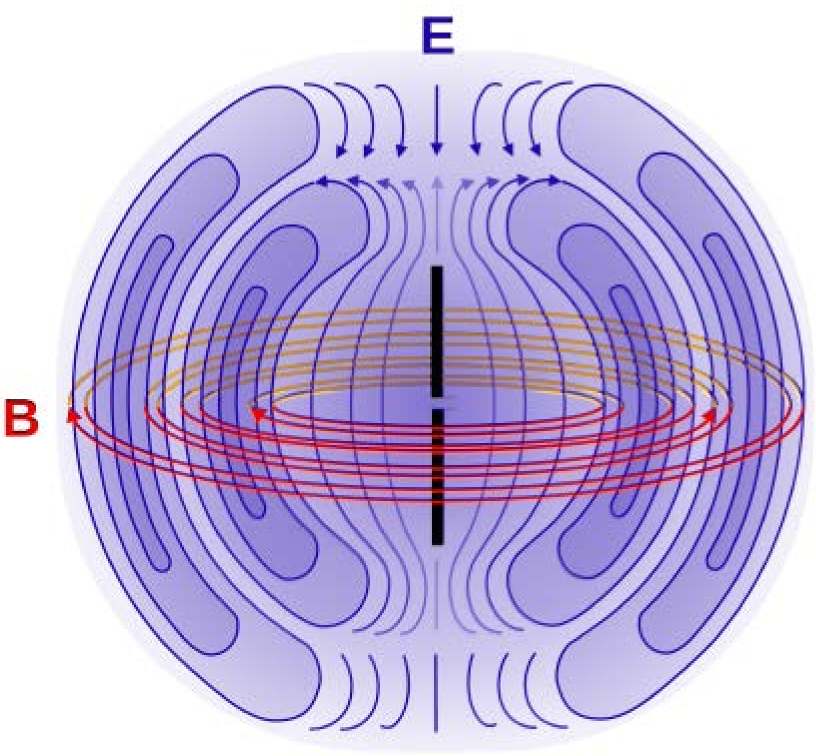
Electric/magnetic dipoles

Electric and magnetic dipole fields have opposite parity:
Magnetic dipoles have even parity and electric dipole fields have odd parity.

$$\Rightarrow \ \pi(M\ell) = (-1)^{\ell+1} \ \ and \ \ \pi(E\ell) = (-1)^{\ell}$$



Classical electrodynamics

- The nucleus is a collection of moving charges, which can induce magnetic/electric fields
- The power radiated into a small area element is proportional to $sin^2(\theta)$
- The average power radiated for an **electric dipole** is:

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^3}d^2$$

- For a **magnetic dipole** is

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^5}\mu^2$$

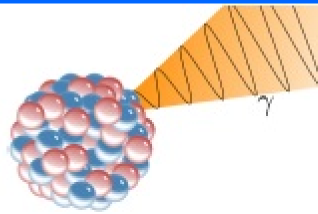
Higher order multipoles

It is possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials.

- ℓ: The index of radiation
  2^ℓ: The multipole order of the radiation

- $\ell = 1 \quad \rightarrow \quad Dipole$
  $\ell = 2 \quad \rightarrow \quad Quadrupole$
  $\ell = 3 \quad \rightarrow \quad Octupole$

- The associated Legendre polynomials $P_{2\ell}\left(cos(\theta)\right)$ are:
  For $\ell = 1$: $P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1)$
  For $\ell = 2$: $P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$



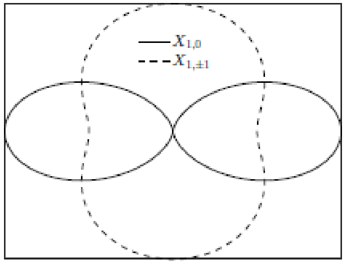
- *The photon is a spin-1 boson*
- Like α-decay and β-decay the emitted γ-ray can carry away units of *angular momentum ℓ*, which has given us different multipolarities for transitions.
- For orbital angular momentum, we can have values $\ell = 0,1,2,3,\cdots$ that correspond to our multipolarity.
- Therefore, our selection rule is:

$$\left|I_i - I_f\right| \leq \ell \leq I_i + I_f$$

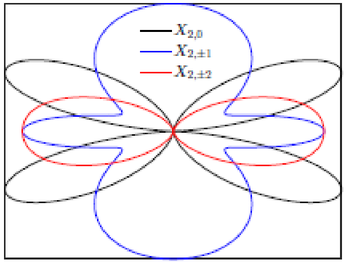
Characteristics of multipolarity selection rules

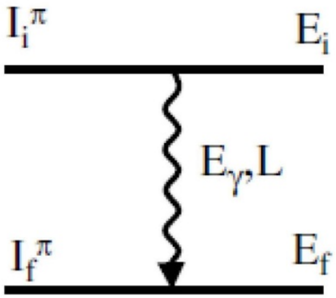
L	multipolarity	$\pi(E\ell)$ / $\pi(M\ell)$	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

$\ell = 1$



$\ell = 2$





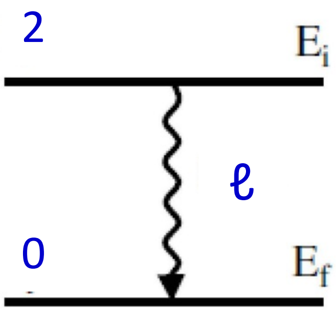
$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

$$\Delta\pi(E\ell) = (-1)^\ell$$

$$\Delta\pi(M\ell) = (-1)^{\ell+1}$$

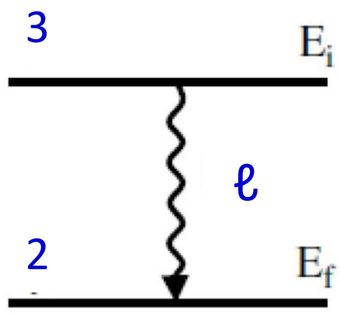
The basics of the situation



$$|2-0| \leq \ell \leq 2+0$$

Here $\Delta I = 2$ and $\ell = 2$
this is a stretched transition

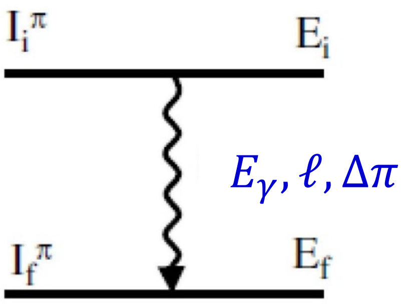
The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$
and the transition can be a mix of 5 multipolarities

The basics of the situation



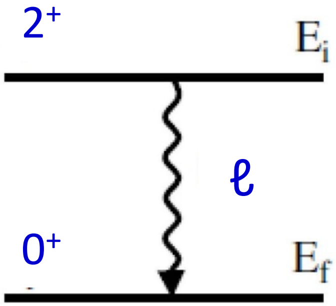
Electromagnetic transitions:

$$\Delta\pi\ (electric) = (-1)^{\ell}$$

$$\Delta\pi\ (magnetic) = (-1)^{\ell+1}$$

$\Delta\pi$	*yes*	E1	M2	E3	M4
	no	M1	E2	M3	E4

The basics of the situation

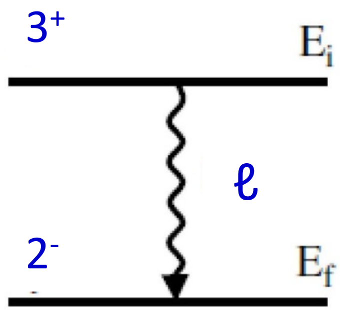


$$|2-0| \leq \ell \leq 2+0$$

$\ell = 2$ and no change in parity

$\Delta\pi$					
	no	~~M1~~	E2	M3	E4

The basics of the situation



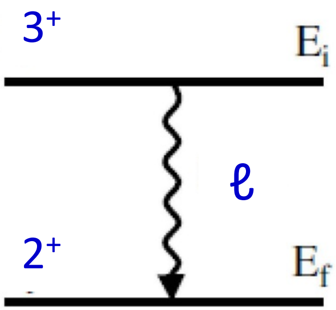
$$|3 - 2| \leq \ell \leq 3 + 2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$	*yes*	E1	M2	E3	M4

mixed E1,M2,E3,M4,E5

The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta J = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$					
	no	M1	E2	M3	E4

mixed M1,E2,M3,E4,M5

The basics of the situation

$3^+ \rightarrow 2^+$: mixed M1,E2,M3,E4,M5

$3^+ \rightarrow 2^-$: mixed E1,M2,E3,M4,E5

In general only the lowest 2 multipoles compete

and (for reasons we will see later)

$\ell + 1$ multipole generally only competes if it is electric:

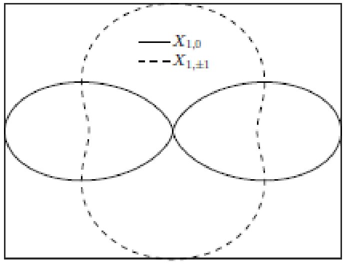
$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (very little M2 admixture)

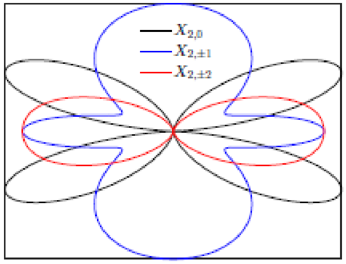
Characteristics of multipolarity

L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

ℓ = 1



ℓ =2



parity: electric multipoles $\pi(E\ell) = (-1)^{\ell}$, magnetic multipoles $\pi(M\ell) = (-1)^{\ell+1}$

The power radiated is proportional to:

$$P(\sigma\ell) \propto \frac{2(\ell+1)\cdot c}{\varepsilon_0 \cdot \ell \cdot [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} |\mathcal{M}(\sigma\ell)|^2$$

where σ means either E or M and $\mathcal{M}(\sigma\ell)$ is the E or M multipole moment of the appropriate kind.

Emission of electromagnetic radiation

$$T(E1; I_i \rightarrow I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \rightarrow I_f)$$

$$T(E2; I_i \rightarrow I_f) = 1.225\ 10^{13}\ E_\gamma^5\ B(E2; I_i \rightarrow I_f)$$

$$T(E3; I_i \rightarrow I_f) = 5.709\ 10^{8}\ E_\gamma^7\ B(E3; I_i \rightarrow I_f)$$

$$T(E4; I_i \rightarrow I_f) = 1.697\ 10^{4}\ E_\gamma^9\ B(E4; I_i \rightarrow I_f)$$

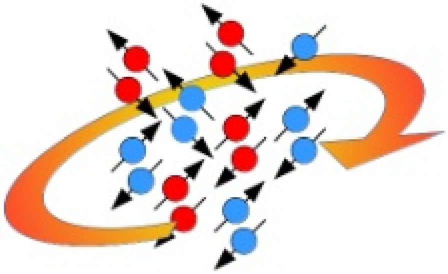
$$T(M1; I_i \rightarrow I_f) = 1.758\ 10^{13}\ E_\gamma^3\ B(M1; I_i \rightarrow I_f)$$

$$T(M2; I_i \rightarrow I_f) = 1.355\ 10^{7}\ E_\gamma^5\ B(M2; I_i \rightarrow I_f)$$

$$T(M3; I_i \rightarrow I_f) = 6.313\ 10^{0}\ E_\gamma^7\ B(M3; I_i \rightarrow I_f)$$

$$T(M4; I_i \rightarrow I_f) = 1.877\ 10^{-6}\ E_\gamma^9\ B(M4; I_i \rightarrow I_f)$$

where $E_\gamma = E_i - E_f$ is the energy of the emitted γ quantum in MeV (E_i, E_f are the nuclear level energies, respectively), and the reduced transition probabilities B(Eℓ) in units of $e^2(\text{barn})^\ell$ and B(Mℓ) in units of $\mu_N^2 = (e\hbar/2m_N c)^2\ \ (fm)^{2\ell-2}$



Nuclear magnetic dipole moments arise from

- intrinsic spin magnetic dipole moments of the protons and neutrons
- circulating currents (motion of the protons)

The nuclear magnetic dipole moment can be written as

$$\vec{\mu} = \frac{\mu_N}{\hbar} \sum_i \left[g_L \vec{L} + g_s \vec{s} \right]$$ summed over all p, n

where $\mu_N = e\hbar/2m_p$ is the Nuclear Magneton.

or $\mu = g_J \mu_N J$ where J total nuclear spin quantum number

g_J nuclear g-factor (analogous to Landé g-factor in atoms)

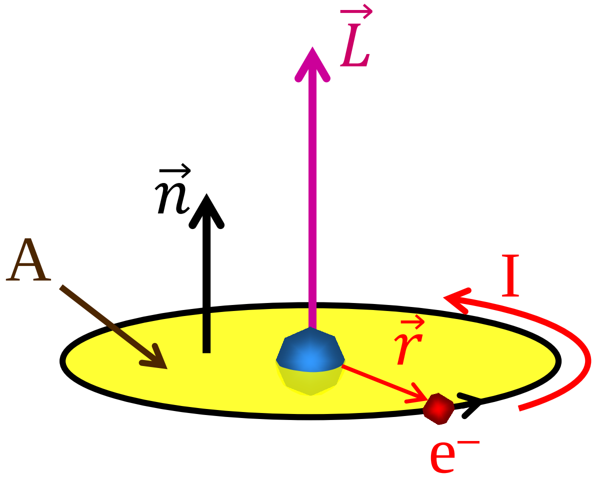
g_J may be predicted using the Nuclear Shell Model (see later), and measured using magnetic resonance (see Advanced Quantum course).

All even-even nuclei have $\mu = 0$ since $J = 0$

Magnetic moment classical description

e^--movement $\Rightarrow$ magnetic moment $\vec{\mu_e}$

$$\left.\begin{aligned}\vec{\mu_e} &= \frac{I}{c}\cdot A\cdot\vec{n}\\ I &= -e\cdot\nu = -e\cdot\frac{v}{2\pi r}\\ A &= \pi\cdot r^2\end{aligned}\right\} \Rightarrow \vec{\mu_e} = -\frac{1}{2}\cdot\frac{e\cdot v\cdot r\cdot\vec{n}}{c}$$



angular momentum: $\vec{L} = \vec{r}\times\vec{p} = r\cdot m_e\cdot v\cdot\vec{n} \Rightarrow \vec{\mu_e} = -\frac{e}{2\cdot m_e\cdot c}\vec{L}$

Quantum mechanical description: angular momentum will be quantized, in units of ħ.

$$\mu_B = \frac{e\cdot\hbar}{2\cdot m_e\cdot c} = 5.788\cdot 10^{-11}\ \frac{MeV}{Tesla}$$

$$\mu_K = \frac{e\cdot\hbar}{2\cdot m_p\cdot c} = 3.152\cdot 10^{-14}\ \frac{MeV}{Tesla}$$

Quark model

	Charge	Mass m_q	Spin	mag. Moment
u-quark	+ 2/3	~ 1/3 M_p	1/2	$2 \cdot \left(\frac{2}{3}\right) \cdot \frac{e \cdot \hbar}{2 \cdot m_q \cdot c} \cdot s = 4 \cdot \mu_K \cdot s$
d-quark	- 1/3	~ 1/3 M_p	1/2	$2 \cdot \left(-\frac{1}{3}\right) \cdot \frac{e \cdot \hbar}{2 \cdot m_q \cdot c} \cdot s = -2 \cdot \mu_K \cdot s$

general coupling rule:

$$g(j_1 \times j_2; J) = \frac{1}{2}(g_1 + g_2) + \frac{j_1(j_1+1) - j_2(j_2+1)}{2 \cdot J \cdot (J+1)} \cdot (g_1 - g_2)$$

Proton (uud) :

$$g(1/2 \times 1/2; 1) = \frac{1}{2}(4+4) + \frac{1/2 \cdot 3/2 - 1/2 \cdot 3/2}{2 \cdot 1 \cdot 2} \cdot (4-4) = 4$$

$$g(1 \times 1/2; 1/2) = \frac{1}{2}(4-2) + \frac{2 - 3/4}{3/2} \cdot (4+2) = \boxed{+6}$$

Neutron (ddu) :

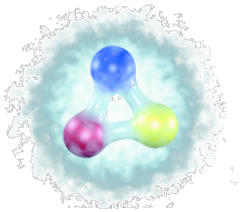
$$g(1/2 \times 1/2; 1) = \frac{1}{2}(-2-2) + \frac{1/2 \cdot 3/2 - 1/2 \cdot 3/2}{2 \cdot 1 \cdot 2} \cdot (-2+2) = -2$$

$$g(1 \times 1/2; 1/2) = \frac{1}{2}(-2+4) + \frac{2 - 3/4}{3/2} \cdot (-2-4) = \boxed{-4}$$

$g_{Dirac} = 2$ prediction of the Dirac-theory for the gyromagnetic factor for spin ½-particles

$g_s^{proton} = +5.58 \Rightarrow$ big deviation from g = 2 $\Leftrightarrow$ no fundamental particle

$g_s^{neutron} = -3.82$



Success of the extreme single-particle shell model

➢ ***Magnetic moments:***

The g-factor g_j is given by: $\overrightarrow{\mu_j} = g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} = g_j \cdot \vec{j}$ $\quad \Rightarrow \quad \overrightarrow{\mu_j} = \left[\left(g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} \right) \cdot \frac{\vec{j}}{|j|} \right] \cdot \frac{\vec{j}}{|j|}$

with $\vec{\ell}^2 = (\vec{j} - \vec{s})^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{s} + \vec{s}^2$ $\quad \vec{s}^2 = \left(\vec{j} - \vec{\ell} \right)^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{\ell} + \vec{\ell}^2$

$$\vec{\mu}_j = \frac{g_\ell \cdot \{j(j+1) + \ell(\ell+1) - 3/4\} + g_s \cdot \{j(j+1) - \ell(\ell+1) + 3/4\}}{2 \cdot j(j+1)} \cdot \vec{j}$$

$$g_j = \frac{1}{2} \cdot (g_\ell + g_s) + \frac{1}{2} \cdot \frac{\ell(\ell+1) - s(s+1)}{2j(j+1)} \cdot (g_\ell - g_s)$$

Simple relation for the g-factor of single-particle states

$$\frac{\mu}{\mu_N} = g_{nucleus} = g_\ell \pm \frac{(g_s - g_\ell)}{2\ell + 1} \quad for \quad j = \ell \pm 1$$

nucleus	state	J^π	μ/μ_N model	μ/μ_N experiment
^{15}N	p-$1p_{1/2}^{-1}$	$1/2^-$	$-0,264$	$-0,283$
^{15}O	n-$1p_{1/2}^{-1}$	$1/2^-$	$+0,638$	$+0,719$
^{17}O	n-$1d_{5/2}$	$5/2^+$	$-1,913$	$-1,894$
^{17}F	p-$1d_{5/2}$	$5/2^+$	$+4,722$	$+4,793$

Success of the extreme single-particle shell model

- ***magnetic moments:***

$$\langle \mu_z \rangle = \begin{cases} \left[g_\ell \cdot \left(j - \frac{1}{2} \right) + \frac{1}{2} \cdot g_s \right] \cdot \mu_N & for \quad j = \ell + 1/2 \\ \frac{j}{j+1} \cdot \left[g_\ell \cdot \left(j + \frac{3}{2} \right) - \frac{1}{2} \cdot g_s \right] \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

- ***g-factor of nucleons:***

proton: $g_\ell = 1$; $g_s = +5.585$

neutron: $g_\ell = 0$; $g_s = -3.82$

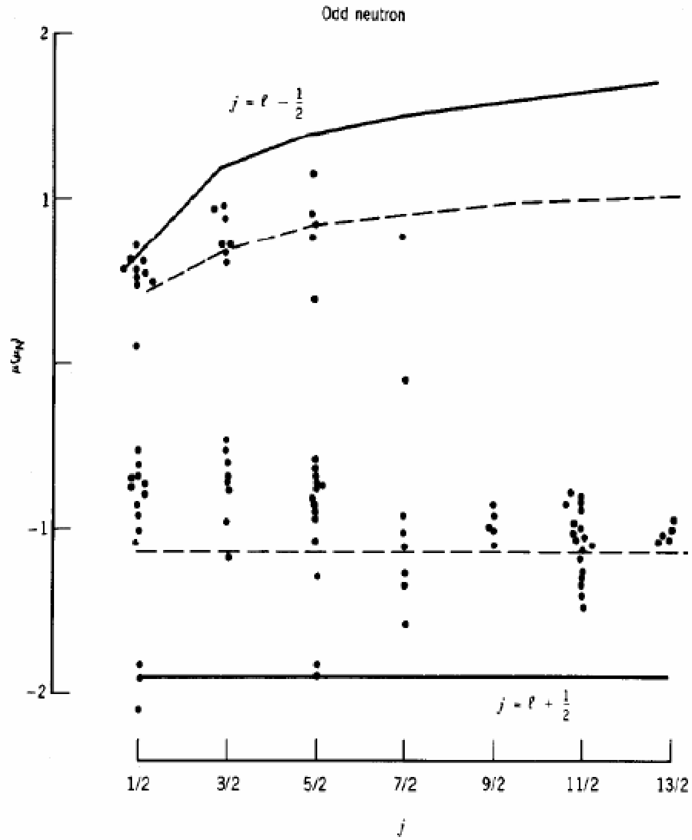
proton:

$$\langle \mu_z \rangle = \begin{cases} (j + 2.293) \cdot \mu_N & for \quad j = \ell + 1/2 \\ (j - 2.293) \cdot \frac{j}{j+1} \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

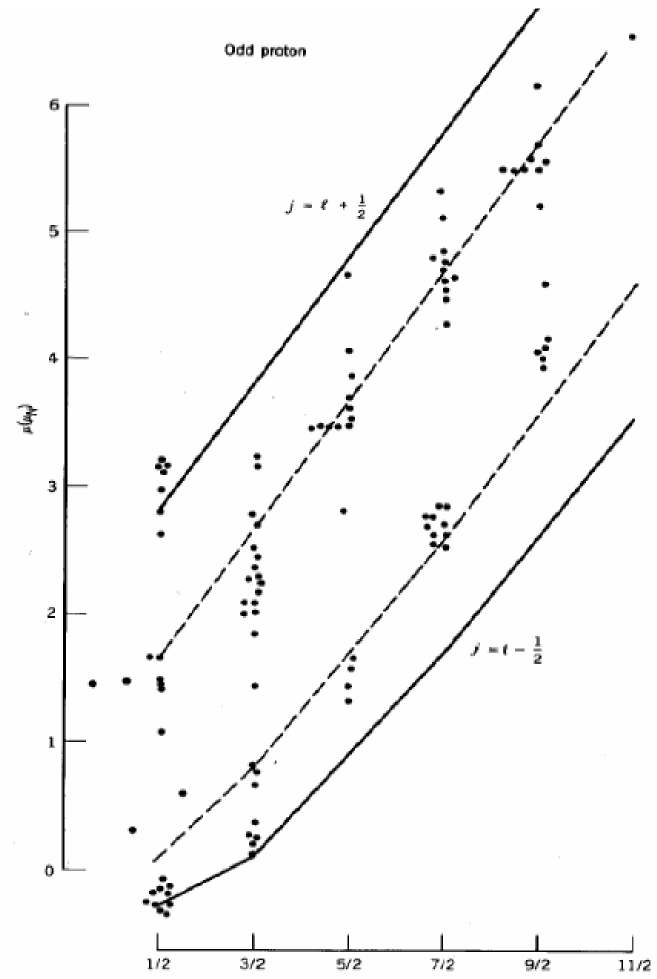
neutron:

$$\langle \mu_z \rangle = \begin{cases} -1.91 \cdot \mu_N & for \quad j = \ell + 1/2 \\ +1.91 \cdot \frac{j}{j+1} \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

magnetic moments: neutron



magnetic moments: proton



Single particle transition (Weisskopf estimate)

$$B(E\lambda; I_i \rightarrow I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} \quad e^2(fm)^{2\lambda}$$

$$B(M\lambda; I_i \rightarrow I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}\left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \quad \mu_N^2(fm)^{2\lambda-2}$$

For the first few values of λ, the Weisskopf estimates are

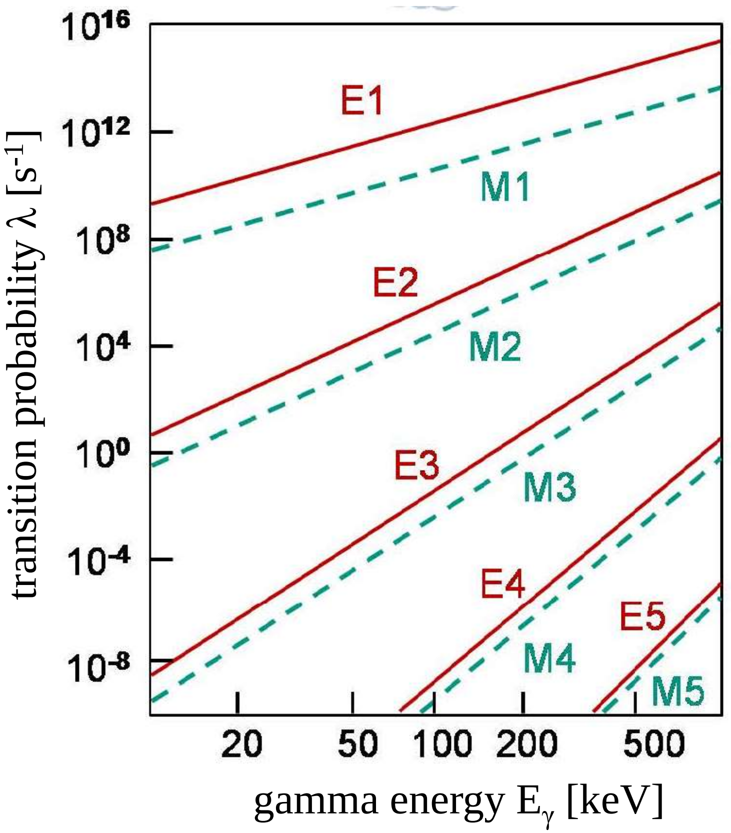
$$B(E1; I_i \rightarrow I_{gs}) = 6.446\ 10^{-4}\ A^{2/3} \quad e^2(barn)$$

$$B(E2; I_i \rightarrow I_{gs}) = 5.940\ 10^{-6}\ A^{4/3} \quad e^2(barn)^2$$

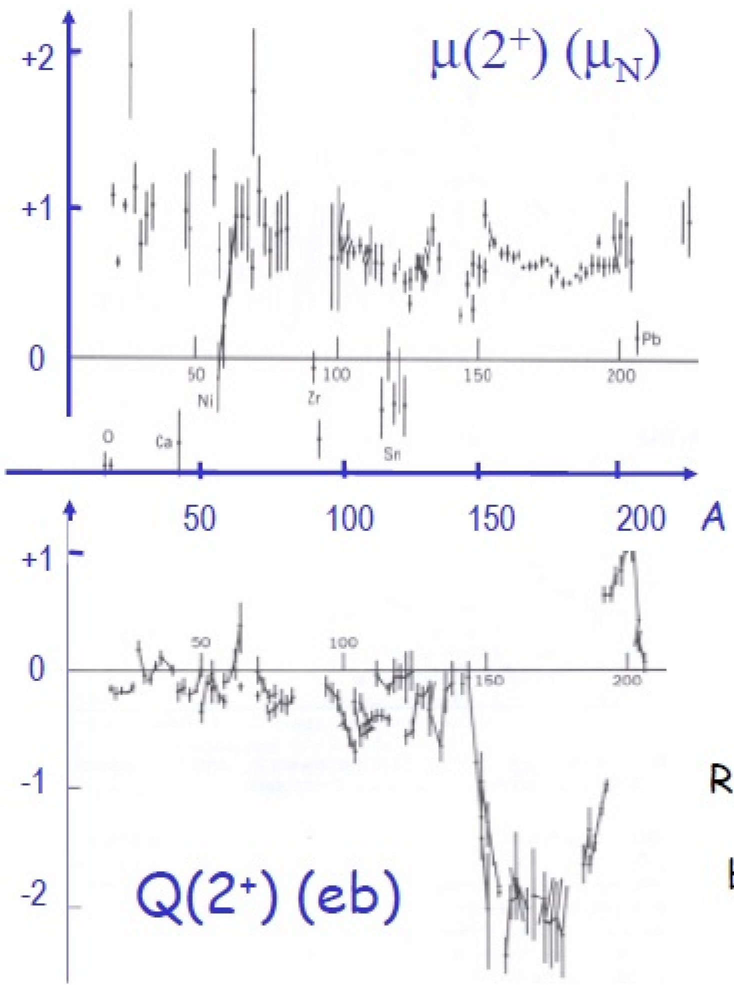
$$B(E3; I_i \rightarrow I_{gs}) = 5.940\ 10^{-8}\ A^{2} \quad e^2(barn)^3$$

$$B(E4; I_i \rightarrow I_{gs}) = 6.285\ 10^{-10}\ A^{8/3} \quad e^2(barn)^4$$

$$B(M1; I_i \rightarrow I_{gs}) = 1.790 \quad \left(\frac{e\hbar}{2Mc}\right)^2$$



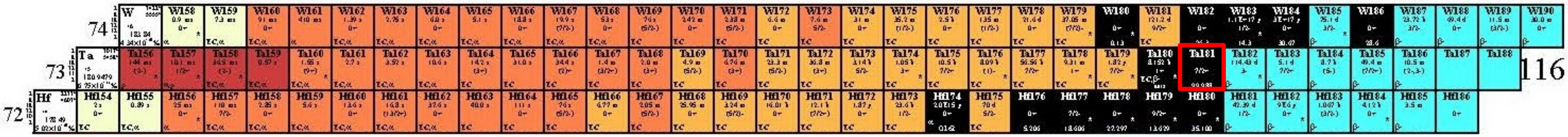
Magnetic moments are not sensitive to deformation or collectivity



No signature in the magnetic moments !

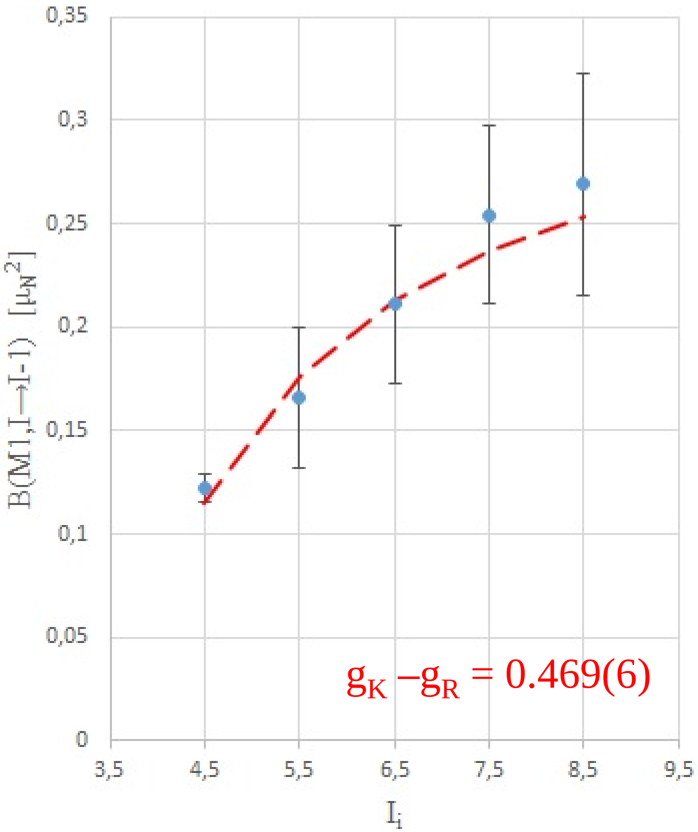
Region of well-deformed nuclei
A = 150 - 200
between Z=50-82, N=82-126
|Q| > 1 eb

Odd-even nucleus: ^{181}Ta



$$\langle I-1, K \| M(M1) \| I, K \rangle = -\sqrt{\frac{3}{4\pi}}\sqrt{\frac{(I+K)(I-K)}{I}} \cdot K \cdot (g_K - g_R)\left[1 + \delta_{K,1/2}(-1)^{I+1/2} b_0\right]\mu_N$$

The quantity b_0 depends on the magnetic decoupling parameter



M. Loewe; dissertation

$$\mu(7/2^+) = 2.3705 \pm 0.0007$$

$$\mu = \frac{K}{I+1} \cdot (g_K - g_R) \cdot K + g_R \cdot I$$

$g_R = 0.313(5)$

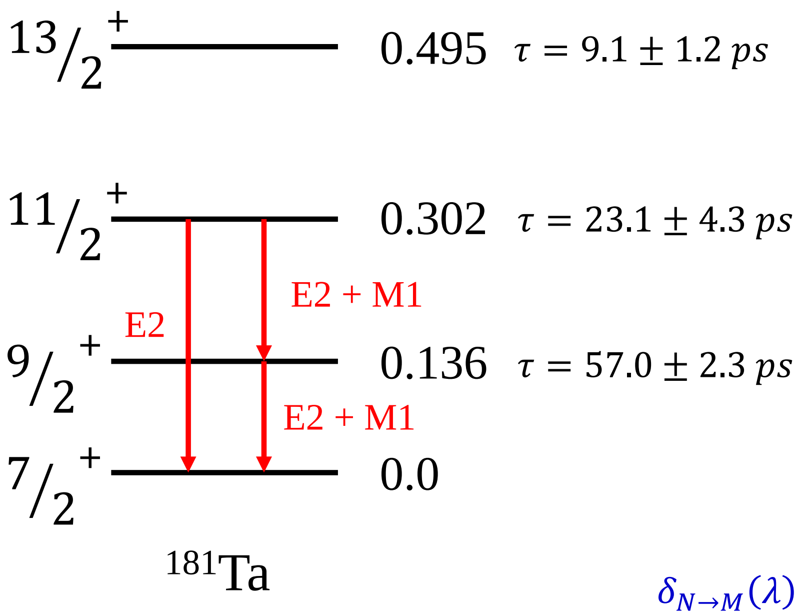
$g_K = 0.782(2)$

$$\mu(9/2^-) = 5.28 \pm 0.09$$

$$^{9/2^-}(g_K - g_R) = \frac{22}{81}\left({}^{9/2^-}\mu - {}^{7/2^+}\mu \frac{9}{7} + {}^{7/2^+}(g_K - g_R)\frac{7}{2} \right)$$

$$^{9/2^-}(g_K - g_R) \approx {}^{7/2^+}(g_K - g_R)\frac{77}{81} + 0.606 = 1.052$$

Odd-even nucleus: ^{181}Ta



$$\tau = \left\{\sum_K \sum_\ell [\varepsilon^2_{N\to K}(\lambda) + \delta^2_{N\to K}(\lambda)]\right\}^{-1}$$

$$\tau = {}^{T_{1/2}}/_{ln2}$$

$$\delta_{N\to M}(\lambda) = \left\{\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{\hbar\omega}{\hbar c}\right)^{2\lambda+1}\right\}^{1/2} \cdot (2I_N+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(\lambda) \| I_N\rangle$$

$$\delta_{N\to M}(E2) = \left\{1.225 \cdot 10^{13} \cdot E_\gamma^5 (MeV)^5\right\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(E2) \| I_N\rangle$$

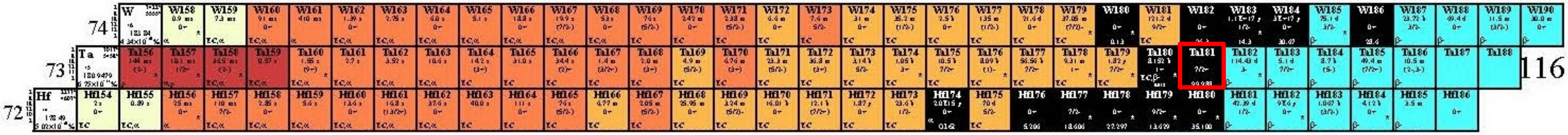
$$\delta_{N\to M}(M1) = \left\{1.758 \cdot 10^{13} \cdot E_\gamma^3 (MeV)^3\right\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(M1) \| I_N\rangle$$

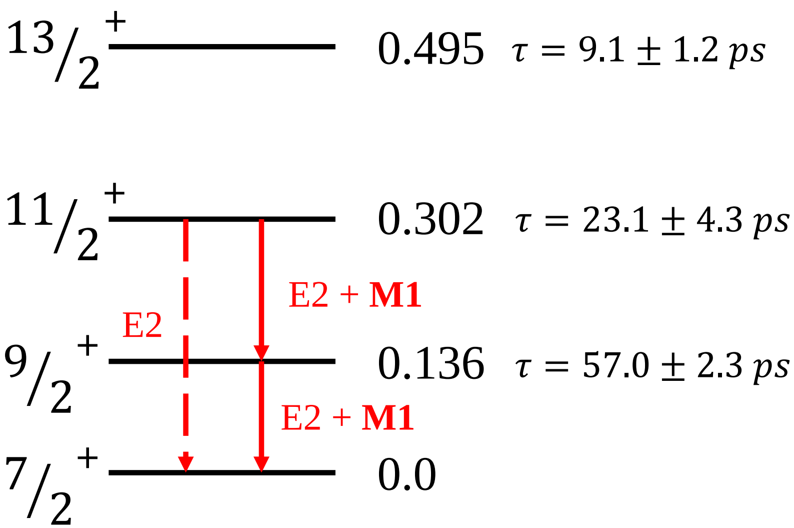
$$\varepsilon^2_{N\to M}(\ell) = \delta^2_{N\to M}(\ell) \cdot \alpha_{N\to M}(\ell)$$

T. Inamura et al., Nucl. Phys. A270 (1976) 255

conversion coefficient.: bricc.anu.edu.au

Odd-even nucleus: ^{181}Ta





^{181}Ta

$$\tau = \left\{ \sum_K \sum_\ell \left[\varepsilon^2_{N\to K}(\lambda) + \delta^2_{N\to K}(\lambda) \right] \right\}^{-1}$$

$$\tau = T_{1/2} / ln2$$

Spin I	E_γ (MeV)	$(2I+1)^{-1/2}$	<I-1//M()//I>	delta	α_T	ε^2	τ (ps)
9/2	0.1365	0.3162	-3.899 (E2)	-29594.	1.1	$9.98\cdot10^8$	533
			1.103 (M1)	73591	1.8	$9.75\cdot10^9$	**58.7**
11/2	0.1654	0.2887	-4.291 (E2)	-48238	0.6	$1.33\cdot10^9$	274
			1.413 (M1)	115044	1.0	$1.38\cdot10^{10}$	32.6
11/2	0.3017	0.2887	1.977 (E2)	99868	0.1	$8.08\cdot10^8$	**24.1**

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Odd proton-even neutron nuclei

		μ	(g_K-g_R)	B(M1)W.u.	(g_K-g_R)
^{153}Eu	$5/2^+$	+1.5324(3)	0.551	0.00608(28)	0.185
^{159}Tb	$3/2^+$	+2.014(4)	1.556	0.173(8)	1.471
^{165}Ho	$7/2^-$	+4.177(5)	1.012	0.275(14)	0.973
^{169}Tm	$1/2^+$	-0.2316(15)		0.0342(8)	
^{175}Lu	$7/2^+$	+2.2327(11)	0.298	0.0354(14)	0.349
^{181}Ta	$7/2^+$	+2.3705(7)	0.352	0.068(4)	0.484
^{185}Re	$5/2^+$	+3.1871(3)	1.217	0.28(5)	1.252
^{187}Re	$5/2^+$	+3.2197(3)	1.242	0.260(18)	1.206