Modern Atomic Physics: Experiment and Theory

Experiments
Lamb shift, hyperfine structure, quasimolecules and MO spectra

Lecture 3

April 29th, 2014

Preliminary plan of the lectures

- 1 15.04.2015 Preliminary Discussion / Introduction

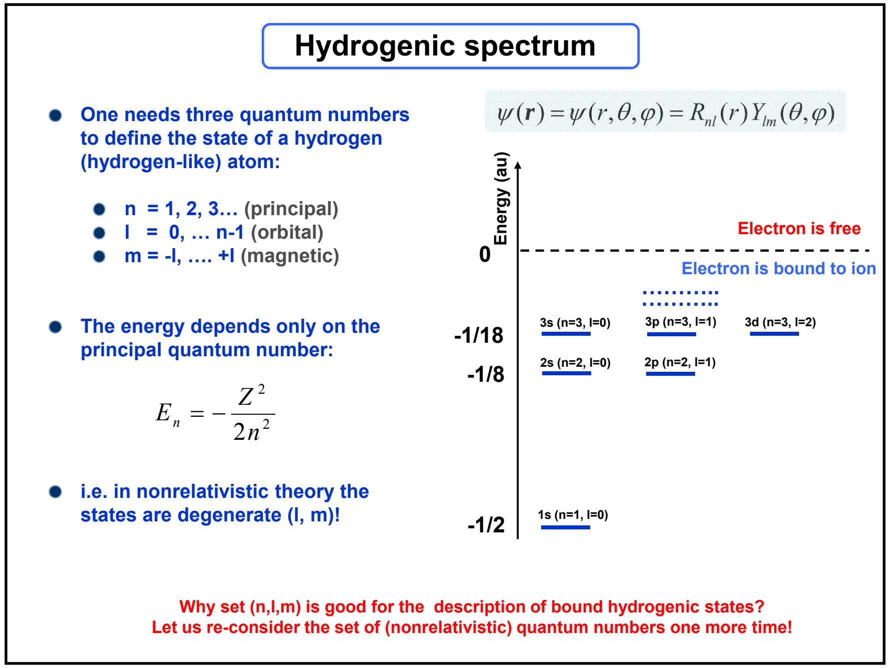
- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (photon, quantum field theory (just few words), Feynman diagrams, QED corrections)

- 6 20.05.2015 Theory (matrix elements and their evaluation, radiative decay and absorption)
- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)

- 8 03.06.2015 Theory (single and multiple scattering, energy loss mechanisms, channeling regime)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ….)

- 10 17.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 11 24.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)

- 12 01.07.2015 Theory (atomic-physics tests of the Standard Model, search for a new physics)
- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)



Hydrogenic spectrum

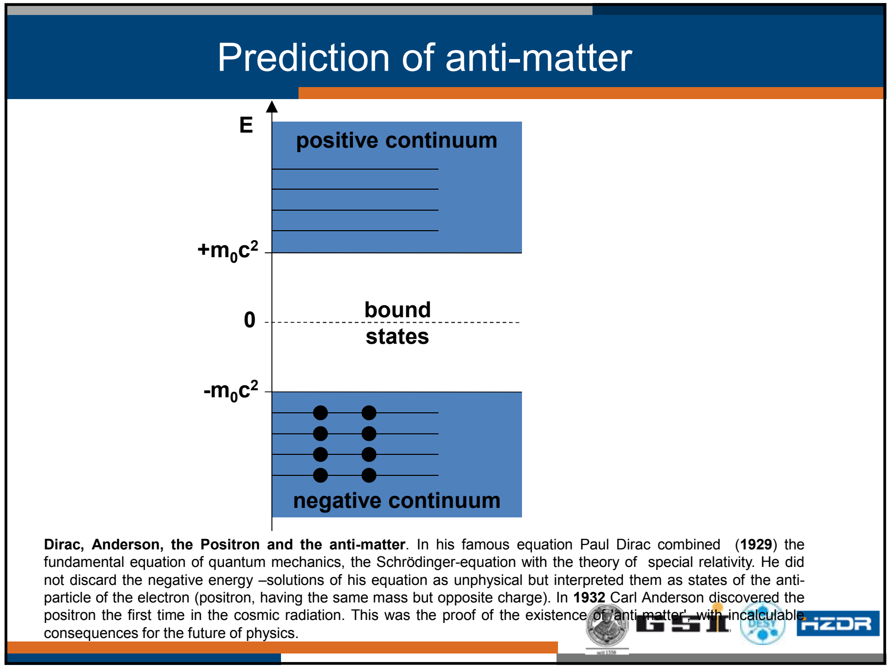
$$\psi(\mathbf{r}) = \psi(r,\theta,\varphi) = R_{nl}(r)\,Y_{lm}(\theta,\varphi)$$

- One needs three quantum numbers to define the state of a hydrogen (hydrogen-like) atom:
  - n = 1, 2, 3… (principal)
  - l = 0, … n-1 (orbital)
  - m = -l, …. +l (magnetic)
- The energy depends only on the principal quantum number:

$$E_n = -\frac{Z^2}{2n^2}$$

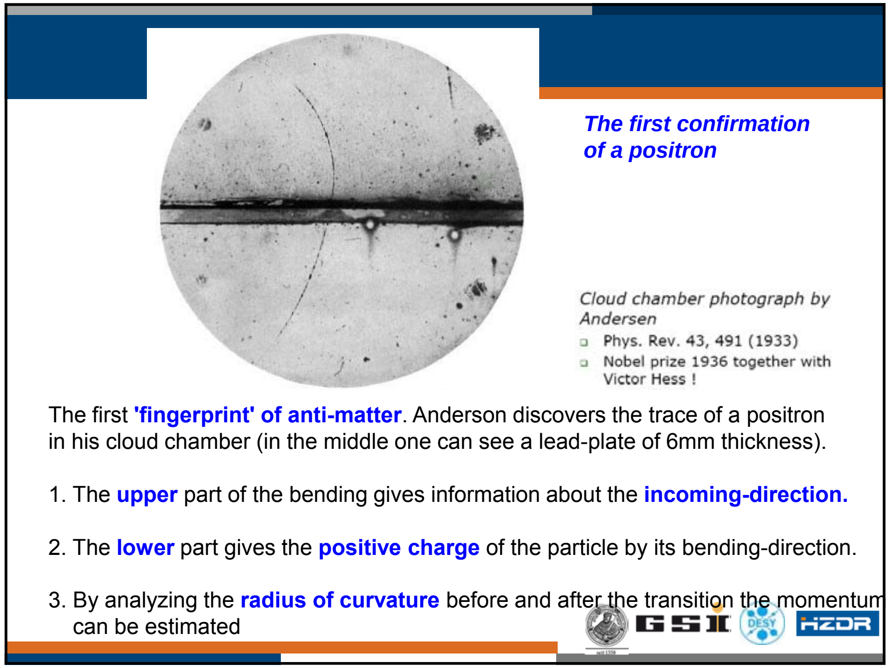
- i.e. in nonrelativistic theory the states are degenerate (l, m)!

Why set (n,l,m) is good for the description of bound hydrogenic states?
Let us re-consider the set of (nonrelativistic) quantum numbers one more time!



Prediction of anti-matter

Dirac, Anderson, the Positron and the anti-matter. In his famous equation Paul Dirac combined (**1929**) the fundamental equation of quantum mechanics, the Schrödinger-equation with the theory of special relativity. He did not discard the negative energy –solutions of his equation as unphysical but interpreted them as states of the anti-particle of the electron (positron, having the same mass but opposite charge). In **1932** Carl Anderson discovered the positron the first time in the cosmic radiation. This was the proof of the existence of 'anti-matter', with incalculable consequences for the future of physics.



The first confirmation of a positron

Cloud chamber photograph by Andersen

- Phys. Rev. 43, 491 (1933)
- Nobel prize 1936 together with Victor Hess !

The first **'fingerprint' of anti-matter**. Anderson discovers the trace of a positron in his cloud chamber (in the middle one can see a lead-plate of 6mm thickness).

1. The **upper** part of the bending gives information about the **incoming-direction.**
2. The **lower** part gives the **positive charge** of the particle by its bending-direction.
3. By analyzing the **radius of curvature** before and after the transition the momentum can be estimated

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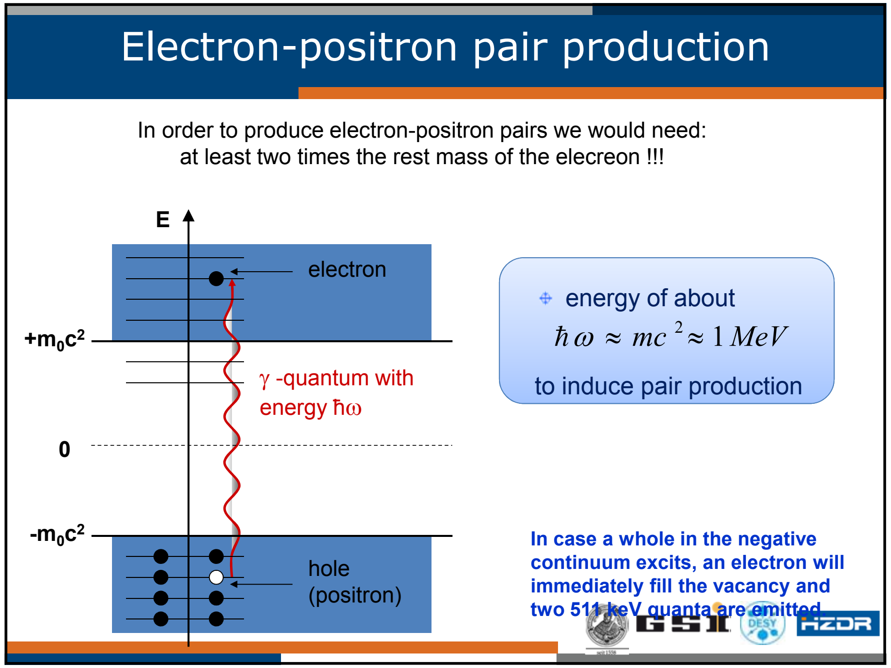


Electron-positron pair production

In order to produce electron-positron pairs we would need:
at least two times the rest mass of the elecreon !!!

- energy of about $\hbar\omega \approx mc^2 \approx 1\,MeV$ to induce pair production

In case a whole in the negative continuum excits, an electron will immediately fill the vacancy and two 511 keV quanta are emitted

Experiments

Lamb shift in one and two-electron ions, hyperfine structure

Content

QED: Radiative correction viewed by an experimentalist

Experiments on Hydrogen

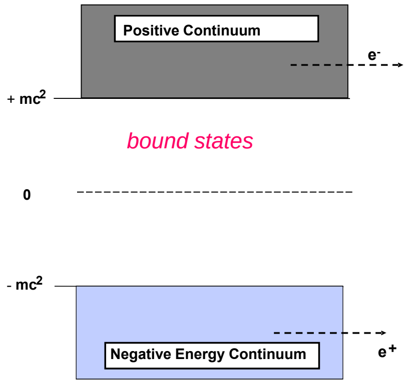
Strong fields: High-Z ions versus exotic atomic systems

The lamb shift in uranium

The hyperfine structure at high-Z

DIRAC Theory

$$E_{nj} = mc^2 \Bigg/ \sqrt{1 + \left(\frac{Z\alpha}{n - |j + 1/2| + \sqrt{(j+1/2)^2 - (Z\alpha)^2}} \right)^2}$$



n: principle quantum number
j: total angular momentum
α: fine structure constant- α = 1/137.036)

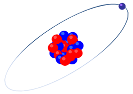
mc²: electron rest mass
(511 keV)

Dirac-Theorie: **(complete relativistic treatment of single electron systems)**

*All levels with the same n and **j** are degenerated.*

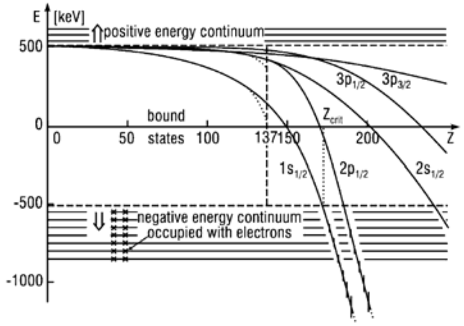
Critical electromagnetic fields

- Let us come back to Dirac energy of a *single* hydrogen-like ion:



$$E_{1s} = mc^2 \sqrt{1 - (Z\alpha)^2}$$

- What happens if we increase the nuclear charge Z?
- If nuclear charge of the ion is greater than Z_{crit} the ionic levels can "dive" into Dirac's negative continuum.
- Physical vacuum becomes unstable: creation of pairs may take place!



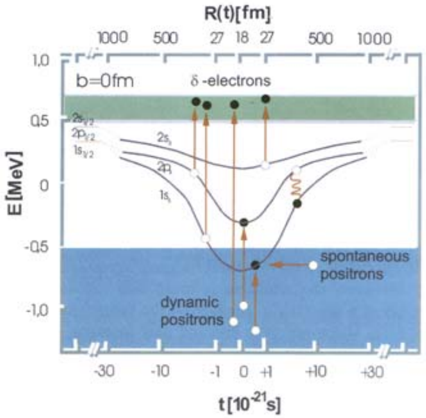
Relativistic quantum numbers and spectroscopic notations

n	j	parity	spectroscopic notation
1	1/2	+	$1s_{1/2}$
2	1/2	+	$2s_{1/2}$
2	1/2	-	$2p_{1/2}$
2	3/2	-	$2p_{3/2}$

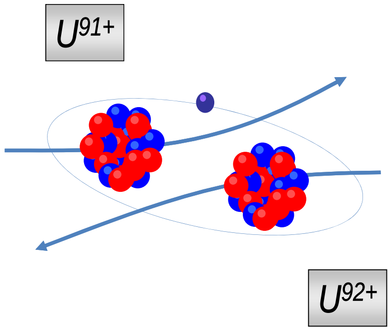
- **Finally, we know how to characterize bound states of (relativistic) hydrogen.**
- **What are the energies of these states?**

Supercritical fields: Formation of Quasi-Molecules

Merged Beams



A. Artemyev, et al. J. Phys. B **43** (2010) 235207

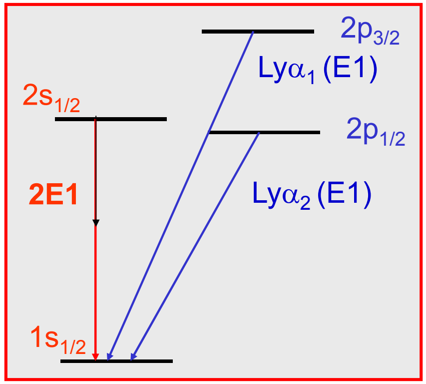


The Lamb shift

Experiment: Lamb-Retherford 1947; Theoretical Explanation: Bethe 1947

This experiment proves that for hydrogen the 2s1/2 and the 2p1/2 ***are not degenerated***

=> ***In contradiction to Dirac-Theory***



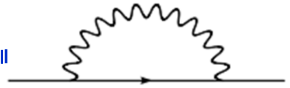
Level sheme for hydrogen

Lamb-Shift
deviations from the Dirac theory for a point like nucleus

The Lamb shift (LS) is essentially a consequence of the interaction of the bound electron with its own radiation field

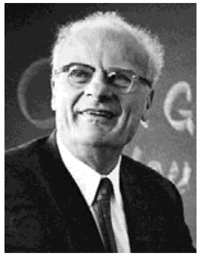
Lamb shift: idea of radiation corrections

- **From the end of 1930th : interaction of electron with radiation field. But which field?**
- **Ideas: electron may interact with its own field. The Coulomb potential is therefore perturbed by a small amount and the degeneracy of the two energy levels is removed.**



- **But: problems with divergence of results!**
- **In 1947 Hans Bethe has shown how to identify the divergent terms and to substract them from the theoretical expression.**

Development of
Quantum Electrodynamics (QED)
and Quantum Field Theory (QFT)



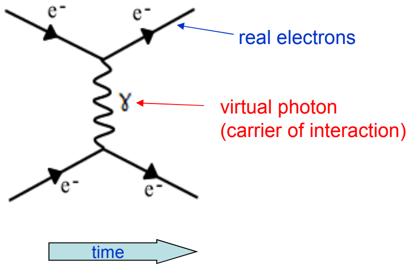
Hans Bethe

Feynman diagrams

Richard Feynman

- **Feynman diagrams are a nice tool to represent exchange forces and, hence, to perform calculations of scattering processes.**

Electron-electron repulsion



From: http://www.slac.stanford.edu/

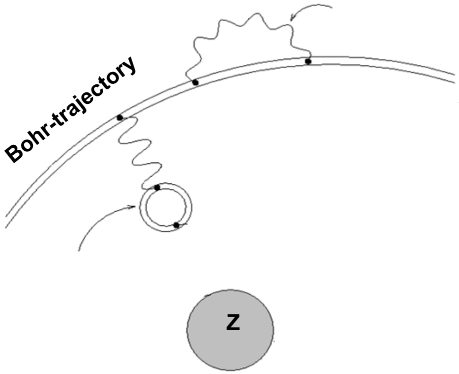
The effects of QED increase with the intensity of the electrical field, the electrons are exposed to. That means the s electrons are most affected.

Lamb-Shift (three effects)

QED

- self-energy

Self Energy



- Vacuum-polarization

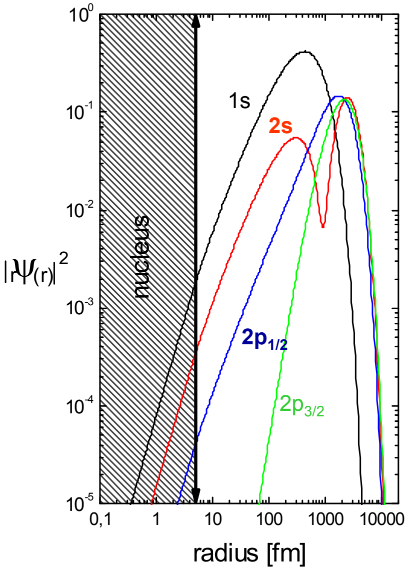
Vacuum Polarization

- size of the nucleus

Lamb Shift: An effect of strong fields

QED corrections
$\Delta E \sim Z^4/n^3$

Z: nuclear charge
n: principal quantum number



Appendix: probability density of the electrons at the nucleus (non-relativistic)

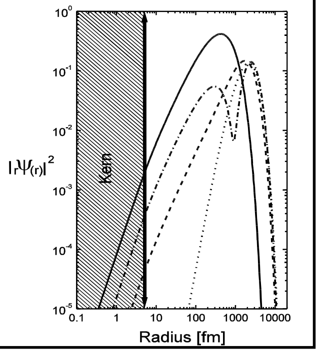
r = Z/n in atomic units. With the normalization condition

$$\int_{-\infty}^{\infty} |\Psi(\mathrm{r})|^2 \mathrm{d}^3\mathrm{r} = 1$$

follows: $\Psi(r) = \left(\frac{Z}{n}\right)^{3/2}$

What is the electron density at the nucleus (r=0)?

for l = 0 $\boxed{|\Psi(r=0)|^2 = \frac{Z^3}{\mathrm{n}^3}}$



Self energy

Self-energy: Describes the emission and reabsorption of a virtual photon (Bethe 1947) ***The Self-energy decreases the binding energy.***

virtual photon

electron in the outer field
(z.B. nucleus-Coulombfield)

Mass-renormalization	$m = m_e - \partial m$	m: measurable electron mass. It already includes the radiation correction. ∂m
	m_e	electron mass of the free electron without interaction with a field.

The self-energy is the most important radiation correction for light systems and decreases the binding energy. Here the electron emits and reabsorbs one and the same virtual photon. In every classical theory the consequence would be an infinitely high mass corresponding to an infinitely high self energy. The first calculations on the self-energy which explains the lamb shift of Hydrogen were made by Bethe. His success is based on an idea of Kramer, called **mass-renormalization**. The interaction with the virtual photon leads to a modification of the electron mass. This effect is always present and the measured electron mass **m** already includes the radiation correction. Therefore the self-energy correction for a bound state can be regarded as the mass difference between the bound electron and a free electron of the mass $\mathbf{m_e}$ that is not liable to any interaction with the electromagnetic field at the time t

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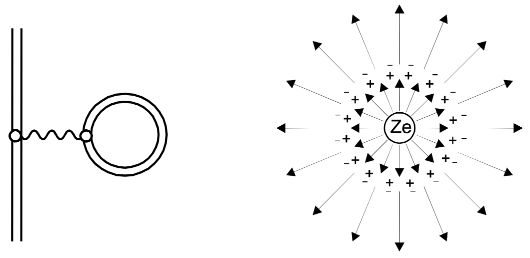
Vacuum Polarization (VP)



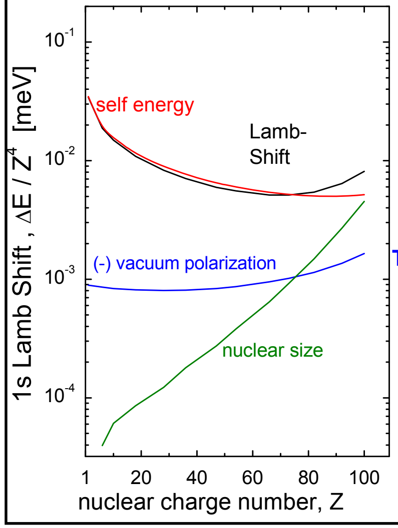
Virtual creation and annihilation of an e^+e^--pair

Charge renormalization

VP acts attractive

Summary of the Lamb Shift Effects

$\Delta E = \alpha/\pi\ (\alpha Z)^4\ F(\alpha Z)\ m_e c^2$



Self Energy

Vacuum Polarization

The size of the individual corrections in detail

	Hydrogen, H	H-like uranium, U^{91+}
$2p_{1/2} \rightarrow 1s_{1/2}$	10.2 eV	97.6 keV
Finestructure	45.4 μeV	4.56 keV
2sLS	4.4 μeV	75.3 eV
1sLS	35.6 μeV	463.95 eV

Bound-State QED: 1s Lamb Shift at High-Z

Sum of all corrections, leading to deviations from the Dirac theory for a point like nucleus

Self energy

Vacuum polarization

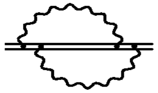
U^{92+}	SE	VP	NS
	355.0 eV	-88.6 eV	198.7 eV

$\Delta E = \alpha/\pi\ (\alpha Z)^4\ F(\alpha Z)\ m_e c^2$

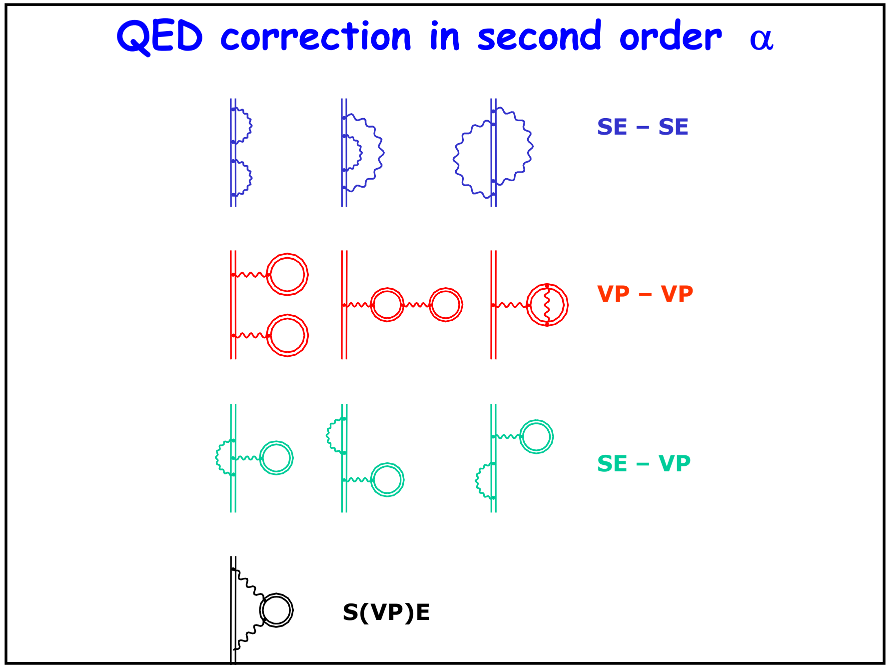
Low Z-Regime: $\alpha Z \ll 1$
F(αZ): series expansion in α**Z**

High Z-Regime: $\alpha Z \approx 1$
F(αZ): series expansion in α**Z** not appropriate

Goal:



±1 eV



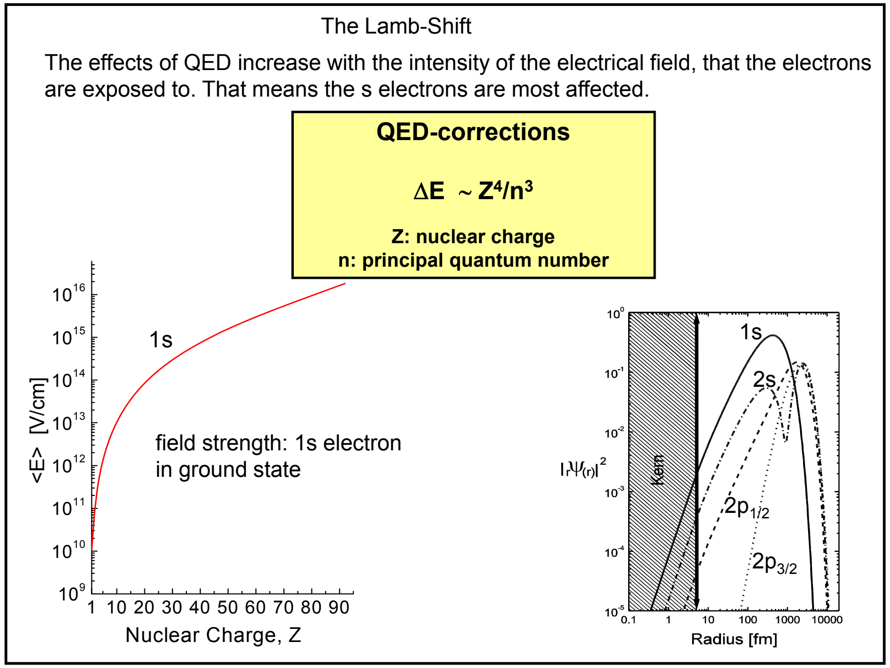
QED correction in second order α

SE – SE

VP – VP

SE – VP

S(VP)E



The Lamb-Shift

The effects of QED increase with the intensity of the electrical field, that the electrons are exposed to. That means the s electrons are most affected.

QED-corrections

$\Delta E \sim Z^4/n^3$

Z: nuclear charge
n: principal quantum number

field strength: 1s electron
in ground state

Test of Bound-State QED at high-Z

Self Energy

Experiments

- 1s Lamb Shift
- **2eQED for He-like ions**
- **2s-2p transitions in Li-like heavy ions**
- **hyperfine-structure**
- **g-factor of bound electrons**

Vacuum Polarization

Content

QED: Radiative correction viewed by an experimentalist

Experiments on Hydrogen

Strong fields: High-Z ions versus exotic atomic systems

The lamb shift in uranium

The hyperfine structure at high-Z

Hydrogen Atom

(http://www.mpq.mpg.de/~haensch/hydrogen/h.html)

f(1S-2S) = 2 466 061 102 474 851(34) Hz

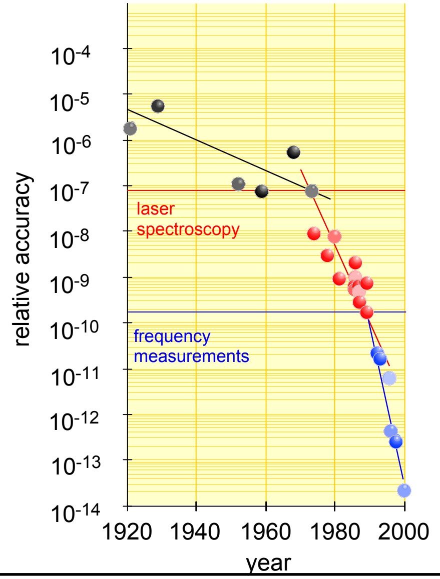
Rydberg Constant

R = 10 973 731.568 525(84) m^{-1}

Lamb Shift

L1S = 8 172.840(22) MHz

Laser Spectroscopy of Hydrogen



The Nobel Prize in Physics 2005

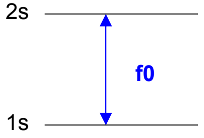
"for his contribution to the quantum theory of optical coherence"

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

Roy J. Glauber	**John L. Hall**	**Theodor W. Hänsch**
◐ 1/2 of the prize	◔ 1/4 of the prize	◔ 1/4 of the prize
USA	USA	Germany
Harvard University Cambridge, MA, USA	University of Colorado, JILA; National Institute of Standards and Technology Boulder, CO, USA	Max-Planck-Institut für Quantenoptik Garching, Germany; Ludwig-Maximilians-Universität Munich, Germany
b. 1925	b. 1934	b. 1941

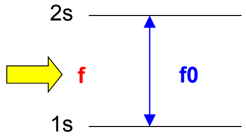
http://nobelprize.org/physics/laureates/2005/index.html

Laser spectroscopy

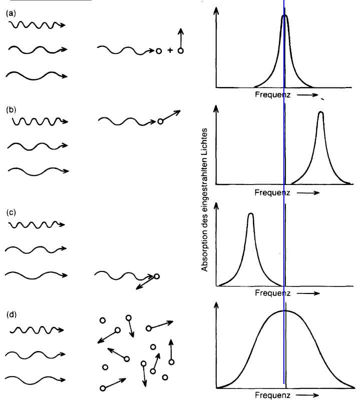


Laser frequency: f

excitation frequency: f_0



v << c



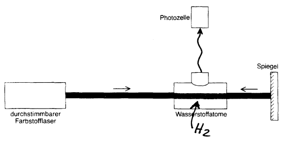
$$f = f_0$$

$$f = f_0(1+\frac{v}{c})$$

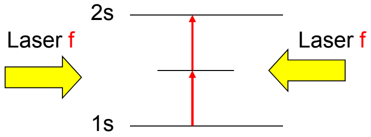
$$f = f_0(1-\frac{v}{c})$$

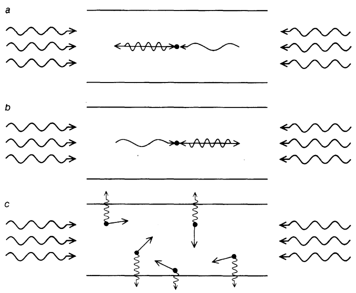
Large Doppler broadening by thermal distribution

Two-photon laser spectroscopy



$\boxed{f_0 = 2 \times f}$





$$f_1 = \frac{f_0}{2}(1+\frac{v}{c})$$

$$f_2 = \frac{f_0}{2}(1-\frac{v}{c})$$

$$\boxed{f\ = f_1 + f_2}$$

Doppler free-Spectroscopy
Condition for Precision-spectroscopy

Detection of fluorescence-radiation
as a function of the laser frequency

Real precision is achieved by relative measurements !!!

Assumption: The corrections (relativistic, QED etc.) are much better known for high n (much smaller !!!) than for n=1 and n=2

Idea: Compare Balmer-β radiation with Lyman-α

Balmer-β: $E_{Ba} = Ry \cdot \left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

Lyman-α: $E_{Ly} = Ry \cdot \left(1 - \frac{1}{2^2}\right)$

$$4 \cdot E_{Ba} = 1 \cdot E_{Ly}$$

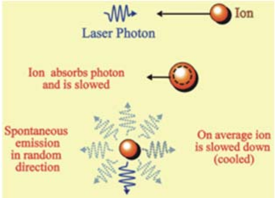
classically correct !!!

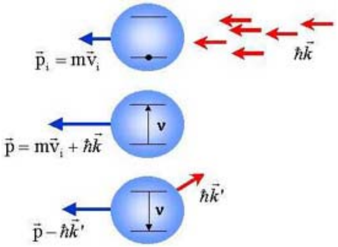
Effectively it is:

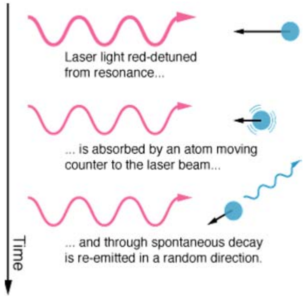
$$\boxed{4 \cdot E_{Ba} = 1 \cdot E_{Ly} + C}$$

E_{Ba} and E_{Ly} are measurement categories in the experiment

C is the parameter for the ground state, one has to determine for checking the theories







http://nobelprize.org/nobel_prizes/physics/laureates/1997/illpres/


The Nobel Prize in Physics 1997

The Nobel Prize in Physics 1997

The Royal Swedish Academy of Sciences has awarded the 1997 Nobel Prize in Physics jointly to

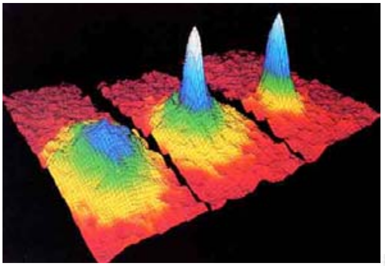
Steven Chu, Claude Cohen-Tannoudji and William D. Phillips

for their developments of methods to cool and trap atoms with laser light.

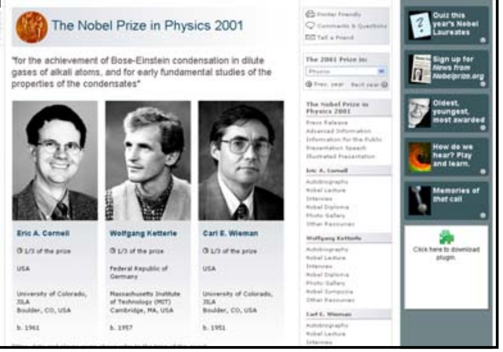
Chilling with light

This year's Nobel laureates in physics have developed methods of cooling and trapping atoms by using laser light. Their research is helping us to study fundamental phenomena and measure important physical quantities with unprecedented precision.

Applications: Bose-Einstein of atoms



http://nobelprize.org/nobel_prizes/physics/laureates/2001/index.html


The Nobel Prize in Physics 2001

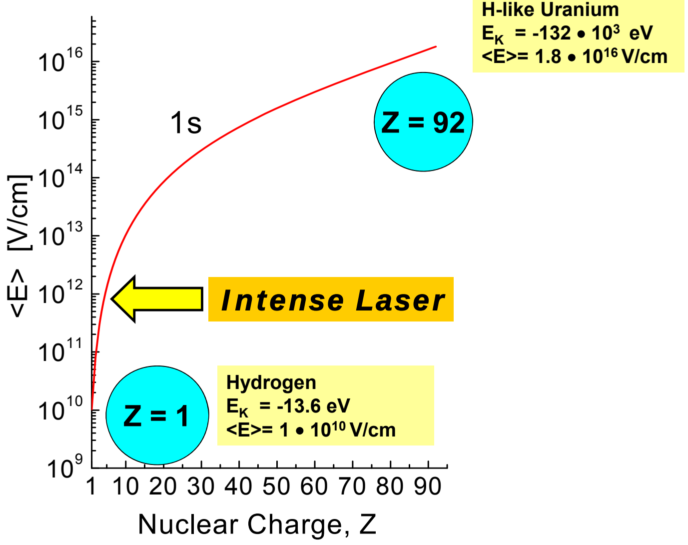
"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

Eric A. Cornell	**Wolfgang Ketterle**	**Carl E. Wieman**
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	Federal Republic of Germany	USA
University of Colorado, JILA Boulder, CO, USA	Massachusetts Institute of Technology (MIT) Cambridge, MA, USA	University of Colorado, JILA Boulder, CO, USA
b. 1961	b. 1957	b. 1951

Content

- **QED: Radiative correction viewed by an experimentalist**
- **Experiments on Hydrogen**
- **Strong fields: High-Z ions versus exotic atomic systems**
- **The Lamb shift of uranium (Z=92)**
- **The hyperfine structure at high-Z**

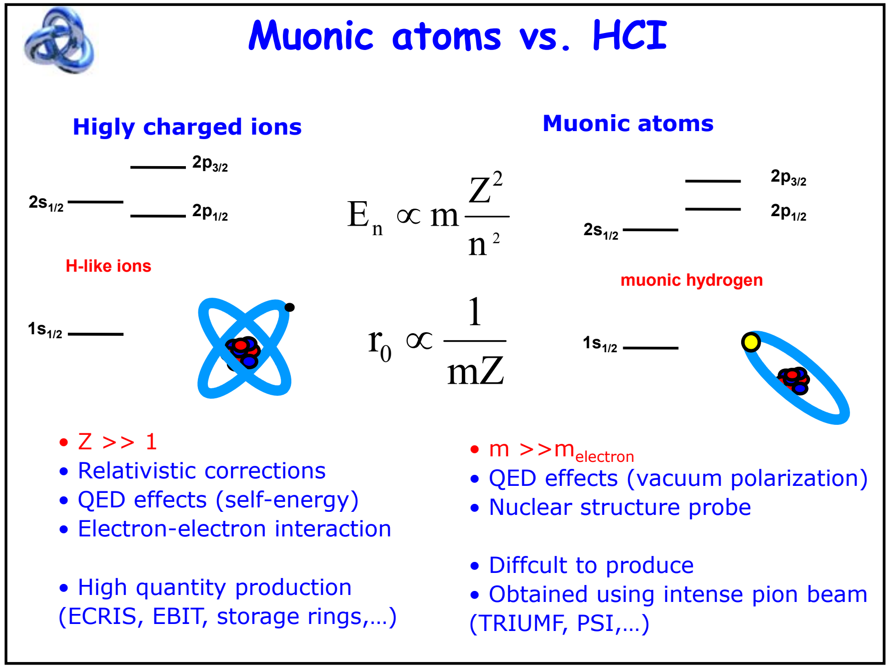
Extremly Strong Coulomb Fields



H-like Uranium
E_K = -132 • 10^3 eV
<E>= 1.8 • 10^{16} V/cm

Hydrogen
E_K = -13.6 eV
<E>= 1 • 10^{10} V/cm

1s-ground state: increase of the electric field strength by six orders of magnitude



Muonic atoms vs. HCI

Higly charged ions

H-like ions

$$E_n \propto m\frac{Z^2}{n^2}$$

$$r_0 \propto \frac{1}{mZ}$$

Muonic atoms

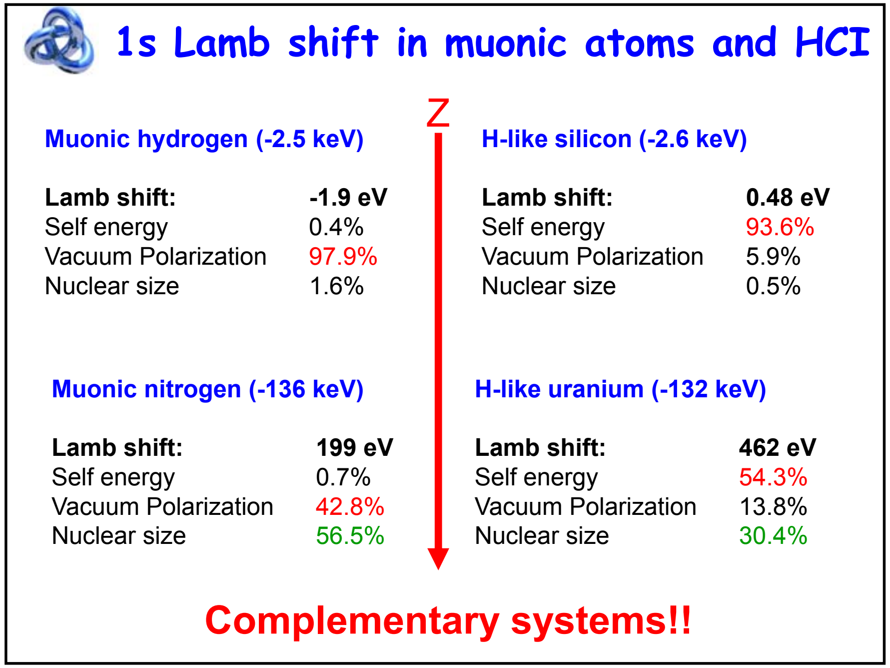
muonic hydrogen

- Z >> 1
- Relativistic corrections
- QED effects (self-energy)
- Electron-electron interaction

- High quantity production (ECRIS, EBIT, storage rings,…)

- m >>$m_{electron}$
- QED effects (vacuum polarization)
- Nuclear structure probe

- Diffcult to produce
- Obtained using intense pion beam (TRIUMF, PSI,…)



1s Lamb shift in muonic atoms and HCI

Muonic hydrogen (-2.5 keV)

Lamb shift:	**-1.9 eV**
Self energy	0.4%
Vacuum Polarization	97.9%
Nuclear size	1.6%

H-like silicon (-2.6 keV)

Lamb shift:	**0.48 eV**
Self energy	93.6%
Vacuum Polarization	5.9%
Nuclear size	0.5%

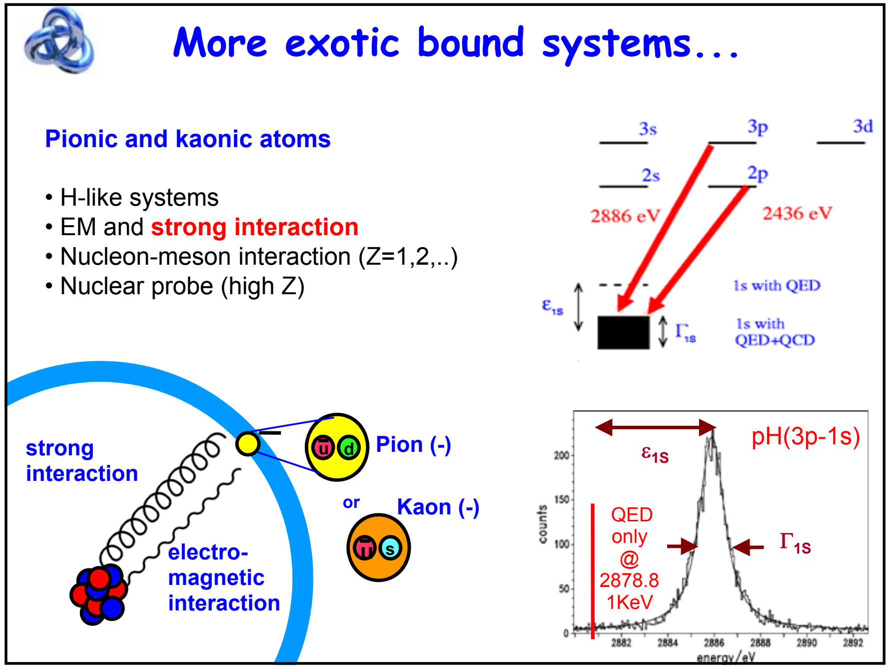
Muonic nitrogen (-136 keV)

Lamb shift:	**199 eV**
Self energy	0.7%
Vacuum Polarization	42.8%
Nuclear size	56.5%

H-like uranium (-132 keV)

Lamb shift:	**462 eV**
Self energy	54.3%
Vacuum Polarization	13.8%
Nuclear size	30.4%

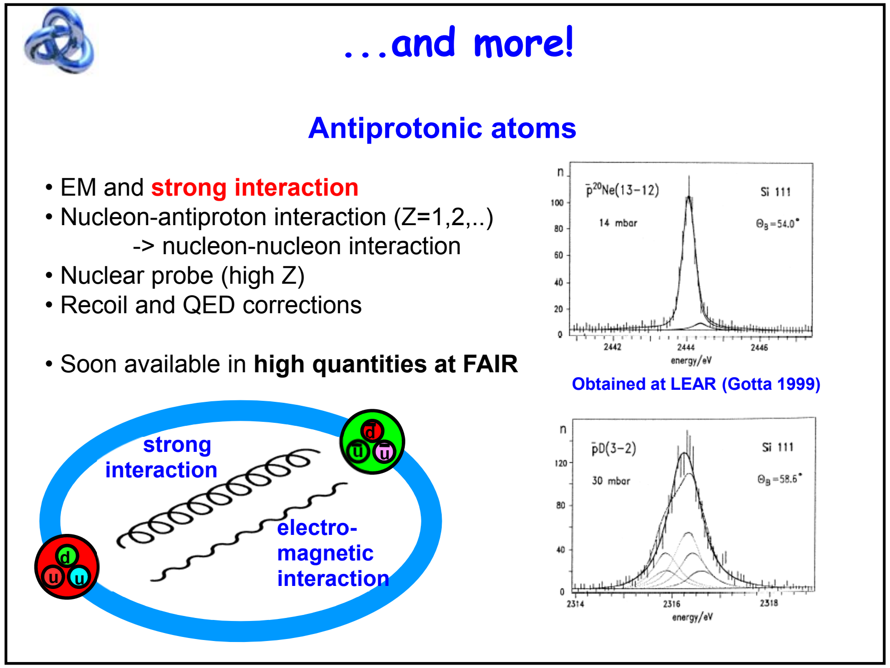
Complementary systems!!



More exotic bound systems...

Pionic and kaonic atoms

- H-like systems
- EM and **strong interaction**
- Nucleon-meson interaction (Z=1,2,..)
- Nuclear probe (high Z)



...and more!

Antiprotonic atoms

- EM and **strong interaction**
- Nucleon-antiproton interaction (Z=1,2,..)
  -> nucleon-nucleon interaction
- Nuclear probe (high Z)
- Recoil and QED corrections

- Soon available in **high quantities at FAIR**

Obtained at LEAR (Gotta 1999)

Content

QED: Radiative correction viewed by an experimentalist

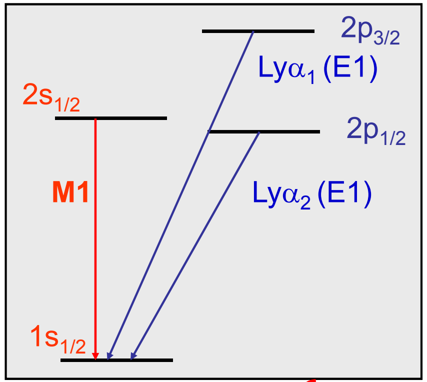
Experiments on Hydrogen

Strong fields: High-Z ions versus exotic atomic systems

The Lamb shift of uranium (Z=92)

The hyperfine structure at high-Z

The Structure of One-Electron Systems



QED corrections

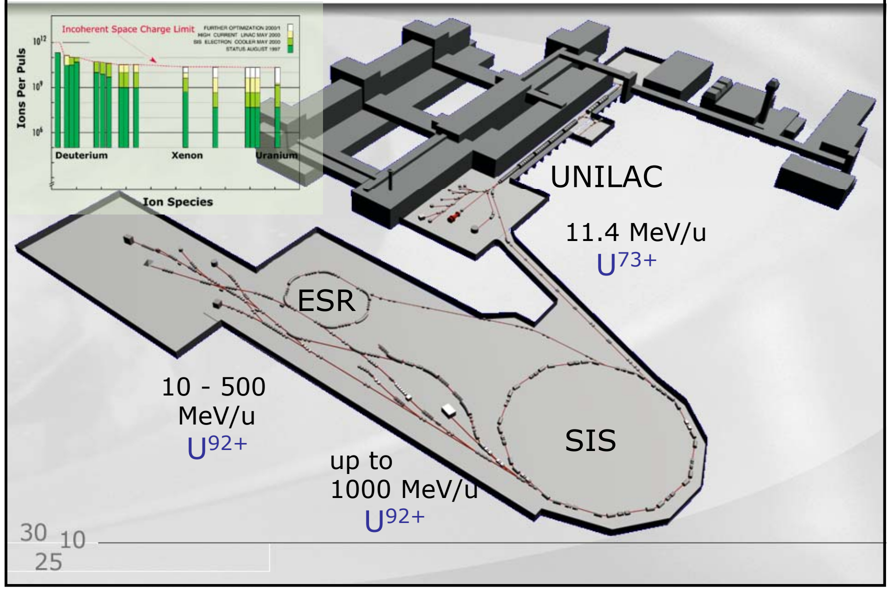
$\Delta E \sim Z^4/n^3$

Z: nuclear charge number
n: prinzipal quantum number

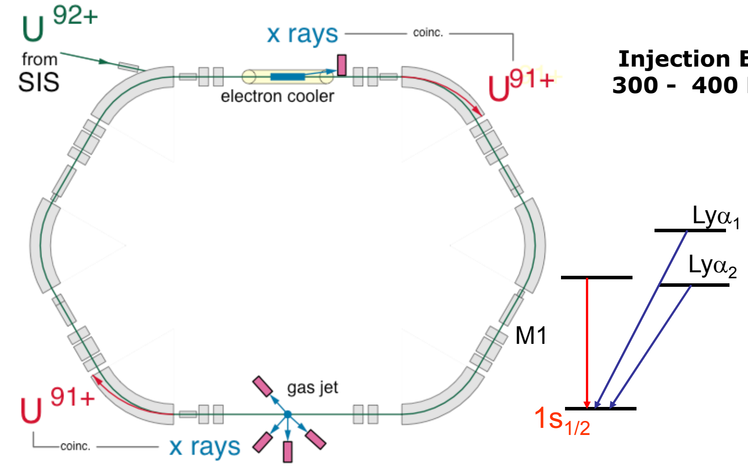
Atomic systems at high-Z

- Large relativistic effects on energy levels and transition rates (e.g. shell and subshell splitting)
- Large QED corrections
- Transition energies close to 100 keV

GSI-Accelerator Facility



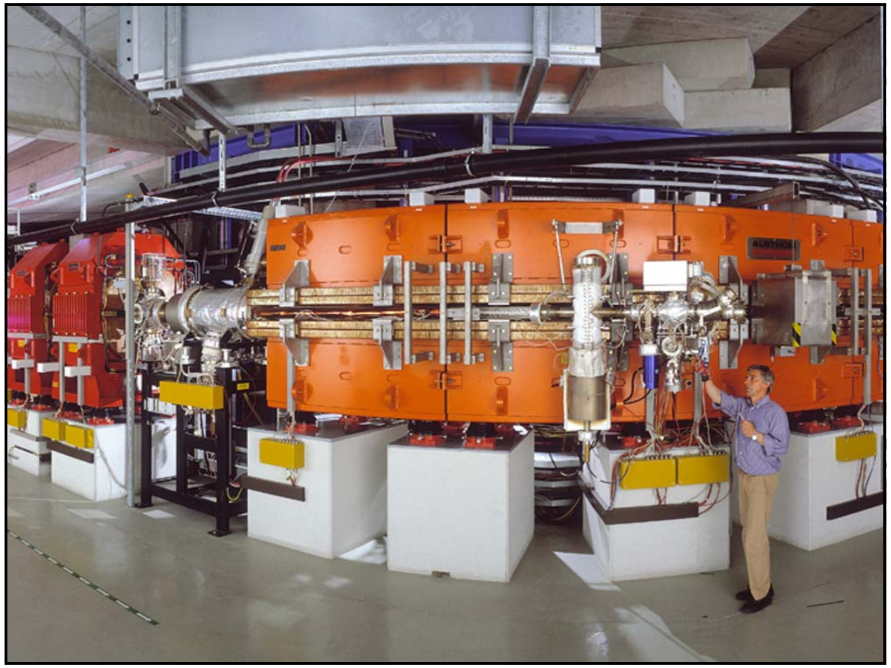
X-Ray Spectroscopy at the ESR Storage Ring



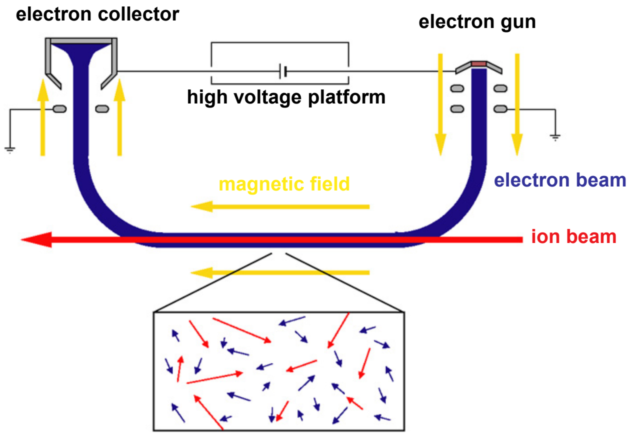
Injection Energy
300 - 400 MeV/U

Circumference: 108 m
Number of Ions: 10^8
Frequency: 10^6 1/s

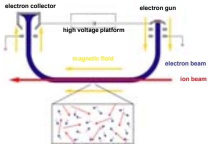
At the ESR, production of characteristic x-rays by electron capture into the bare ions (electron cooler or jet-target)



Storage Rings: Cooled Ion Beams



The Electron Cooler



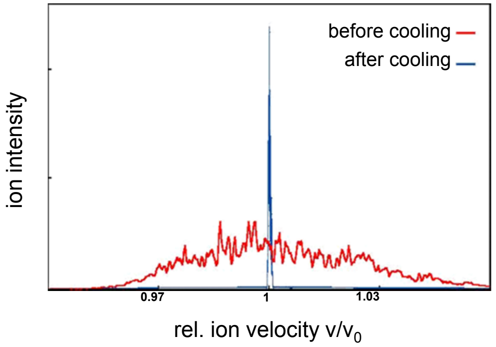
Electron Cooler

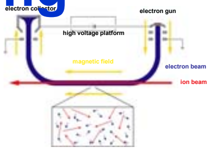
2.5 m interaction zone

Voltage: 5 to 200 kV

Current: 10 to 1000 mA

The Effect of Cooling





Ions interact 10^6 1/s with a collinear beam of cold electrons

Properties of the cold ions

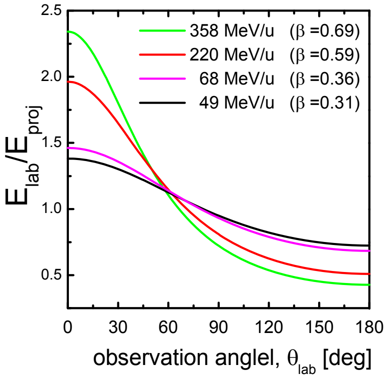
Momentum spread	$\Delta p/p$: $10^{-4} - 10^{-5}$
Diameter	2 mm

The Experimental Challenge

Relativistic Doppler-Transformation

$$E_{lab} = \frac{E_{proj}}{\gamma \cdot (1 - \beta \cdot \cos\theta_{lab})}$$

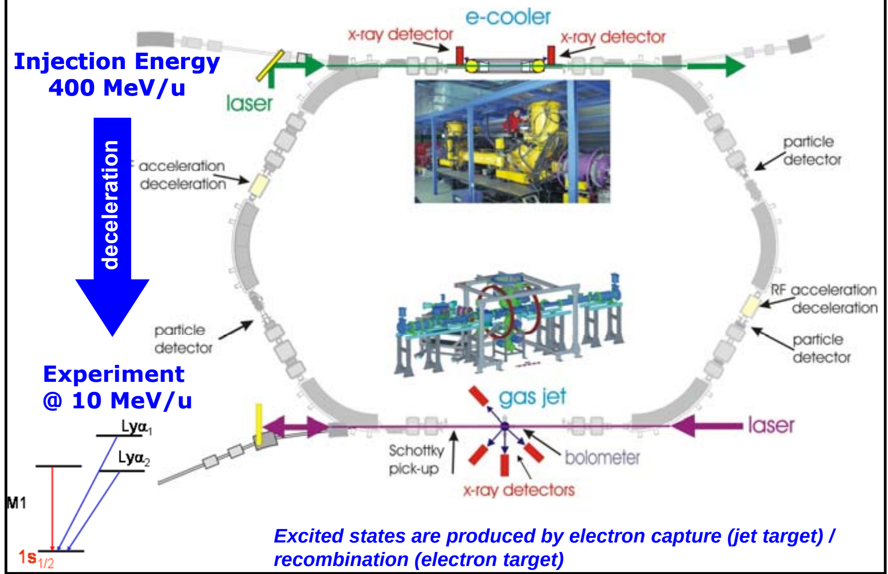
E_{lab}: Photon energy in the laboratory system
E_{proj}: Photon energy in the emitter system



Doppler-Correction: *Strong dependence on velocity and the observation angle* θ_{LAB}

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}; \beta = \frac{v}{c}$$

X-Ray Spectroscopy at the ESR Storage Ring



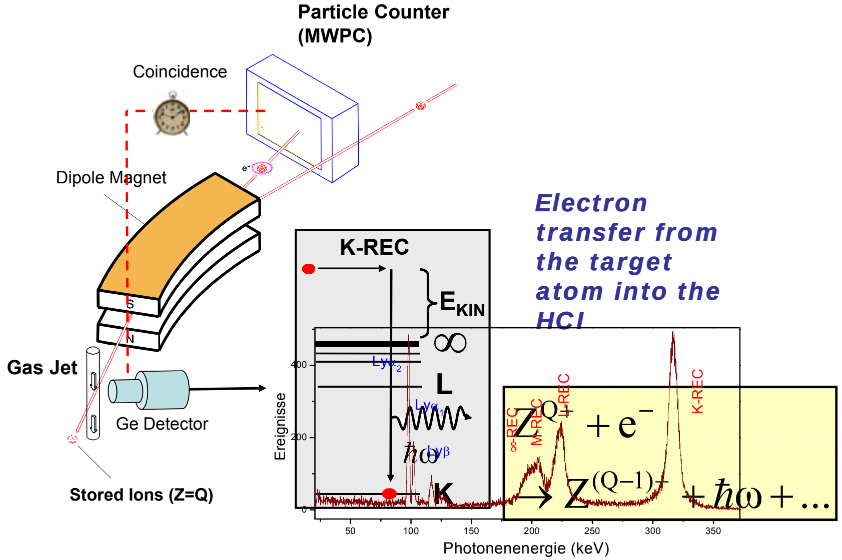
Excited states are produced by electron capture (jet target) / recombination (electron target)

Principle of LS Experiments

1. *Production of bare ions*
2. *Cooling*
3. *Electron capture in excited states (jet-target or electron cooler)*
4. *Detection of x-rays*
5. *Doppler correction*
6. *Comparison with Dirac theory*

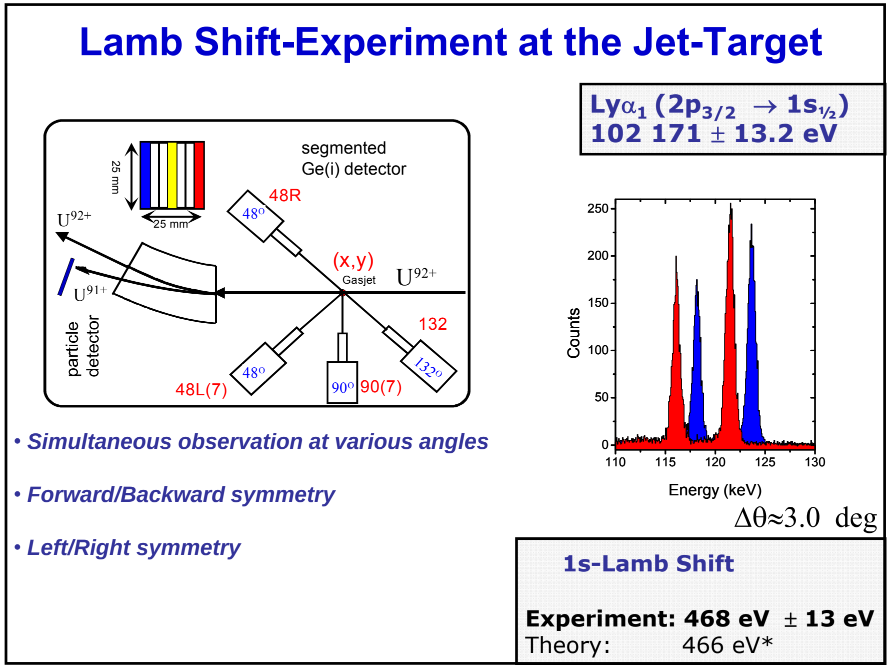
→ **1s Lamb Shift**

Experiments at the Jet-Target



$$Z^{Q+} + e^{-} \rightarrow Z^{(Q-1)+} + \hbar\omega + \ldots$$

Electron transfer from the target atom into the HCI



Lamb Shift-Experiment at the Jet-Target

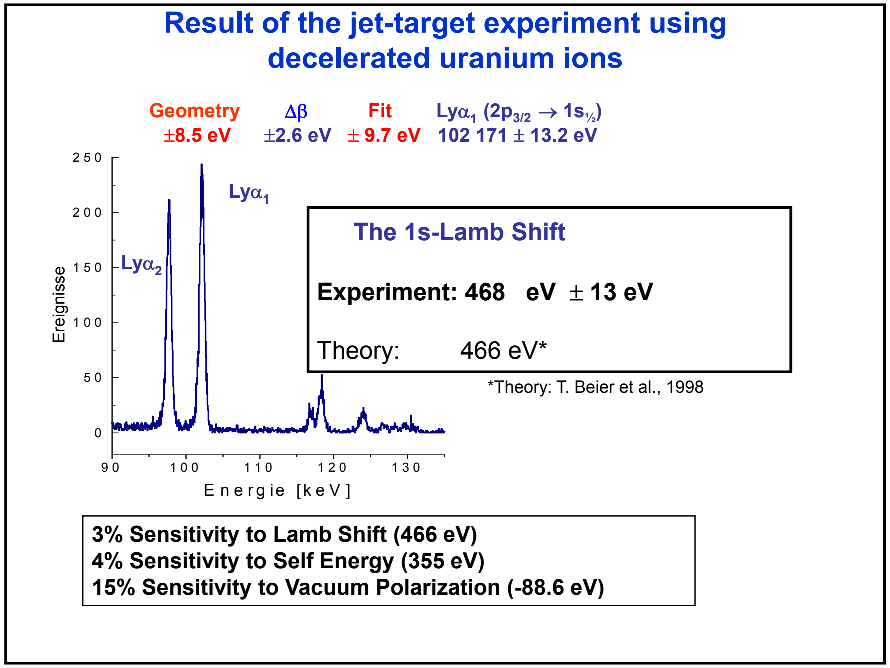
Lyα$_1$ (2p$_{3/2}$ → 1s$_{1/2}$)
102 171 ± 13.2 eV

- *Simultaneous observation at various angles*
- *Forward/Backward symmetry*
- *Left/Right symmetry*

$\Delta\theta \approx 3.0$ deg

1s-Lamb Shift

Experiment: 468 eV ± 13 eV
Theory: 466 eV*



Result of the jet-target experiment using decelerated uranium ions

Geometry	Δβ	Fit	Lyα$_1$ (2p$_{3/2}$ → 1s$_{1/2}$)
±8.5 eV	±2.6 eV	± 9.7 eV	102 171 ± 13.2 eV

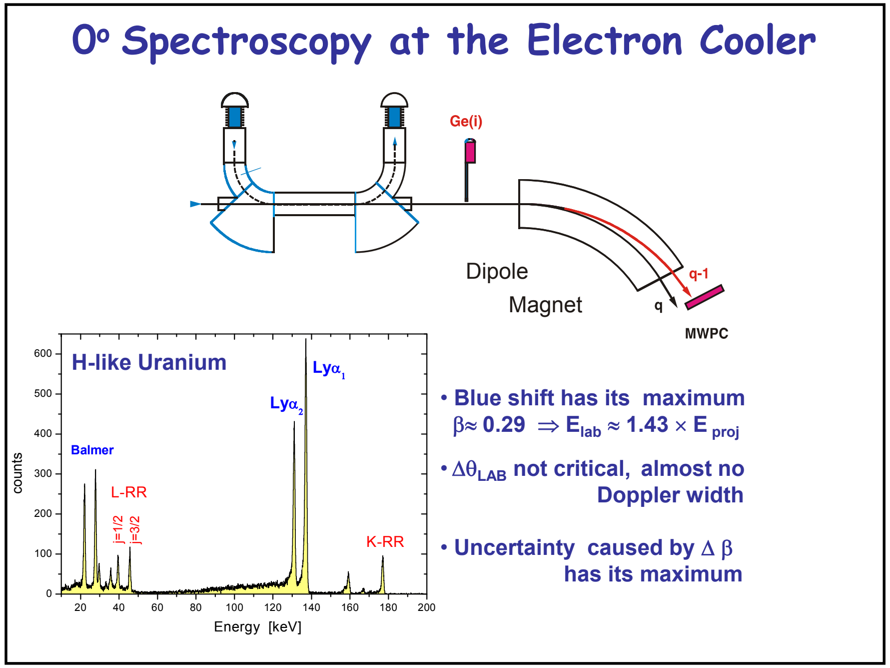
The 1s-Lamb Shift

Experiment: 468 eV ± 13 eV

Theory: 466 eV*

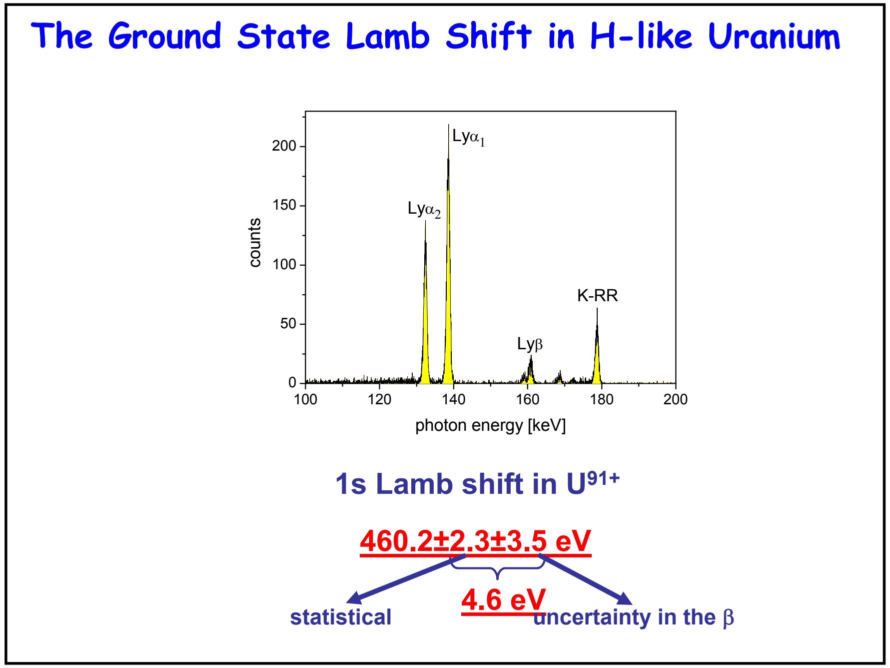
*Theory: T. Beier et al., 1998

3% Sensitivity to Lamb Shift (466 eV)
4% Sensitivity to Self Energy (355 eV)
15% Sensitivity to Vacuum Polarization (-88.6 eV)



0° Spectroscopy at the Electron Cooler

- **Blue shift has its maximum $\beta \approx 0.29 \Rightarrow E_{lab} \approx 1.43 \times E_{proj}$**
- **$\Delta\theta_{LAB}$ not critical, almost no Doppler width**
- **Uncertainty caused by $\Delta \beta$ has its maximum**



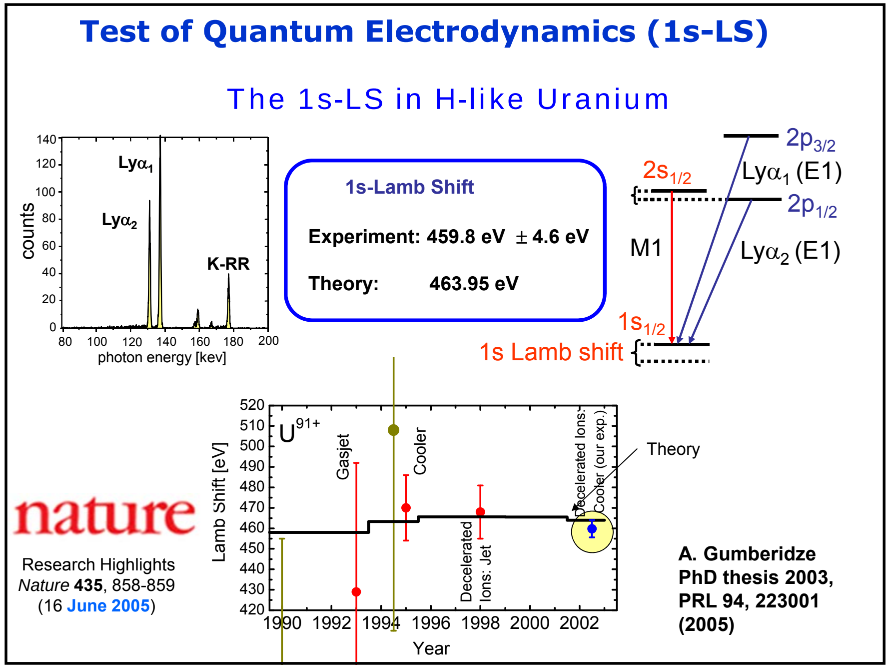
The Ground State Lamb Shift in H-like Uranium

1s Lamb shift in U^{91+}

$$\underbrace{460.2 \pm 2.3 \pm 3.5}_{4.6\ \text{eV}}\ \text{eV}$$

statistical

uncertainty in the β



Test of Quantum Electrodynamics (1s-LS)

The 1s-LS in H-like Uranium

1s-Lamb Shift

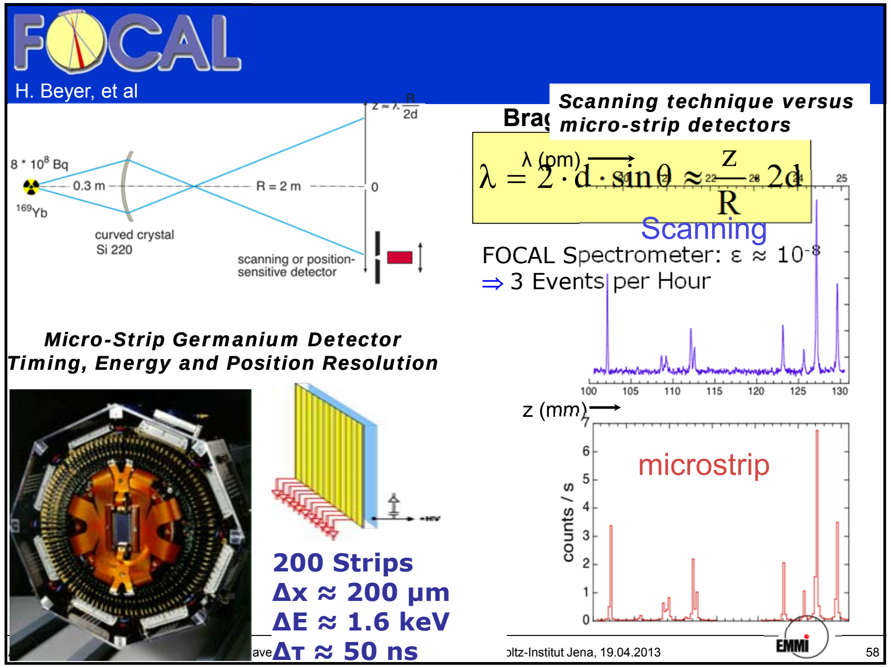
Experiment: 459.8 eV ± 4.6 eV

Theory: 463.95 eV

nature

Research Highlights
Nature **435**, 858-859
(16 **June 2005**)

A. Gumberidze
PhD thesis 2003,
PRL 94, 223001
(2005)



FOCAL

H. Beyer, et al

Scanning technique versus micro-strip detectors

Brag

$$\lambda = 2 \cdot d \cdot \sin\theta \approx \frac{Z}{R} 2d$$

Scanning

FOCAL Spectrometer: ε ≈ 10^{-8}
⇒ 3 Events per Hour

Micro-Strip Germanium Detector
Timing, Energy and Position Resolution

200 Strips
Δx ≈ 200 µm
ΔE ≈ 1.6 keV
Δτ ≈ 50 ns

ave

oltz-Institut Jena, 19.04.2013

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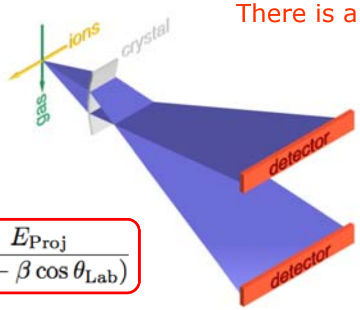
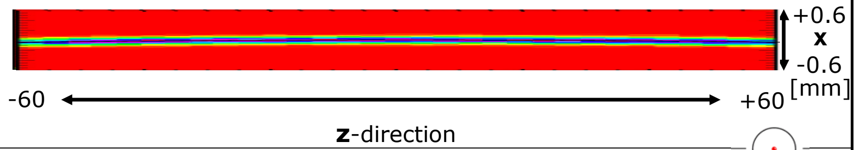
There is a need for 2D detectors !



$$E_{\mathrm{Lab}} = \frac{E_{\mathrm{Proj}}}{\gamma(1-\beta\cos\theta_{\mathrm{Lab}})}$$

fast moving source

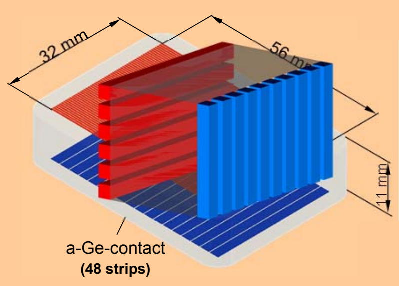
stationary source

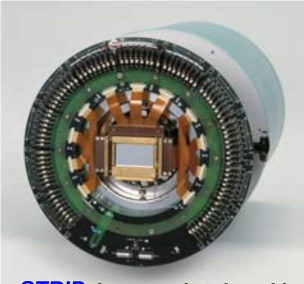


Prototype 2D μ*STRIP* X-Ray Detector

2D µSTRIP planar detector systems for precision
x-ray spectroscopy experiments (FOCAL)

energy resolution – timing - 2D position sensitiviy





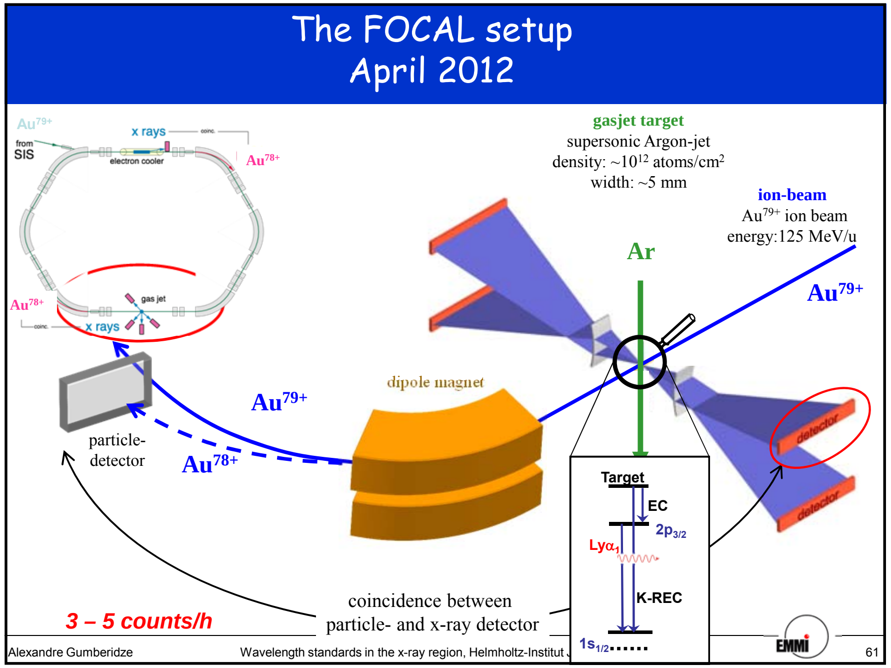
µSTRIP **detector developed by**

front: 128 strips pitch ~250µm

back: 48 strips pitch ~1167µm

equivalent to 6144 pixel

2D µSTRIP germanium detector



The FOCAL setup
April 2012

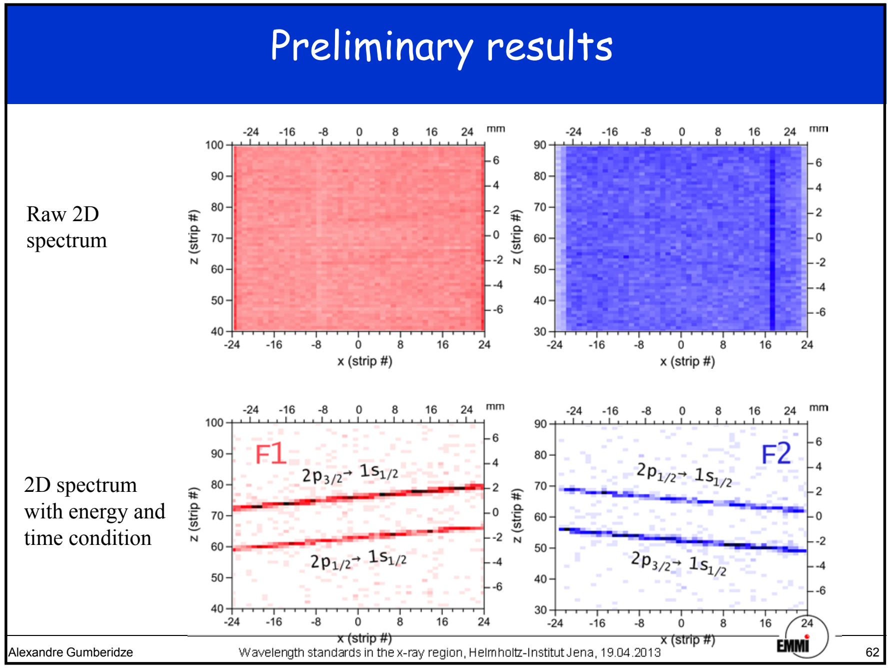
gasjet target
supersonic Argon-jet
density: ~10^{12} atoms/cm^2
width: ~5 mm

ion-beam
Au^{79+} ion beam
energy:125 MeV/u

particle-
detector

coincidence between
particle- and x-ray detector

3 – 5 counts/h



Preliminary results

Raw 2D
spectrum

2D spectrum
with energy and
time condition

Content

QED: Radiative correction viewed by an experimentalist

Experiments on Hydrogen

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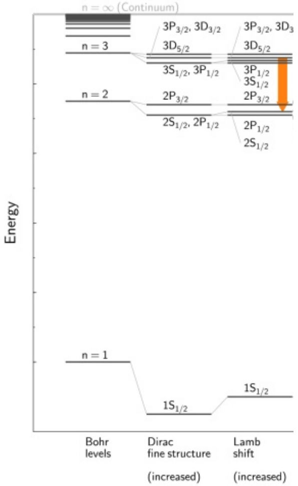
The Lamb shift of uranium (Z=92)

The hyperfine structure at high-Z

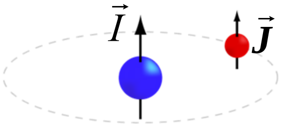
Energy levels of hydrogen atom: From Schrödinger to Dirac to QED



- Do we expect some more effects which can influence the level structure?
- Actually yes!

Hyperfine interaction

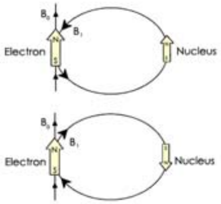
- Hyperfine structure is a perturbation in the energy levels of atoms/ions due to the magnetic dipole-dipole interaction, arising from the interaction of the nuclear magnetic moment with the magnetic field of the electron.



From: http://www.wikipedia.org

- Coupling of electron and nuclear momenta give rise to the total angular momentum of the system:

$$\vec{F} = \vec{J} + \vec{I}$$



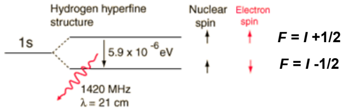
- The HF splitting is:

$$\Delta E_{HF} \approx a\left(F(F+1) - I(I+1) - J(J+1)\right)$$

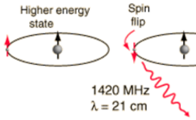
- Levels are spitted one more time!

Hyperfine interaction

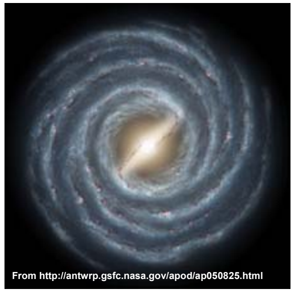
- For the case of ground $1s_{1/2}$ state (j=1/2) of hydrogen, HF interaction results in splitting of energy level into two levels.



- One can observe transition between two HF levels: famous 21 cm line in astrophysics!



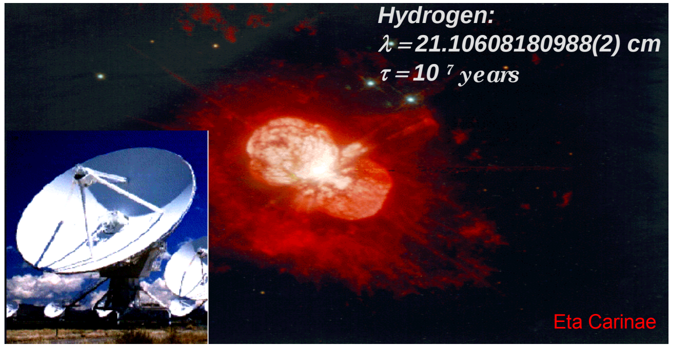
From: http://http://hyperphysics.phy-astr.gsu.edu

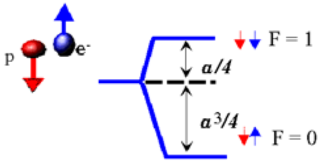


From http://antwrp.gsfc.nasa.gov/apod/ap050825.html

- 21 cm radiation is used, for example, to measure radial velocities of the spiral arms of the Milky Way.

Hyperfine Structure at High-Z





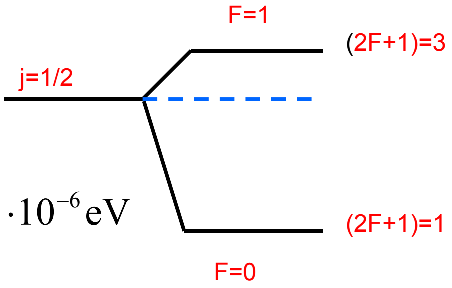
Hydrogen Atom: Groundstate

$$\Delta E^{1}{}_{HFS} = +\frac{A}{4}$$

for j=1/2, s_p=1/2, F=1

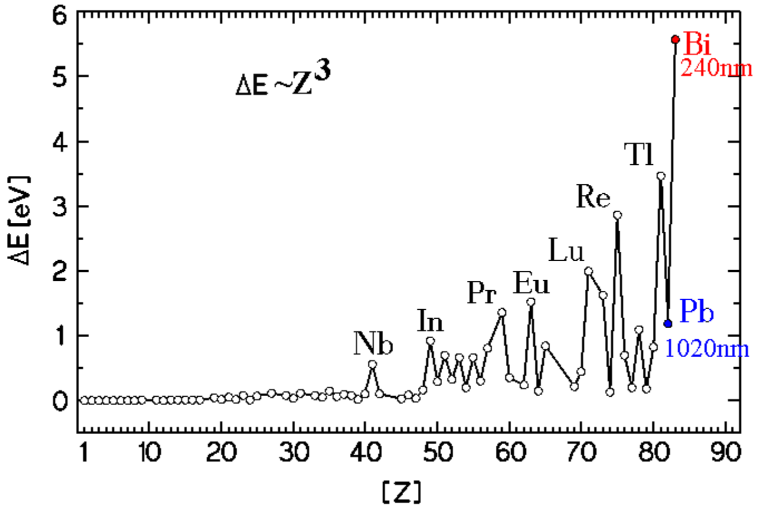
$$\Delta E^{0}{}_{HFS} = -\frac{3A}{4}$$

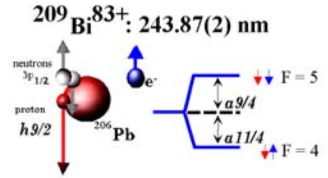
for j=1/2, s_p=1/2, F=0



$$\Delta E_{HFS} \propto 6 \cdot 10^{-6}\,\mathrm{eV}$$

Hyperfine Structure at High-Z





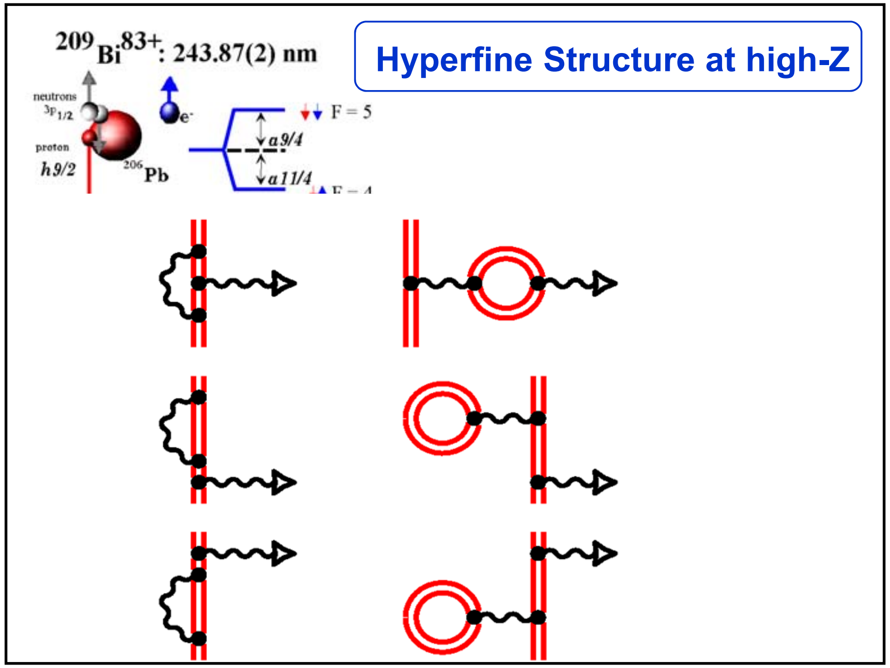
HFS: Test of QED in strong magnetic fields (high-Z)

strong magnetic fields
≈ 10^{12} Gauss

Fermiformula for the HFS

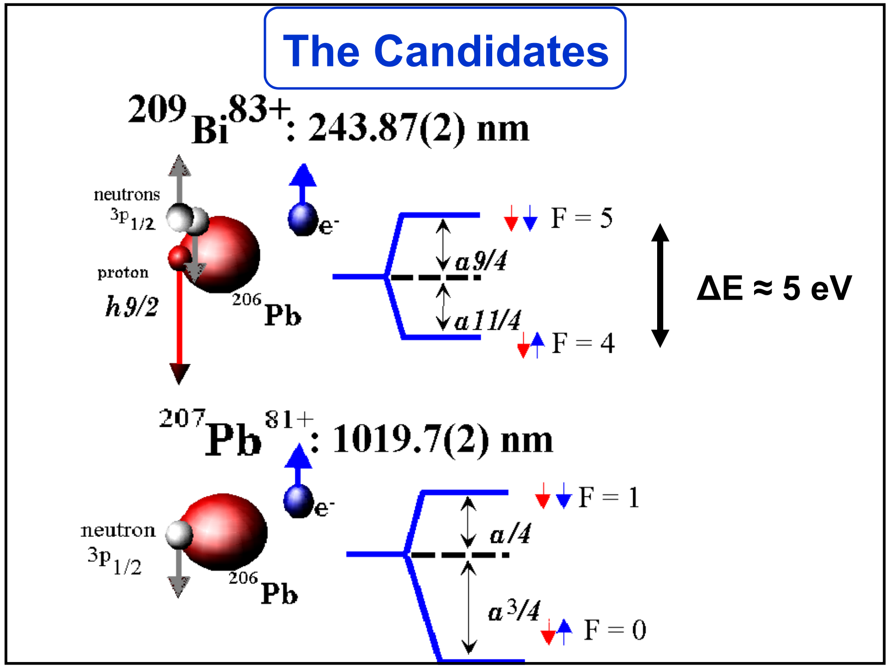
$$\Delta\nu = \left(I + \frac{1}{2}\right) \cdot \frac{8}{3}\frac{h \cdot c}{n^3} \cdot Z^3 \cdot Ry \cdot \alpha^2 \cdot g_I \cdot \frac{M}{m_e} \times$$

• $F(j,Z)$	relativistic correction
• $(1-\delta)$	Nuclear-Charge distribution (Breit-Rosental-Corwford-Effect)
• $(1-\varepsilon)$	current distribution (Bohr-Weisskopf-Effect)
• QED	???



Hyperfine Structure at high-Z

$^{209}Bi^{83+}$: 243.87(2) nm

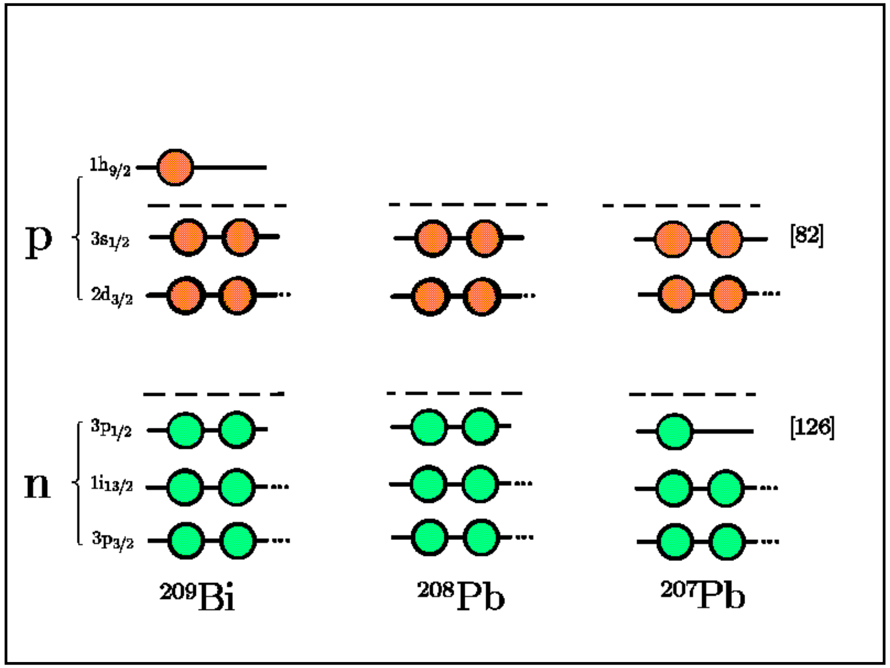


The Candidates

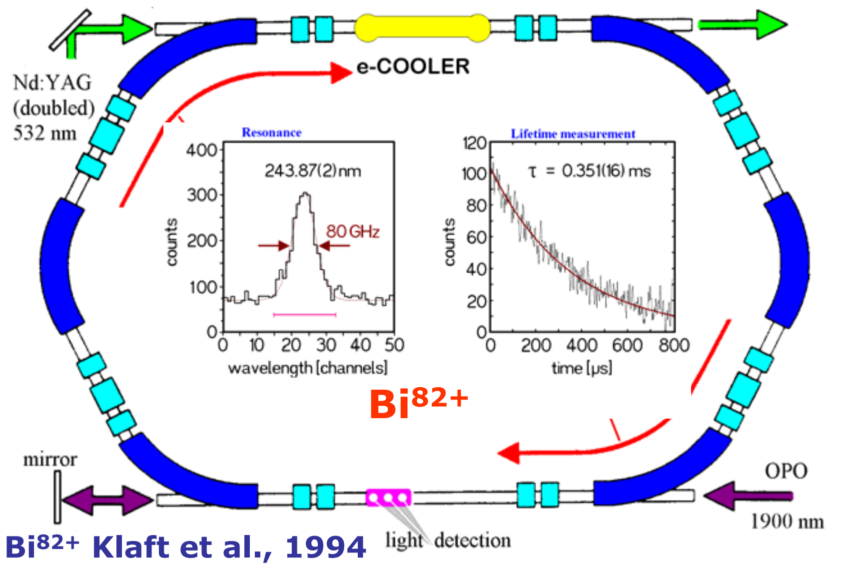
$^{209}Bi^{83+}$: 243.87(2) nm

ΔE ≈ 5 eV

$^{207}Pb^{81+}$: 1019.7(2) nm



Laser Experiments at the ESR



Bi^{82+} Klaft et al., 1994

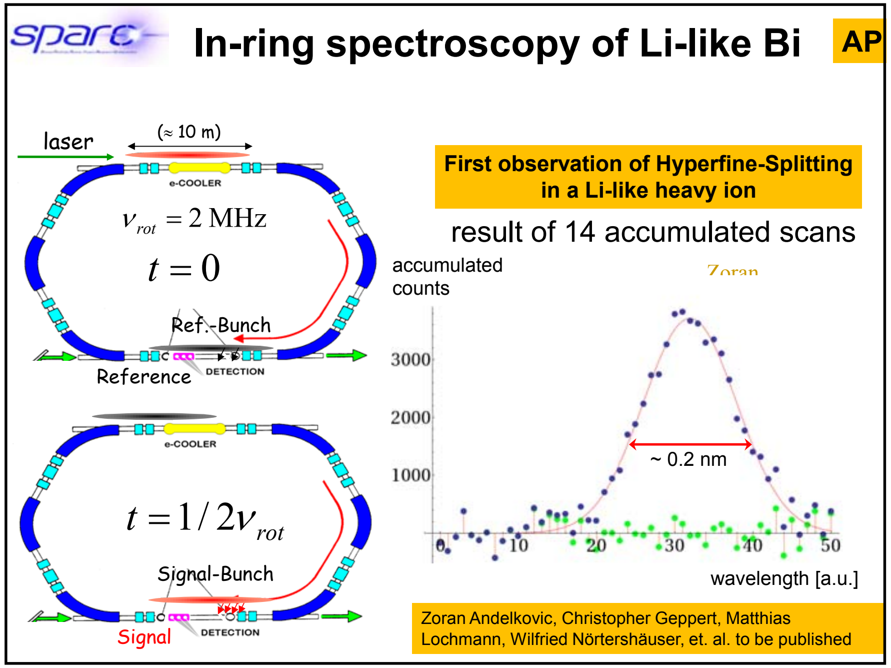
We have a problem !

	$^{209}Bi^{82+}$	$^{207}Pb^{81+}$
RMS radius	*5.519 fm*	*5.497 fm*
Magn.Mom.(corr.)	*4.1106 n.m.*	*0.58219 n.m.*
Point nucl.(Dirac)	212.320 nm	880.017 nm
Breit-Schawlow	+ 26.561(50) nm	+109.64(1) nm
Ext. Nucleus	***238.888(50) nm***	***989.65(1) nm***
Bohr-Weisskopf	+ 5.025(330) nm	+ 29.5(20) nm
Theory, no QED	***243.91(38) nm***	***1019.1(21) nm***
Vac. polarisation	- 1.64 nm	- 6.83 nm
Self energy	+ 2.86 nm	+ 11.9 nm
Total QED	+ 1.22 nm	+ 5.08 nm
Theory incl.QED	***245.13(58) nm***	***1024.2(25)nm***
Experimental	**243.87(2) nm**	**1019.5(2)**

?

In-ring spectroscopy of Li-like Bi

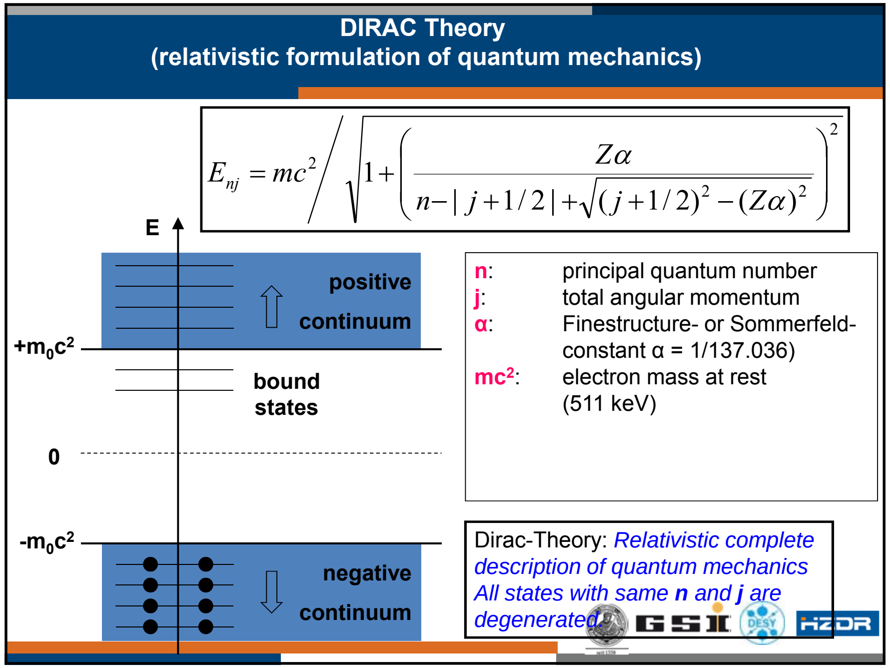
AP



First observation of Hyperfine-Splitting in a Li-like heavy ion

result of 14 accumulated scans

Zoran Andelkovic, Christopher Geppert, Matthias Lochmann, Wilfried Nörtershäuser, et. al. to be published



DIRAC Theory
(relativistic formulation of quantum mechanics)

$$E_{nj} = mc^2 \Big/ \sqrt{1+\left(\frac{Z\alpha}{n-|j+1/2|+\sqrt{(j+1/2)^2-(Z\alpha)^2}}\right)^2}$$

n: principal quantum number
j: total angular momentum
α: Finestructure- or Sommerfeld-constant α = 1/137.036)
mc^2: electron mass at rest (511 keV)

Dirac-Theory: *Relativistic complete description of quantum mechanics*
*All states with same **n** and **j** are degenerated*