QCD, Chiral Symmetry and Hadrons in Matter

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Strong interaction at low and high energies

Quantum chromodynamics

QCD Lagrangian (here restricted to up and down quarks)

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \bar{q}_j \left(i\gamma_\mu \partial^\mu - \mathcal{M}_q \right) q_j + \bar{q}_j \, g \gamma_\mu A^\mu_a (\lambda^a)_{jk} \, q_k$$

with

- matter (quark) fields $q_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}$ and gauge (gluon) fields A^μ_a
- quark-gluon (and gluon-gluon) coupling constant g
  $\rightarrow \quad \alpha_s = \frac{g^2}{4\pi}$
- current quark mass (matrix)

$$\mathcal{M}_q(\mu \approx 2\,\mathrm{GeV}) \approx \begin{pmatrix} 3 & 0 \\ 0 & 6 \end{pmatrix} \mathrm{MeV}$$

Search for single quarks

- What distinguishes a quark from observable hadrons?

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- "Quark has color"(?)

Search for single quarks

- What distinguishes a quark from observable hadrons?
- "Quark has color"(?)
- But: How to measure color?

Non-trivial degrees of freedom

- fundamental degrees of freedom: quarks, i.e.
  - very light objects with
  - fractional (electric and baryonic) charge

$\hookrightarrow$ (so far) not observed as single states in nature

$\longrightarrow$ confinement

Count fundamental degrees of freedom

- for latter use: count light degrees of freedom: in total 40

quarks

$$\text{(particles+antiparticles)} \times \text{spin} \times \text{flavor} \times \text{color}$$

$$= \quad 2 \times 2 \times 2 \times 3 \quad = \quad 24$$

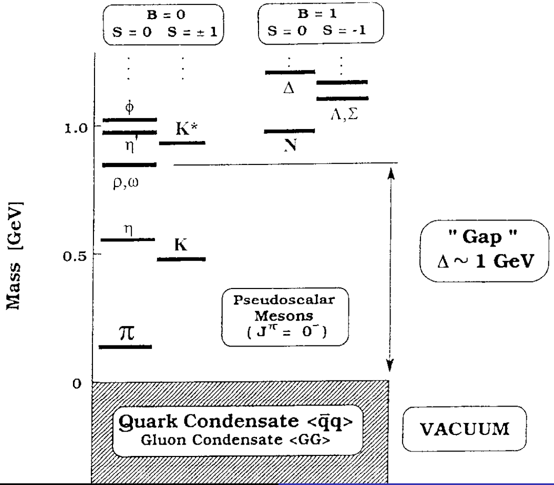
$$\text{polarization} \times \text{color} \quad = \quad 2 \times 8 \quad = \quad 16$$

gluons

Strong interaction at low energies

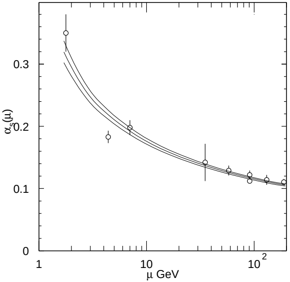
- hadronic world:
  - states with integer electric and baryonic charges
  - rather heavy states (as compared to current quark masses)
  - comparatively light pseudoscalars (as compared to other hadrons) $\longrightarrow$ pions
  - no degenerate (single particle) states with opposite parity (at least not for lightest states pions, rhos, omegas, nucleons, ...)
- for latter use: count light degrees of freedom:
  in total 3 (as compared to 40!)
  including strangeness: 8 (as compared to 52!)

Hadron spectrum



Strong interaction at high energies

small strong coupling constant (asymptotic freedom)



Particle Data Group

- use QCD perturbation theory for quantities insensitive to low energies

- note:
large coupling at low energies
$\hookrightarrow$ suggestive: cannot simply read off relevant degrees of freedom from Lagrangian
$\longrightarrow$ compatible with
(but no proof of) confinement

Symmetries of QCD

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Importance of symmetries

Which features can we expect from symmetries?

- quantum mechanical example: central potential $V(|\vec{r}|)$ (e.g. hydrogen atom)

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Translation to field theory

- conservation laws:
  - conserved charges
  - selection rules
- degeneracy:
  - states with same mass

Explicit symmetry breaking

- quantum mechanical example: central potential $V(|\vec{r}|)$ (e.g. hydrogen atom)

$\hookrightarrow$ rotational invariance

$\hookrightarrow$ conservation of angular momentum

$\hookrightarrow$ degenerate energy levels

- switch on small external magnetic field

Explicit symmetry breaking

- quantum mechanical example: central potential $V(|\vec{r}|)$ (e.g. hydrogen atom)

$\hookrightarrow$ rotational invariance

$\hookrightarrow$ conservation of angular momentum

$\hookrightarrow$ degenerate energy levels

- switch on small external magnetic field

$\hookrightarrow$ breaks rotational invariance explicitly

$\hookrightarrow$ energy levels slightly split up

$\hookrightarrow$ approximately degenerate energy levels

$\hookrightarrow$ systematics in splitting pattern

Global symmetries of QCD

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \bar{q}_{jf}\, i\gamma_\mu \partial^\mu\, q_{jf} + \bar{q}_{jf}\, g\gamma_\mu A^\mu_a (\lambda^a)_{jk}\, q_{kf} - \bar{q}_{jf}\, (\mathcal{M}_q)_{fg}\, q_{jg}$$

- baryon number conservation:
  can change phase of all quarks simultaneously
  $\rightarrow U_B(1) \rightsquigarrow$ baryons cannot decay into mesons
- approximate symmetry for $m_q \rightarrow 0$: chiral symmetry
  can mix flavors and helicities $\rightarrow SU_L(N_f) \times SU_R(N_f)$
  $\rightsquigarrow$ (approximate) flavor multiplets + parity doublets (?)
  symmetry very good for $N_f = 2$, reasonable for $N_f = 3$
- approximate symmetry for $m_q \rightarrow \infty$: center symmetry
  connected to confinement, complicated

Flavor symmetry

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{q}_f \left(i\gamma_\mu \partial^\mu + g\gamma_\mu A^\mu \right) q_f - \bar{q}_f \left(\mathcal{M}_q \right)_{fg} q_g$$

- (indices denote now flavor, not color)
- If all quark masses were the same: $\mathcal{M}_q \to m_q \mathbb{1}$

$\hookrightarrow$ $\bar{q}_f \left(\ldots \right) q_f$ does not change under transformations $SU_V(N_f)$:

$$q_f \to [\exp(i\Theta_a \tau_a)]_{fg}\, q_g \,, \qquad \bar{q}_f \to \bar{q}_g\, [\exp(-i\Theta_a \tau_a)]_{gf}$$

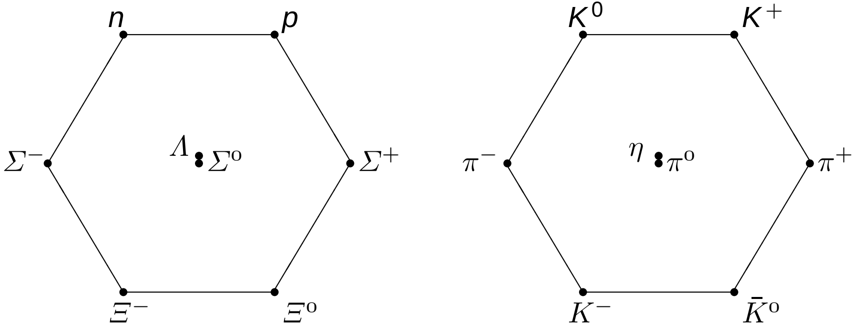
$\hookrightarrow$ isospin (flavor) conservation

$\hookrightarrow$ degenerate states (multiplets), i.e.
states with equal mass and different isospin (flavor)

- indeed for $N_f = 2$: approximately degenerate states:
  (n, p), (π^-, π^0, π^+), (K^0, K^+), $(K^-, \bar{K}^0)$, ...
- quark masses not the same, but difference small (?)
  (difference = explicit symmetry breaking)

Flavor multiplets

- for $N_f = 3$: systematic splitting pattern



- allows for predictions $\rightsquigarrow \Omega^-$

Small quark mass difference?

- $m_u \approx 3\,\text{MeV}$, $m_d \approx 6\,\text{MeV} \Rightarrow \Delta m = m_d - m_u \approx 3\,\text{MeV}$
- What is small compared to what?

Small quark mass difference?

- $m_u \approx 3\,\text{MeV}$, $m_d \approx 6\,\text{MeV} \Rightarrow \Delta m = m_d - m_u \approx 3\,\text{MeV}$
- What is small compared to what?
- $(m_d - m_u)/m_{u,d}$ not small, but $(m_u - m_d)/M_h$ small

↪ isospin multiplets do not emerge because up and down quark masses are similar on an absolute scale, but because both are very small on a hadronic scale

↪ worth to study limit of massless quarks

↪ more symmetries ahead

Chiral symmetry

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{q}_f \left(i\gamma_\mu \partial^\mu + g\gamma_\mu A^\mu \right) q_f - \bar{q}_f \left(\mathcal{M}_q \right)_{fg} q_g$$

- neglect quark mass term (and recall $\bar{q} = q^\dagger \gamma_0$)

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + q^\dagger_{fs} (\gamma_0 \gamma_\mu)_{st} \left(i\partial^\mu + gA^\mu \right) q_{ft}$$

- (indices denote now flavor and spinor)
- γ_5 commutes with combination $\gamma_0\gamma_\mu$
  (but not with γ_0 alone $\to$ mass term breaks chiral sym.)

$\hookrightarrow$ $\mathcal{L}_0$ does not change under transformations $SU_A(N_f)$:

$$q_{fs} \to \left[\exp(i\tilde{\Theta}_a \tau_a \gamma_5) \right]_{fgst} q_{gt} \,, \quad q^\dagger_{fs} \to q^\dagger_{gt} \left[\exp(-i\tilde{\Theta}_a \tau_a \gamma_5) \right]_{gfts}$$

Chiral transformations — the formal stuff

- take flavor transformations together $SU_V(N_f) \times SU_A(N_f)$

$$q \to \exp(i\Theta_a \tau_a)\, \exp(i\tilde{\Theta}_a \tau_a \gamma_5)\, q$$

- introduce left- and right-handed quarks
  $q = q_L + q_R = \frac{1}{2}(1-\gamma_5)\, q + \frac{1}{2}(1+\gamma_5)\, q$

$$q_{R,L} \to \exp(i\Theta_a \tau_a)\, \exp(\pm i\tilde{\Theta}_a \tau_a)\, q_{R,L}$$

- note: $\gamma_5\, q_{L,R} = \pm q_{L,R}$ and $(\gamma_5)^2 = 1$

Chiral transformations — the formal stuff

$$q_{R,L} \to \exp(i\Theta_a \tau_a)\, \exp(\pm i\tilde{\Theta}_a \tau_a)\, q_{R,L}$$

- 1. choose $\Theta_{Ra} := \Theta_a/2 = \tilde{\Theta}_a/2$

$$q_R \to \exp(i\Theta_{Ra}\tau_a)\, q_R \,, \qquad q_L \to q_L$$

- 2. choose $\Theta_{La} := \Theta_a/2 = -\tilde{\Theta}_a/2$

$$q_R \to q_R \,, \qquad q_L \to \exp(i\Theta_{La}\tau_a)\, q_L$$

- formally: $SU_V(N_f) \times SU_A(N_f) = SU_L(N_f) \times SU_R(N_f)$
- can perform flavor transformations separately for left- and right-handed quarks — without changing the physics (chiral symmetry)

Left- and right-handed states

- chirality: spin points in or against flight direction

- meaningful (Lorentz invariant) concept (only) for massless particles
- otherwise: boost from system slower than particle into system faster than particle
  $\longrightarrow$ characterizes system, not particle

Left- and right-handed flavored quarks

$$
\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + q^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,q \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + q_L^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,q_L + q_R^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,q_R \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + u_L^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,u_L + u_R^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,u_R \\
&\quad + d_L^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,d_L + d_R^\dagger\,\gamma_0\gamma_\mu\,(i\partial^\mu + gA^\mu)\,d_R + \dots
\end{aligned}
$$

- massless QCD contains $2 \times N_f$ identical copies of quarks

↪ consequences:

- in interactions quarks keep their chirality and their flavor
- interaction is blind to flavor and chirality
  (always the same interaction)

Chiral multiplets

- Remember: flavor symmetry, e.g. for $N_f = 2$

$\hookrightarrow$ conservation of isospin

$\hookrightarrow$ degenerate states (multiplets), i.e.
states with equal mass and different isospin

- now: chiral symmetry

Chiral multiplets

- Remember: flavor symmetry, e.g. for $N_f = 2$

$\hookrightarrow$ conservation of isospin

$\hookrightarrow$ degenerate states (multiplets), i.e. states with equal mass and different isospin

- now: chiral symmetry

$\hookrightarrow$ separate conservation of left- and right-handed isospin

$\hookrightarrow$ degenerate states (multiplets), i.e. states with equal mass and different isospin and different handedness(?)

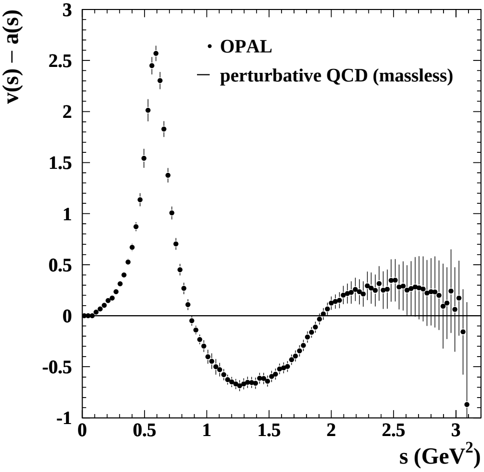
$\hookrightarrow$ weak interaction couples to $V - A$; no difference between V and A in massless QCD(?)

$\hookrightarrow$ degenerate spectra of V and A(?)

Absence of chiral multiplets

- $SU_V(N_f)$ transformations mix flavors
- $SU_A(N_f)$ transformations mix flavors and flip parity (γ_5)
- expect: (approx.) degenerate partners with opposite parity
- but: $N(940) \leftrightarrow N^*(1535)$, $\rho(770) \leftrightarrow a_1(1240)$, ...
- expect: degenerate spectra of V and A, e.g. in τ decay
- but: $\rightsquigarrow$ fig.

One of the clearest signs of chiral symmetry breaking



v: $\tau \to \nu_\tau + m\pi$ (m even)

a: $\tau \to \nu_\tau + n\pi$ (n odd)

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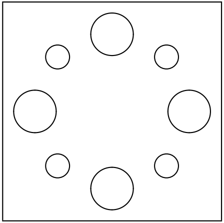
Chiral symmetry breaking (χSB)

$\longrightarrow$ experimental findings can be explained by spontaneous symmetry breaking

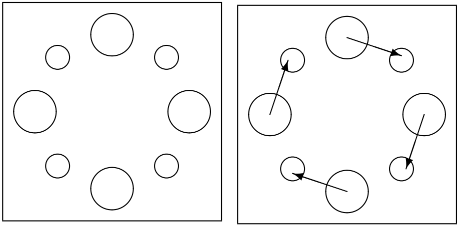
- definition: Lagrangian has symmetry which ground state (vacuum) does not have
- in the following: four levels of sophistication:
  - dinner table with salad plates
  - Heisenberg magnet
  - simple scalar field theory (exercises)
  - chiral symmetry breaking (χSB)

Dinner table with salad plates



parity invariance

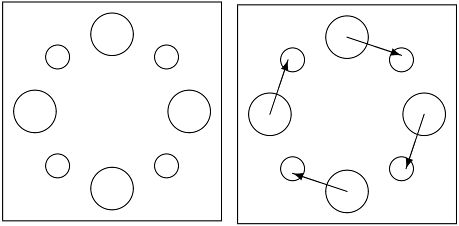
Dinner table with salad plates



parity invariance

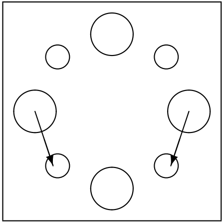
broken

Dinner table with salad plates



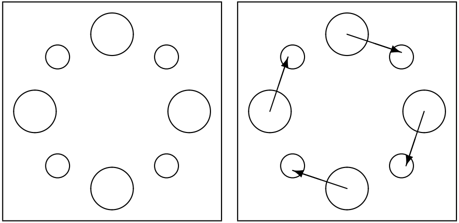
parity invariance

broken



Dinner table with salad plates

“heated” system: hungry guests, end of dinner announced



parity invariance   broken

Dinner table with salad plates

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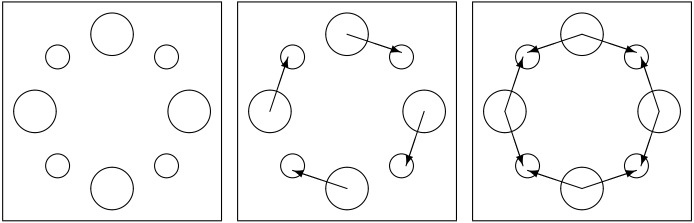


parity invariance broken restoration

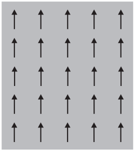
Heisenberg magnet

- interaction between microscopic magnetic dipoles (spins) does not prefer any direction

$$H_{\text{int}} = g \sum_{i \neq j} \vec{s}_i \cdot \vec{s}_j$$

$\longrightarrow$ rotational invariance

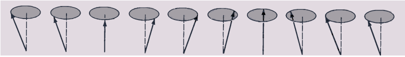
- in contrast ground state (unexcited solid state) has preferred direction



$\longrightarrow$ breaking of rotational invariance

Features of the Heisenberg magnet

- gapless excitation spectrum: spin waves



$\longrightarrow$ Goldstone bosons

- Why is it gapless?
- study (infinitely) long wavelength limit and vanishing frequency

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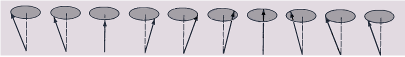
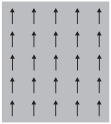
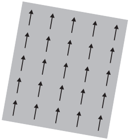


$\longrightarrow$ Goldstone bosons

- Why is it gapless?
- study (infinitely) long wavelength limit and vanishing frequency

$\hookrightarrow$ spin wave corresponds to (adiabatic) rotation of whole solid state





- does not cost energy

Features of the Heisenberg magnet II

- macroscopic magnetization $\vec{M} = \langle \vec{s}_i \rangle$
- $\vec{M}$ can be measured in the presence of an external magnetic field $\vec{B}$:

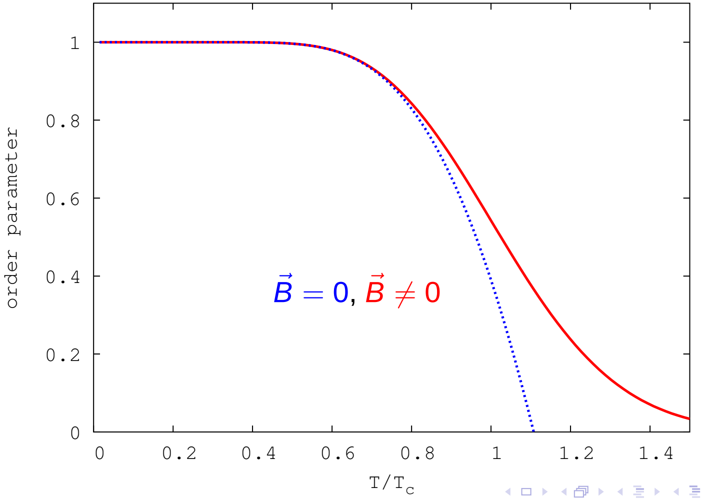
$$H_{\text{int}} = g \sum_{i \neq j} \vec{s}_i \cdot \vec{s}_j + \vec{B} \cdot \sum_i \vec{s}_i$$

- $\vec{B}$ breaks rotational invariance explicitly
- presence of $\vec{B}$: excitation spectrum no longer gapless; however, gap scales with $\vec{B}$
- if system is heated, $\vec{M}$ vanishes above a critical value

$\longrightarrow$ phase transition, symmetry restoration

$\longrightarrow$ $\vec{M}$ is order parameter

Temperature dependence of order parameter

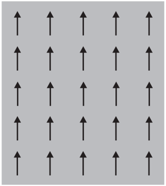


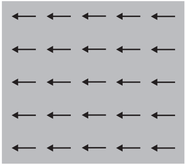
Is symmetry only hidden?

- spontaneous symmetry breaking is also called "hidden symmetry"
- Is symmetry still there?

Is symmetry only hidden?

- spontaneous symmetry breaking is also called "hidden symmetry"
- Is symmetry still there?
- rotation of solid state:
  not the same system, but same properties

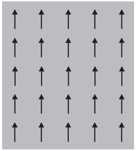




Symmetry is only hidden

- Is symmetry still there?
- symmetry suggests degenerate states?

$\hookrightarrow$ study e.g. phonon excitation:



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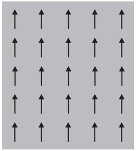

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$\hookrightarrow$ study e.g. phonon excitation:

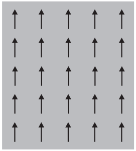
$\longrightarrow$ 1. no symmetry breaking:

- excitations in x and y direction cost same energy

$\longrightarrow$ 2. symmetry breaking:

- excitations in x and y direction do not cost the same energy

Symmetry is only hidden



- Is symmetry still there?
- symmetry suggests degenerate states?

$\hookrightarrow$ study e.g. phonon excitation:

$\longrightarrow$ 1. no symmetry breaking:

- excitations in x and y direction cost same energy

$\longrightarrow$ 2. symmetry breaking:

- excitations in x and y direction do not cost the same energy
- but: excitation in x direction costs same energy as rotation plus excitation in y direction

$\hookrightarrow$ recall: rotation = long-wavelength spin wave

Degenerate states and broken/hidden symmetry

1. no symmetry breaking:
   degeneracy at level of single excitations/particles
2. symmetry breaking: degeneracy of
   excitation and {excitation plus (soft) spin wave}

$\hookrightarrow$ approximate degeneracy in presence of explicit symmetry breaking

Translation to QCD: quark condensate

- $\vec{M} := \langle \vec{s}_i \rangle \neq 0$

$\hookrightarrow$ non-trivial expectation value with respect to ground state

$\rightsquigarrow$ vacuum expectation value

$\hookrightarrow$ for which operator?

$\longrightarrow$ recall explicit symmetry breaking term

$$H_{\rm ex} = \vec{B} \cdot \sum \vec{s}_i$$

$\rightsquigarrow$ look at term in QCD Lagrangian which breaks chiral symmetry explicitly:

$$\mathcal{L} = \mathcal{L}_0 - m_u \bar{u}u - m_d \bar{d}d - \dots$$

$\hookrightarrow$ quark condensate $\langle \bar{q}q \rangle \approx -(240\,\mathrm{MeV})^3 \times N_f$

Translation to QCD: pions

- gapless excitation spectrum $\equiv$ massless states (Goldstone bosons)
- for finite $\vec{B} \neq 0$ (explicit symmetry breaking):

$\hookrightarrow$ spin wave excitation no longer exactly gapless

$\hookrightarrow$ but gap small, scales with $\vec{B}$

$\rightsquigarrow$ for finite $m_q \neq 0$ (explicit symmetry breaking):

$\hookrightarrow$ Goldstone bosons are no longer exactly massless, but light

$\longrightarrow$ pions

$\hookrightarrow$ Gell-Mann–Oakes–Renner relation (here $N_f = 2$)

$$m_\pi^2 f_\pi^2 = -\bar{m}_q \langle \bar{q}q \rangle + o(m_q^2)$$

- note: in principle conceivable: $\langle \bar{q}q \rangle = 0$ and $m_\pi^2 \sim m_q^2$

Translation to QCD: chiral restoration

- order parameter $\langle \vec{M} \rangle$ drops with temperature

$\hookrightarrow$ restoration of rotational invariance above Curie temperature

$\hookrightarrow$ but not completely in presence of external $\vec{B}$ field

$\rightsquigarrow$ order parameter $\langle \bar{q}q \rangle$ drops with temperature

$\hookrightarrow$ chiral symmetry restoration

$\hookrightarrow$ but not completely in presence of finite current quark masses

$\longrightarrow$ might change order of phase transition ($\rightsquigarrow$ later)

Translation to QCD: degenerate states

- {excitation plus spinwave} is degenerate to excitation
- only approximate in presence of external $\vec{B}$

$\rightsquigarrow$ multiplets with different parity:

$\hookrightarrow$ state plus pion is chiral partner to state

$\hookrightarrow$ e.g. $m_N \approx m_N + m_\pi$ (soft pion)

Further remarks:

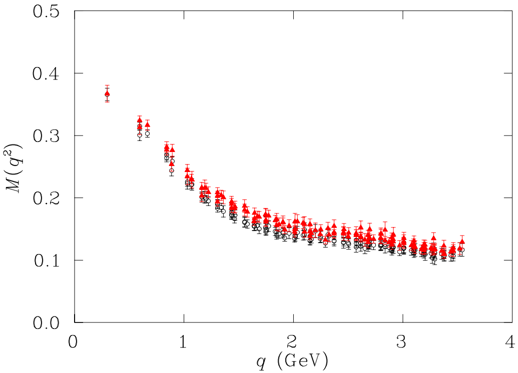
- in QCD-inspired models χSB accompanied by generation of large constituent quark masses $M \approx 300\,\mathrm{MeV} \gg m_q$
- center symmetry:

↪ order parameter: Polyakov loop $\langle L \rangle \sim e^{-E_{\mathrm{quark}}/T}$, i.e. $\langle L \rangle = 0 \Leftrightarrow$ energy of single quark $E_{\mathrm{quark}} \to \infty$ (confinement)

↪ symmetry unbroken at low and broken at high temperatures

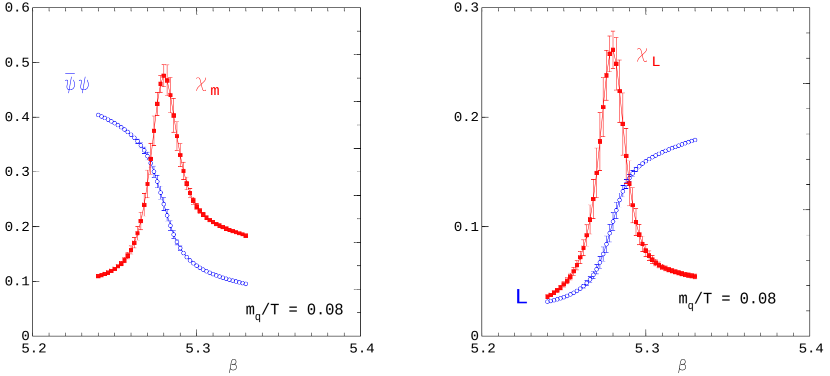
- symmetry restoration of chiral symmetry and breaking of center symmetry seems to appear at same temperature

Constituent quark mass on the lattice



P. O. Bowman, U. M. Heller and A. G. Williams, Phys. Rev. D **66**, 014505 (2002) [arXiv:hep-lat/0203001]

Temperature dependence of chiral condensate and Polyakov loop (from lattice QCD)



F. Karsch, Lect. Notes Phys. **583**, 209 (2002), hep-lat/0106019

Symmetry pattern of QCD

symmetry	$SU_V(2)$	$SU_A(2)$	center symmetry
vacuum	unbroken	broken	unbroken
high temperature	unbroken	unbroken	broken
multiplets	$(n,p),\dots$	$(N,\{N,\pi\}),\dots$	—
order parameter	—	$\langle\bar{q}q\rangle$	$\langle L\rangle$

Local color symmetry

- so far: global symmetries considered
- so far: color (and gluons) did not play any role

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu}(x) F^{\mu\nu}_a(x) + \bar{q}_j(x) \left(i\gamma_\mu [D^\mu(x)]_{jk} - \mathcal{M}_q \delta_{jk} \right) q_k(x)$$

- with $D^\mu(x) = \partial^\mu_x - igA^\mu_a(x)\lambda^a$, $F^{\mu\nu}(x) = \frac{i}{g}[D^\mu(x), D^\nu(x)]$
- (indices denote color again, not flavor or spinor)
- Lagrangian invariant with respect to local transformations in color space, $U(x) := \exp(i\Theta_a(x)\lambda_a) \in SU_c(3)$

$$q_j(x) \to [U(x)]_{jk} q_g(x) \,, \qquad \bar{q}_j(x) \to \bar{q}_k(x)[U^{-1}(x)]_{kj} \,,$$

$$[A^\mu(x)]_{jm} := A^\mu_a(x)\,[\lambda^a]_{jm} \to [U(x)]_{jk} \left[A^\mu(x) + \frac{i}{g}\partial^\mu_x \right]_{kl} [U^{-1}(x)]_{lm}$$

Consequences of local color symmetry

- only objects which are invariant under local (gauge) transformations are observable

$\hookrightarrow$ color white states

$\longrightarrow$ natural explanation for appearance of quark-antiquark and three-quark states

$\hookrightarrow$ indeed: $\bar q_{jfs} q_{jgt} \to \bar q_{jfs} U^{-1}_{jk} U_{kl} q_{lgt} = \bar q_{jfs} q_{jgt} \rightsquigarrow$ white

$$\epsilon_{jkl}\, q_{jfs}\, q_{kgt}\, q_{lhu} \to \ldots = \underbrace{\det U}_{=1}\, \epsilon_{jkl}\, q_{jfs}\, q_{kgt}\, q_{lhu} \rightsquigarrow \text{white}$$

- confinement: Why can one not construct a white state from a single quark and infinitely many gluons?

$\hookrightarrow$ at least natural: such a state should be heavy

Consequences of local color symmetry

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{q}\, (i\gamma_\mu D^\mu - \mathcal{M}_q)\, q$$

- coupling constant g appears in $D^\mu = \partial^\mu - igA^\mu$ and in $F^{\mu\nu} = \frac{i}{g}[D^\mu, D^\nu]$ ($\rightsquigarrow$ gluon-gluon coupling)

$\hookrightarrow$ gauge invariance holds only if same g appears in both expressions

$\hookrightarrow$ {quark type a}-gluon coupling = gluon-gluon coupling = = {quark type b}-gluon coupling

$\hookrightarrow$ only one universal coupling constant

$\hookrightarrow$ only few parameters: one coupling, few quark masses

$\longrightarrow$ high predictive power

Note on electric charges

- only one universal coupling constant in QCD
- in principle different in QED: proton charge could be distinct from positron charge (no photon-photon coupling)

$\hookrightarrow$ universal coupling is property of non-abelian gauge theories

$\hookrightarrow$ grand unified theories use non-abelian gauge groups to explain agreement between proton and positron charge

Full chiral group

- so far I have cheated!

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + q^\dagger_{fs} (\gamma_0 \gamma_\mu)_{st} \, (i\partial^\mu + gA^\mu) \, q_{ft}$$

- $\mathcal{L}_0$ not only invariant under $q_{fs} \to \exp(i\Theta)\, q_{fs}$,

$$q_{fs} \to [\exp(i\Theta_a \tau_a)]_{fg} \, q_{gs} \quad \text{and} \quad q_{fs} \to \left[\exp(i\tilde{\Theta}_a \tau_a \gamma_5)\right]_{fgst} q_{gt}$$

Full chiral group

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$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + q^\dagger_{fs} (\gamma_0 \gamma_\mu)_{st} \left(i\partial^\mu + g A^\mu \right) q_{ft}$$

- $\mathcal{L}_0$ not only invariant under $q_{fs} \to \exp(i\Theta)\, q_{fs}$,

$$q_{fs} \to [\exp(i\Theta_a \tau_a)]_{fg}\, q_{gs} \quad \text{and} \quad q_{fs} \to \left[\exp(i\tilde{\Theta}_a \tau_a \gamma_5) \right]_{fgst} q_{gt}$$

- but also $q_{fs} \to \left[\exp(i\tilde{\Theta}\gamma_5) \right]_{st} q_{ft}$
- the latter mixes only handedness without mixing flavors
- full chiral group:

$\hookrightarrow$ $U_V(N_f) \times U_A(N_f) = U_B(1) \times SU_V(N_f) \times SU_A(N_f) \times U_A(1)$

Status of axial symmetry

- symmetry with respect to axial transformations might be
  - fully realized

  $\hookrightarrow$ parity partners?

  $\hookrightarrow$ no
  - hidden, i.e. spontaneously broken

  $\hookrightarrow$ flavor singlet Goldstone boson?

  $\hookrightarrow$ $N_f = 2$: no isoscalar pion (η too heavy)

  $\hookrightarrow$ $N_f = 3$: no light flavor singlet (η' too heavy)

Status of axial symmetry

- symmetry with respect to axial transformations might be
  - fully realized
  - $\hookrightarrow$ parity partners?
  - $\hookrightarrow$ no
  - hidden, i.e. spontaneously broken
  - $\hookrightarrow$ flavor singlet Goldstone boson?
  - $\hookrightarrow$ $N_f = 2$: no isoscalar pion (η too heavy)
  - $\hookrightarrow$ $N_f = 3$: no light flavor singlet (η' too heavy)
- solution: axial symmetry is a symmetry of chromodynamics

$\longrightarrow$ but not of quantum chromodynamics

$\hookrightarrow$ anomaly

What is an anomaly?

- definition:
  - classical system has symmetry
  - symmetry spoiled by quantization
  - quantum system does not have symmetry any more
- illustrative example from (quantum) mechanics:
  Suppose we know all solutions of equation of motion

$$\frac{d^2}{dt^2}x(t) = -k_1 x(t) - \lambda_1 [x(t)]^3$$

Mechanical system

- Suppose we know all solutions of equation of motion

$$\frac{d^2}{dt^2}x(t) = -k_1 x(t) - \lambda_1 [x(t)]^3 \tag{1}$$

- consider in addition:

$$\frac{d^2}{dt^2}y(t) = -k_2 y(t) - \lambda_2 [y(t)]^3 \tag{2}$$

- How to find solution for (2)?

Mechanical system

- Suppose we know all solutions of equation of motion

$$\frac{d^2}{dt^2}x(t) = -k_1 x(t) - \lambda_1 [x(t)]^3 \tag{1}$$

- consider in addition:

$$\frac{d^2}{dt^2}y(t) = -k_2 y(t) - \lambda_2 [y(t)]^3 \tag{2}$$

- How to find solution for (2)?
- rescaling: solve (1) and take $y(t) = \alpha x(\beta t)$

$$\frac{d^2}{dt^2}y(t) = -\beta^2 k_1 y(t) - \frac{\lambda_1 \beta^2}{\alpha^2}[y(t)]^3 \tag{3}$$

- choose $\beta^2 = \frac{k_2}{k_1}$, $\frac{\beta^2}{\alpha^2} = \frac{\lambda_2}{\lambda_1}$

Mechanical system

- present symmetry: scale invariance
- every equation of type

$$\frac{d^2}{dt^2}x(t) = -kx(t) - \lambda[x(t)]^3$$

can be rescaled

$$y(t) = \sqrt{\frac{\lambda}{k}}\,x(t/\sqrt{k})$$

and transformed into

$$\frac{d^2}{dt^2}y(t) = -y(t) - [y(t)]^3$$

Quantum mechanical system

- present symmetry: scale invariance

$$y(t) = \sqrt{\frac{\lambda}{k}}\, x(t/\sqrt{k})$$

- introduce momenta: $p = m\dot{x}$, $q = m\dot{y} = (\sqrt{\lambda}/k)\, p$
- quantum mechanics: uncertainty relation

$$\Delta x \, \Delta p \approx \hbar$$

Quantum mechanical system

- present symmetry: scale invariance

$$y(t) = \sqrt{\frac{\lambda}{k}}\, x(t/\sqrt{k})$$

- introduce momenta: $p = m\dot{x}$, $q = m\dot{y} = (\sqrt{\lambda}/k)\, p$
- quantum mechanics: uncertainty relation

$$\Delta x\, \Delta p \approx \hbar$$

- but:

$$\Delta y\, \Delta q \approx \frac{\lambda}{\sqrt{k}^3}\, \hbar \neq \hbar$$

- describes different world
- quantization condition breaks scale invariance

Symmetry status of full chiral group

- $U_L(N_f) \times U_R(N_f) = U_B(1) \times SU_V(N_f) \times SU_A(N_f) \times U_A(1)$
- $U_B(1)$ realized, baryon number conservation
- $SU_V(N_f)$ realized, flavor conservation, multiplets
- $SU_A(N_f)$ hidden/spontaneously broken,
  $N_f^2 - 1$ Goldstone bosons: pions (kaons, η)
- $U_A(1)$ not realized at quantum level, anomaly $\rightsquigarrow \eta'$ heavy

Systematic approaches to QCD

Systematic approaches

- perturbative QCD:
  approximate full QCD by Taylor expansion in α_s
  $\hookrightarrow$ applicable for high energies only
- lattice QCD:
  approximate full continuum QCD by (full!) QCD on a grid
  $\hookrightarrow$ finite grid point distance $a \neq 0$ and finite volume $V \neq \infty$
- chiral perturbation theory:
  approximate full QCD by effective theory of degrees of freedom relevant at low energies
  $\hookrightarrow$ pions (and nucleons)

QCD perturbation theory

- free quarks and gluons as basic objects

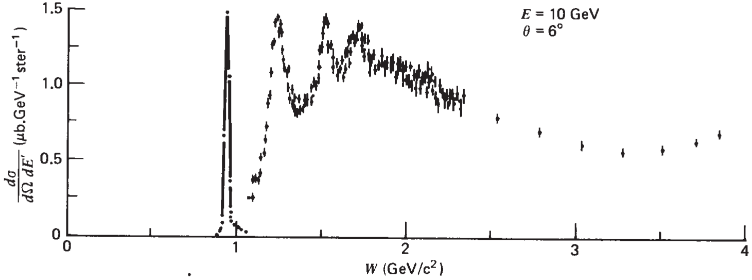
$\hookrightarrow$ expansion in terms of α_s

$\hookrightarrow$ i.e. approximate full QCD by Taylor expansion

$\rightsquigarrow$ applicable for high energies only $\rightarrow$ fig.

- in contrast: QED applicable in entire (nowadays) accessible energy regime
  - coupling constant much smaller
  - running much weaker
  - coupling grows with energy

Electron-nucleon scattering



QCD on a grid

- approximate full continuum QCD by (full!) QCD on a grid, i.e. with finite grid point distance $a \neq 0$ and finite volume $V \neq \infty$

$\hookrightarrow$ numerically expensive

$\hookrightarrow$ e.g. calculations for different number of grid points

$\hookrightarrow$ study scaling behavior (surface versus volume effects)

- lattice QCD not restricted to small coupling

$\rightsquigarrow$ not restricted to high or low energies

$\hookrightarrow$ but: restricted to static qantities

Lattice QCD

- calculate multi-dimensional integral

$$\int \mathcal{D}[A_\mu, \bar{q}, q]\, \mathcal{O}\, \frac{1}{N} \exp\left(-S^{\mathrm{E}}_{\mathrm{QCD}}\right)$$

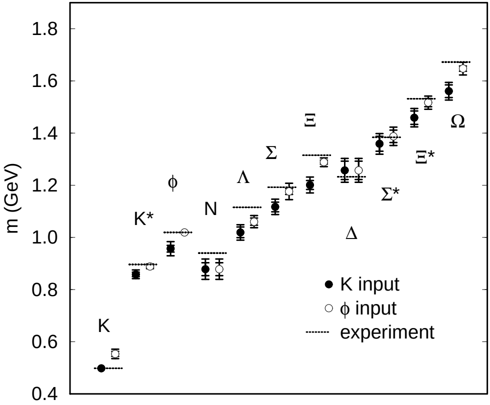
with

$$N = \int \mathcal{D}[A_\mu, \bar{q}, q] \exp\left(-S^{\mathrm{E}}_{\mathrm{QCD}}\right)$$

by Monte-Carlo algorithm

- necessary: positive weight function $\widehat{=}$ probability for throwing a dice

Hadron masses from lattice QCD



S. Aoki *et al.* [CP-PACS Collaboration], Phys. Rev. D **67**, 034503 (2003) [arXiv:hep-lat/0206009]

Problems for lattice QCD

- light states: large Compton wave length does not fit on finite grid (oversimplified)

$\longrightarrow$ in practice: put in too heavy current quark masses

$\longrightarrow$ get out pions which are too heavy (χSB)

- finite chemical potential μ:
  e.g. surplus of baryons over antibaryons

$\longrightarrow$ nuclear matter

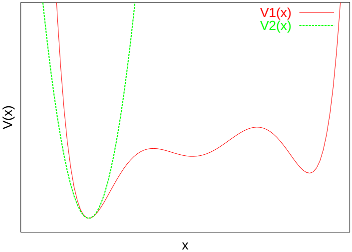
$\hookrightarrow$ reason: for $\mu \neq 0$ no positively definite weight function for Monte-Carlo algorithm

QCD at low energies

- idea of an effective field theory:
  - use (only) relevant degrees of freedom
  - write down all possible interactions
  - no expansion in powers of coupling constants
  - instead expansion in powers of energy/momenta
- chiral perturbation theory: approximate full QCD by effective theory of interacting Goldstone bosons
  - chiral symmetry reduces free parameters
  - Taylor expansion in terms of energies and masses
  - applicable for low energies where non-Goldstone bosons are not excited
  - nucleons can be included, other states complicated

Effective theory

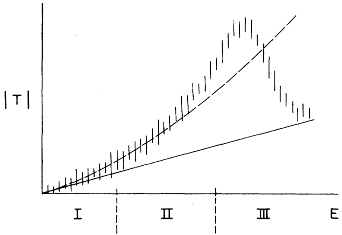
- consider particle P in complicated potential V_1 but with small energy



- complicated potential V_1 and quadratic potential V_2 are in general completely different but effectively agree for particle P
- note: even if we do not know anything about V_1 we can determine V_2 with a few measurements

Effective field theory

- want to describe resonant scattering $\longrightarrow$



- consider Lagrangian ($M^2 \gg m^2$)

$$\mathcal{L}[\phi,\chi] = -\frac{1}{2}\chi\,(\Box + M^2)\chi - \frac{1}{2}\phi\,(\Box + m^2)\phi - \frac{1}{2}g\chi\phi^2$$

Integrating out degrees of freedom

$$\mathcal{L}[\phi,\chi] = -\frac{1}{2}\chi(\Box + M^2)\chi - \frac{1}{2}\phi(\Box + m^2)\phi - \frac{1}{2}g\chi\phi^2 =$$

$$-\frac{1}{2}\left(\chi + \frac{1}{2}g\phi^2\frac{1}{\Box + M^2}\right)(\Box + M^2)\left(\chi + \frac{1}{2}g\frac{1}{\Box + M^2}\phi^2\right)$$

$$-\frac{1}{2}\phi(\Box + m^2)\phi + \frac{1}{8}g^2\phi^2\frac{1}{\Box + M^2}\phi^2$$

at low energies:

$$\mathcal{L} \to -\frac{1}{2}\phi(\Box + m^2)\phi + \frac{g^2}{8M^2}\phi^2\sum_{n=0}^{\infty}\left(\frac{-\Box}{M^2}\right)^n\phi^2$$

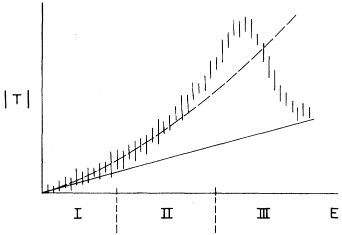
Effective Lagrangians

- at very low energies (region I):

$$\mathcal{L}_{\rm int} \cong \frac{g^2}{8M^2}\phi^4 =: \mathcal{L}_{\rm eff}^{(1)}$$

- at low energies (region II):

$$\mathcal{L}_{\rm int} \cong \mathcal{L}_{\rm eff}^{(1)} - \frac{g^2}{8M^4}\phi^2\Box\phi^2 =: \mathcal{L}_{\rm eff}^{(2)}$$



$\hookrightarrow$ can construct effective Lagrangians

$\hookrightarrow$ look rather different from $\mathcal{L}$

$\hookrightarrow$ but contain same physics

$\hookrightarrow$ in specific kinematic regions

Effective Lagrangians II

$$\mathcal{L}_{\text{eff}}^{(1)} = \frac{g^2}{8M^2}\phi^4$$

$$\mathcal{L}_{\text{eff}}^{(2)} = \mathcal{L}_{\text{eff}}^{(1)} - \frac{g^2}{8M^4}\phi^2\Box\phi^2$$

- if we don't know $\mathcal{L}$ exactly:

$\hookrightarrow$ write more general $\mathcal{L}_{\text{eff}}^{(2)} = c_1\phi^4 + c_2\phi^2\Box\phi^2$

$\hookrightarrow$ determine c_1, c_2 by measurement

$\hookrightarrow$ determine g, M from c_1, c_2

$\hookrightarrow$ i.e. from low energy scattering

$\hookrightarrow$ cf. electroweak theory and Fermi's theory

Problems

1. finite convergence radius: $p^2 < M^2$
   $\hookrightarrow$ cannot get resonance with low energy expression
2. in our example

$$\mathcal{L}^{(1)}_{\text{eff}} = \frac{g^2}{8M^2}\phi^4 \,, \qquad \mathcal{L}^{(2)}_{\text{eff}} = \mathcal{L}^{(1)}_{\text{eff}} - \frac{g^2}{8M^4}\phi^2 \Box \phi^2$$

if, however, nothing is known about $\mathcal{L}$

$\hookrightarrow$ write down most general form for $\mathcal{L}^{(1)}_{\text{eff}}$, $\mathcal{L}^{(2)}_{\text{eff}}$:

$$\mathcal{L}^{(1)}_{\text{eff}} = f_0(\phi^2)$$

$$\mathcal{L}^{(2)}_{\text{eff}} = \mathcal{L}^{(1)}_{\text{eff}} + f_1(\phi^2)\left[\partial_\mu f_2(\phi^2)\right]\left[\partial^\mu f_3(\phi^2)\right]$$

with f_i arbitrary functionals $\rightarrow$ less predictive

A hint for importance of symmetries

- most general form for $\mathcal{L}_{\text{eff}}^{(1)}$, $\mathcal{L}_{\text{eff}}^{(2)}$:

$$\mathcal{L}_{\text{eff}}^{(1)} = f_0(\phi^2)$$

$$\mathcal{L}_{\text{eff}}^{(2)} = \mathcal{L}_{\text{eff}}^{(1)} + f_1(\phi^2) \left[\partial_\mu f_2(\phi^2)\right] \left[\partial^\mu f_3(\phi^2)\right]$$

- why not ϕ^3?

↪ original Lagrangian symmetric $\phi \to -\phi$

↪ symmetries help (a lot) to restrict form of $\mathcal{L}_{\text{eff}}$

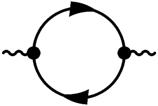
↪ another example: photon-photon scattering at low energies

Photon-photon interaction at low energies

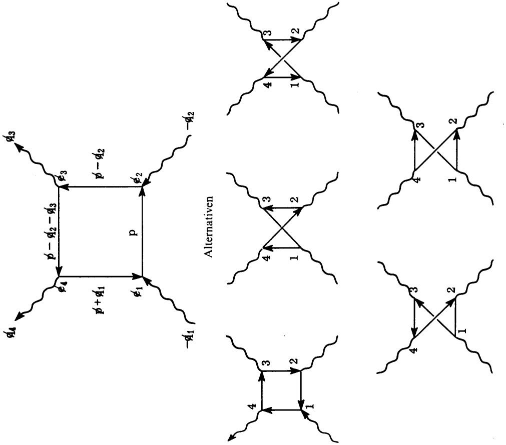
- for $p^2 \ll m_e^2$:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} Z_\gamma F_{\mu\nu} F^{\mu\nu} + a\, F_{\mu\nu} \Box F^{\mu\nu} + b\, (\partial^\mu F_{\mu\nu})(\partial_\alpha F^{\alpha\nu}) + c\, (F_{\mu\nu} F^{\mu\nu})^2 + d\, F_{\mu\nu} F^{\nu\alpha} F_{\alpha\beta} F^{\beta\mu}$$

- correct up to corrections $o(\partial^6)$, i.e. $o((p^2/m_e^2)^3)$
- note: coefficients pure numbers here, not functionals
- calculation of low-energy coefficients:

1. $Z_\gamma - 1$, a, b from  $+ \, o(\alpha^2)$
2. c, d from ▷ fig. $+ \, o(\alpha^3)$

Photon-photon scattering in QED



Use of effective theories

- if one does not know $\mathcal{L}_{\mathrm{QED}}$ or is too lazy/busy/... to calculate diagrams

$\hookrightarrow$ determine coefficients by a few measurements

$\hookrightarrow$ predictive power at low energies

$\longrightarrow$ two possibilities of application:

1. microscopic theory known: often more efficient:
   - calculate only low-energy coefficients in microscopic theory
   - calculate e.g. scattering in effective theory
2. microscopic theory unknown:
   - determine low-energy coefficients by a few measurements
   - predictions for other experiments possible

Chiral perturbation theory

- QCD Lagrangian coupled to external currents:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_0 + \bar{q}\left(-s + i\gamma_5 p + \gamma_\mu v^\mu + \gamma_\mu \gamma_5 a^\mu\right) q$$

- note: current quark masses incorporated in s
- at low energies only Goldstone bosons Φ

$\hookrightarrow$ $\mathcal{L}_{\rm eff}$ functional of Φ
and of external currents s, p, v_μ, a_μ (flavor matrices)

- recall chiral circle

$$U = \exp(i\Phi/F_\pi)$$

and introduce $\chi := 2B_0(s + ip)$, $v_\mu^{L,R} := v_\mu \pm a_\mu$ and corresponding covariant derivative ∇_μ and field strength $F_{\mu\nu}^{L,R}$

Lowest orders of chiral perturbation theory

- formulate effective Lagrangians
- sizes of external currents treated as
  $\chi \sim m_q \sim M_\pi^2 = o(\partial^2)$, $v_\mu^{L,R} \sim o(\partial)$
- e.g. satisfied up to corrections $o(\partial^6)$

Lowest orders of chiral perturbation theory

- formulate effective Lagrangians
- sizes of external currents treated as $\chi \sim m_q \sim M_\pi^2 = o(\partial^2)$, $v_\mu^{L,R} \sim o(\partial)$
- e.g. satisfied up to corrections $o(\partial^6)$

$\hookrightarrow$ $\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_1 + \mathcal{L}_2$ with

$$\mathcal{L}_1 = \frac{1}{4} F_\pi{}^2 \, \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger) = O(\partial^2)$$

and already rather lengthy $\mathcal{L}_2 = O(\partial^4)$:

$$
\begin{aligned}
\mathcal{L}_2 \; = \; & L_1[\mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U)]^2 + L_2 \mathrm{tr}(\nabla_\mu U^\dagger \nabla_\nu U)\mathrm{tr}(\nabla^\mu U^\dagger \nabla^\nu U) \\
& + L_3 \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U \nabla_\nu U^\dagger \nabla^\nu U) \\
& + L_4 \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U)\mathrm{tr}(\chi^\dagger U + \chi U^\dagger) \\
& + L_5 \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U\,(\chi^\dagger U + \chi U^\dagger)) \\
& + L_6[\mathrm{tr}(\chi^\dagger U + \chi U^\dagger)]^2 + L_7[\mathrm{tr}(\chi^\dagger U - \chi U^\dagger)]^2 \\
& + L_8 \mathrm{tr}(\chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger) \\
& - i L_9 \mathrm{tr}(F^{\mathrm{R}}_{\mu\nu} \nabla^\mu U \nabla^\nu U^\dagger + F^{\mathrm{L}}_{\mu\nu} \nabla^\mu U^\dagger \nabla^\nu U) \\
& + L_{10} \mathrm{tr}(U^\dagger F^{\mathrm{R}}_{\mu\nu} U F^{\mathrm{L}\mu\nu})
\end{aligned}
$$

Low-energy constants

- "chiral coefficients" to be determined experimentally (or by a model or by lattice QCD):
  - $O(\partial^2)$: only two: F_π, B_0
  - $O(\partial^4)$: already ten: L_{1-10}
  - $O(\partial^6)$: ≈ 100
- note: $O(\partial^6)$ irrelevant at low enough energies

$\hookrightarrow$ predictive at low energies

$\hookrightarrow$ uneconomical at higher energies

$\hookrightarrow$ breakdown in resonance region

Analyze lowest order Lagrangian (for three flavors)

- free part of $\mathcal{L}_1 = \frac{1}{4} F_\pi{}^2 \, \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger)$:

↪ expand $U = \exp(i\Phi/F_\pi)$ up to second order in the fields with

$$\Phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

↪ switch off external currents (ignore up-down mass diff.)

$$\chi \to 2B_0 \mathcal{M}_q \approx 2B_0 \begin{pmatrix} m_q & 0 & 0 \\ 0 & m_q & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Thus: $\mathcal{L}_1 \quad \to \quad \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + \partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta$

$$+\,\partial_\mu K^+\partial^\mu K^- + \partial_\mu K^0\partial^\mu \bar{K}^0$$

$$-\,B_0 m_q\,(\pi^{0\,2} + \frac{1}{3}\,\eta^2 + 2\,\pi^+\pi^- + K^+K^- + K^0\bar{K}^0)$$

$$-\,B_0 m_s\,(K^+K^- + K^0\bar{K}^0 + \frac{2}{3}\,\eta^2)$$

$$\hookrightarrow \qquad \begin{aligned} m_\pi^2 &= B_0\,2m_q \\ m_K^2 &= B_0\,(m_q + m_s) \\ m_\eta^2 &= B_0\,(\frac{2}{3}m_q + \frac{4}{3}m_s) \end{aligned}$$

i.e. $B_0 = -\langle\bar{u}u\rangle/{F_\pi}^2$

$$m_\pi^2 = B_0 \, 2 m_q$$
$$m_K^2 = B_0 \, (m_q + m_s)$$
$$m_\eta^2 = B_0 \, (\frac{2}{3} m_q + \frac{4}{3} m_s)$$

$\hookrightarrow$ predictions for quark mass ratio and for η mass (without knowledge of B_0):

$$\frac{m_s}{m_q} = 2 \frac{m_K^2}{m_\pi^2} - 1 \approx 23$$

$$m_\eta^2 = \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 \qquad \Rightarrow \qquad m_\eta \approx 560 \, \text{MeV}$$

(Experiment $m_\eta \approx 550\,\text{MeV}$)

Analyze lowest order Lagrangian

- e.g. pion decay and pion-pion interactions contained in

$$\mathcal{L}_1 = \frac{1}{4} F_\pi{}^2 \, \mathrm{tr}(\nabla_\mu U^\dagger \nabla^\mu U + \chi^\dagger U + \chi U^\dagger)$$

- pion decay (without too much details):

$\hookrightarrow$ expand $U = \exp(i\Phi/F_\pi)$ up to linear order in the fields

$\hookrightarrow$ switch off external currents except for left-handed external field (W boson) contained in $\nabla_\nu \sim \partial_\nu - v^L_\nu = \partial_\nu - v_\nu + a_\nu$

$\hookrightarrow$ ignore quark masses: $\chi \to 0$

$\to$ $\mathcal{L}_1 \sim F_\pi \, \pi \, (\partial^\nu a_\nu)$

$\hookrightarrow$ pion decays with strength F_π

$\hookrightarrow$ “pion decay constant”

Analyze lowest order Lagrangian

- pion-pion interactions:

↪ expand $U = \exp(i\Phi/F_\pi)$ up to fourth order in the fields

↪ switch off external currents and ignore quark masses

→ $\mathcal{L}_1 \sim \frac{1}{F_\pi{}^2} [\pi, \partial_\mu \pi] [\pi, \partial^\mu \pi]$

↪ same strength determines pion decay and (lowest-order) pion-pion interaction

↪ "Weinberg-Tomozawa interaction"

↪ exists also for interaction of Goldstone bosons with arbitrary hadron

↪ strength still given by pion decay constant

Systematic approaches — features and shortcomings

- lattice QCD can be systematically improved by smaller grid distance, smaller quark masses, ...

$\hookrightarrow$ but: numerically expensive

- QCD perturbation theory can be systematically improved by calculating next order in α_s

$\hookrightarrow$ but: more and more diagrams

- chiral perturbation theory can be systematically improved by calculating next order in energies/momenta

$\hookrightarrow$ but: more and more possible interactions/diagrams

Why we need models

- lattice QCD not applicable for dynamical processes (scattering)
- QCD perturbation theory only applicable above resonance region
- chiral perturbation theory breaks down in resonance region

$\longrightarrow$ need hadronic models

$\hookrightarrow$ symmetries help again to reduce possible interactions, correlate masses, coupling constants, ...

$\hookrightarrow$ but: models unsystematic (no systematic way to improve them)

Phases of QCD and order parameters

Strongly interacting matter

with rising temperature/density

- hadrons overlap

$\hookrightarrow$ hard to tell which quark belongs to which hadron

- complementary picture: hadrons interact more and more

$\hookrightarrow$ movability of single quarks rises

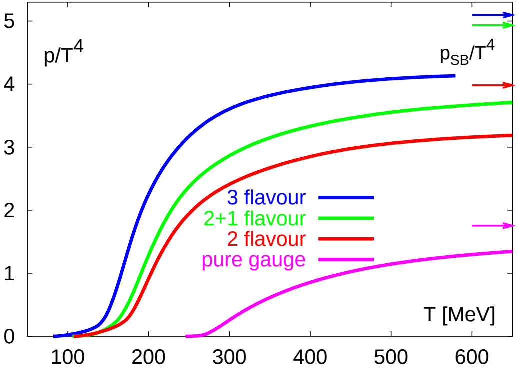
$\longrightarrow$ quarks (and gluons) become relevant degrees of freedom (d.o.f.)

$\hookrightarrow$ complete change of properties (3 versus 40 d.o.f.)

$\longrightarrow$ (phase) transition

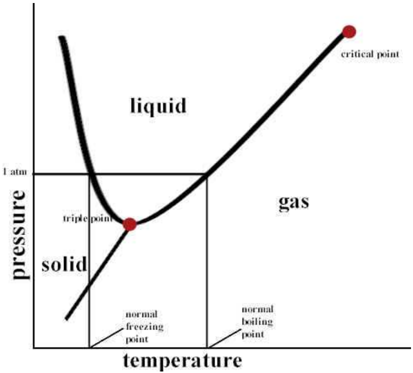
reasonable: expect to see precursors of this transition already in hadronic medium

Pressure from lattice QCD



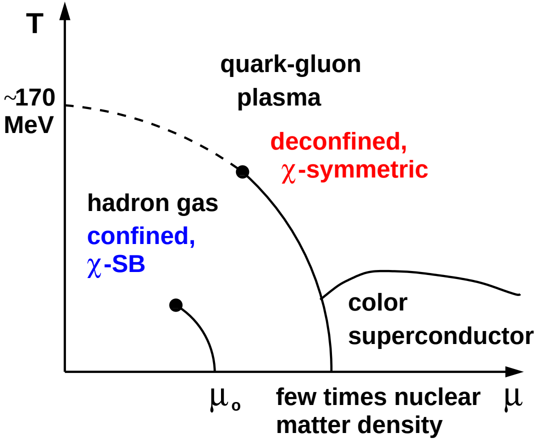
F. Karsch, Lect. Notes Phys. **583**, 209 (2002), hep-lat/0106019

Phase diagram of water



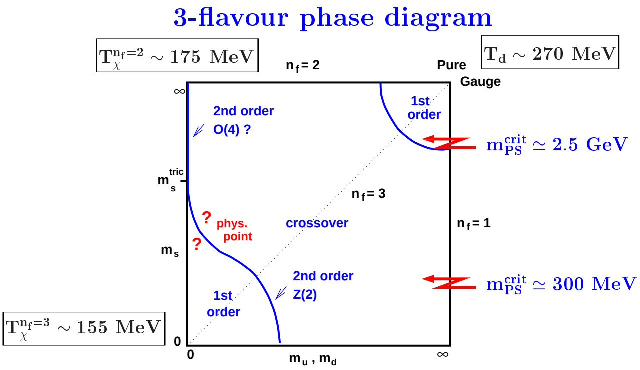
wikipedia

Sketch of phase diagram



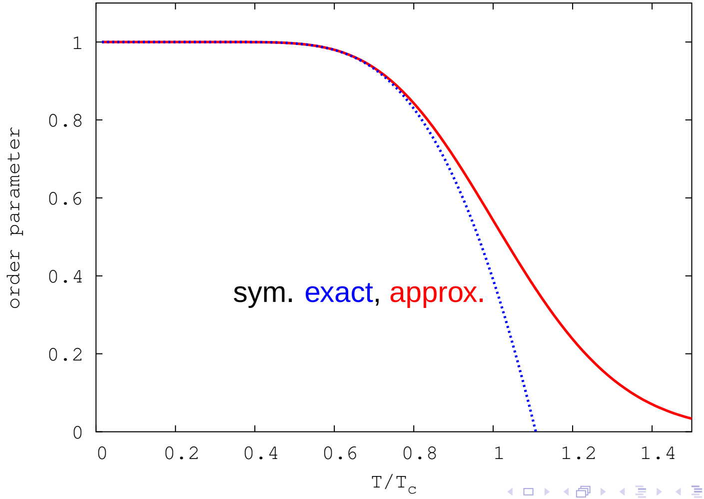
F. Karsch,
hep-lat/0601013

Order of phase transition — mass dependence

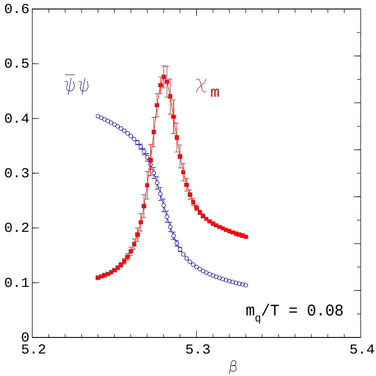


F. Karsch, Lect. Notes Phys. **583**, 209 (2002), hep-lat/0106019

Temperature dependence of order parameter



Two-quark condensate from lattice QCD



F. Karsch, Lect. Notes Phys. **583**, 209 (2002), hep-lat/0106019

Quark condensate in a hadronic medium

Why the lattice results are not enough:

- finite temperatures: pions most abundant (lightest states)

$\hookrightarrow$ recall: pions too heavy on the lattice

- low baryonic densities

$\hookrightarrow$ recall: finite chemical potential on the lattice in infant stadium

$\leadsto$ need hadronic theories/models (e.g. χPT)

- low temperatures: pion gas
- low baryonic densities: Fermi gas of nucleons
- higher temperatures/densities: models!

Quark condensate at low temperatures

- in general:

$$\langle\Omega|\,\bar{u}u\,|\Omega\rangle = -\frac{T}{V}\,\frac{d\ln Z}{dm_u} = -\frac{dP}{dm_u}$$

- low temperatures $\hat{=}$ low pion densities:

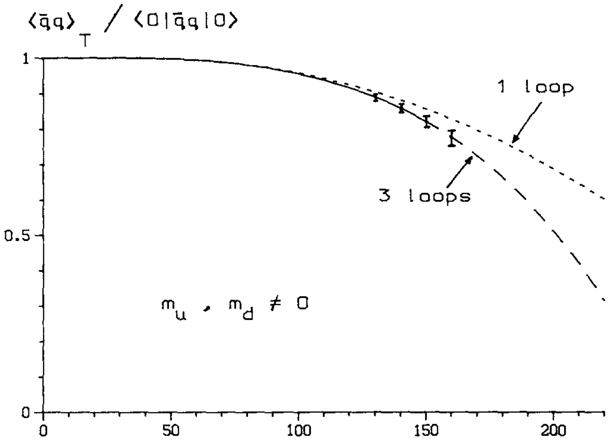
$\hookrightarrow$ contributions of massive states suppressed by $\exp(-M/T)$

$$\frac{\langle\bar{u}u\rangle_{\text{pionic med.}}}{\langle\bar{u}u\rangle_{\text{vac}}} \approx \frac{\langle\bar{d}d\rangle_{\text{pionic med.}}}{\langle\bar{d}d\rangle_{\text{vac}}} \approx 1 - \frac{\rho_\pi}{F_\pi^2} + \dots \quad \rightsquigarrow \text{fig.}$$

with (scalar) pion density

$$\rho_\pi = 3\int\frac{d^3k}{(2\pi)^3\,2E_k}\,\frac{1}{e^{E_k/T}-1} \xrightarrow{M_\pi\to 0} \frac{1}{8}\,T^2$$

Quark condensate in pion gas (Gerber/Leutwyler 1989)



Quark condensate at low baryon densities

- in general:

$$\langle\Omega|\, \bar{u}u\, |\Omega\rangle = -\frac{T}{V}\frac{d\,\ln Z}{dm_u} = -\frac{dP}{dm_u}$$

- low baryonic densities (at $T = 0$):
  - no mesons at all
  - no baryons for low enough chemical potential
  - nucleon-nucleon correlations $\sim \rho_N^2$

$$\frac{\langle\bar{u}u\rangle_{\text{nucl. med.}}}{\langle\bar{u}u\rangle_{\text{vac}}} \approx \frac{\langle\bar{d}d\rangle_{\text{nucl. med.}}}{\langle\bar{d}d\rangle_{\text{vac}}} \approx 1 - \frac{\rho_s\sigma_N}{F_\pi^2 M_\pi^2} \approx 1 - \frac{1}{3}\frac{\rho_N}{\rho_0}$$

with (scalar) nucleon density

$$\rho_s = 4\int\frac{d^3k}{(2\pi)^3}\frac{m_N}{\sqrt{\vec{k}^2+m_N^2}}\,\Theta(k_F - |\vec{k}|) \approx \rho_N = 4\int\frac{d^3k}{(2\pi)^3}\,\Theta(k_F - |\vec{k}|)$$

Complete set of states and correlations

- in general:

$$\langle \mathcal{O} \rangle (T, \mu) := \frac{\mathrm{Tr}\left(\mathcal{O} e^{-\beta (H - \mu N)}\right)}{\mathrm{Tr}\left(e^{-\beta (H - \mu N)}\right)}$$

Tr corresponds to summation over all *n-body states*

- at low densities: restrict to one-body states (and vacuum)

$$\langle \mathcal{O} \rangle (T, \mu) \approx \langle 0 | \mathcal{O} | 0 \rangle + \rho_\pi \langle \pi | \mathcal{O} | \pi \rangle + \rho_N \langle N | \mathcal{O} | N \rangle + \rho_{\bar{N}} \langle \bar{N} | \mathcal{O} | \bar{N} \rangle$$

- at higher densities: have to consider correlations

$\hookrightarrow$ $\langle \pi\pi | \mathcal{O} | \pi\pi \rangle$, $\langle \pi N | \mathcal{O} | \pi N \rangle$, ...

$\hookrightarrow$ in general complicated

Correlations and resonances

- at higher densities: have to consider correlations

$\hookrightarrow$ in general complicated

- most important correlations at finite temperature:

$\hookrightarrow$ formation of resonances (Dashen/Ma/Bernstein, 1969)

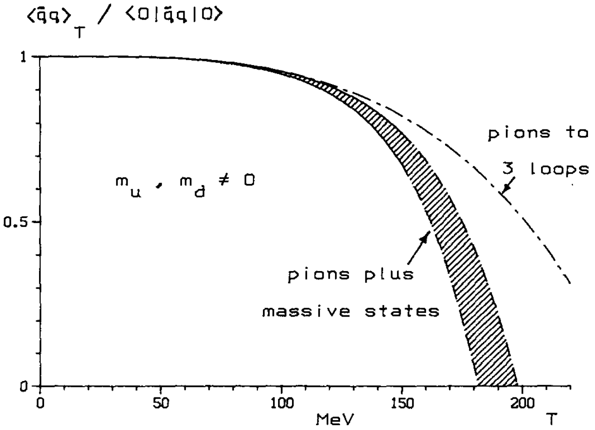
$\hookrightarrow$ consider gas of resonances:

$$\langle \mathcal{O} \rangle (T, \mu) \approx \langle 0 | \mathcal{O} | 0 \rangle + \sum_X \rho_X \langle X | \mathcal{O} | X \rangle$$

- important note: This is not a good approximation at low temperature and high chemical potential

$\hookrightarrow$ nucleon-nucleon correlations most important there

Quark condensate in resonance gas (Gerber/Leutwyler 1989)



Resonance gas — can that be all?

maybe yes; (speculative) explanation:

- change of vacuum structure possibly triggered by excluded volume (percolation)
- medium constituents carry chirally restored phase in their interior
- outside: chirally broken phase
- increasing density $\rightsquigarrow$ percolation
- purely geometrical effect
- covered by "linear-density approximation" (but: density of resonances)

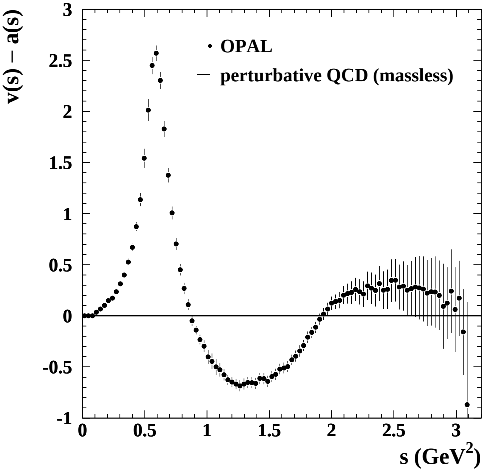
What is “wrong” with the two-quark condensate?

- no direct relation to observables
(GOR: $F_\pi^2 M_\pi^2 \sim m_q \langle \bar{q}q \rangle$, but also quark mass is not directly observable (scale dependent))
- chiral restoration $\Rightarrow \langle \bar{q}q \rangle = 0$

$\hookrightarrow$ but: $\langle \bar{q}q \rangle = 0 \not\Rightarrow$ chiral restoration

$\rightsquigarrow$ study also other order parameters

One of the clearest signs of chiral symmetry breaking



v: $\tau \to \nu_\tau + m\pi$ (m even)

a: $\tau \to \nu_\tau + n\pi$ (n odd)

Eur. Phys. J. C7 (1999) 571

Weinberg sum rules

- moments of difference $v - a$:

$$\int_0^{\infty} ds\, [v(s) - a(s)] = F_\pi^2$$

$$\int_0^{\infty} ds\, s\, [v(s) - a(s)] = 0$$

- in practice: replace $\infty \to s_0$

$\hookrightarrow$ at large s: $v(s)$ and $a(s)$ given by perturbative QCD

$\rightsquigarrow$ $v - a \approx 0$

Generalized Weinberg sum rules

$$\int_0^{s_0} ds\, [v(s) - a(s)] = F_\pi^2$$

$$\int_0^{s_0} ds\, s\, [v(s) - a(s)] = 0$$

$$\int_0^{s_0} ds\, s^2\, [v(s) - a(s)] = \langle \mathcal{O}_{\chi \mathrm{SB}} \rangle_{\mu=\sqrt{s_0}}$$

with four-quark condensate $\langle \mathcal{O}_{\chi \mathrm{SB}} \rangle$ =

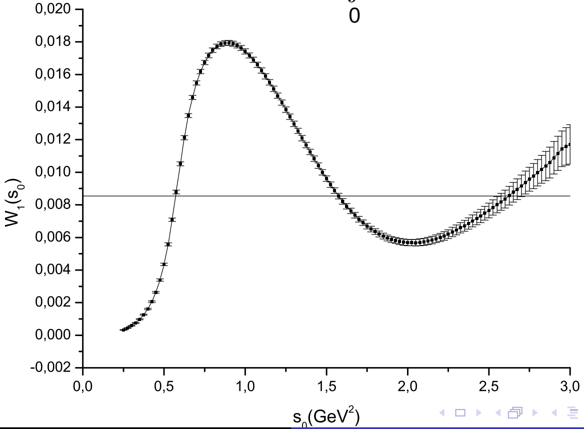
$$-\tfrac{1}{2}\,\pi \alpha_s \left\langle (\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d)^2 - (\bar{u}\gamma_\mu\lambda^a u - \bar{d}\gamma_\mu\lambda^a d)^2 \right\rangle$$

Weinberg sum rules in practice

- How large must s_0 be? $\rightsquigarrow$ $$\int_0^{s_0} ds\,[v(s) - a(s)] = F_\pi^2$$

Weinberg sum rules in practice

- How large must s_0 be? $\rightsquigarrow$ $\int_0^{s_0} ds\,[v(s) - a(s)] = F_\pi^2$



Weighted Weinberg sum rules

standard

- $$\int_0^{s_0} ds\,[v(s)-a(s)] = F_\pi^2$$
- $$\int_0^{s_0} ds\,s\,[v(s)-a(s)] = 0$$
- $$\int_0^{s_0} ds\,s^2\,[v(s)-a(s)] = \\ = \langle \mathcal{O}_{\chi\mathrm{SB}} \rangle$$

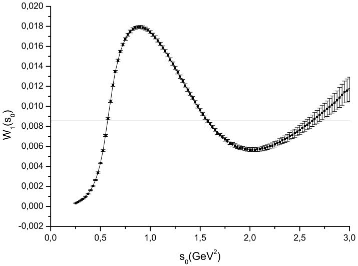
weighted

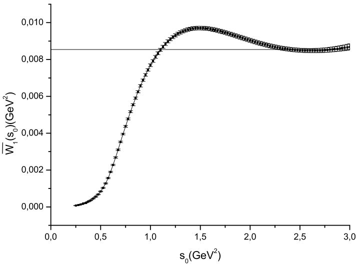
- $$\int_0^{s_0} ds\left(1-\frac{s}{s_0}\right)[v(s)-a(s)] = \\ = F_\pi^2$$
- $$\int_0^{s_0} ds\,s\,(s-s_0)\,[v(s)-a(s)] = \\ = \langle \mathcal{O}_{\chi\mathrm{SB}} \rangle$$

Weighted Weinberg sum rules II

$$\int_0^{s_0} ds\, [v(s) - a(s)] = F_\pi^2$$

$$\int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right) [v(s) - a(s)] = F_\pi^2$$





slow convergence

good convergence

Bordes/Dominguez/Penarrocha/Schilcher,
JHEP 02 (2006) 037, hep-ph/0511293

Other order parameters of chiral symmetry breaking

- F_π^2 and $\langle \mathcal{O}_{\chi\mathrm{SB}} \rangle$ connected to observable quantities (at least in vacuum)
- $F_\pi^2(T,\mu)$ known in linear order in pion or nucleon density

$$F_\pi^2(\rho_\pi(T)) \approx F_\pi^2 \left(1 - \frac{4\rho_\pi}{3F_\pi^2}\right) \qquad \text{Gasser/Leutwyler, 1986}$$

$$F_\pi^2(\rho_N(\mu)) \approx F_\pi^2 \left(1 - 0.52\,\frac{\rho_N}{\rho_0}\right) \qquad \text{Meißner/Oller/Wirzba, 2002}$$

- beyond?
- even more complicated for $\langle \mathcal{O}_{\chi\mathrm{SB}} \rangle(T,\mu)$ (SL, hep-ph/0604058)

Order parameters, summary

- Expect sizable changes in hadronic medium, especially at finite density
- On the other hand: not every in-medium interaction has to do with the symmetries, but might change the hadronic properties as well!
- How does all this change the properties of hadrons?
- What can be observed?

In-medium changes of hadrons

Classical Resonance

- equation of motion for damped harmonic oscillator, externally driven

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = e^{-i\omega t}$$

→ solution = response of system to external excitation

$$x(t) = x_0 \, e^{-i\omega t}$$

with ω-dependent coefficient

$$x_0 = \frac{1}{-\omega^2 - i\gamma\omega + \omega_0^2}$$

- note: there is additional contribution dying out with $e^{-\gamma t}$

Response function

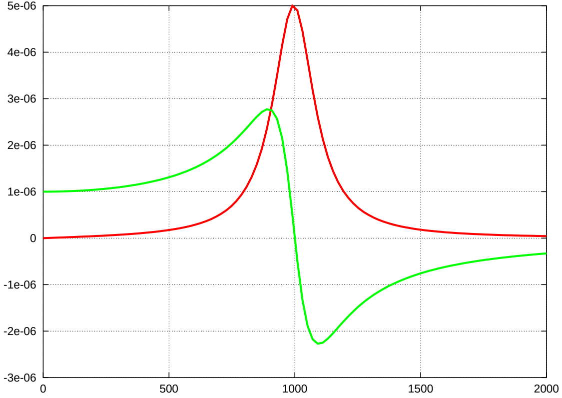
- split in real and imaginary part (for later use):

$$\mathrm{Re}\chi_0 = \frac{\omega_0^2 - \omega^2}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}$$

$$\mathrm{Im}\chi_0 = \frac{\gamma\omega}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}$$

- note: all knowledge about system, i.e. ω_0 and γ, can be deduced from $\mathrm{Im}\chi_0$ alone

Response function



$\mathrm{Im}\chi_0$

$\mathrm{Re}\chi_0$

Resonant Scattering

- scatter two particles to form a resonance

$\hookrightarrow$ i.e. deposit energy E ($\hat{=}$ external excitation)

- resonance can decay ($\hat{=}$ friction term $\sim \gamma$)

$\longrightarrow$ translation (de Broglie):
  - $\omega^2 \rightarrow E^2 = (\hbar\omega)^2 = s$ (cms)
  - $\omega_0^2 \rightarrow m_R^2$ resonance mass
  - $\gamma \rightarrow \Gamma$ resonance width

- important difference: width Γ depends on energy $\rightsquigarrow \Gamma(s)$

$\hookrightarrow$ reason: available phase space for resonance decay energy dependent
  - note: construction of ω-dependent γ also possible for oscillator case $\rightsquigarrow$ *retarded damping*

Precursor to spectral function

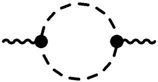
- recall

$$x_0 = \frac{1}{-\omega^2 - i\gamma\omega + \omega_0^2}\,, \quad \mathrm{Im} x_0 = \frac{\omega\gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}$$

$\hookrightarrow$ (not quite the) **SPECTRAL FUNCTION**

$$\mathcal{A}(s) \approx \frac{\sqrt{s}\,\Gamma}{(s - m_R^2)^2 + s\Gamma^2} = \mathrm{Im}\,\frac{-1}{s - m_R^2 + i\sqrt{s}\,\Gamma}$$

- field theory: Γ connected to *self energy* diagram:

$\Pi(s) =$  , $\Gamma = -\frac{\mathrm{Im}\Pi}{\sqrt{s}}$

Definition of spectral function

$\hookrightarrow$ also real part of Π enters definition of

SPECTRAL FUNCTION:

$$\mathcal{A}(s) = \frac{-\mathrm{Im}\Pi(s)}{|s - m_R^2 - \Pi(s)|^2} = \frac{\sqrt{s}\,\Gamma}{(s - m_R^2 - \mathrm{Re}\Pi)^2 + s\Gamma^2}$$

- note: real part of Π can **shift peak position** m_R
- response function ($\hat{=} x_0$): *Green function* or *propagator*

$$G(s) = \frac{1}{s - m_R^2 - \Pi(s)}$$

- obviously: $\mathcal{A} = -\mathrm{Im}G$

Unitarity, analyticity and Dispersion Relations

- spectral function tells how single quantum state is distributed over possible energies

↪ normalization condition: $$\int_0^\infty \frac{ds}{\pi} \mathcal{A}(s) = 1$$

- as for oscillator case: $\mathcal{A}$ completely determines resonance

↪ G can be calculated from $\mathcal{A}$
  - since $G^{(\mathrm{ret})}$ is analytic function in upper half of complex energy plane

↪ dispersion relation: $$G(s) = -\int_0^\infty \frac{ds'}{\pi} \frac{\mathcal{A}(s')}{s' - s - i\epsilon}$$

What changes in a medium?

1. **need spectral and statistical information**
   - spectral: distribution of one state over possible energies
   - statistical: how many states are there?

2. **appearance of new channels**

$\hookrightarrow$ will be discussed in a moment

How to get the statistical information

- general non-equilibrium situation:

$\longrightarrow$ have to determine both informations and their evolution in time ($\rightarrow$ e.g. transport theory)

- equilibrium:

$\longrightarrow$ maximal entropy requirement fixes statistical distribution

$\hookrightarrow$ number of states at given four-momentum:

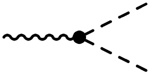
$$\mathcal{A}(E,\vec{p})\,\frac{1}{e^{\frac{1}{T}(E-\mu)}\pm 1}$$

with temperature T and chemical potential μ
(for nuclear matter ($T=0$): $\pm\mathcal{A}(E,\vec{p})\,\Theta(\mu-E)$)

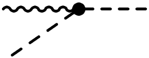
$\hookrightarrow$ $\mathcal{A}$ contains all information (in equilibrium!)

Appearance of new channels in a medium

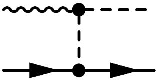
- vacuum: probe can only decay



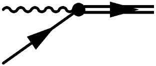
- medium: scattering with constituents of medium
(e.g. pions of heat bath, nucleons of nuclear matter)



Landau damping



(inelastic)
scattering

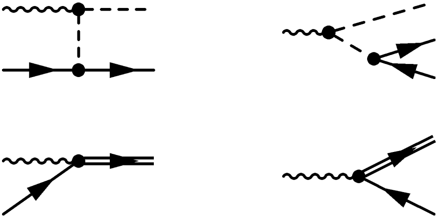


resonance
formation

Unified language for vacuum and medium

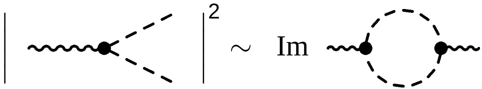
- Feynman:
  incoming particle equivalent to outgoing antiparticle (hole) with negative energy (traveling backwards in time)

$\hookrightarrow$ scattering is decay into particle(s) and hole(s)

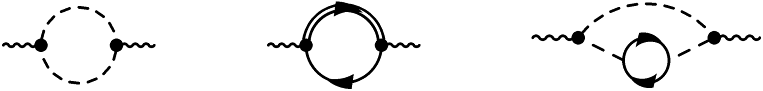


$\hookrightarrow$ excitation of nucleon-hole and resonance-hole states

Cutkosky rules

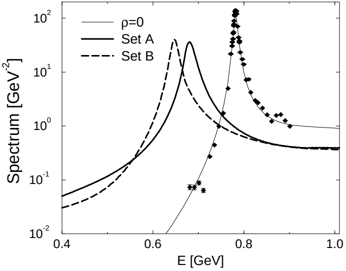


↪ have to calculate self energies like



What changes in a medium?

- imaginary parts of self energies change width:
  - decays Bose-enhanced or Pauli-blocked
  - new “decay” channels
- real parts shift peak position



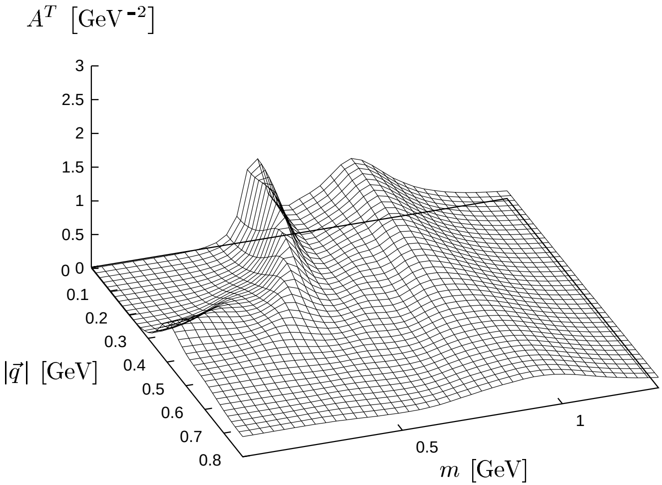
- Klingl/Waas/Weise
  NPA 650 (1999) 299

What changes in a medium?

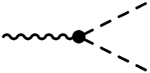
- vacuum: it does not matter whether probe is moving (Lorentz invariance)
- medium: it DOES matter whether probe is moving with respect to other scatterers

$\hookrightarrow$ explicit dependence on E, $\vec{q}$, not only on $s = E^2 - \vec{q}^2$

$\hookrightarrow$ independent variables: E, $|\vec{q}|$ or $m := \sqrt{s}$, $|\vec{q}|$

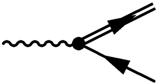

A^T [GeV^{-2}]
3
2.5
2
1.5
1
0.5
0 0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$|\vec{q}|$ [GeV]
0.5
1
m [GeV]

Appearance of new structures in a medium

- vacuum decay 

$\hookrightarrow$ outgoing states can have arbitrary momenta

$\leadsto$ structureless

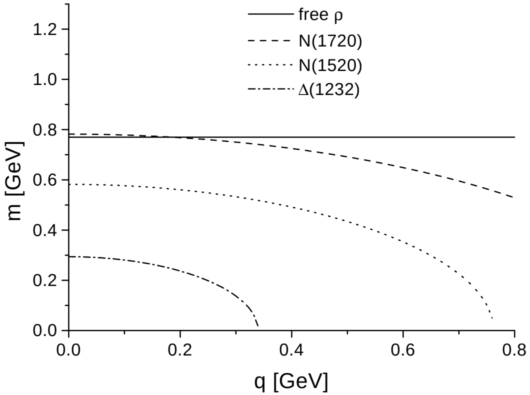
- medium “decay” in particle-hole 

$\hookrightarrow$ outgoing hole = incoming particle restricted in momentum (by temperature or chemical potential)

$\leadsto$ structure in self energy

$\leadsto$ resonance-hole branches

Resonance-hole branches

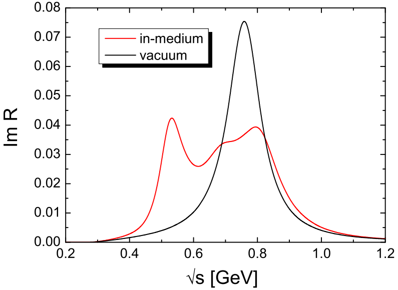


Appearance of new structures in a medium

- structure in self energy

⇝ structure in spectral function

↪ e.g. additional peaks, bumps, shoulders



Low-density approximation

- central quantity: (in-medium) spectral function for hadron H

$$\mathcal{A}(q) = -\mathrm{Im} D(q) = -\mathrm{Im}\frac{1}{q^2 - m_H^2 - \Pi(q)}$$
$$= \frac{-\mathrm{Im}\Pi(q)}{[q^2 - m_H^2 - \mathrm{Re}\Pi(q)]^2 + [\mathrm{Im}\Pi(q)]^2}$$

- decomposition: $\Pi(q) = \Pi_{\mathrm{vac}}(q) + \Pi_{\mathrm{med}}(q)$
- linear-density ("ρT") approximation for (in-medium) self energy

$$\Pi_{\mathrm{med}}(q) = \sum_X \rho_X T_{XH}(q)$$

with medium constituents X (e.g. N, π)

Forward scattering amplitude

$$\Pi_{\mathrm{med}}(q) = \sum_X \rho_X T_{XH}(q)$$

- T_{XH}: (vacuum) forward scattering amplitude for $X + H$ with medium constituents X (e.g. N, π)
- underlying idea: probe (H) scatters on single medium constituents
- “trivial” in-medium effect
- only vacuum quantity (scattering amplitude) enters
- imaginary part of T from inelasticities

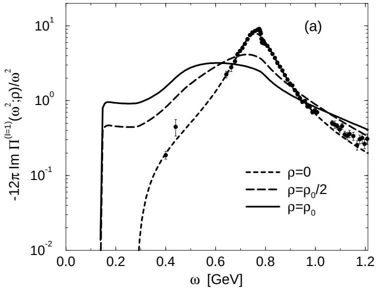
$\rightsquigarrow$ data for backward reactions, if H unstable

Unstable probe

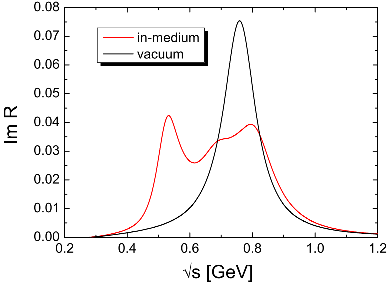
- everything well under control for low densities?
- in principle yes: need “only” vacuum scattering amplitudes T_{XH}
- in practice no: H can be unstable

$\rightsquigarrow$ no H beam, no direct access on scattering amplitude

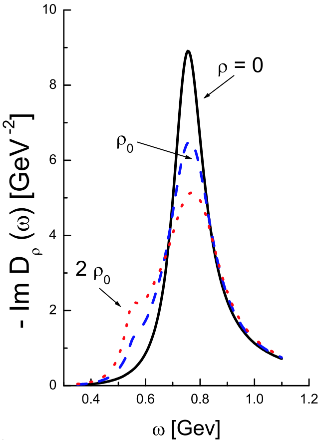
- sizable model dependences
- e.g. for ρ meson in cold nuclear matter $\rightsquigarrow$ figs.

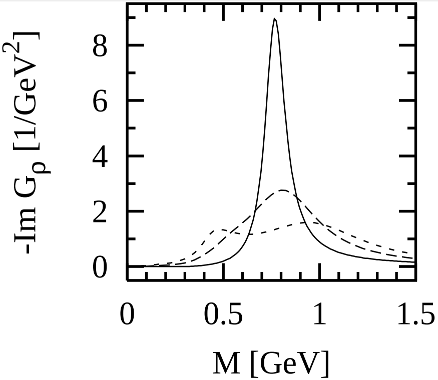


Klingl/Kaiser/Weise,
NPA 624 (1997) 527
(note: log plot!)



Post/Leupold/Mosel,
NPA 741 (2004) 81





Lutz/Wolf/Friman,
NPA 706 (2002) 431

- vacuum: solid, ρ_0: long dashed, $2\rho_0$: short dashed

Urban/Buballa/Rapp/Wambach,
NPA 641 (1998) 433

Interest in vector mesons

- Why are we interested in ρ mesons — more generally: in vector mesons?

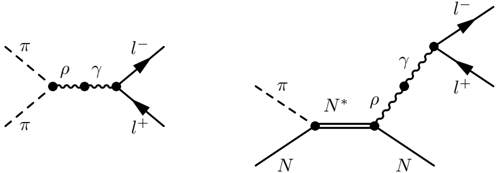
$\hookrightarrow$ neutral vector mesons couple directly to photons $\rightsquigarrow$ fig.

$\hookrightarrow$ dilepton decay channel

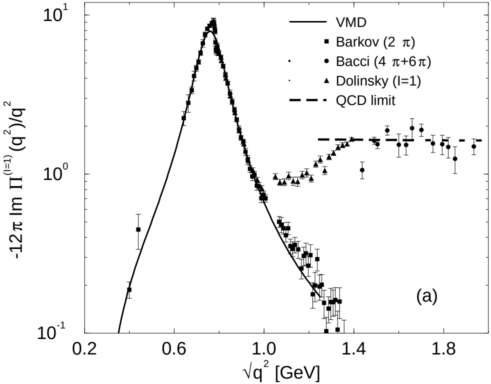
$\hookrightarrow$ information from strongly interacting dense matter

$\hookrightarrow$ in the following: focus on vector mesons

$\hookrightarrow$ but: a lot of considerations apply also to other hadrons



$$\frac{\sigma(e^+e^- \to \text{hadrons}, I=1)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Klingl/Kaiser/Weise, NPA 624 (19997) 527, hep-ph/9704398

How fancy is the linear-density approximation?

- simple toy model for dilepton production $\sim n_B(q)\mathcal{A}_\rho(q)/q^2$
  - mediated by ρ meson (vector meson dominance)
  - ρ meson couples to 2π and resonance-hole (RN^{-1})

$\rightsquigarrow$ $\mathcal{A}_\rho(q) =$

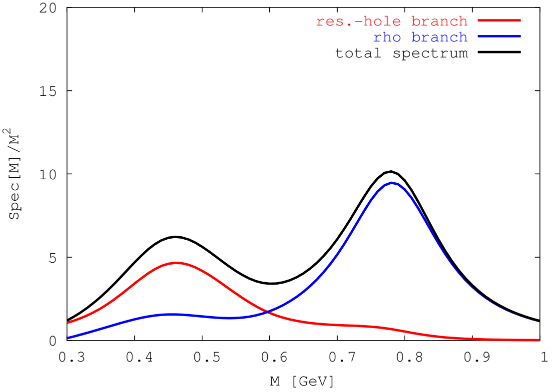
$$\frac{-\mathrm{Im}\Pi_{2\pi}(q) - \mathrm{Im}\Pi_{RN^{-1}}(q)}{[q^2 - m_\rho^2 - \mathrm{Re}\Pi_{2\pi}(q) - \mathrm{Re}\Pi_{RN^{-1}}(q)]^2 + [\mathrm{Im}\Pi_{2\pi}(q) + \mathrm{Im}\Pi_{RN^{-1}}(q)]^2}$$

- recall: $\Pi_{RN^{-1}} = \rho_N T_{\rho N \to R \to \rho N}$
- appearance of density in denominator causes non-elementary effect: resummation

$\hookrightarrow$ spectral function not simply given by “vac.”+ $\{$term linear in density$\}$

In-medium ρ meson spectral information

Decomposition



How fancy is the linear-density approximation?

$\rightsquigarrow$ $\mathcal{A}_\rho(q) =$

$$\frac{-\mathrm{Im}\Pi_{2\pi}(q) - \mathrm{Im}\Pi_{RN^{-1}}(q)}{[q^2 - m_\rho^2 - \mathrm{Re}\Pi_{2\pi}(q) - \mathrm{Re}\Pi_{RN^{-1}}(q)]^2 + [\mathrm{Im}\Pi_{2\pi}(q) + \mathrm{Im}\Pi_{RN^{-1}}(q)]^2}$$

- corresponding elementary reactions:

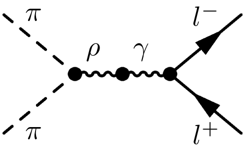
$$\frac{-\mathrm{Im}\Pi_{2\pi}(q) - \mathrm{Im}\Pi_{RN^{-1}}(q)}{[q^2 - m_\rho^2 - \mathrm{Re}\Pi_{\mathrm{vac}}(q)]^2 + [\mathrm{Im}\Pi_{\mathrm{vac}}(q)]^2}$$

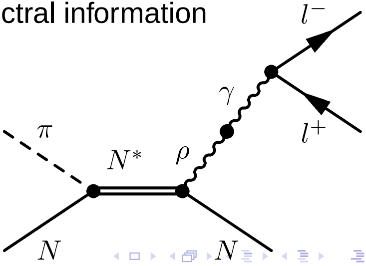
Interpretation of elementary processes

$$\frac{-\mathrm{Im}\Pi_{2\pi}(q) - \mathrm{Im}\Pi_{RN^{-1}}(q)}{[q^2 - m_\rho^2 - \mathrm{Re}\Pi_{\mathrm{vac}}(q)]^2 + [\mathrm{Im}\Pi_{\mathrm{vac}}(q)]^2}$$

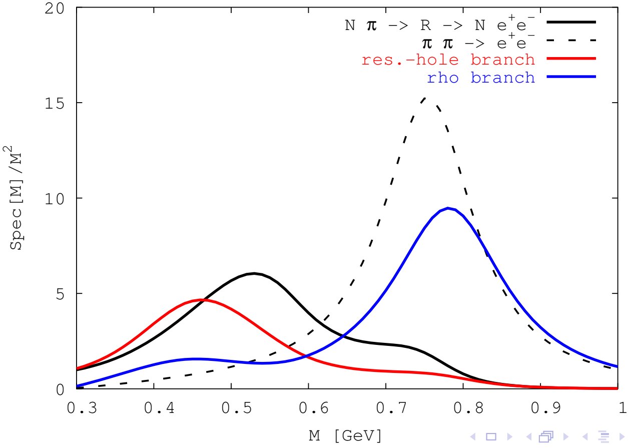
$$= \frac{\mathrm{Im}\Pi_{2\pi}(q)}{\mathrm{Im}\Pi_{\mathrm{vac}}(q)} \mathcal{A}_\rho^{\mathrm{vac}} + \frac{\mathrm{Im}\Pi_{RN^{-1}}(q)}{\mathrm{Im}\Pi_{\mathrm{vac}}(q)} \mathcal{A}_\rho^{\mathrm{vac}}$$

- i.e. branching ratios times spectral information





Elementary reactions versus full in-medium spectrum (at $\vec{q} = 0$!)



Conclusions from simple toy model

- structures already present in elementary reactions
- “denominator effect”: level repulsion and overall depletion
- elementary reactions should be measured

⇝ πN to dileptons, not only NN
(in latter resonance structure more smeared out, phase space)

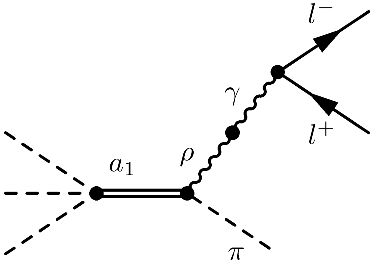
- note: “elementary” reactions are genuine in-medium
(π in initial state)

What is “chiral mixing”?

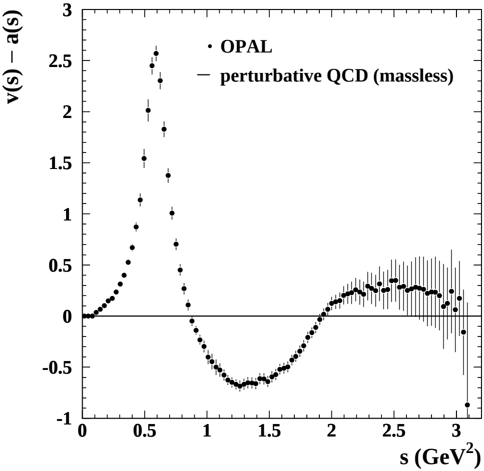
- recall τ decay:
  - ρ meson appears in vector current v
  - a_1 meson in axial-vector current a
- χSB dictates coupling strength of π-v-a

$\rightarrow$ χSB dictates coupling strength of π-ρ-a_1

- chiral mixing:



Resonances in $v - a$

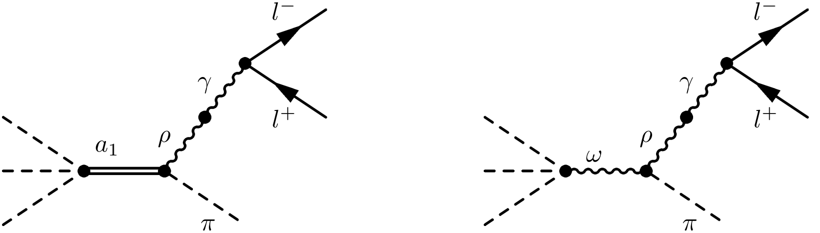


v: peak of ρ

a: bump of a_1

Chiral and non-chiral mixing

- chiral mixing (s-wave)
- similar non-chiral effect (p-wave)



- What is the important aspect about chiral mixing?

↪ Fancy effect predicted by chiral symmetry restoration?

↪ No! Effect is standard

- the important aspect (cf. also kaon potentials): strength of chiral mixing dictated by χSB

Linear-density approximation and beyond

- underlying idea: probe (H) scatters on single medium constituents
- “trivial” in-medium effect

↪ only vacuum quantity (scattering amplitude) enters

↪ already resummation by “denominator effect”

- works if density is not too large
- break down depends on probe and medium
- what comes beyond?

↪ hadronic language:
n-body scattering amplitudes with $n > 2$

↪ i.e. probe scatters on correlated n-body states

- becomes uneconomical

Beyond linear-density approximation

- what comes beyond?

1. hadronic language:
   n-body scattering amplitudes with $n > 2$

$\hookrightarrow$ i.e. probe scatters on correlated *n*-body states

1a consider most important correlations: resonances

1b resummations: selfconsistent calculations

2. connection to in-medium change of condensates

$\hookrightarrow$ additional effects on top or only different language?

Symmetry changes and/or many-body effects?

study hadronic probe in a hadronic medium

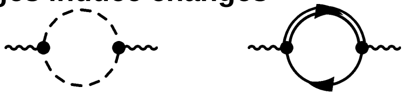
1. hadronic many-body effects, many-body calculations (spectral functions)
   $\longrightarrow$ fix input from elementary scattering (if possible ...)
2. phase transitions, changes in symmetries (chiral symmetry, deconfinement, ...)
   $\longrightarrow$ change of underlying vacuum structure

- Does 1 happen on top of 2?
- Double counting?
- Does 1 imply 2?
- Are there in-medium changes of hadronic properties which cannot be traced back to hadronic interactions?

Beyond low densities: Selfconsistent calculations

— changes induce changes

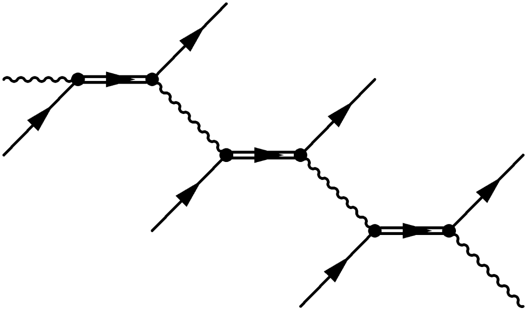
so far:



but also the other states get medium modified
self energies:

self consistency might be important

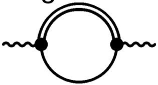
$\hookrightarrow$ possible: inclusion (*resummation*) of classes of multi-scattering events:



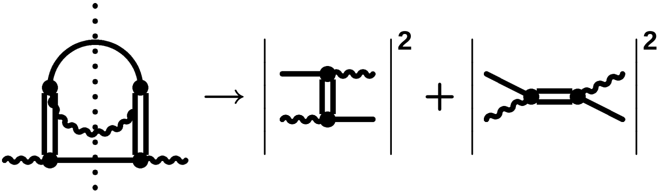
...

included and not included diagrams

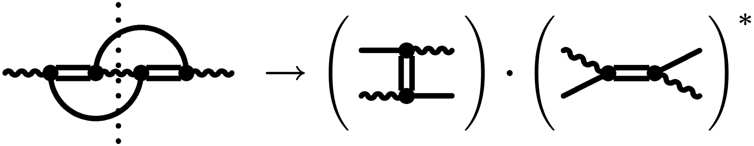
basic diagram



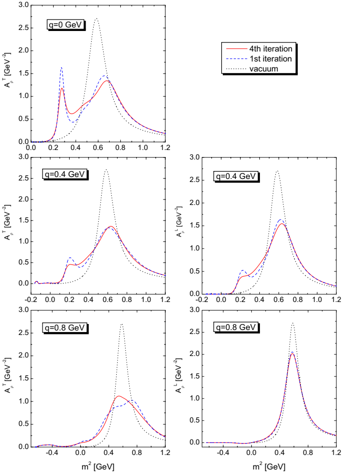
correction for propagator (included)

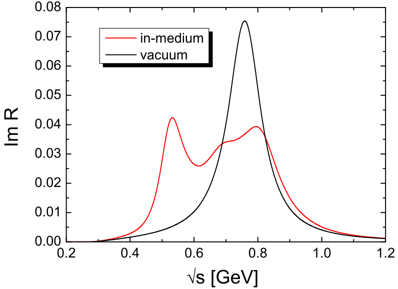


correction for vertex (not included) $\rightarrow$ intimate connection to interferences

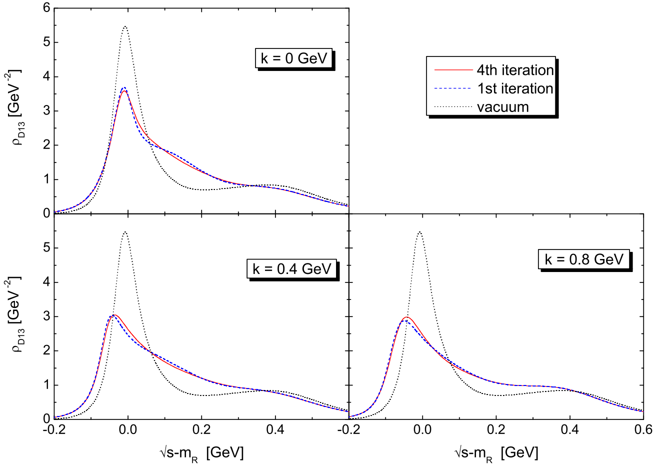


explicit example (M. Post, PhD thesis)





Back reaction on the D_{13}



Selfconsistent calculations, summary

- stuctures already there on elementary level
- some reshuffling of strength
- not everything can be resummed
- have to decide what is important $\rightarrow$ model dependence
- check importance of resummations by studying also elementary level
- "elementary" is not superposition of *NN* or *NA* reactions
- elementary input required: e.g. for dileptons:
  1. $\pi\pi$ (known from inverse reaction)
  2. *NN* (measured/measurable)
  3. πN (important to measure)

Dropping mass scenarios

- basic qualitative idea:

$\hookrightarrow$ recall finding from quark models:

  - χSB related to generation of constituent quark masses

  $\hookrightarrow$ $M \approx 300\,\text{MeV} \gg m_q$

  $\hookrightarrow$ roughly explains masses of nucleon, vector mesons

- medium: chiral restoration $\rightsquigarrow$ $M \to m_q$

$\hookrightarrow$ precursor: M drops $\rightsquigarrow$ hadron masses drop

Quantitative picture

- propose model which links elementary hadronic parameters (bare masses, coupling constants) e.g. with quark condensate (Brown/Rho)

$$\frac{m_{H,\mathrm{med.}}}{m_{H,\mathrm{vac.}}} = \left(\frac{\langle\bar{q}q\rangle_{\mathrm{med.}}}{\langle\bar{q}q\rangle_{\mathrm{vac.}}}\right)^{\alpha}$$

- α might be density/temperature dependent
- includes effects beyond linear-density approximation
- oversimplified?

$\hookrightarrow$ universal law at low densities in conflict with low-density theorem

$\rightsquigarrow$ should be fused with standard many-body effects(?)

The same unanswered questions:

- Should one fuse dropping mass scenario with standard many-body effects?
- double counting?
- different, economic language for hadronic higher-order many-body effects?
- alternative: resummation techniques, self consistency?
- or additional effects on top of hadronic effects?

$\hookrightarrow$ propose model and check against data

How to observe mass shifts?

- masses of resonances seen in scattering phase shifts

$\hookrightarrow$ hard to scatter inside a medium (medium has to stay intact)

- detectors outside of medium

$\hookrightarrow$ stable states change their mass back when leaving the medium

interesting probe: vector mesons

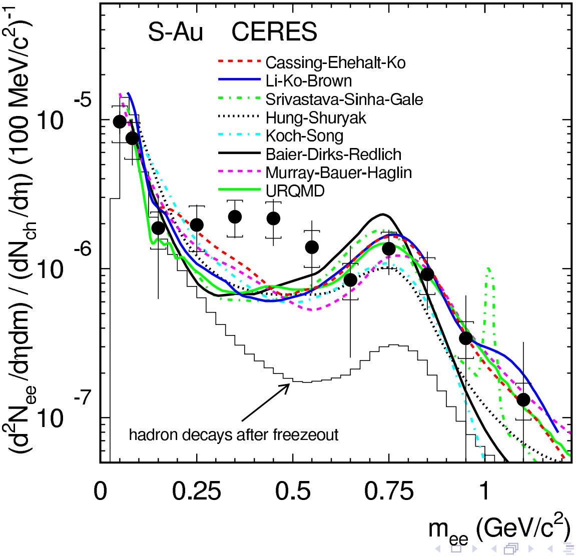
- interact strongly

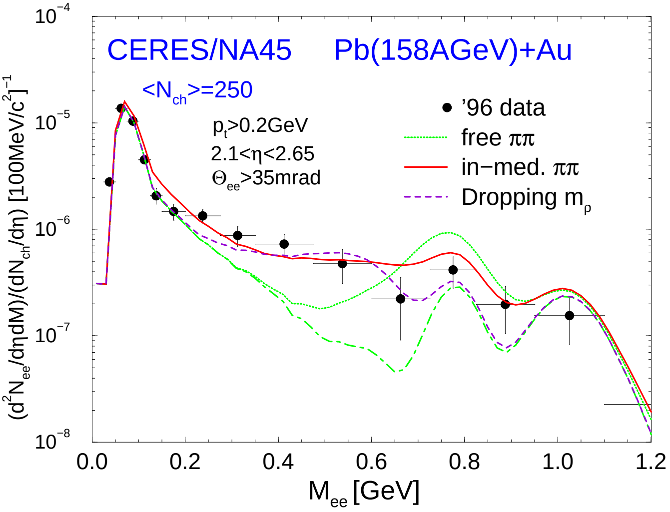
$\longrightarrow$ sensitive to in-medium changes

- couple directly to dileptons

$\hookrightarrow$ the latter leave medium without further interaction

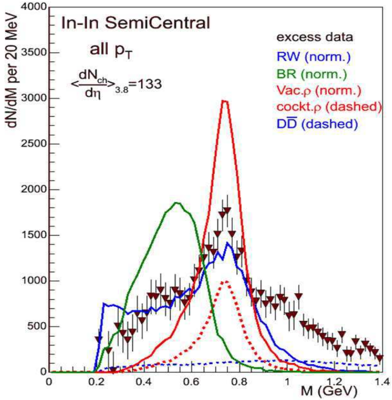
observe increasing strength at lower invariant mass
$\longrightarrow$ but: dropping or broadening?


S-Au CERES
Cassing-Ehehalt-Ko
Li-Ko-Brown
Srivastava-Sinha-Gale
Hung-Shuryak
Koch-Song
Baier-Dirks-Redlich
Murray-Bauer-Haglin
URQMD
hadron decays after freezeout
$(d^2N_{ee}/d\eta dm) / (dN_{ch}/d\eta)$ (100 MeV/c^2)$^{-1}$
10^{-5}
10^{-6}
10^{-7}
0
0.25
0.5
0.75
1
m_{ee} (GeV/c^2)


CERES/NA45 Pb(158AGeV)+Au
<N_ch>=250
p_t>0.2GeV
2.1<η<2.65
Θ_ee>35mrad
'96 data
free ππ
in−med. ππ
Dropping m_ρ
(d²N_ee/dηdM)/(dN_ch/dη) [100MeV/c²]⁻¹
M_ee [GeV]
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Data quality makes a difference

NA60



dropping mass scenario
hadronic model

Other approaches and concepts

- many models on the market

$\hookrightarrow$ instead of overview:

$\hookrightarrow$ select some related to vector mesons or to chiral symmetry

1. vector meson dominance
2. QCD sum rules
3. hidden local symmetry
4. chiral quartets

Vector meson dominance (VMD)

- in general: (vector meson V_μ, photon A_μ)

$$\mathcal{L}_{\text{int}} = g_1 V_\mu j_\pi^\mu + g_2 V_\mu j_N^\mu + g_3 V_\mu j_{RN^{-1}}^\mu + \ldots - \frac{e\, M_V^2}{g} V^\mu A_\mu + \tilde{g}_1 A_\mu j_\pi^\mu + \tilde{g}_2 A_\mu j_N^\mu + \tilde{g}_3 A_\mu j_{RN^{-1}}^\mu + \ldots$$

- strict VMD: all hadronic interactions mediated by vector mesons

$$\mathcal{L}_{\text{int}} = -\frac{e\, M_V^2}{g} V^\mu A_\mu + g_1 V_\mu j_\pi^\mu + g_2 V_\mu j_N^\mu + g_3 V_\mu j_{RN^{-1}}^\mu + \ldots$$

$\rightsquigarrow$ less parameters, more predictive power

Vector meson dominance (VMD)

- strict VMD seems to work well for meson decays

$$\mathcal{L}_{\text{int}} = -\frac{e\,M_V^2}{g}\,V^\mu A_\mu + g_1 V_\mu j_\pi^\mu + g_2 V_\mu j_N^\mu + g_3 V_\mu j_{RN^{-1}}^\mu + \dots$$

- strict VMD has less parameters, more predictive power

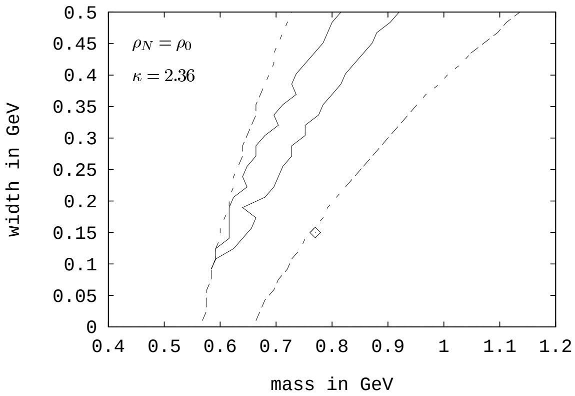
↪ e.g. fix g_3 from decay $R \to \gamma N$

⇝ prediction for $R \to \gamma^* N \to e^+ e^- N$

↪ need data on resonance decays into photons and into dileptons

QCD sum rules

- no prediction for mass shift
- but constraints for hadronic models
- relation to four-, not two-quark condensates

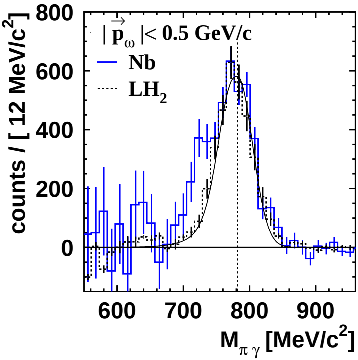


Leupold/Peters/Mosel
NPA 628 (1998) 311

Hidden local symmetry

- vector mesons treated as gauge bosons of local chiral symmetry
- ⇝ vector meson masses generated by chiral symmetry breaking (Higgs mechanism)
- ⇝ vector mesons become massless at chiral restoration
- ⇝ dropping masses
- but only for vector mesons, not for all hadrons (maybe for nucleon as chiral soliton???)
- ω meson is not necessary as gauge boson, but in SU(3) member of vector meson nonet
- note: also here relation to four-, not two-quark condensates

Experimental significance for dropping ω mass



Trnka et al (CBELSA/TAPS), PRL 94 (2005) 192303

Chiral quartets of baryons

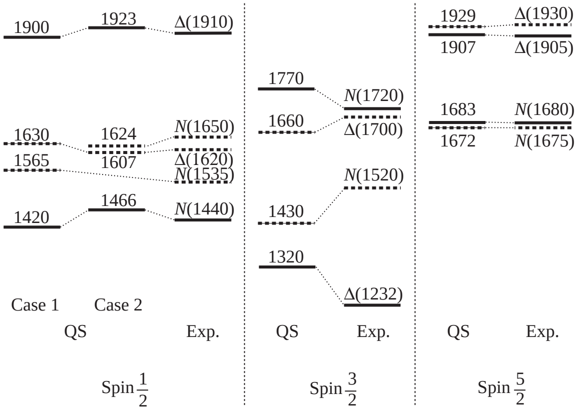
- for linear realization of chiral symmetry:

$\rightsquigarrow$ sort baryons in chiral multiplets,
e.g. $\Delta(1232)$, $N(1520)$, $\Delta(1700)$, $N(1720)$

$\rightsquigarrow$ mass splitting by symmetry breaking

- Jido/Hatsuda/Kunihiro, PRL 84 (2000) 3252
- degeneracy at chiral restoration
- observable?

Chiral quartets



Jido/Hatsuda/Kunihiro, PRL 84 (2000) 3252

Summary

want to learn about

- symmetry pattern of QCD
- many-body effects
- nature of hadrons

need

- models which incorporate as much as possible input from
  - QCD (chiral symmetry, . . .)
  - data on elementary scattering
- decisive experiments which can rule out models