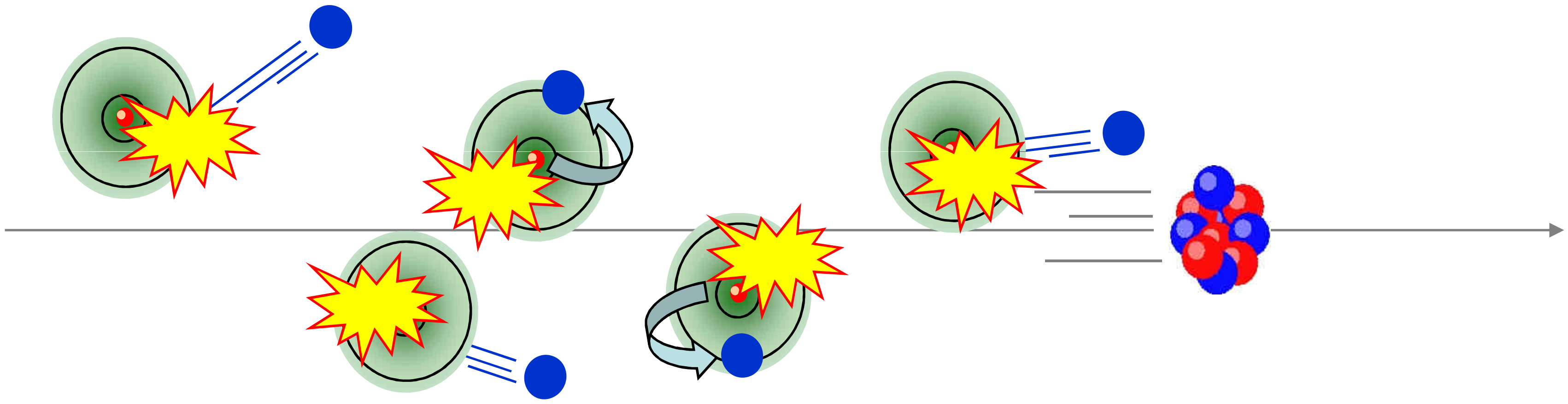
- For example: Famous Rutherford experiment for α-particle scattering

Ernest Rutherford

Heavy particles: Basic interaction processes

- **Finally, we consider now a <u>model</u> when a charged particle transverses matter and it loses energy mainly through collisions with target electrons.**

- **Collision of heavy particles with atoms in media may result in excitation and ionization of the atomic electrons.**



- **In every collision ion loses small fraction of its energy ⇨ its decelerates!**

- **Can we estimate the energy loss in such a single collision?**

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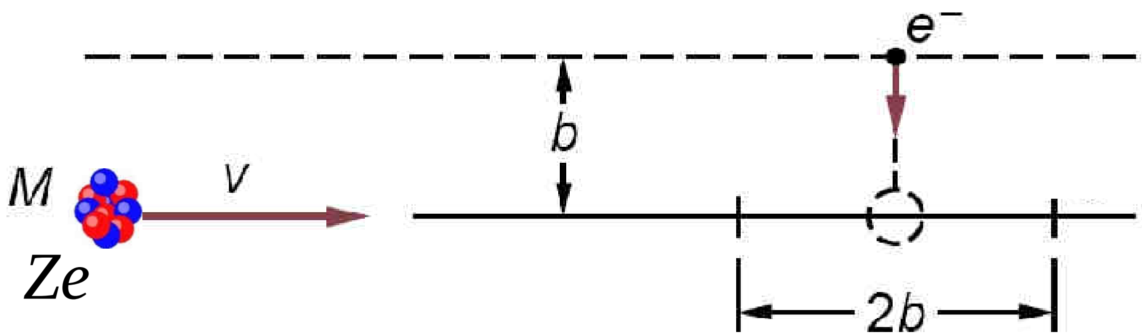
Energy loss of charged particle in matter

(a little bit of theory (1))

- **Let us start with a simple classical model.**



charge of particle: Ze

Picture from: http://www.whfreeman.com/MODPHYSICS/

- **The (maximal) force between particle and electron:** $F = \frac{Ze^2}{b^2}$

- **For simplicity we assume that force has the constant value for the time it takes the particle to pass the electron which is the order of** $\Delta t \approx 2b / \mathrm{v}$

- **Momentum given to electron:** $\Delta p \approx F\ \Delta t = \frac{2Ze^2}{b\mathrm{v}}$

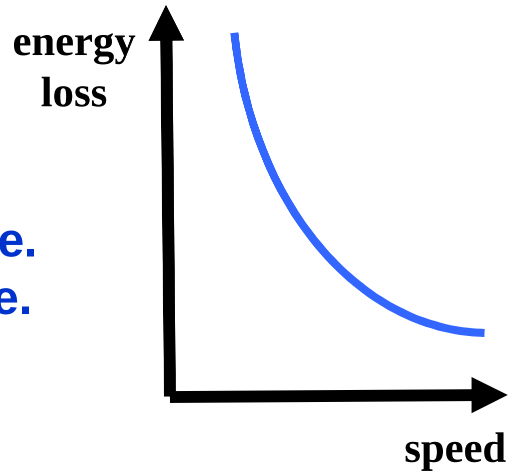
The energy given to electron: $\Delta E \approx \frac{(\Delta p)^2}{2m_e} = \frac{2Z^2e^4}{m_e b^2 \mathrm{v}^2}$

Energy loss of charged particle in matter

(a little bit of theory (2))

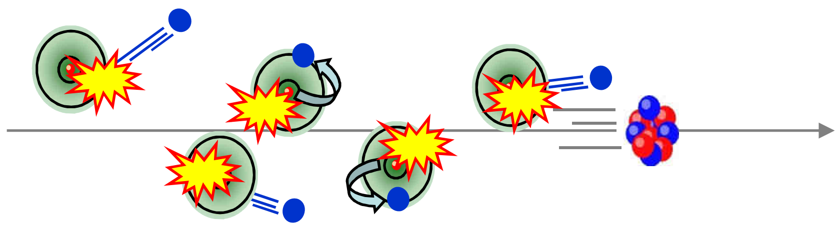
- **The energy given to an electron (and, hence, lost by the particle) in one encounter is:**

$$\Delta E \approx \frac{2Z^2 e^4}{m_e b^2 v^2} \propto \frac{Z^2}{v^2}$$



- **Let us make some intermediate observations:**
  - **Energy loss increases with charge of the particle.**
  - **Energy loss decreases with speed of the particle.**

- **The results we obtained so far are for single encounter. Can we learn something for the "real" situation when particle interacts with many electrons (sitting in may atoms)?**



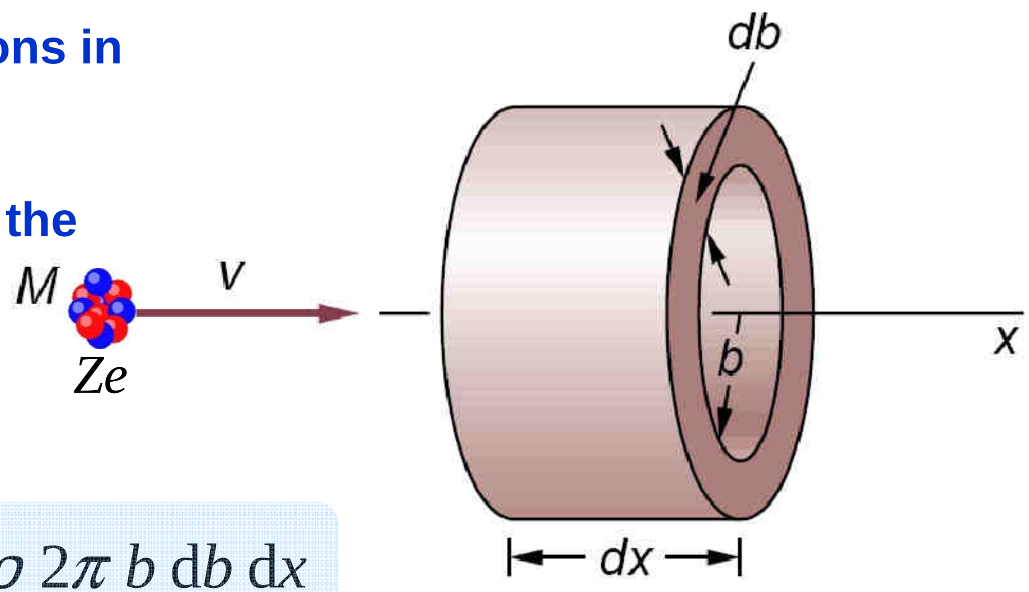
Energy loss of charged particle in matter

(a little bit of theory (3))

- **There are $N = nV$ electrons in volume V.**

- **For the cylindrical shell on the picture:**

$$V = 2\pi\, b\, \mathrm{d}b\, \mathrm{d}x$$



Picture from: http://www.whfreeman.com/MODPHYSICS/

- **Finally:** $N = Z_T (N_A / A)\rho\, 2\pi\, b\, \mathrm{d}b\, \mathrm{d}x$

charge of target atoms

- **The energy loss to these electrons:**

$$-\mathrm{d}E = \frac{2Z^2 e^4}{m_e \mathrm{v}^2 b^2} Z_T (N_A / A)\rho\, 2\pi\, b\, \mathrm{d}b\, \mathrm{d}x$$

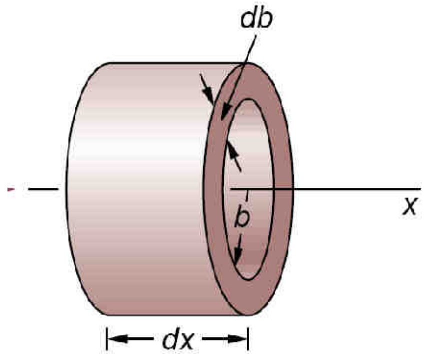
- **Integrating over impact parameter *b*:**

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{4\pi Z^2 e^4 (Z_T / A) N_A \rho}{m_e \mathrm{v}^2} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Energy loss of charged particle in matter

(a little bit of theory (4))

- **Energy loss:** $$-\frac{\mathrm{d}E}{\mathrm{d}x}=\frac{4\pi Z^2 e^4 (Z_T/A) N_A \rho}{m_e \mathrm{v}^2}\ln\left(\frac{b_{\max}}{b_{\min}}\right)$$



- **Is not defined in the case:** $b_{\max}\to\infty,\quad b_{\min}\to 0$
- **We shall put some limitations on the impact parameter.**

Limitation for $b_{\max}$

- **In reality the atomic electrons are not free but bound to target atoms.**
- **Therefore, we can not transmit to them infinitely small portion of energy.**

Limitation for $b_{\min}$

- **Maximum energy we can transmit to an electron is $2m_e\mathrm{v}^2$ (like in classical case).**

Bethe-Bloch equation

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Bethe-Bloch equation

- **By taking into account arguments from above and by considering relativistic corrections we may obtain:**

$$-\frac{dE}{dx} = \frac{4\pi Z^2 e^4 (Z_T / A) N_A \rho}{m_e v^2}\left[\ln\left(\frac{2 m_e c^2 \beta^2}{\bar{I}}\right) - \ln\left(1-\beta^2\right) - \beta^2\right]$$

Hans Bethe

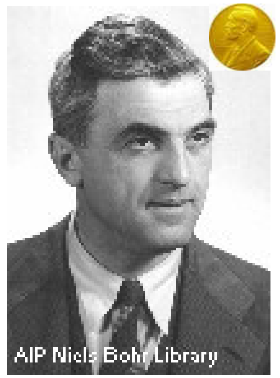
- **Here** $\beta = \frac{v}{c}$ **and** $\bar{I}$ **is the mean ionization potential:** $\bar{I} \approx 13.5 \times Z_T\ \mathrm{eV}$

Annalen der Physik, Volume 397, Issue 3

Zur Theorie des Durchgangs schneller Korpuskularstrahlen durch Materie

Von H. Bethe

Der unelastische Zusammenstoß einer schnellen geladenen Partikel (Elektron, Proton, α-Teilchen) mit einem Atom wird nach der wellenmechanischen Theorie von Born behandelt. Ein sehr einfaches Verfahren für die Auswertung der in die Theorie eingehenden Matrixelemente wird angegeben (§ 3) und die engen Beziehungen zur Intensität des Comptoneffekts festgestellt (§ 5). Die Theorie wird für Zusammenstöße mit Wasserstoffatomen im einzelnen und für kompliziertere Atome soweit wie möglich durchgeführt, es werden berechnet: die Winkel-



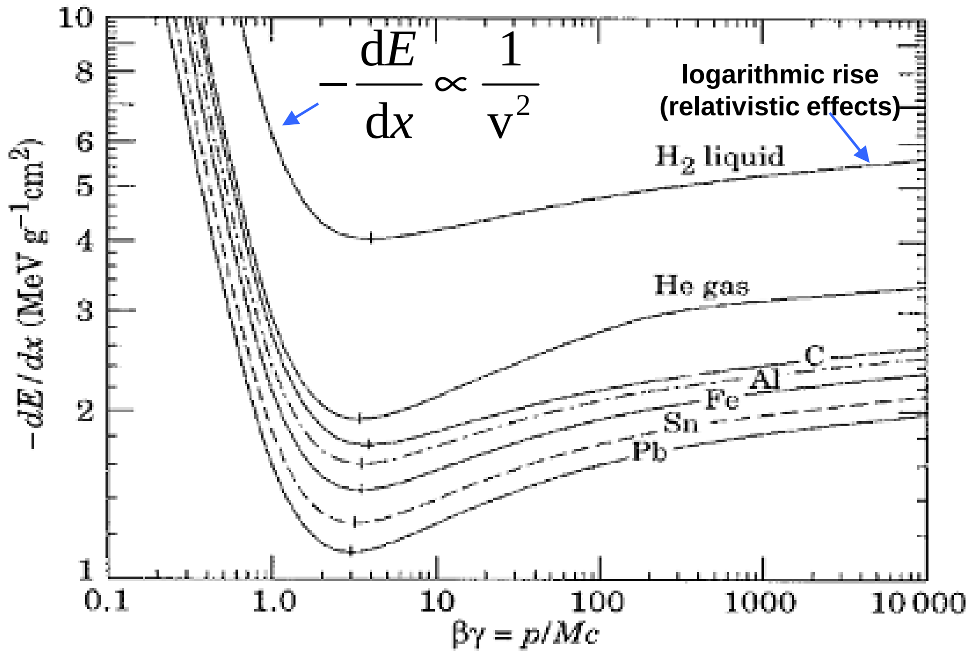
Felix Bloch

Bethe-Bloch equation

$$-\frac{dE}{dx} = \frac{4\pi Z^2 e^4 (Z_T / A) N_A \rho}{m_e v^2}\left[\ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}}\right) - \ln\left(1-\beta^2\right) - \beta^2\right]$$

- **Important consequences:**
  - **The energy loss is roughly proportional to the density of material.**
  - **The energy loss does not depend on the mass of the incident charged particle.**
  - **The energy loss depends on the charge of the target also through the mean ionization potential.**

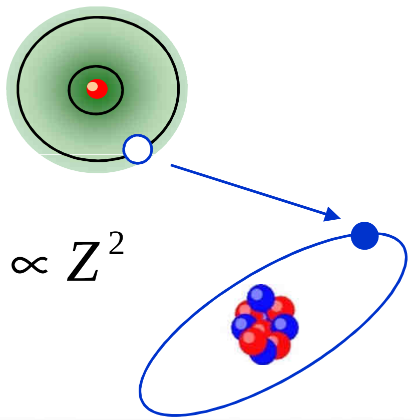
Range of dE/dx for muons



From: mxp.physics.umn.edu

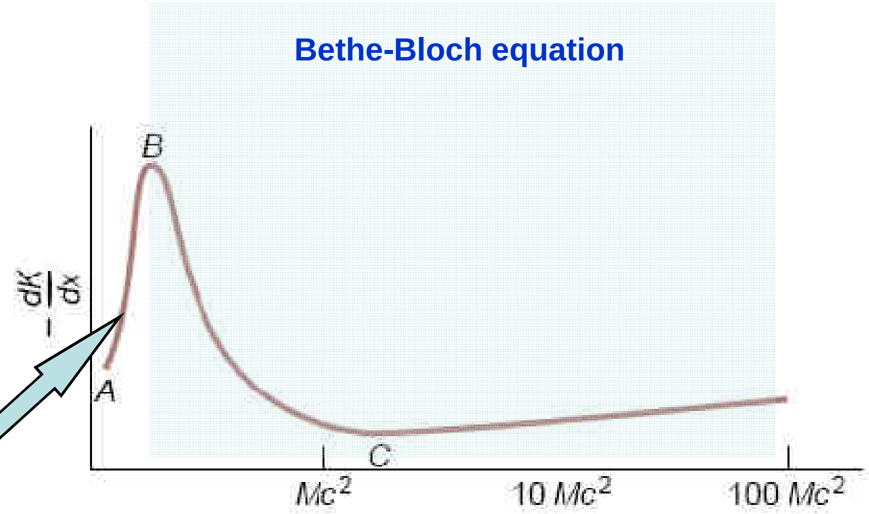
Bethe-Bloch equation

- **At low collision energies Bethe-Bloch equation fails to describe the energy loss of the particles.**
- **At low energies the energy loss function is more complicated.**



$$-\frac{\mathrm{d}E}{\mathrm{d}x} \propto Z^2$$

Attached electrons will "screen" the charge of the particle and, hence, decrease the energy loss.



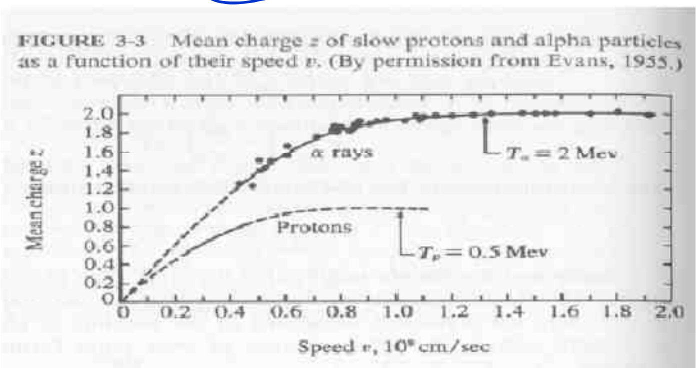
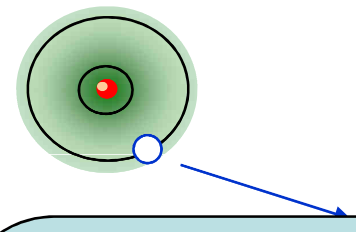


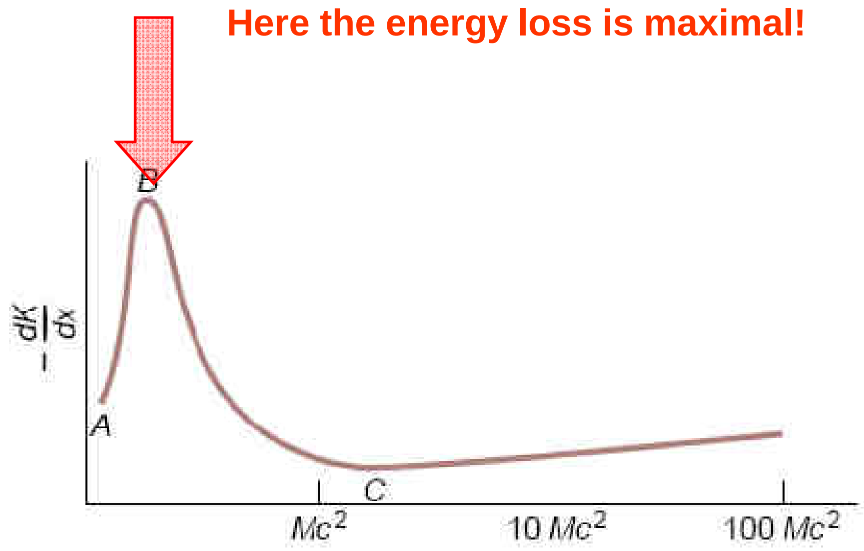
FIGURE 3-3 Mean charge z of slow protons and alpha particles as a function of their speed v. (By permission from Evans, 1955.)

- **Particles may start capturing electrons from the target atoms and begin to lose their charge.**
- **More sophisticated methods are required to analyze this region of energies.**

Bethe-Bloch equation

- **At low collision energies Bethe-Bloch equation fails to describe the energy loss of the particles.**
- **At low energies the energy loss function is more complicated.**





$-\frac{dE}{d}$

…apturing … rget atoms … eir charge.

…methods are … his region of

Imagine particle penetrating the solid.

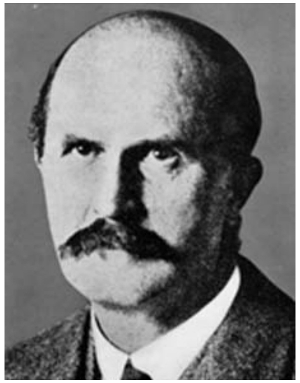
When the energy of the particle is low?

Obviously: in the end of the path (before stopping).

Heavy charged particle deposits a lot of energy before stopping!

Bragg peak

- **Since the energy loss of heavy charged particles is larger for smaller energies most of the energy is deposited by particle before stopping.**
- **This effect has an important applications for cancer therapy!**
- **A beam of heavy charged particles can be used to destroy cancer cells at given depth in the body without destroying healthy cells.**



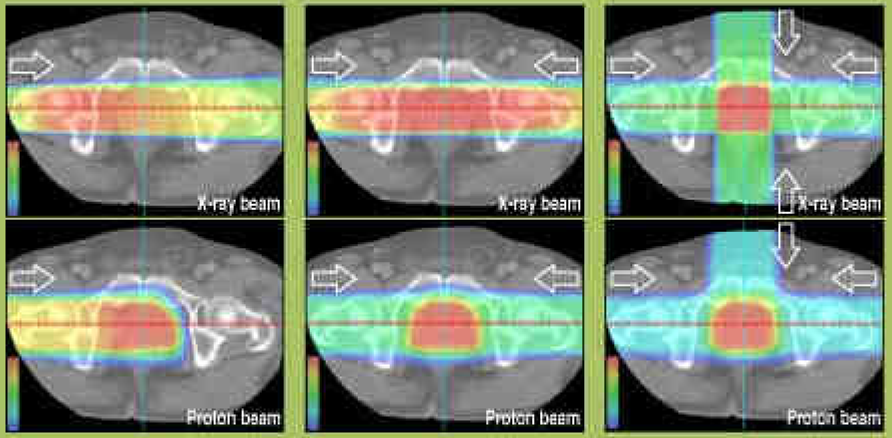
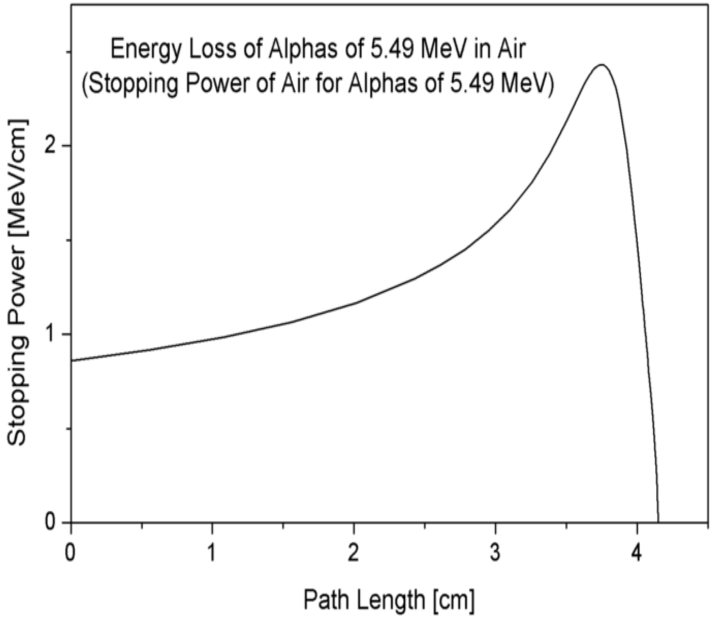
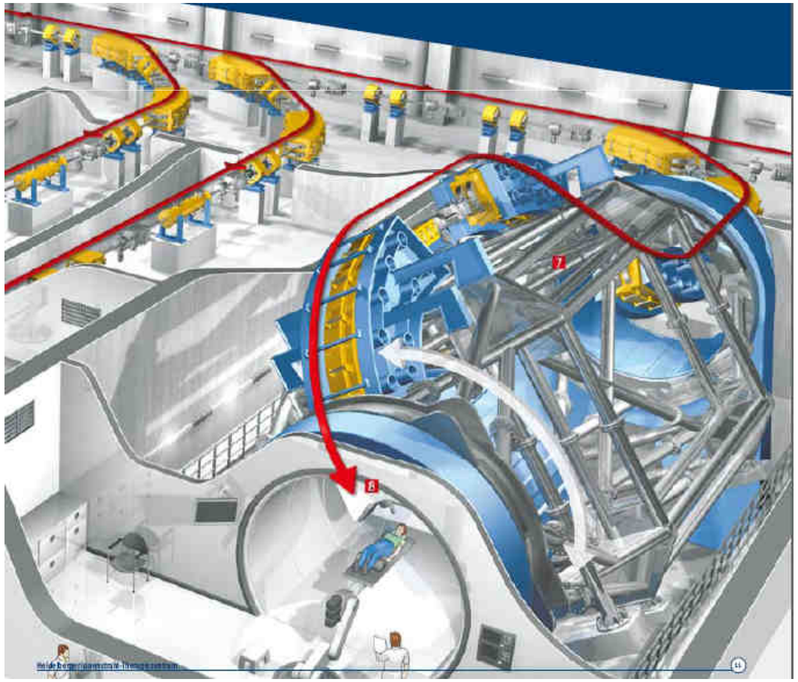


Fig. X-ray beam vs. Proton beam for prostate cancer



Cancer treatment

HIT
Heidelberger Ionenstrahl-Therapiezentrum
UniversitätsKlinikum Heidelberg
IonenStrahl TherapieZentrum



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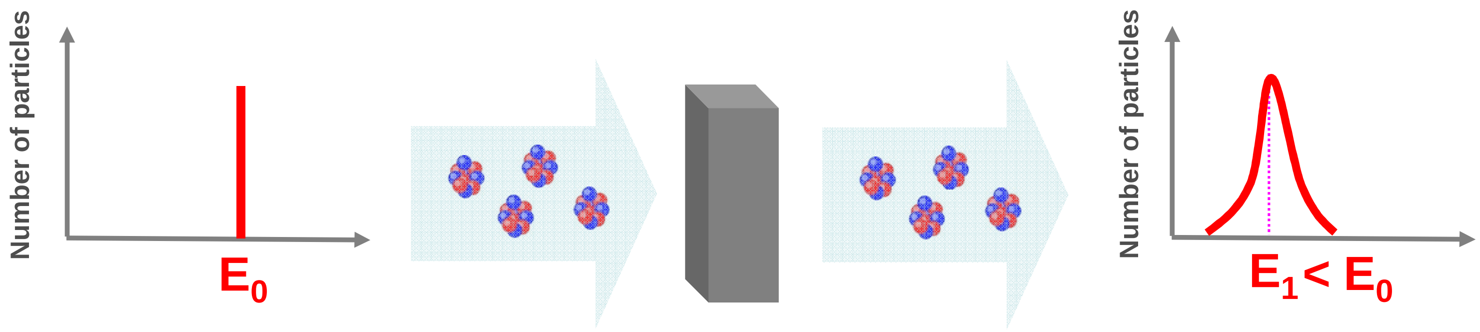
Energy loss straggling

- **Let us return one more time to the Bethe-Bloch formula:**

$$-\frac{dE}{dx} = \frac{4\pi Z^2 e^4 (Z_T / A) N_A \rho}{m_e v^2}\left[\ln\left(\frac{2m_e c^2 \beta^2}{\bar{I}}\right) - \ln\left(1-\beta^2\right) - \beta^2\right]$$

  - **Due to the statistical nature of ionization energy loss, large fluctuations can occur in the amount of energy deposited by a particle traversing an absorber element.**



Lev Landau

- **The fluctuations of energy loss by ionization of a charged particle in a thin layer of matter was first described theoretically by Landau.**

Landau distribution

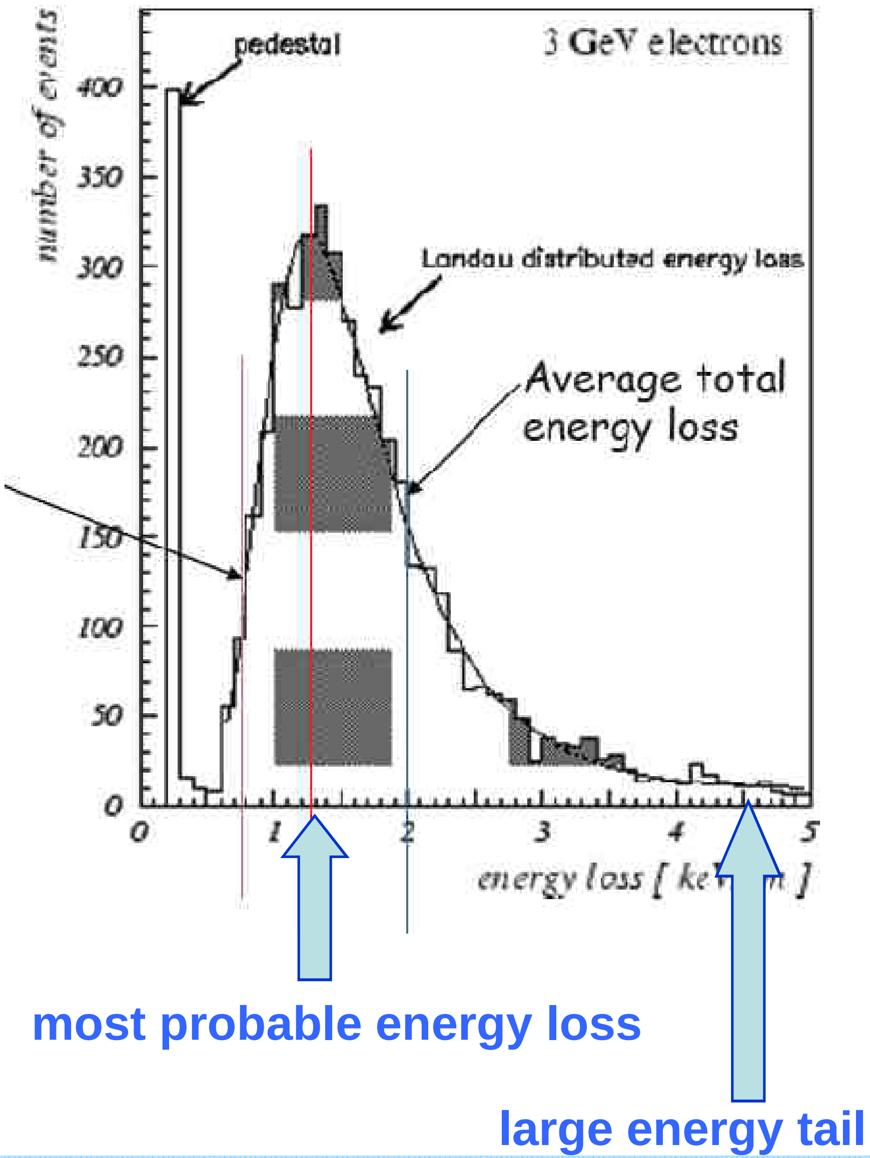
$$p(x) = \frac{1}{\pi}\int_0^\infty e^{-t\log t - xt}\sin(\pi t)\,dt.$$

Landau distribution

$$p(x) = \frac{1}{\pi}\int_0^\infty e^{-t\log t - xt}\sin(\pi t)\,dt.$$

- **The Landau distribution is defined in terms of the normalized deviation from the "most probable energy loss", which is associated with the peak of the distribution.**

- **Large-energy tail is associated to the small number of individual collisions, each with a small probability of transferring comparatively large amounts of energy.**



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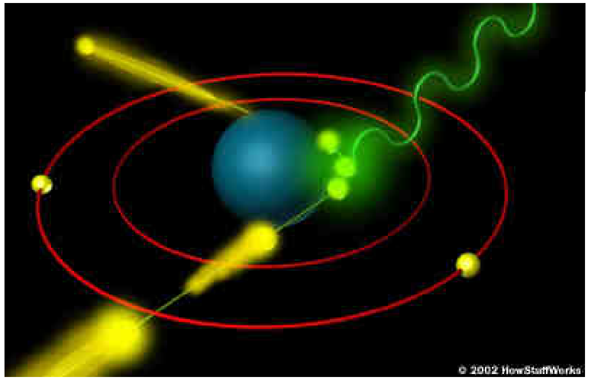
Electron passing the matter

(difference from heavy particles)

- **Similar to heavy particles, electrons also interact through Coulomb forces with the atomic electrons.**
- **But! Due to their small mass the radiation loses become important for them.**



We already discussed that radiation loses by heavy ions are very small.

How can we understood it?

Electron passing the matter

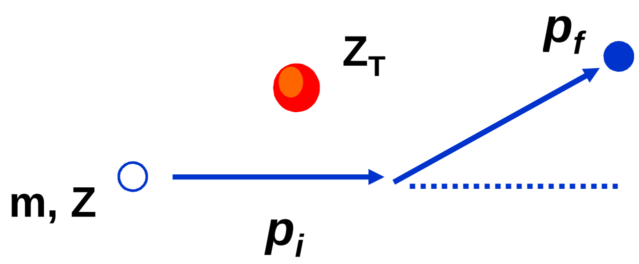
(difference from heavy particles)

- **Similar to heavy particles, electrons also interact through Coulomb forces with the atomic electrons.**
- **But! Due to their small mass the radiation loses become important for them.**

- **Let us estimate scattering angle in single collision:**

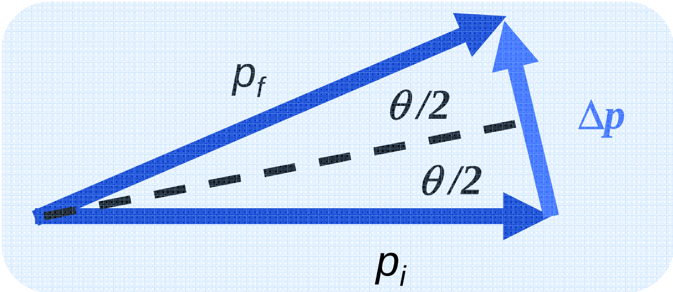
$$\sin\left(\frac{\theta}{2}\right) \approx \frac{\Delta p}{2m\mathrm{v}} \approx \frac{ZZ_T e^2}{m\mathrm{v}^2}\frac{1}{b}$$

mass of scattering particle!



- **Proton mass: $m_p = 1836\ m_e$!**

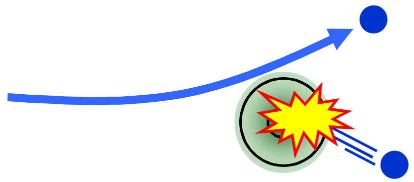
In collision electron deviates at much greater angles than the heavy particles!



Ionization vs. radiation loses (1)

- **In contrast to heavy particles, for electrons we have to take into account both ionization (collision) and radiation losses.**

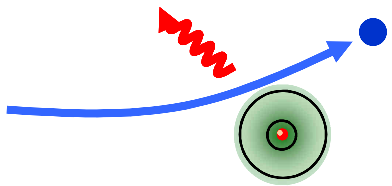
- **Ionization (collision) loses:**



$$-\frac{dE}{dx}_{ion} = \frac{2\pi e^4 Z_T (N_A / A)\rho}{m_e v^2}\left[\ln\left(\frac{m_e c^2 E}{\bar{I}^2}\frac{\beta^2}{2(1-\beta^2)}\right) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)\ln 2 + 1 - \beta^2 + \frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right]$$

(up to small correction this is similar to Bethe-Bloch formula)

- **Radiation loses:** $-\frac{dE}{dx}_{rad} = E / X_0$

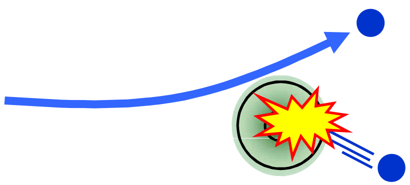


  - **where X_0 is the so-called radiation length:** $1/X_0 = \frac{4\alpha N_A Z(Z+1)\, r_e^2 \log(183\, Z^{-1/3})}{A}$

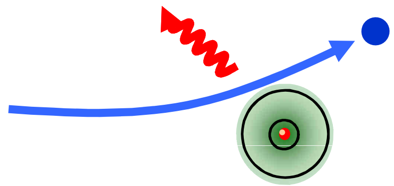
Radiation length is the mean path length required to reduce the energy of electron by the factor 1/e, or 0.368, as they pass through matter.

Ionization vs. radiation loses (2)

- **Ionization loses are dominant for relatively small energies of electron and decrease like $1/v^2$.**
- **Radiation loses become dominant for high energies of electron and enhance linearly with energy E.**



$$\left(\frac{dE}{dx}_{rad}\right) / \left(\frac{dE}{dx}_{ion}\right) \approx \frac{Z_T E}{700}$$



E – electron kinetic energy in MeV

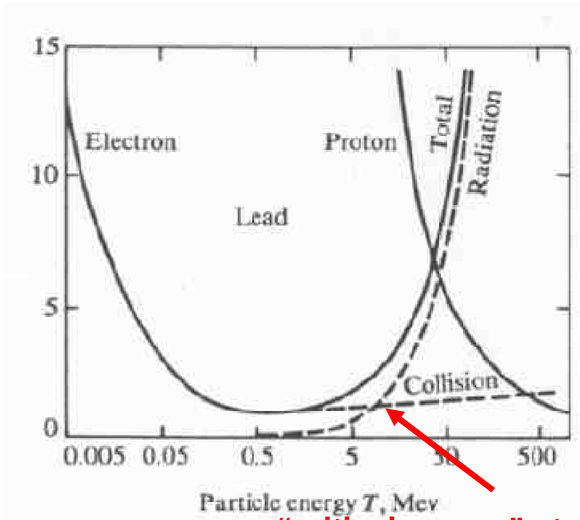


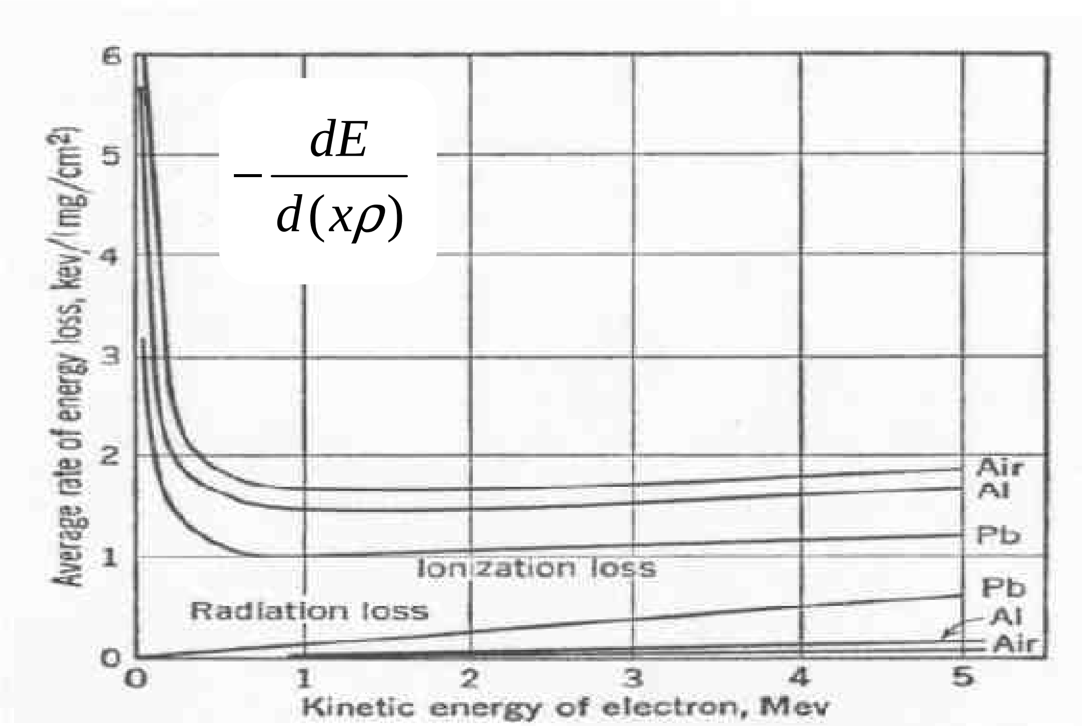
FIGURE 3-7 Energy loss of electrons and protons in lead. (From W. Heitler, 1954, as adapted by Burcham, 1963, by permission.)

"critical energy" at which ionization and radiation losses are equal

Ionization vs. radiation loses (3)

$$\left(\frac{dE}{dx}_{rad}\right)\Big/\left(\frac{dE}{dx}_{ion}\right) \approx \frac{Z_T E}{700}$$

E – electron kinetic energy in MeV



From: R. D. Evan, "*The atomic nucleus*"

target	Critical energy (MeV)
C	119
Air	83
Al	55
Fe	24
Cu	21.5
Pb	9

For heavier targets radiation losses become important for lower electron energies!

Plan of lecture

- **Motivation: Why to study interaction of particles with matter?**
- **Structure of matter (very briefly)**
- **Interaction of positively charged particles with matter**
  - **Energy loss**
  - **Bethe-Bloch equation**
  - **Energy straggling**
- **Interaction of electrons with matter**
  - **Radiation and ionization losses**
  - **Track patterns**
- **Channeling regime and its applications**

Track patterns: Electrons vs. heavy particles

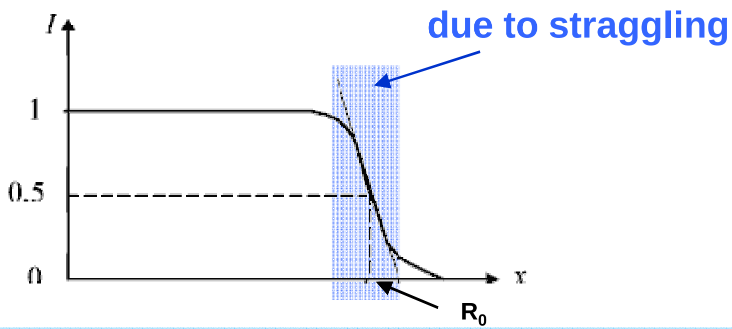
- **Another important consequence of an electron's small mass and, hence, large deflections in collisions is *zigzag*-like path.**

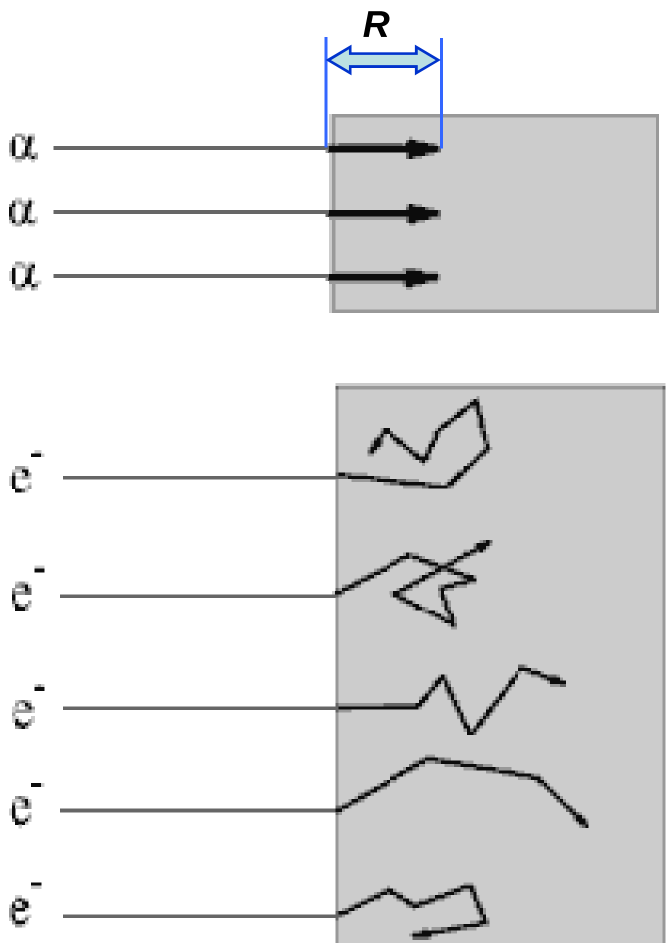
heavy particles

- **Transverse in almost straight line.**
- **One may introduce range R:**

$$R = \int_0^E \left(- \frac{\mathrm{d}E}{\mathrm{d}x} \right)^{-1} \mathrm{d}E$$

- **Which could be also understood as a path length.**





Track patterns: Electrons vs. heavy particles

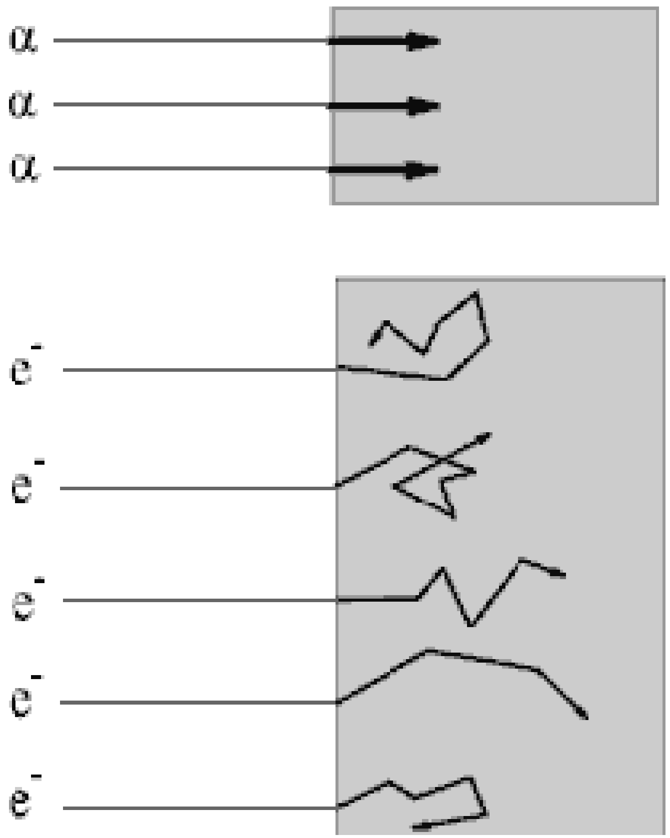
- **Another important consequence of an electron's small mass and, hence, large deflections in collisions is *zigzag*-like path.**

electrons

- **Transverse in zigzag line.**
- **Path length is much longer than the range *R*.**
- **One may introduce effective range, which is for *Al* target is given by:**

$$R = 0.4\,E^{1.4} \quad \text{at } E < 0.8\,\text{MeV}$$

$$R = 0.54\,E - 0.133 \quad \text{at } E > 0.8\,\text{MeV}$$



Plan of lecture

- **Motivation: Why to study interaction of particles with matter?**
- **Structure of matter (very briefly)**
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  - **Energy loss**
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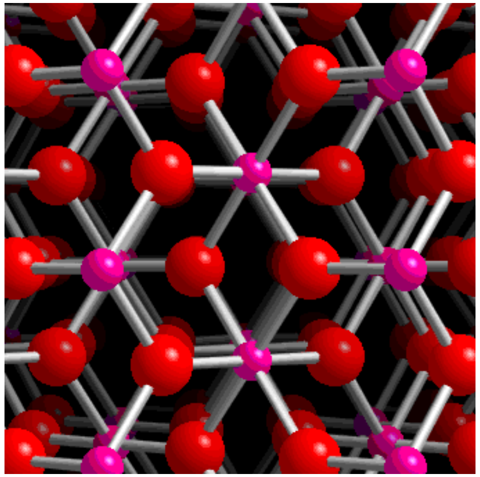
- **Interaction of electrons with matter**
  - **Radiation and ionization losses**
  - **Track patterns**
- **Channeling regime and its applications**

Particle passing crystals...

- **A crystal is a solid in which the constituent atoms, molecules, or ions are packed in a <u>regularly order</u>, repeating pattern extending in all three spatial dimensions.**

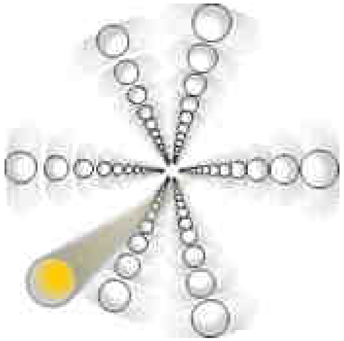


- **Example: crystal structure of manganese(IV) oxide (MnO_2).**

From: www.haverford.edu

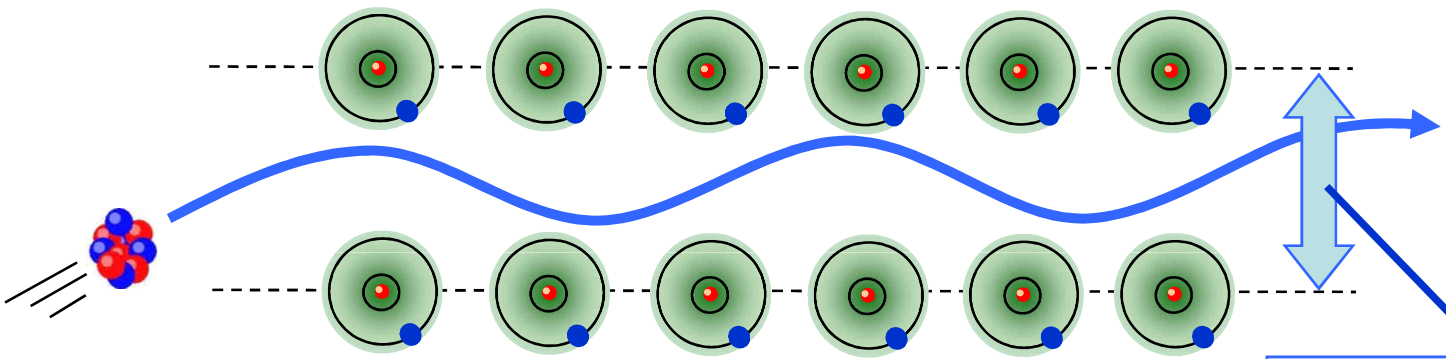
- **We may study, therefore, moving of particles in a regularly ordered matter!**

Idea of channeling of particles in crystals!

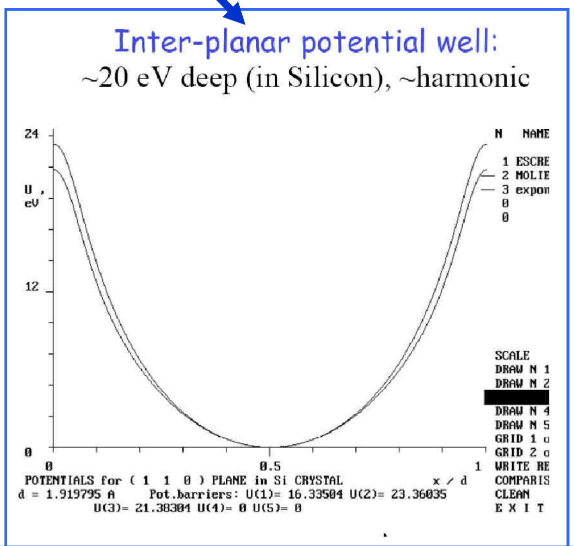


Channeling

- **If (positively charge) particles enter a crystal at an appropriate angle (!) the scattering events are correlated and the particles will move along the crystal planes.**

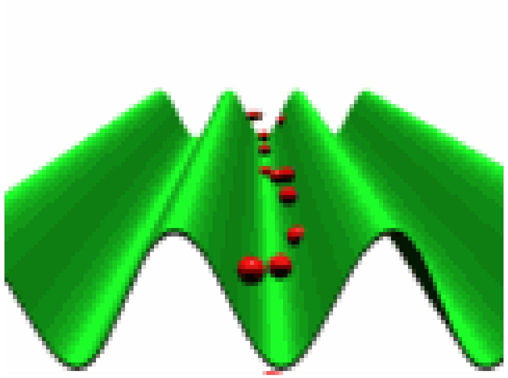


- **Positively-charged particles are repulsed from the nuclei of the plane, and after entering the space between two neighboring planes, they will be repulsed from the second plane.**

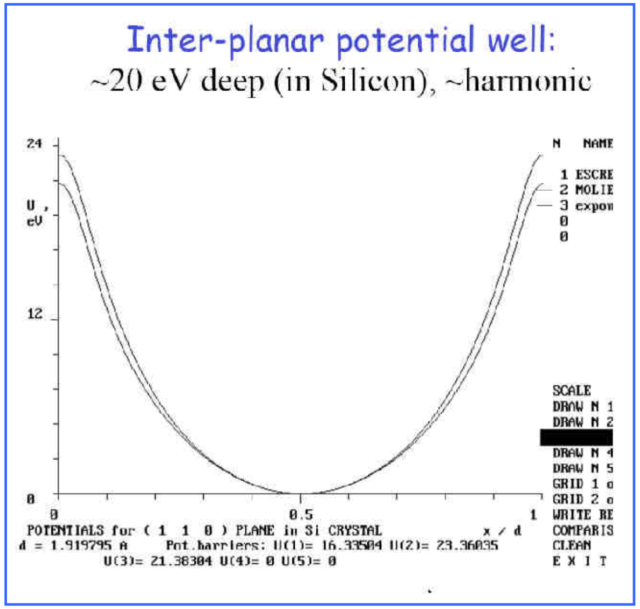


Channeling

- **Positively-charged particles are repulsed from the nuclei of the plane, and after entering the space between two neighboring planes, they will be repulsed from the second plane.**



Animation from: "Scientific American"



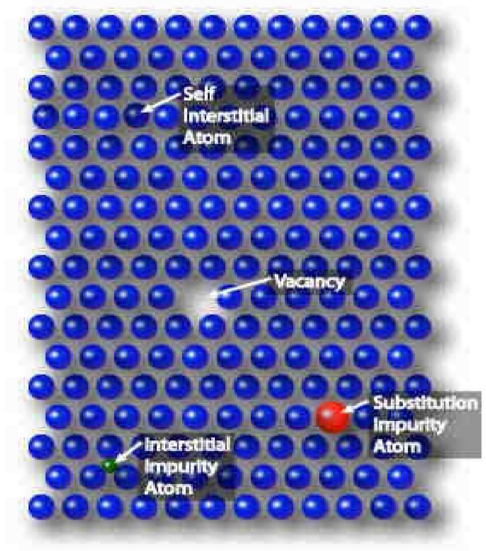
- **Electron density inside of the channel is smaller than near the crystal planes and, therefore, channeling particle loses less energy.**

$$-\frac{\mathrm{d}E}{\mathrm{d}x} \propto \rho$$

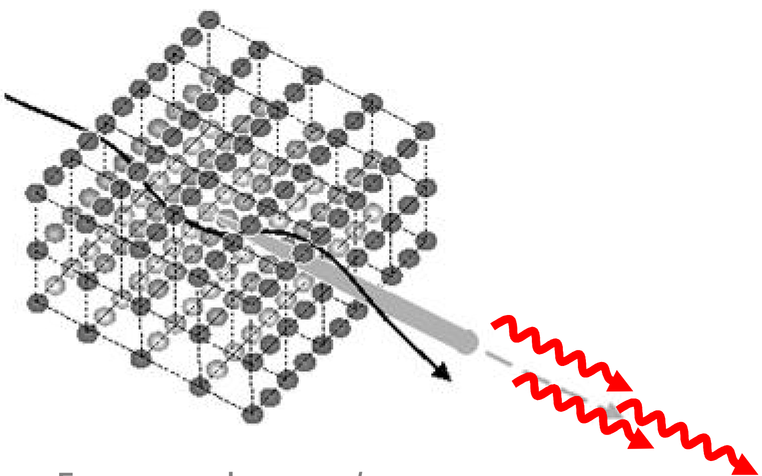
- **Particle in "channeling regime" may travel much longer if compared to particle penetrating amorphous solid.**

Applications of channeling

- **Channeling may be used as a toll for the analysis of the properties of crystal lattices and (searching for defects).**



From: www.ndt-ed.org



From: www.phys.rsu.ru/

- **The applications include also the channeling radiation for enhanced production of high energy gamma rays.**

Plan of lectures

- 1 15.04.2015 Preliminary Discussion / Introduction

- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (bound-state solutions of Dirac equation, quantum numbers)
- 6 20.05.2015 Theory (bound-state Dirac wavefunctions, QED corrections)

- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)
- 8 03.06.2015 Theory (description of the light-matter interaction)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ….)
- 10 17.06.2015 Theory (interaction of charged particles with matter)

- 11 24.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 12 01.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)

- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)