## Modern Atomic Physics: Experiment and Theory

# Experiments Hyrogen, Dirac theory 

## Lecture 2

April 22 ${ }^{\text {nd }} 2014$

## Lectures in Internet

Find zipped .PPT \& .PDF files with the lectures at:

## http://web-docs.gsi.de/~stoe_exp/lectures/lectures.php

(password: dirac2015)


## Preliminary plan of the lectures

- 1 15.04.2015 Preliminary Discussion / Introduction
- 2 22.04.2015 Experiments (discovery of the positron, formation of antihydrogen, ...)
- 3 29.04.2015 Experiments (Lamb shift, hyperfine structure, quasimolecules and MO spectra)
- 4 06.05.2015 Theory (from Schrödinger to Dirac equation, solutions with negative energy)
- 5 13.05.2015 Theory (photon, quantum field theory (just few words), Feynman diagrams, QED corrections)
- 6 20.05.2015 Theory (matrix elements and their evaluation, radiative decay and absorption)
- 7 27.05.2015 Experiment (photoionization, radiative recombination, ATI, HHG...)
- 8 03.06.2015 Theory (single and multiple scattering, energy loss mechanisms, channeling regime)
- 9 10.06.2015 Experiment (Kamiokande, cancer therapy, ....)
- 10 17.06.2015 Experiment (Auger decay, dielectronic recombination, double ionization)
- 11 24.06.2015 Theory (interelectronic interactions, extension of Dirac (and Schrödinger) theory for the description of many-electron systems, approximate methods)
- 12 01.07.2015 Theory (atomic-physics tests of the Standard Model, search for a new physics)
- 13 08.07.2015 Experiment (Atomic physics PNC experiments (Cs,...), heavy ion PV research)
General
NIST Physical Reference Data - X-Ray and Gamma-Ray Data ..... http://physics.nist.gov/PhysRefData/contents-xray.html
Fundamental Physical Constantshttp://physics.nist.gov/PhysRefData/contents-constants.html
Atomic Spectroscopic Data
X-Ray World Wide Web Server

X-ray Emission Lines

## Electron Binding Energies

http://xray.uu.se/hypertext/XREmission.html

http://xray.uu.se/hypertext/EBindEnergies.htm|

## Berkeley National Laboratory

Table of I sotopes
Atomic Data
Elemental Physical Properties
http://ie.lbl.gov/education/isotopes.htm
http://ie.lbl.gov/atomic/atom.htm
http://ie.Ibl.gov/elem/elem.htm (pdf download possible)

## CODATA Internationally recommended values of the Fundamental Physical Constants

http://physics.nist.gov/cuu/Constants/index.htm|
I nstitute of Chemistry, Free University Berlin

Fundamental Physical Constants
Conversion of Units
http://www.chemie.fu-berlin.de/chemistry/general/constants en.html http://www.chemie.fu-berlin.de/chemistry/general/units en.html
Periodic tables (professional edition)
http://www.webelements.com/

## Korea Atomic Energy Research Institute

Table of Nuclides
http://atom.kaeri.re.kr/ton/nuc6.html
Center for Synchrotron Radiation Research and Instrumentation, Chicago, United States
Periodic Table of Elements - X-ray properties
http://www.csrri.iit.edu/periodic-table.html

## Contents

- Summary: The hydrogen atom in a non-relativistic view
- Stern-Gerlach Experiment - The Spin of the electron
- Dirac - The effect of relativity on the atomic structure
- Cosmic Rays
- The discovery of the positron
- First production of antihydrogen
- Positron-Emissions-Tomographie (PET)


# Hydrogen atom 

## Hydrogen

$\Delta E / E \approx 10^{-14}$

## U/tracold \& Trapped $\bar{p}$

## Hydrogen

## Antihydrogen



Same Structure?


CPT I nvariance dat CéfN

- H14
$\Delta E / E \approx 10^{\text {in traps: } 2002}$


## 1) A SRIEF




## + INTO THE ANTIWORLD +

Dirac's dramatic discovery of Antimatter


## The Nobel Prize in Physics 1933

"for the discovery of new productive forms of atomic theory"


Paul Adrien Maurice Dirac
(1) $1 / 2$ of the prize

United Kingdom
University of Cambridge Cambridge, United Kingdom
b. 1902
d. 1984

Erwin Schrödinger
(1) $1 / 2$ of the prize

Austria
Berlin University Berlin, Germany
b. 1887
d. 1961


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| The 1933 Prize in: |
| Physics |
| () Prev. year Next year (©) |

The Nobel Prize in Physics 1933

Presentation Speech

Erwin Schrödinger
Biography
Nobel Lecture
Documentary
Banquet Speech
Other Resources

Paul A.M. Dirac
Biography
Nobel Lecture
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Banquet Speech
Other Resources


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## ze.org

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ze in Physics, Nobel Prize in Chemistry, Nobel Prize in Medicine, Nobel Prize in Literature, Nobel Peace Prize, Prize in Economics

## The Nobel Prize in Physics 1936

"for his discovery of cosmic radiation"


Victor Franz Hess
"for his discovery of the positron"


Carl David Anderson
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The 1936 Prize in:
Physics
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The Nobel Prize in Physics 1936

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## The Nobel Prize in Physics 1959

"for their discovery of the antiproton"


## Emilio Gino Segrè

(1) $1 / 2$ of the prize

USA
University of California Berkeley, CA, USA
b. 1905
(in Tivoli, Italy)
b. 1920
d. 2006


## Owen Chamberlain

(1) $1 / 2$ of the prize

USA

University of California Berkeley, CA, USA
Printer Friendly

The 1959 Prize in:
Physics
(9) Prev. year Next year (1)

The Nobel Prize in Physics 1959

Presentation Speech

Emilio Segrè
Biography
Nobel Lecture
Banquet Speech

Owen Chamberlain
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Try the Nobel Prize Quiz!
-
"for his contribution to the quantum theory of optical coherence"


Roy J. Glauber
(1) $1 / 2$ of the prize USA

Harvard University Cambridge, MA, USA
"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"


John L. Hall
D) $1 / 4$ of the prize USA

University of Colorado, JILA; National Institute of Standards and Technology Boulder, CO, USA
b. 1934


Theodor W. Hänsch
(1) $1 / 4$ of the prize

Germany
Max-Planck-Institut für Quantenoptik
Garching, Germany;
Ludwig-
Maximilians-
Universität
Munich, Germany
b. 1941
http://nobelprize.org/physics/laureates/2005/index.html

## The Spectrum of Hydrogen

- One needs three quantum numbers to define the state of hydrogen (hudrogen-like) atom:
- $n=1,2,3 \ldots$ (principal)
- $\mathrm{I}=0, \ldots \mathrm{n}-1$ (orbital)
- $m=-1, \ldots+1$ (magnetic)
- The energy depends only on the principal quantum number:

$$
E_{n}=-\frac{Z^{2}}{2 n^{2}}
$$

- i.e. in nonrelativistic theory the states are degenerate ( $\mathrm{I}, \mathrm{m}$ )!
$\psi(\boldsymbol{r})=\psi(r, \theta, \varphi)=R_{n l}(r) Y_{l m}(\theta, \varphi)$



## Hydrogen

Energy eigenvalue of the hydrogen atom

$$
\mathrm{E}_{\mathrm{n}}=-\frac{\mathrm{me}^{4}}{32 \pi^{2} \varepsilon_{0}^{2} \hbar^{2} \mathrm{n}^{2}}=\frac{-13.6}{\mathrm{n}^{2}} \mathrm{eV}
$$

$$
\begin{gathered}
\text { Schrödinger equation } \quad \mathrm{U} \propto-\frac{1}{\mathrm{r}} \\
-\frac{\hbar^{2}}{2 \mathrm{~m}}\left(\frac{\partial^{2}}{\partial \mathrm{x}^{2}}+\frac{\partial^{2}}{\partial \mathrm{y}^{2}}+\frac{\partial^{2}}{\partial \mathrm{z}^{2}}\right) \psi_{\mathrm{nlm}}+\mathrm{U} \psi_{\mathrm{nlm}}=\mathrm{E}_{\mathrm{n}} \psi_{\mathrm{nlm}}
\end{gathered}
$$

The solution (energy) for a central Coulomb-potential only depends on the quantum number $n$, but not on I or $m$. States with the same $n$ are degenerated, what means they have the same energy.
(In many-electron atoms the degeneracy disappears because of a non-central Coulomb-potential.)

## Atomic Units

$$
\text { for electrons } \quad m_{e}=1 \Rightarrow v=p
$$

scaling properties $\quad r=\frac{n^{2}}{Z} \quad v=\frac{Z}{n} \quad E=\frac{1}{2} Z^{2}$

$$
\text { fine-structure constant } \quad \mathrm{C}=\frac{1}{\alpha}=137.036
$$

$$
\begin{gathered}
\alpha=\left(\frac{\mathrm{e}^{2}}{\hbar \cdot c}\right)_{\text {gauss }} ; \alpha=\left(\frac{e^{2}}{4 \pi \cdot \varepsilon_{0} \cdot \hbar \cdot c}\right)_{S I} \\
\alpha=1 / 137.03599911(46)
\end{gathered}
$$

## Atomic Units

Atomic Units

| $\hbar=1$ | atomic Planck constant |
| :--- | :--- |
| $\mathrm{m}_{\mathrm{e}}=1$ | atomic mass unit |
| $e=1$ | atomic charge unit |
| $4 \pi \varepsilon_{0}=1$ | dielectric constant |

## SI-Units

1.05 * $10^{-34} \mathrm{Js}$
9.1 * $10^{-31} \mathrm{~kg}$
1.6 * 10-19 C

The Bohr-radius defines the atomic length unit

$$
a_{0}=0,53 \cdot 10^{-8} \mathrm{~cm}: \quad 1 \text { a.u. }
$$

The atomic energy unit is 27.21 eV and is called Hartree

For the ionization-energy of the hydrogen atom follows

$$
\text { 1/2 Hartree = } 13.6 \text { eV = } 1 \text { Rydberg }
$$

## The hydrogen spectrum



A lot of stars have spectra which are identical to the absorption spectrum of hydrogen. In 1885 Balmer developed an empirical formula to calculate the frequency of these lines

$$
\begin{equation*}
\nu_{m}=R\left(\frac{1}{4}-\frac{1}{m^{2}}\right) \quad \lambda_{m}=\frac{\mathrm{c}}{\nu_{m}} \tag{1}
\end{equation*}
$$

where $\mathrm{m} \geq 3$ and R are constants (Rydberg-frequency). This formula describes for $m=3,4, \ldots$ a continuous serial of lines of the frequencies $v_{m}$ (resp. the wavelengths $\lambda_{m}$ ) known as Balmer-series. In general these lines are described in the following way:
$H_{\alpha}(m=3), H_{\beta}(m=4), \ldots$

## The spectrum of atomic hydrogen



solar spectrum (top) with absorption-lines of sodium ( $D$ ) und hydrogen, in comparison to calibration lines of some elements


Spectrum of Sirius depending on the wavelenght [in $\left.A=10^{-8} \mathrm{~cm}\right]$ with a multitude of hydrogen $(\mathrm{H})$-absorption lines from the Balmer-series.

## Magnetic moments

## Orbital magnetic dipole moment



In classical electrodynamics:

vector area
of the
current loop

## Spin magnetic moment

classical picture


$$
|\mu|=\boldsymbol{I} \cdot \boldsymbol{A}=\frac{q}{T} \pi r^{2}=\frac{q v}{2 \pi r} \pi r^{2}=\frac{q}{2 m} m v r=\frac{q}{2 m} L
$$

In quantum mechanics, for electron: $q=-e$


$$
\begin{aligned}
\hat{\boldsymbol{\mu}}_{l}=-\mu_{0} \hat{\boldsymbol{L}} / \hbar, & \mu_{0}=\frac{e \hbar}{2 m_{e}} \\
& \text { Bohr magneton }
\end{aligned}
$$



Gyromagnetic rato


M FEBRUAR 1922 WURDE IN DIESEM GEBAUDE DES PHYSKALISCHEN VEREINS, FRANKFURT AM MAIN, YON OITO STERN UND WALTHER GERLACH DIE FUNDAMENTALE ENTDECKUNG DER RAUMOUANTISIERUNG. OER MAGNETISCHEN MOMENTE IN ATOMEN GEMACHT AUF DEM STERN GERLACH-EXPERMMENT BEPI IU GEMACHT PHYSKRLISCH TECHNISCHE ENTWICKI BERUHEN WICHTIGE WE KERNSPNRESONANZMETHODE LUNGEN DES 20 JHDTS OITO STER WURDE 1943 FUR DIESE ENT OTDECKUNA
DER NOBEIPREIS VERLEHEN

## The z-component of the angular momentum

example: d-state with
(Stern-Gerlach experiment 1922)
$\mathrm{n}=3$
$L_{z}=2 \hbar$
$L_{z}=\hbar$
$L_{z}=0$
$L_{z}=-\hbar$
$L_{z}=-2 \hbar$

$$
\vec{\mu}=\frac{\mathrm{e}}{2 \mathrm{~m}}(\mathrm{~m} \overrightarrow{\mathrm{v}} \times \overrightarrow{\mathrm{r}})=\mathrm{e} / 2 \mathrm{~m} \cdot \overrightarrow{\mathrm{~L}}
$$

$$
\text { Magnetic moment }=\text { current } x \text { area }
$$



In a magnetic field $\vec{B}=B_{z} \cdot \vec{e}$ is the magnetic energy of an electron

$$
E=\vec{\mu} \cdot \vec{B}=-\mu_{z} B_{z}=-\frac{e}{2 m} L_{z} B_{z}
$$

Is $B_{z}$ inhomogeneous ( $\partial / \partial z B_{z} \neq 0$ ), the electron feels a force proportional to $L_{z}$

$$
F_{z} \propto-L_{z} \frac{U}{\partial z} B_{z}
$$

## Stern-Gerlach Experiment

Stern and Gerlach used silver atoms ( $\mathrm{Ag}, \mathrm{Z}=47$ ) electron configuration: ${ }_{36} \mathrm{Kr}+4 \mathrm{~d}^{10}+5 \mathrm{~s}^{1}$; accordingly one valence electron in the 5 s -shell

$$
F_{z} \propto-L_{z} \partial / \partial z B_{z}
$$



Stern-Gerlach-experiment: In an inhomogeneous magnetic field a beam of silver atoms is diverted and splitted into two beam parts. The magnetic field possesses a gradient of $10 \mathrm{~T} / \mathrm{cm}$ and a length of 10 cm .

Stern and Gerlach assumed L=1 for the electron and therefore expected a splitting into three parts with

$$
m_{z}=-1,0,1
$$



Observed intensity of the silver atom beam as a function of the distance to the beam axis: with (dashed line) and without (solid line) magnetic field

Only two lines were observed !!!

existance of a quantization direction


Contrary to the expectation, an even splitting was observed

From today's point of view it is known that the assumption $\mathbf{L}=\mathbf{1}$ for the valence electron in the silver atom was wrong. The $47^{\text {th }}$ electron occupies the 5 s -shell and therefore $\mathrm{L}=0$.
Assuming this, a single spot would have been expected instead of two!
In 1925 Goudsmit, Uhlenbeck and Pauli found the solution to this problem by postulating the 'exclusion principle'

Besides the known quantum numbers n , $\mathrm{I}, \mathrm{m}$ there must be a fourth quantum number

## Stern-Gerlach Experiment: The experimental result




## DIRAC theory <br> (relativistic formulation of quantum mechanics)

Schrödinger's wave function (1926) was the first 'highlight" of the new quantum mechanics. But there was still a problem: the theory of special relativity was not included.

Hamilton-operator of a free electron according to Dirac

$$
\begin{gathered}
\mathrm{H}=\alpha \cdot \mathrm{p}+\beta m_{e} \\
\mathrm{H}|\Psi>=E| \Psi>
\end{gathered}
$$

with the operators $\alpha$ and $\beta$ ( $4 \times 4$ matrix). The corresponding eigenvalue-equation is:
with the two solutions

$$
\mathrm{E}=+\mathrm{c} \sqrt{\left(\mathrm{p}^{2}+\mathrm{m}^{2} \mathrm{c}^{2}\right)}
$$

$$
\mathrm{E}=-\mathrm{c} \sqrt{\left(\mathrm{p}^{2}+\mathrm{m}^{2} \mathrm{c}^{2}\right)}
$$

## Unexpected Antiparticles (Dirac)

1928
Since half the solutions must be rejected as referring to the charge $+e$ on the electron, the correct number will be left to account for duplexity phenomena.

## 1930

would fill it, and will thus correspond to its possessing a charge $+e$. We are therefore led to the assumption that the holes in the distribution of negativeenergy electrons are the protons. When an electron of positive energy drops into

## 1931

nearly all, of the negative-energy states for electrons are occupied. A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We should not expect to find any of

Presumably the protons will have their own negative-energy states, all of which normally are occupied, an unoccupied one appearing as an anti-proton.

## Prediction of anti-matter



Dirac, Anderson, the Positron and the anti-matter. In his famous equation Paul Dirac combined (1929) the fundamental equation of quantum mechanics, the Schrödinger-equation with the theory of special relativity. He did not discard the negative energy -solutions of his equation as unphysical but interpreted them as states of the antiparticle of the electron (positron, having the same mass but opposite charge). In 1932 Carl Anderson discovered the positron the first time in the cosmic radiation. This was the proof of the existence of 'anti-matter', with incalculable consequences for the future of physics.

## Energy spectrum of the Dirac particle

- For the free particles we found: $\quad E_{ \pm}(p)= \pm \sqrt{\left(m_{e} c^{2}\right)^{2}+(p c)^{2}}$
- Energy of positive energy particles: $\quad E_{+}(p)>m_{e} c^{2}$
- Energy of negative energy particles: $\quad E_{-}(p)<-m_{e} c^{2}$


Where is the problem here?

## Dirac sea

- In 1930 Paul Dirac have proposed a theoretical model of the vacuum as an infinite sea of particles possessing negative energy.

- Since all the states in Dirac sea are occupied "our" electron can not go down from the domain of positive energies. (Pauli principle.)


## Cosmic Rays and the Discovery of Positrons

The Nobel Prize in Physics 1936
Victor F. Hess, Carl D. Anderson

| The Nobel Prize in Physics 1936 | v |
| :---: | :---: |
| Victor F. Hess | - |
| Carl D. Anderson | $\checkmark$ |



Victor Franz Hess


Carl David Anderson

The Nobel Prize in Physics 1936 was divided equally between Victor Franz Hess "for his discovery of cosmic radiation" and Carl David Anderson "for his discovery of the positron".

## Origin of Cosmic Rays

- The Sun
(mostly low energy)
- Supernovae
- Gamma Ray Bursts


Extra-solar cosmic rays also known as Galactic Cosmic Rays, originate and are accelerated to nearly the speed of light by supernovae explosions.

- High-speed proton's and electrons, primary particles, strike Earth's upper atmosphere. Nuclear collisions produce a shower of secondary particles, called' muons; $1 t$ is mainly these muons that . are observed on Earth's surfaçe.


## Discovery of Cosmic Rays

- From 1911 to 1913 Victor Hess measured radiation levels at various altitudes (up to 4500 m ) in the Earth's atmosphere.
- Radiation levels increased with altitude!
- This radiation was called "Cosmic Radiation" later became "Cosmic Rays".
- Nobel Award in 1936.


## Discovery of Cosmic Rays

## 1084

 Hess, Durchdringende Strahlung bei sieben Freiballonfahrten. Physik. Zeitschr. XIII, 1912.wird sie aber gewi $B$ gerne übernehmen; er hat auch einige meiner früheren Blitzaufnahmen übernommen.

Aus der Abteilung für Geophysik, Meteorologie und Erdmagnetismus:
ViktorF.Hess(Wien), Über Beobachtungen der durchdringenden Strahlung bei sieben Freiballonfahrten.
ser Behandlung zeigte Apparat I eine normale Ionisation von ca. 16 Ionen, Apparat 2 eine solche von ca. II Ionen pro ccm und Sek. Die Firma Günther \& Tegetmeyer in Braunschweig hat an den Apparaten noch eine weitere wesentliche Verbesserung angebracht: bisher erfolgte die Scharfeinstellung auf die Fäden durch alleiniges Verschieben des. Okulars, was mit nicht unbeträchtlichen Änderungen der Vergrößerung verbunden war und bei wiederholter Einstellung Ablesungsdifferenzen bis zu 0,5 be- 7. Fahrt (7. August I9I2).

Ballon: „Böhmen" ( 1680 cbm Wasserstoff). Meteorolog. Beobachter: E. Wolf.

Führer: Hauptmann W. Hoffory. ${ }^{\text {- }}$ Luftelektr. Beobachter: V. F. Hess, ei

| Nr. | Zeit | Mittlere Höhe |  | Beobachtete Strahlung |  |  |  | Temp. | Relat. Feucht. Proz. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Apparat I | Apparat 2 | - Apparat 3 |  |  |  |
|  |  | m | m | $q_{1}$ | $q_{2}$ | 23 | reduz. $2_{3}$ |  |  |
| 1 | $15^{\text {h }} 15-\mathrm{rch} 15$ | 156 | $\bigcirc$ | 17,3 | 12,9 | - | - | - | - |
| 2 | 16 h 15-17h 15 | 156 | $\bigcirc$ | I5,9 | II, 0 | 18,4 | 18:4 | I $1 / 2 \mathrm{Tag}$ | dem Auf |
| 3 | $17 \mathrm{~h} 15-18 \mathrm{~h} 15$ | 156 | $\bigcirc$ | 15,8 | II, 2 | ${ }^{1} 7,5$ | ${ }^{1} 7,5$ | stiege | Wien) |
| 4 | 6h $45-7 \mathrm{~h} 45$ | 1700 | 1400 | 15,8 | 14,4 | 21,1 | 25.3 | $+6.40$ | 60 |
| 5 | $7^{\text {h }} 45-8 \mathrm{~h} 45$ | 2750 | 2500 | 173 | 12,3 | 22,5 | 31,2 | $+1,40$ | 41 |
| 6 | $8 \mathrm{~h} 45-9^{\text {h }} 45$ | 3850 | 3600 | 19,8 | 16,5 | 2I,S | 35,2 | $-6,80$ $-9,80$ | 64 40 |
| 7 | $9^{\text {h }} 45-10^{\text {h }} 4.5$ | $4800$ | [4700 | 40,7 | 31.8 | - | - | -9,80 | 40 |
| 8 | Ioh $45-1 \mathrm{I}^{\text {h }}$ I5 | 4400 | 4200 | 28, 1 | 22,7 | - | - | - | - |
| 9 | IIh I5-IIh 45 | 1300 | 1200 | (9,7) | 11,5 | - | - | 150 |  |
| 10 | Ifh 45-12h 10 | 250 | 150 | I 1,9 | 10,7 | - | - | +16,0 |  |
| II | $12 \mathrm{~h} 25-13^{\text {h }} 12$ | 140 | - | 15,0 | I 1, 6 | - | - | (nach de Pieskow, | andung in andenburg) |

## Discovery

waren nicht möglich, da der Ballon infolge der Abkühlung des Gases zum Niedergehen gezwungen wurde.

Es wurde also eine Vergrößerung der StrahIung in ca. 2000 m gefunden. Da kein Einfluß der Verfinsterung auf die durchdringende Strahlung zu bemerken war, werden wir schlieBen dürfen, da $B$ selbst, wenn ein Teil der

Strahlung kosmischen Ursprungs sein sollte, er kaum von der Sonne ausgeht, wenigstens solange man eine direkte, geradlinig sich ausbreitende $\gamma$-Strahlung im Auge hat. Diese Anschauung wird noch dadurch bekräftigt, daB ich bei den späteren Fahrten im Ballon nie einen ausgeprägten Unterschied der Strahlung bei Tag und bei Nacht gefunden habe.

## Hess determined that "essentially, the sun could not be the source of cosmic rays, at least as far as the undeflected (by the solar eclipse) rays were concerned."

## Production of Positrons in the Atmosphere

## Cascades

Development of cosmic-ray air showers

$$
\begin{array}{ll}
\pi^{+} \rightarrow \mu^{+}+\nu_{\mu} \\
\pi^{-} \rightarrow \mu^{-}+\bar{\nu}_{\mu} \\
& \\
& \\
& \begin{array}{l}
\mu^{+} \rightarrow \mathrm{e}^{+}+v_{\mathrm{e}}+v_{\mu} \\
\mu^{-} \rightarrow \mathrm{e}^{-}+v_{\mathrm{e}}+v_{\mu}
\end{array}
\end{array}
$$

Direct Production by Gammas


## Discovery of the positron (Carl David Anderson 1905 - 1991)

Detector (cloud chamber: Wilson 1910)
Cloud chamber is filled with over-saturated watersteam, which condensates along the track of an energetic, ionizing particle. In addition a strong magnetic field $B$ is applied to the cloud chamber. A charged particle will be forced on a circular, in general case an ellipsoidal track by the $B$ field which crosses the interaction plane perpendicularly.

Cloud Chamber Reveals Tracks of Charged Particles


Charged particle in magnetic field:
(momentum mv and $\mathbf{B}$ are perpendicular)

$\mathrm{B} \rho$ magnetic ridigity
detector in magnetic field: particles are moving on a circular track

## Issues to be considered by the experiment

1. What can be used as a source for positrons?
2. How is it possible to determine the signature of the charge?
(Did the particle come from the 'top' or from 'bottom'?)
3. How can the 'new' positron with electron mass $\boldsymbol{m}_{\mathrm{e}}$ and positive charge $\mathbf{q}=+\mathbf{e}$ be distinguished from a proton in case, only momentum-measurement is possible?
Solutions:
4. Cosmic radiation
5. Cloud chamber: separated by a lead-plate of 6 mm thickness, which extenuates the energy of the cosmic particle. As a result, the radius of curvature has to be smaller before passing the plate then afterwards. This gives the incoming-direction.
6. for protons and positrons with given momentum mv the range of coverage in the cloud chamber differs a lot!

## The first confirmation of a positron

Cloud chamber photograph by Andersen

- Phys. Rev. 43, 491 (1933)
- Nobel prize 1936 together with Victor Hess !

The first 'fingerprint' of anti-matter. Anderson discovers the trace of a positron in his cloud chamber (in the middle one can see a lead-plate of 6 mm thickness).

1. The upper part of the bending gives information about the incoming-direction.
2. The lower part gives the positive charge of the particle by its bending-direction.
3. By analyzing the radius of curvature before and after the transition the momentum can be estimated

## Electron-positron pair production

In order to produce electron-positron pairs we would need:
at least two times the rest mass of the elecreon !!!


+ energy of about $\hbar \omega \approx m c^{2} \approx 1 \mathrm{MeV}$
to induce pair production

In case a whole in the negative continuum excits, an electron will immediately fill the vacancy and two 511 keV quanta are emitted.

## Application in tomography



Figure 1.1. Positron emission and annihilation.

Production of a positron during $\beta+$ decay and annihilation afterwards


Positron emission tomography (PET):
$\beta+$-active $\mathbf{C -}$-, O-, or Fluor-nuclei are injected into the brain. There the local brain activity can be measured by detecting the collinear 511 keV photons of the electronpositron elimination-radiation

## PET - camera made of segmented (position sensitive) $\gamma$-detectors



## Matter $\Leftrightarrow$ Energy E=mc²

Collision processes of high-energetic particles (cm system), particle production (antiproton production)

$$
p+p=>p+p+(p+p)
$$

i) Initial

E1


$\stackrel{\mathrm{E} 2}{\stackrel{\text { target }}{ } \quad$|  (total momentum  |
| :--- |
| $\left.\mathrm{p}_{1}+\mathrm{p}_{2}=0\right)$ |$}$

ii) interaction $\infty$ total energy
E1+E2
iii) final


$$
\begin{array}{lll}
\mathrm{CM} & \stackrel{\mathrm{v}=0.9 \mathrm{c}}{\rightarrow} & \mathrm{v}=-0.9 \mathrm{c} \\
\mathrm{LAB} & \mathrm{O}_{\mathrm{V}=0.994 \mathrm{c}} \rightarrow 0
\end{array}
$$

$$
\begin{aligned}
& \mathrm{E}^{2}=\left(\mathrm{m}_{0} \mathrm{c}^{2}\right)^{2}+\mathrm{c}^{2} \mathrm{p}^{2} \\
& m \cdot c^{2}=\sqrt{E^{2}-\vec{p}^{2} \cdot c^{2}}
\end{aligned}
$$

Comparing the center of mass energy with the lab energy at these high energies,

$$
\mathrm{E}_{\mathrm{lab}}=\left(\mathrm{m}+\mathrm{m}_{0}\right) \mathrm{c}^{2}
$$

$$
\mathrm{E}_{\mathrm{cm}}^{2}-\mathrm{c}^{2} \mathrm{p}_{\mathrm{cm}}^{2}=\mathrm{E}_{\mathrm{lab}}^{2}-\mathrm{c}^{2} \mathrm{p}_{\mathrm{lab}}^{2}
$$

$$
\mathrm{E}_{\mathrm{cm}}^{2}=\mathrm{E}_{\mathrm{lab}}^{2}-\mathrm{c}^{2} \mathrm{p}_{\mathrm{lab}}^{2} ; \text { but } \mathrm{p}_{\mathrm{cm}}=0
$$

$$
\mathrm{E}_{\mathrm{cm}}^{2}=\mathrm{m}^{2} \mathrm{c}^{4}+2 \mathrm{mc}^{2} \mathrm{~m}_{0} \mathrm{c}^{2}+\mathrm{m}_{0}^{2} \mathrm{c}^{4}-\mathrm{p}_{\mathrm{lab}}^{2} \mathrm{c}^{2} ; \quad \mathrm{p}_{\mathrm{lab}} \approx \mathrm{mc}
$$

$$
\mathrm{E}_{\mathrm{cm}}^{2}=\mathrm{m}_{0} \mathrm{c}^{2}\left(2 \mathrm{mc}^{2}+\mathrm{m}_{0} \mathrm{c}^{2}\right)
$$

$$
\mathrm{E}_{\mathrm{cm}}^{2}=2 \mathrm{~m}_{0} \mathrm{c}^{2} \mathrm{mc}^{2} ; \mathrm{m} \gg \mathrm{~m}_{0}
$$

$$
\mathrm{E}_{\mathrm{cm}} \approx \sqrt{2 \mathrm{~m}_{0} \mathrm{c}^{2} \mathrm{E}_{\mathrm{lab}}}
$$

BEVALAC / Berkeley
particle production

## $\mathrm{E}_{\mathrm{cm}} \approx \sqrt{2 \mathrm{~m}_{0} \mathrm{c}^{2} \mathrm{E}_{\mathrm{lab}}}$

$$
\mathrm{E}_{\mathrm{cm}} \geq 4 m_{0} c^{2} \Rightarrow E_{c m}^{2} \approx 16 m_{0}^{2} c^{4}=2 m_{0} c^{2} E_{l a b}
$$

$$
8 m_{0} c^{2}=E_{l a b}
$$

## Discovery of the Antiproton

- Bevatron 5.6 GeV
- Just at threshold!
- Discrimination against $\pi^{-}$: measure
- Momentum
- Magnets: 1.19 GeV
- Velocity
- TOF 51 vs. 40 ns
- Cerenkov counter veto
- 60 events in 1955
- $\Delta m / m_{p} \sim 5 \%$
- O. Chamberlain, E. Segre, C. Wiegand, T. Ypsilantis, Phys. Rev. 100, 947 (1955)
- Nobelprize Chamberlain \& Segre 1959

$$
p+p=>p+p+(p+p)
$$

## Principle of Antiproton Production


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1. Consider the process of electron-positron pair production in the field of bare uranium nucleus ( $Z=92$ ). Estimate the minimal energy needed for such a process assuming that the electron is created in the ground, 1 s , state of the resulting hydrogen-like ion. Give the result in eV.
2. The (classical) velocity of the electron in the groud state of hydrogen atom and moving around the nucleus is, according to Bohr model, $\mathrm{v}=$ $0.007^{*} \mathrm{c}$ where c is the speed of light. Estimate the velocity of electron moving in the field of uranium nucleus ( $Z=92$ ).
3. Consider boron-like magnesium-ion $\mathrm{Mg}^{7+}$ (nuclear charge $\mathrm{Z}=12$, number of electrons $\mathrm{N}=5$ ) in which one of electrons is in the Rydberg state with $n=50$. Find ionization energy for the outer (Rydberg) electron.
4. Assume that you are observing radiative decay between 4d and 3p states of atomic hydrogen. How many lines in the spectrum will you observe if you place the atoms in a magnetic field?
