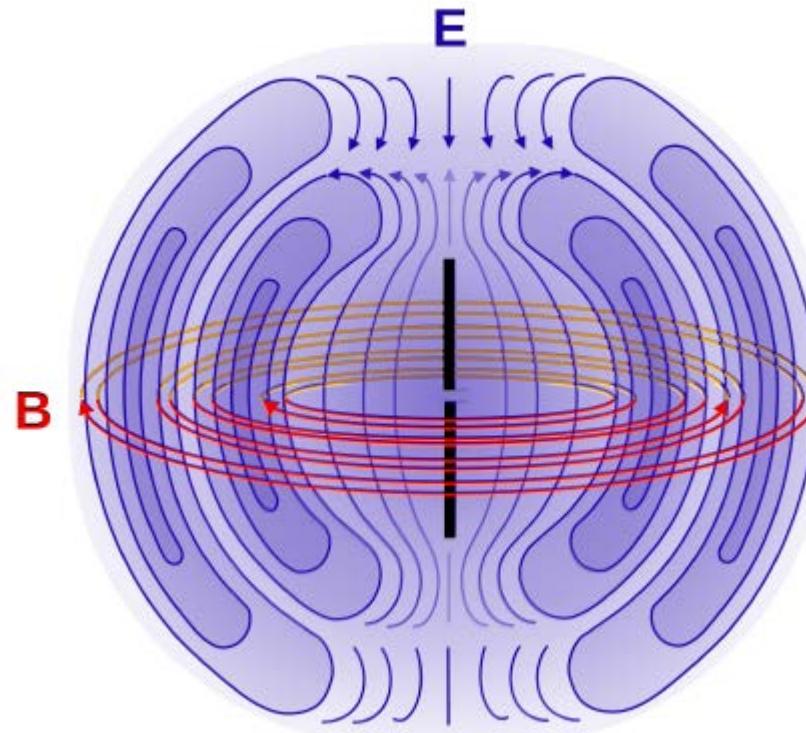


Electric/magnetic dipoles

Electric and magnetic dipole fields have opposite parity:
Magnetic dipoles have even parity and electric dipole fields have odd parity.

$$\Rightarrow \pi(M\ell) = (-1)^{\ell+1} \text{ and } \pi(E\ell) = (-1)^\ell$$



Classical electrodynamics

- ❖ The nucleus is a collection of moving charges, which can induce magnetic/electric fields
- ❖ The power radiated into a small area element is proportional to $\sin^2(\theta)$
- ❖ The average power radiated for an **electric dipole** is:

$$P = \frac{1}{12\pi\epsilon_0} \frac{\omega^4}{c^3} d^2$$

- ❖ For a **magnetic dipole** is

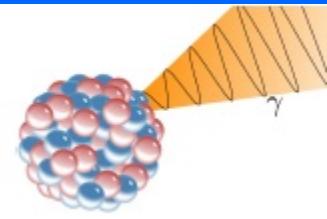
$$P = \frac{1}{12\pi\epsilon_0} \frac{\omega^4}{c^5} \mu^2$$

Higher order multipoles

It is possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials.

- ℓ : The index of radiation
 2^ℓ : The multipole order of the radiation
- $\ell = 1 \rightarrow \text{Dipole}$
 $\ell = 2 \rightarrow \text{Quadrupole}$
 $\ell = 3 \rightarrow \text{Octupole}$
- The associated Legendre polynomials $P_{2\ell}(\cos(\theta))$ are:
For $\ell = 1$: $P_2 = \frac{1}{2}(3 \cdot \cos^2(\theta) - 1)$
For $\ell = 2$: $P_4 = \frac{1}{8}(35\cos^4(\theta) - 30\cos^2(\theta) + 3)$

Nuclear excited state decay angular momentum in γ -decay



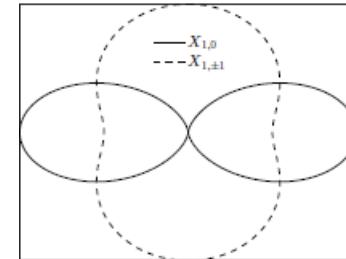
- ❖ *The photon is a spin-1 boson*
- ❖ Like α -decay and β -decay the emitted γ -ray can carry away units of *angular momentum ℓ* , which has given us different multipolarities for transitions.
- ❖ For orbital angular momentum, we can have values $\ell = 0, 1, 2, 3, \dots$ that correspond to our multipolarity.
- ❖ Therefore, our selection rule is:

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

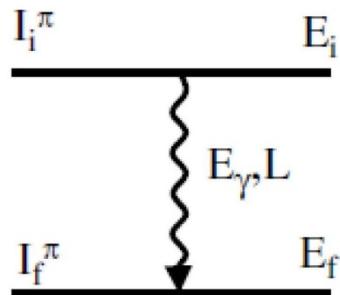
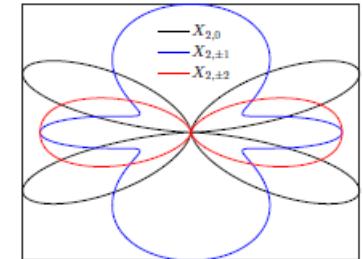
Characteristics of multipolarity selection rules

L	multipolarity	$\pi(E\ell) / \pi(M\ell)$	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

$\ell = 1$



$\ell = 2$



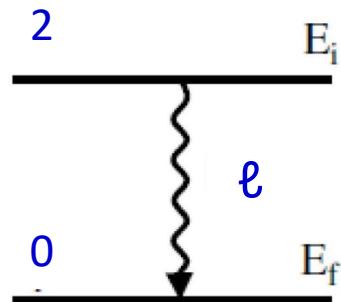
$$E_{\gamma} = E_i - E_f$$

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

$$\Delta\pi(E\ell) = (-1)^{\ell}$$

$$\Delta\pi(M\ell) = (-1)^{\ell+1}$$

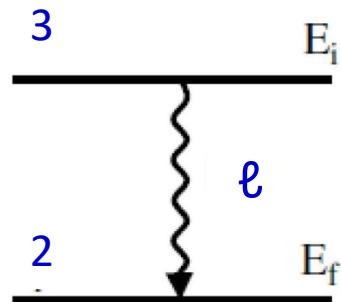
The basics of the situation



$$|2 - 0| \leq \ell \leq 2 + 0$$

Here $\Delta I = 2$ and $\ell = 2$
this is a stretched transition

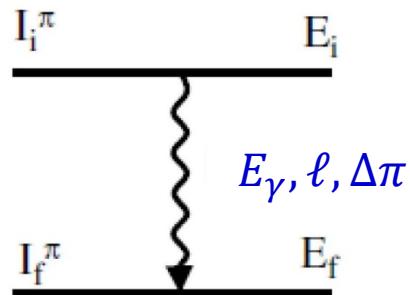
The basics of the situation



$$|3 - 2| \leq \ell \leq 3 + 2$$

Here $\Delta I = 1$ but $\ell = 1, 2, 3, 4, 5$
and the transition can be a mix of 5 multipolarities

The basics of the situation



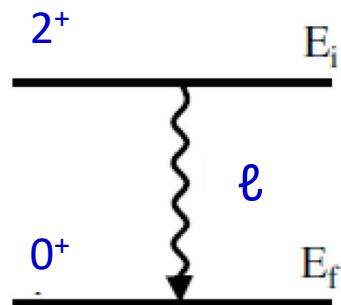
Electromagnetic transitions:

$$\Delta\pi \text{ (electric)} = (-1)^\ell$$

$$\Delta\pi \text{ (magnetic)} = (-1)^{\ell+1}$$

$\Delta\pi$	yes	E1	M2	E3	M4
	no	M1	E2	M3	E4

The basics of the situation

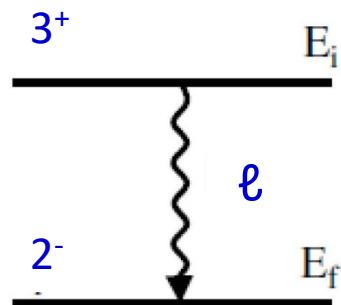


$$|2 - 0| \leq \ell \leq 2 + 0$$

$\ell = 2$ and no change in parity

$\Delta\pi$					
	no	M1	E2	M3	E4

The basics of the situation



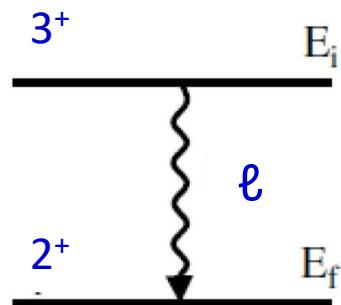
$$|3 - 2| \leq \ell \leq 3 + 2$$

Here $\Delta I = 1$ but $\ell = 1, 2, 3, 4, 5$

$\Delta\pi$	yes	E1	M2	E3	M4

mixed E1,M2,E3,M4,E5

The basics of the situation



$$|3 - 2| \leq \ell \leq 3 + 2$$

Here $\Delta J = 1$ but $\ell = 1, 2, 3, 4, 5$

$\Delta\pi$					
	no	M1	E2	M3	E4

mixed M1,E2,M3,E4,M5

The basics of the situation

$3^+ \rightarrow 2^+$: mixed M1,E2,M3,E4,M5

$3^+ \rightarrow 2^-$: mixed E1,M2,E3,M4,E5

In general only the lowest 2 multipoles compete

and (for reasons we will see later)

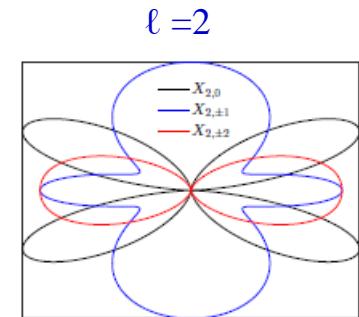
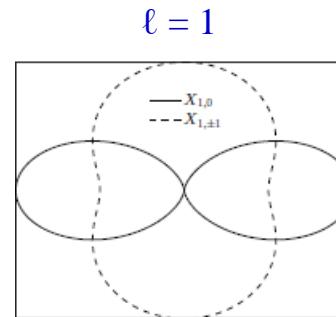
$\ell + 1$ multipole generally only competes if it is electric:

$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (very little M2 admixture)

Characteristics of multipolarity

L	multipolarity	$\pi(E\ell) / \pi(M\ell)$	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			



parity: electric multipoles $\pi(E\ell) = (-1)^\ell$, magnetic multipoles $\pi(M\ell) = (-1)^{\ell+1}$

The power radiated is proportional to:

$$P(\sigma\ell) \propto \frac{2(\ell+1) \cdot c}{\epsilon_0 \cdot \ell \cdot [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} |\mathcal{M}(\sigma\ell)|^2$$

where σ means either E or M and $\mathcal{M}(\sigma\ell)$ is the E or M multipole moment of the appropriate kind.

Emission of electromagnetic radiation

$$T(E1; I_i \rightarrow I_f) = 1.590 \cdot 10^{17} E_{\gamma}^3 B(E1; I_i \rightarrow I_f)$$

$$T(E2; I_i \rightarrow I_f) = 1.225 \cdot 10^{13} E_{\gamma}^5 B(E2; I_i \rightarrow I_f)$$

$$T(E3; I_i \rightarrow I_f) = 5.709 \cdot 10^8 E_{\gamma}^7 B(E3; I_i \rightarrow I_f)$$

$$T(E4; I_i \rightarrow I_f) = 1.697 \cdot 10^4 E_{\gamma}^9 B(E4; I_i \rightarrow I_f)$$

$$T(M1; I_i \rightarrow I_f) = 1.758 \cdot 10^{13} E_{\gamma}^3 B(M1; I_i \rightarrow I_f)$$

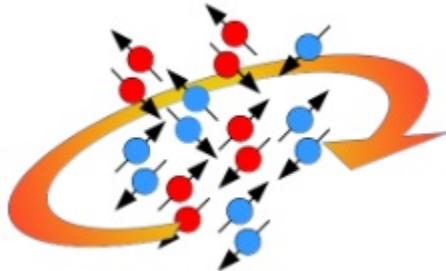
$$T(M2; I_i \rightarrow I_f) = 1.355 \cdot 10^7 E_{\gamma}^5 B(M2; I_i \rightarrow I_f)$$

$$T(M3; I_i \rightarrow I_f) = 6.313 \cdot 10^0 E_{\gamma}^7 B(M3; I_i \rightarrow I_f)$$

$$T(M4; I_i \rightarrow I_f) = 1.877 \cdot 10^{-6} E_{\gamma}^9 B(M4; I_i \rightarrow I_f)$$

where $E_{\gamma} = E_i - E_f$ is the energy of the emitted γ quantum in MeV (E_i, E_f are the nuclear level energies, respectively), and the reduced transition probabilities $B(E\ell)$ in units of $e^2(\text{barn})^{\ell}$ and $B(M\ell)$ in units of $\mu_N^2 = (e\hbar/2m_Nc)^2 (fm)^{2\ell-2}$

Nuclear shape magnetic moments



- Nuclear magnetic dipole moments arise from
- intrinsic spin magnetic dipole moments of the protons and neutrons
 - circulating currents (motion of the protons)

The **nuclear magnetic dipole moment** can be written as

$$\vec{\mu} = \frac{\mu_N}{\hbar} \sum_i [g_L \vec{L} + g_s \vec{s}] \quad \text{summed over all } p, n$$

where $\mu_N = eh/2m_p$ is the Nuclear Magneton.

or $\mu = g_J \mu_N J$ where J total nuclear spin quantum number
 g_J nuclear g -factor (analogous to Landé g -factor in atoms)

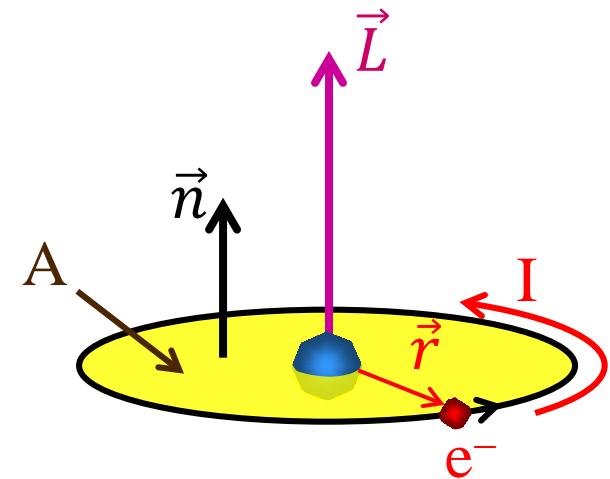
g_J may be predicted using the Nuclear Shell Model (see later), and measured using magnetic resonance (see Advanced Quantum course).

All even-even nuclei have $\mu = 0$ since $J = 0$

Magnetic moment classical description

e^- -movement \Rightarrow magnetic moment $\vec{\mu}_e$

$$\left. \begin{aligned} \vec{\mu}_e &= \frac{I}{c} \cdot A \cdot \vec{n} \\ I &= -e \cdot v = -e \cdot \frac{v}{2\pi r} \\ A &= \pi \cdot r^2 \end{aligned} \right\} \Rightarrow \vec{\mu}_e = -\frac{1}{2} \cdot \frac{e \cdot v \cdot r \cdot \vec{n}}{c}$$



angular momentum: $\vec{L} = \vec{r} \times \vec{p} = r \cdot m_e \cdot v \cdot \vec{n} \Rightarrow \vec{\mu}_e = -\frac{e}{2 \cdot m_e \cdot c} \vec{L}$

Quantum mechanical description: angular momentum will be quantized, in units of \hbar .

$$\mu_B = \frac{e \cdot \hbar}{2 \cdot m_e \cdot c} = 5.788 \cdot 10^{-11} \frac{MeV}{Tesla}$$

$$\mu_K = \frac{e \cdot \hbar}{2 \cdot m_P \cdot c} = 3.152 \cdot 10^{-14} \frac{MeV}{Tesla}$$



Quark model

	Charge	Mass m_q	Spin	mag. Moment
<i>u-quark</i>	+ 2/3	~ 1/3 M _p	1/2	$2 \cdot \left(\frac{2}{3}\right) \cdot \frac{e \cdot \hbar}{2 \cdot m_q \cdot c} \cdot s = 4 \cdot \mu_K \cdot s$
<i>d-quark</i>	- 1/3	~ 1/3 M _p	1/2	$2 \cdot \left(-\frac{1}{3}\right) \cdot \frac{e \cdot \hbar}{2 \cdot m_q \cdot c} \cdot s = -2 \cdot \mu_K \cdot s$

general coupling rule:

$$g(j_1 \times j_2; J) = \frac{1}{2} (g_1 + g_2) + \frac{j_1(j_1+1) - j_2(j_2+1)}{2 \cdot J \cdot (J+1)} \cdot (g_1 - g_2)$$

Proton (uud) :

$$g(1/2 \times 1/2; 1) = \frac{1}{2} (4+4) + \frac{1/2 \cdot 3/2 - 1/2 \cdot 3/2}{2 \cdot 1 \cdot 2} \cdot (4-4) = 4$$

$$g(1 \times 1/2; 1/2) = \frac{1}{2} (4-2) + \frac{2-3/4}{3/2} \cdot (4+2) = \boxed{+6}$$

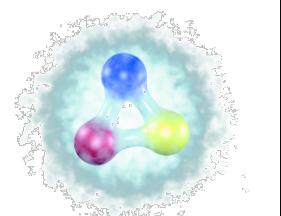
Neutron (ddu) :

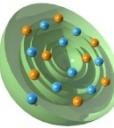
$$g(1/2 \times 1/2; 1) = \frac{1}{2} (-2-2) + \frac{1/2 \cdot 3/2 - 1/2 \cdot 3/2}{2 \cdot 1 \cdot 2} \cdot (-2+2) = -2$$

$$g(1 \times 1/2; 1/2) = \frac{1}{2} (-2+4) + \frac{2-3/4}{3/2} \cdot (-2-4) = \boxed{-4}$$

$g_{\text{Dirac}} = 2$ prediction of the Dirac-theory for the gyromagnetic factor for spin 1/2-particles

$g_s^{\text{proton}} = +5.58 \Rightarrow$ big deviation from $g = 2 \Leftrightarrow$ no fundamental particle
 $g_s^{\text{neutron}} = -3.82$





Success of the extreme single-particle shell model

➤ Magnetic moments:

The g-factor g_j is given by:

$$\vec{\mu}_j = g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} = g_j \cdot \vec{j} \quad \Rightarrow \quad \vec{\mu}_j = \left[(g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s}) \cdot \frac{\vec{j}}{|\vec{j}|} \right] \cdot \frac{\vec{j}}{|\vec{j}|}$$

$$\text{with } \vec{\ell}^2 = (\vec{j} - \vec{s})^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{s} + \vec{s}^2 \quad \vec{s}^2 = (\vec{j} - \vec{\ell})^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{\ell} + \vec{\ell}^2$$

$$\vec{\mu}_j = \frac{g_\ell \cdot \{j(j+1) + \ell(\ell+1) - 3/4\} + g_s \cdot \{j(j+1) - \ell(\ell+1) + 3/4\}}{2 \cdot j(j+1)} \cdot \vec{j}$$

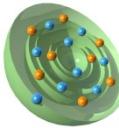
$$g_j = \frac{1}{2} \cdot (g_\ell + g_s) + \frac{1}{2} \cdot \frac{\ell(\ell+1) - s(s+1)}{2j(j+1)} \cdot (g_\ell - g_s)$$

Simple relation for the g-factor
of single-particle states

$$\frac{\mu}{\mu_N} = g_{nucleus} = g_\ell \pm \frac{(g_s - g_\ell)}{2\ell + 1} \quad \text{for } j = \ell \pm 1$$

nucleus	state	J^π	μ/μ_N model	μ/μ_N experiment
^{15}N	p-1p $^{-1}_{1/2}$	1/2 $^-$	-0,264	-0,283
^{15}O	n-1p $^{-1}_{1/2}$	1/2 $^-$	+0,638	+0,719
^{17}O	n-1d $_{5/2}$	5/2 $^+$	-1,913	-1,894
^{17}F	p-1d $_{5/2}$	5/2 $^+$	+4,722	+4,793

Success of the extreme single-particle shell model



➤ **magnetic moments:**

$$\langle \mu_z \rangle = \begin{cases} g_\ell \cdot \left(j - \frac{1}{2} \right) + \frac{1}{2} \cdot g_s \cdot \mu_N & \text{for } j = \ell + 1/2 \\ \frac{j}{j+1} \cdot \left[g_\ell \cdot \left(j + \frac{3}{2} \right) - \frac{1}{2} \cdot g_s \right] \cdot \mu_N & \text{for } j = \ell - 1/2 \end{cases}$$

➤ **g-factor of nucleons:**

proton: $g_\ell = 1$; $g_s = +5.585$

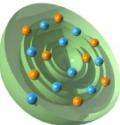
neutron: $g_\ell = 0$; $g_s = -3.82$

proton:

$$\langle \mu_z \rangle = \begin{cases} (j + 2.293) \cdot \mu_N & \text{for } j = \ell + 1/2 \\ (j - 2.293) \cdot \frac{j}{j+1} \cdot \mu_N & \text{for } j = \ell - 1/2 \end{cases}$$

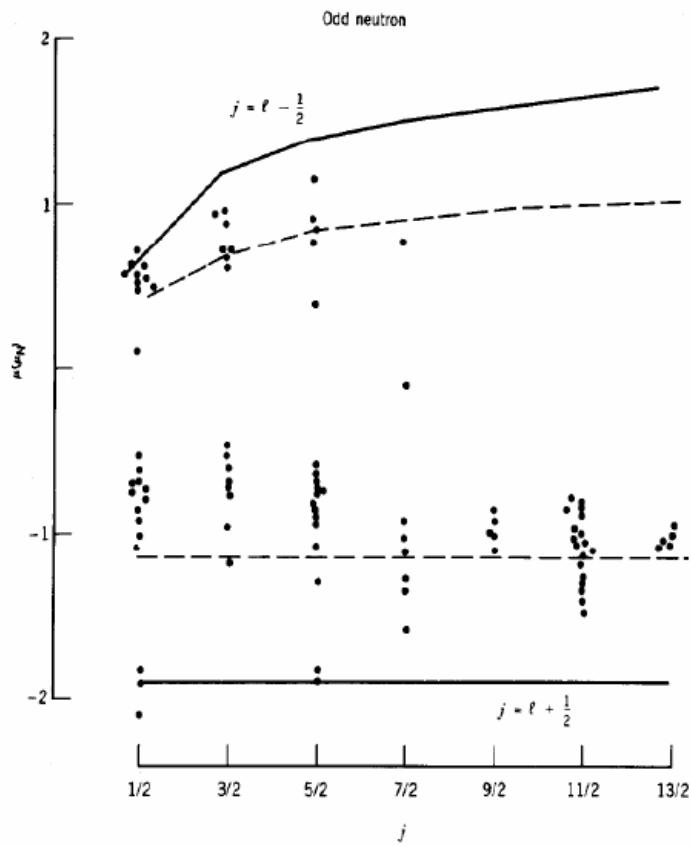
neutron:

$$\langle \mu_z \rangle = \begin{cases} -1.91 \cdot \mu_N & \text{for } j = \ell + 1/2 \\ +1.91 \cdot \frac{j}{j+1} \cdot \mu_N & \text{for } j = \ell - 1/2 \end{cases}$$

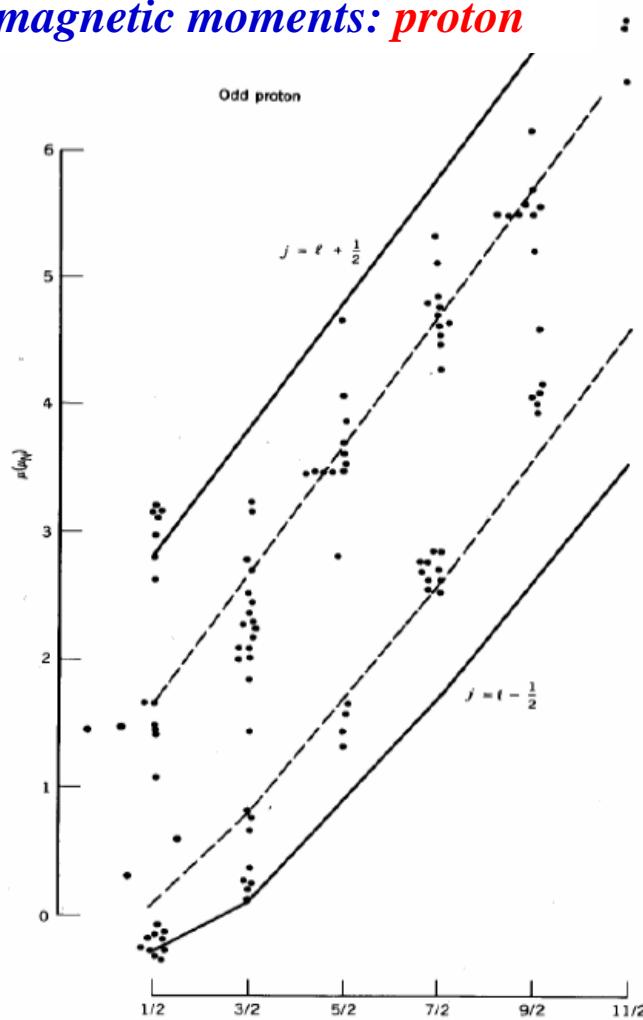


Magnetic moments: Schmidt lines

magnetic moments: neutron



magnetic moments: proton



Single particle transition (Weisskopf estimate)

$$B(E\lambda; I_i \rightarrow I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi} \left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} e^2 (fm)^{2\lambda}$$

$$B(M\lambda; I_i \rightarrow I_{gs}) = \frac{10}{\pi} (1.2)^{2\lambda-2} \left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \mu_N^2 (fm)^{2\lambda-2}$$

For the first few values of λ , the Weisskopf estimates are

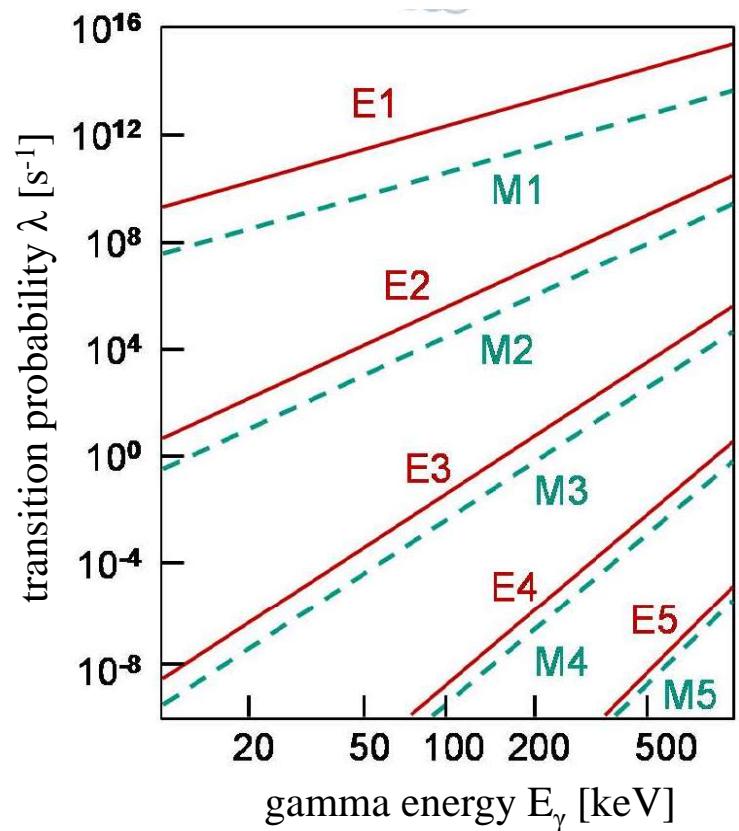
$$B(E1; I_i \rightarrow I_{gs}) = 6.446 \cdot 10^{-4} A^{2/3} e^2 (barn)$$

$$B(E2; I_i \rightarrow I_{gs}) = 5.940 \cdot 10^{-6} A^{4/3} e^2 (barn)^2$$

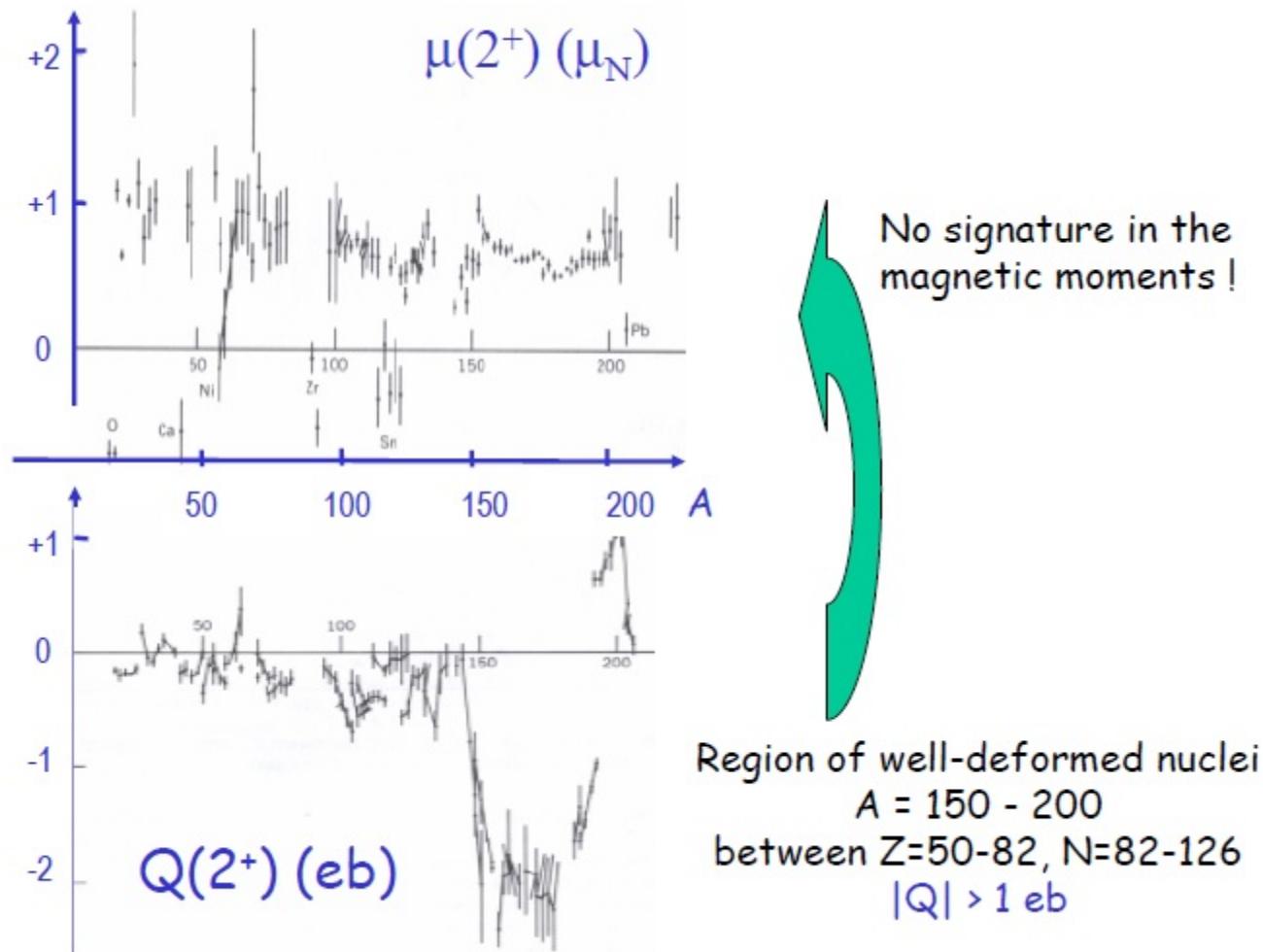
$$B(E3; I_i \rightarrow I_{gs}) = 5.940 \cdot 10^{-8} A^2 e^2 (barn)^3$$

$$B(E4; I_i \rightarrow I_{gs}) = 6.285 \cdot 10^{-10} A^{8/3} e^2 (barn)^4$$

$$B(M1; I_i \rightarrow I_{gs}) = 1.790 \left(\frac{e\hbar}{2Mc}\right)^2$$



Magnetic moments are not sensitive to deformation or collectivity

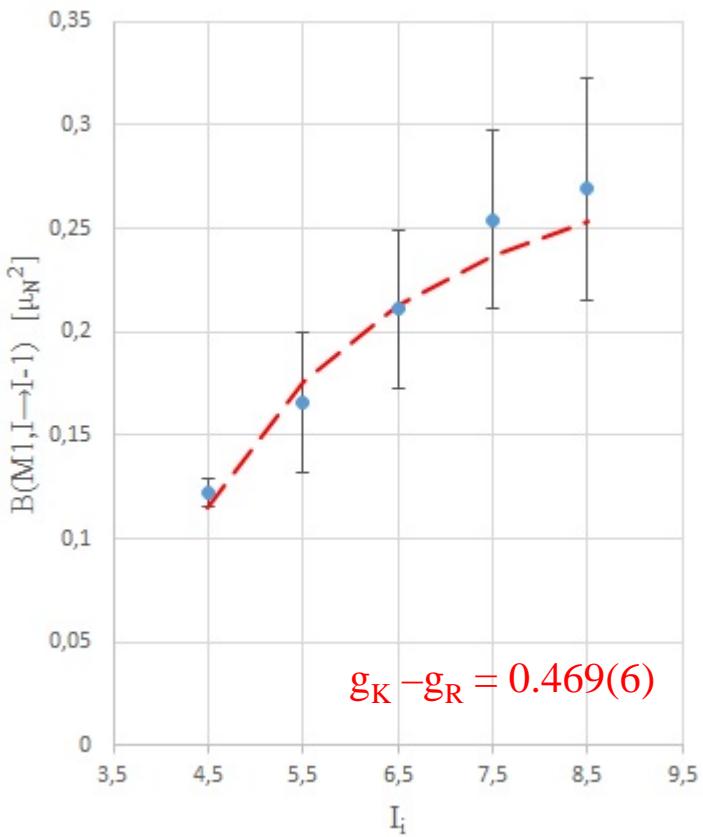


Odd-even nucleus: ^{181}Ta

74	W	^{159}W	^{158}W	^{159}W	^{160}W	^{161}W	^{162}W	^{163}W	^{164}W	^{165}W	^{166}W	^{167}W	^{168}W	^{169}W	^{170}W	^{171}W	^{172}W	^{173}W	^{174}W	^{175}W	^{176}W	^{177}W	^{178}W	^{179}W	^{180}W	^{181}W	^{182}W	^{183}W	^{184}W	^{185}W	^{186}W	^{187}W	^{188}W			
73	Ta	^{154}Ta	^{155}Ta	^{156}Ta	^{157}Ta	^{158}Ta	^{159}Ta	^{160}Ta	^{161}Ta	^{162}Ta	^{163}Ta	^{164}Ta	^{165}Ta	^{166}Ta	^{167}Ta	^{168}Ta	^{169}Ta	^{170}Ta	^{171}Ta	^{172}Ta	^{173}Ta	^{174}Ta	^{175}Ta	^{176}Ta	^{177}Ta	^{178}Ta	^{179}Ta	^{180}Ta	^{181}Ta	^{182}Ta	^{183}Ta	^{184}Ta	^{185}Ta	^{186}Ta	^{187}Ta	^{188}Ta
72	Hf	^{154}Hf	^{155}Hf	^{156}Hf	^{157}Hf	^{158}Hf	^{159}Hf	^{160}Hf	^{161}Hf	^{162}Hf	^{163}Hf	^{164}Hf	^{165}Hf	^{166}Hf	^{167}Hf	^{168}Hf	^{169}Hf	^{170}Hf	^{171}Hf	^{172}Hf	^{173}Hf	^{174}Hf	^{175}Hf	^{176}Hf	^{177}Hf	^{178}Hf	^{179}Hf	^{180}Hf	^{181}Hf	^{182}Hf	^{183}Hf	^{184}Hf	^{185}Hf	^{186}Hf	^{187}Hf	^{188}Hf
71	Lu	^{154}Lu	^{155}Lu	^{156}Lu	^{157}Lu	^{158}Lu	^{159}Lu	^{160}Lu	^{161}Lu	^{162}Lu	^{163}Lu	^{164}Lu	^{165}Lu	^{166}Lu	^{167}Lu	^{168}Lu	^{169}Lu	^{170}Lu	^{171}Lu	^{172}Lu	^{173}Lu	^{174}Lu	^{175}Lu	^{176}Lu	^{177}Lu	^{178}Lu	^{179}Lu	^{180}Lu	^{181}Lu	^{182}Lu	^{183}Lu	^{184}Lu	^{185}Lu	^{186}Lu	^{187}Lu	^{188}Lu

$$\langle I-1, K | M(M1) | I, K \rangle = -\sqrt{\frac{3}{4\pi}} \sqrt{\frac{(I+K)(I-K)}{I}} \cdot K \cdot (g_K - g_R) [1 + \delta_{K,1/2} (-1)^{I+1/2} b_0] \mu_N$$

The quantity b_0 depends on the magnetic decoupling parameter



M. Loewe; dissertation

$$\mu(7/2^+) = 2.3705 \pm 0.0007$$

$$\mu = \frac{K}{I+1} \cdot (g_K - g_R) \cdot K + g_R \cdot I$$

$$g_R = 0.313(5)$$

$$g_K = 0.782(2)$$

$$\mu(9/2^-) = 5.28 \pm 0.09$$

$$^{9/2^-}(g_K - g_R) = \frac{22}{81} \left(^{9/2^-} \mu - ^{7/2^+} \mu \frac{9}{7} + ^{7/2^+} (g_K - g_R) \frac{7}{2} \right)$$

$$^{9/2^-}(g_K - g_R) \approx ^{7/2^+} (g_K - g_R) \frac{77}{81} + 0.606 = 1.052$$

Odd-even nucleus: ^{181}Ta

74	^{181}W	$^{181}\text{W158}$	$^{181}\text{W159}$	$^{181}\text{W160}$	$^{181}\text{W161}$	$^{181}\text{W162}$	$^{181}\text{W163}$	$^{181}\text{W164}$	$^{181}\text{W165}$	$^{181}\text{W166}$	$^{181}\text{W167}$	$^{181}\text{W168}$	$^{181}\text{W169}$	$^{181}\text{W170}$	$^{181}\text{W171}$	$^{181}\text{W172}$	$^{181}\text{W173}$	$^{181}\text{W174}$	$^{181}\text{W175}$	$^{181}\text{W176}$	$^{181}\text{W177}$	$^{181}\text{W178}$	$^{181}\text{W179}$	$^{181}\text{W180}$	$^{181}\text{W181}$	$^{181}\text{W182}$	$^{181}\text{W183}$	$^{181}\text{W184}$	$^{181}\text{W185}$	$^{181}\text{W186}$	$^{181}\text{W187}$	$^{181}\text{W188}$	$^{181}\text{W189}$			
73	^{181}Ta	$^{181}\text{Ta156}$	$^{181}\text{Ta157}$	$^{181}\text{Ta158}$	$^{181}\text{Ta159}$	$^{181}\text{Ta160}$	$^{181}\text{Ta161}$	$^{181}\text{Ta162}$	$^{181}\text{Ta163}$	$^{181}\text{Ta164}$	$^{181}\text{Ta165}$	$^{181}\text{Ta166}$	$^{181}\text{Ta167}$	$^{181}\text{Ta168}$	$^{181}\text{Ta169}$	$^{181}\text{Ta170}$	$^{181}\text{Ta171}$	$^{181}\text{Ta172}$	$^{181}\text{Ta173}$	$^{181}\text{Ta174}$	$^{181}\text{Ta175}$	$^{181}\text{Ta176}$	$^{181}\text{Ta177}$	$^{181}\text{Ta178}$	$^{181}\text{Ta179}$	$^{181}\text{Ta180}$	$^{181}\text{Ta181}$	$^{181}\text{Ta182}$	$^{181}\text{Ta183}$	$^{181}\text{Ta184}$	$^{181}\text{Ta185}$	$^{181}\text{Ta186}$	$^{181}\text{Ta187}$	$^{181}\text{Ta188}$		
72	^{181}Hf	$^{181}\text{Hf154}$	$^{181}\text{Hf155}$	$^{181}\text{Hf156}$	$^{181}\text{Hf157}$	$^{181}\text{Hf158}$	$^{181}\text{Hf159}$	$^{181}\text{Hf160}$	$^{181}\text{Hf161}$	$^{181}\text{Hf162}$	$^{181}\text{Hf163}$	$^{181}\text{Hf164}$	$^{181}\text{Hf165}$	$^{181}\text{Hf166}$	$^{181}\text{Hf167}$	$^{181}\text{Hf168}$	$^{181}\text{Hf169}$	$^{181}\text{Hf170}$	$^{181}\text{Hf171}$	$^{181}\text{Hf172}$	$^{181}\text{Hf173}$	$^{181}\text{Hf174}$	$^{181}\text{Hf175}$	$^{181}\text{Hf176}$	$^{181}\text{Hf177}$	$^{181}\text{Hf178}$	$^{181}\text{Hf179}$	$^{181}\text{Hf180}$	$^{181}\text{Hf181}$	$^{181}\text{Hf182}$	$^{181}\text{Hf183}$	$^{181}\text{Hf184}$	$^{181}\text{Hf185}$	$^{181}\text{Hf186}$	$^{181}\text{Hf187}$	$^{181}\text{Hf188}$

$$13/2^+ \quad 0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$$

$$\tau = \left\{ \sum_K \sum_\ell [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

$$11/2^+ \quad 0.302 \quad \tau = 23.1 \pm 4.3 \text{ ps}$$

$$\tau = T_{1/2} / \ln 2$$

$$9/2^+ \quad 0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$$

$$7/2^+ \quad 0.0$$

^{181}Ta

$$\delta_{N \rightarrow M}(\lambda) = \left\{ \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\lambda+1} \right\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(\lambda) || I_N \rangle$$

$$\delta_{N \rightarrow M}(E2) = \{1.225 \cdot 10^{13} \cdot E_\gamma^5 (\text{MeV})^5\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(E2) || I_N \rangle$$

$$\delta_{N \rightarrow M}(M1) = \{1.758 \cdot 10^{13} \cdot E_\gamma^3 (\text{MeV})^3\}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M || \mathcal{M}(M1) || I_N \rangle$$

$$\varepsilon_{N \rightarrow M}^2(\ell) = \delta_{N \rightarrow M}^2(\ell) \cdot \alpha_{N \rightarrow M}(\ell)$$

T. Inamura et al., Nucl. Phys. A270 (1976) 255

conversion coefficient.: bricc.anu.edu.au

Odd-even nucleus: ^{181}Ta

74	^{181}W	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190		
	1.22(24) 1.32(24)	0.5 ms 0+	7.3 ms 0+	9.1 ms 0+	4.0 ms 0+	1.39 s 0+	2.78 s 0+	5.1 s 0+	1.83 s (0.9 s) 0+	5.1 s 0+	1.93 s (0.9 s) 0+	5.3 s 0+	2.43 s (0.9 s) 0+	2.38 s (0.9 s) 0+	2.4 s (0.9 s) 0+	2.3 s (0.9 s) 0+																				
73	^{181}Ta	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188	116	
	1.04 ms (0.9 s) 0+	1.01 ms (0.9 s) 0+	0.45 ms (0.9 s) 0+	0.57 s 0+	1.35 s (0.9 s) 0+	2.7 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+	1.04 s 0+				
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188
	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+	0.39 s 0+				

$$13/2^+ \quad 0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$$

$$\tau = \left\{ \sum_K \sum_\ell [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

$$11/2^+ \quad 0.302 \quad \tau = 23.1 \pm 4.3 \text{ ps}$$

$$\tau = T_{1/2} / \ln 2$$

$$9/2^+ \quad 0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$$

$$7/2^+ \quad 0.0 \quad \text{E2 + M1}$$

^{181}Ta

Spin I	E_γ (MeV)	$(2I+1)^{-1/2}$	$\langle I-1/M(0)/I \rangle$	delta	α_T	ε^2	τ (ps)
9/2	0.1365	0.3162	-3.899 (E2)	-29594.	1.1	$9.98 \cdot 10^8$	533
			1.103 (M1)	73591	1.8	$9.75 \cdot 10^9$	58.7
11/2	0.1654	0.2887	-4.291 (E2)	-48238	0.6	$1.33 \cdot 10^9$	274
			1.413 (M1)	115044	1.0	$1.38 \cdot 10^{10}$	32.6
11/2	0.3017	0.2887	1.977 (E2)	99868	0.1	$8.08 \cdot 10^8$	24.1

T. Inamura et al., Nucl. Phys. A270 (1976) 255

Oddproton-evenneutron nuclei

		μ	$(g_K - g_R)$	$B(M1)W.u.$	$(g_K - g_R)$
^{153}Eu	$5/2^+$	+1.5324(3)	0.551	0.00608(28)	0.185
^{159}Tb	$3/2^+$	+2.014(4)	1.556	0.173(8)	1.471
^{165}Ho	$7/2^-$	+4.177(5)	1.012	0.275(14)	0.973
^{169}Tm	$1/2^+$	-0.2316(15)		0.0342(8)	
^{175}Lu	$7/2^+$	+2.2327(11)	0.298	0.0354(14)	0.349
^{181}Ta	$7/2^+$	+2.3705(7)	0.352	0.068(4)	0.484
^{185}Re	$5/2^+$	+3.1871(3)	1.217	0.28(5)	1.252
^{187}Re	$5/2^+$	+3.2197(3)	1.242	0.260(18)	1.206