Gas Gain Reduction due to Space Charge in Wire Chambers

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Abstract

We present mathematical expressions for the effective voltage drop due to localized space charge for the Multi Wire Proportional Chamber and the Drift Tube.

Key words: space charge effect, voltage drop, gain reduction, rate effects, localized irradiation
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1 Introduction

The experiments at present high energy colliders like the LHC use large area particle detectors of up to several thousand square meters surface at high particle rates up to a few MHz/cm^2. For this purpose, well known wire chamber geometries like the Multi Wire Proportional Chamber (MWPC) and the Drift Tube are employed. A high rate limit of these detectors is given by the space charge due to the positive ions moving in the chamber volume. A uniform illumination of the detectors results in a uniform space charge, which allows a simple calculation of the effective voltage drop on the amplification wires [1]. In case of localized irradiation, the voltage drop is not uniform across the chamber and the average gas gain drop due to the space charge is lower. To calculate the voltage drop in that case, more detailed mathematical expressions are needed.

In Ref. [2] a rather demanding method to calculate the voltage drop in an MWPC due to irradiation with a rectangular, uniform beam is presented. We prefer to calculate the voltage drop by integration of Green's functions for the relevant geometries. In this publication we present solutions for the voltage drop for the MWPC and Drift Tube geometries. In case of the MWPC

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we assume uniform space charge densities of rectangular or circular shape, but also a circular space charge with Gaussian density in radial direction. For the Drift Tube geometry we assume uniform and Gaussian space charge distributions along the wire direction. The presented formulas allow a straight forward evaluation using numerical integration or summation.

As shown in Ref. [1], the space charge due to positive ions is uniform in the entire detector volume in case the count rate of particles is uniform along the wires. In case the count rate is concentrated along a short section of wire, the space charge will have limited extend in wire direction. Nevertheless, in that case the space charge density can still be assumed constant to first order.

The effective voltage drop at a given position along a wire due to a charge distribution $\rho(\vec{x})$ in the chamber volume is to first order given by the potential at the wire position due to $\rho(\vec{x})$ in absence of the wires. Knowing the Green's function G of the Laplace equation for the required boundary conditions, this voltage drop is therefore given by

$$\Delta V(x,y,z) = \frac{1}{\varepsilon_0} \iiint G(x,y,z,x',y',z') \times \rho(x',y',z')\, dx'\, dy'\, dz' \quad (1)$$

in carthesian coordinates and

$$\Delta V(r,\phi,z) = \frac{1}{\varepsilon_0} \iiint G(r,\phi,z,r',\phi',z') \times \rho(r',\phi',z')\, r'\, dr'\, d\phi'\, dz' \quad (2)$$

in cylinder coordinates. Since we are interested in the voltage drop at the wire position, the coordinates x, y, z (or r, ϕ, z, respectively) are defined by that wire position. In the following sections we will give the Green's functions for the MWPC and Drift Tube geometries together with the resulting formulas for the effective voltage drop.

2 Formulas for MWPC

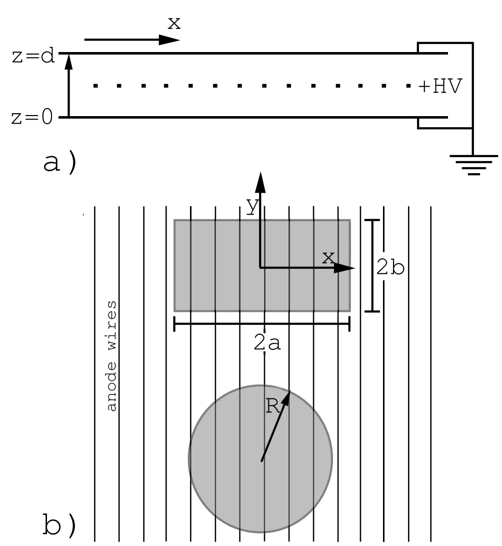

Fig. 1. Multi Wire Proportional Chamber (MWPC). Positive high voltage is applied to the amplification wires, while the cathode planes are grounded. a) Chamber cross section; b) Top view with rectangular and circular space charge.

Fig. 1 shows a schematic view of an MWPC. In absence of the wires it represents a simple parallel plate geometry with two grounded plates at $z = 0$ and $z = d$.

2.1 Rectangular Irradiation

For a beam of rectangular shape the space charge (with density ρ_0) in the MWPC is to first order uniform in the volume $-a < x' < a$, $-b < y' < b$ and $0 < z' < d$, as indicated in Fig. 1. The voltage drop can be calculated using the Green's function for the parallel plate geometry in cartesian coordinates [3,4]

$$\begin{aligned} &G(x, y, z, x', y', z') \\ &\quad = \frac{1}{\pi^2} \int_0^\infty \int_0^\infty dk_1\, dk_2 \\ &\quad \times\ \cos(k_1(x - x')) \\ &\quad \times\ \cos(k_2(y - y')) \\ &\quad \times\ \frac{\sinh(kz_<) \sinh(k(d - z_>))}{k\ \sinh(kd)} \end{aligned} \tag{3}$$

with $k^2 = k_1^2 + k_2^2$, $z_< = \min(z, z')$ and $z_> = \max(z, z')$. Inserting Eq. (3) into Eq. (1) and performing the integrations over x', y' and z' yields [5]

$$\begin{aligned} &\Delta V(x, y, z) \\ &\quad = \frac{4\rho_0}{\pi^2 \varepsilon_0} \int_0^\infty \int_0^\infty dk_1 dk_2 \\ &\quad \times\ \sin(k_1 a)\ \sin(k_2 b) \\ &\quad \times\ \cos(k_1 x)\ \cos(k_2 y) \\ &\quad \times\ \frac{b_{\mathrm{MWPC}}(k, z)}{k_1 k_2 k^2}\ , \end{aligned} \tag{4}$$

with [6]

$$\begin{aligned} b_{\mathrm{MWPC}}(k, z) &= 1 - \cosh(kz) \\ &\quad + \sinh(kz) \tanh\frac{kd}{2}\ . \end{aligned} \tag{5}$$

Because $b_{\mathrm{MWPC}}(k, z)$ approaches 1 for large k_i, the integrand in Eq. (4) decreases like $1/k_i^2$. Thus the integration limits can be chosen such that computing time can be saved while still obtaining precise results. It has been verified that the voltage drop calculated according to Eq. (4) matches the one calculated with Eq. (16) from Ref. [2]. For $a, b \to \infty$ Eq. (4) transforms into

$$\Delta V_0(z) = \frac{\rho_0}{2\varepsilon_0}(zd - z^2)\ , \tag{6}$$

which can also be derived in a straight forward way by applying Gauss' law. For the traditional MWPC, where the wires are centered between the cathode planes, we have $z = d/2$ and find

$$\Delta V_0(z) = \rho_0 d^2/(8\varepsilon_0)\ . \tag{7}$$

In line with Matthieson ([2], Eq. (26)) we define the modifying factor

$$d_m = \frac{\Delta V(x, y, z)}{\Delta V_0(z)}\ . \tag{8}$$

Fig. 2a shows that for larger irradiation areas the voltage drop in the center of that area approaches the case for uniform irradiation, as expected. We assumed an MWPC with gap $d = 0.6\,\mathrm{cm}$ and a rectangular irradiation spot, where one edge is kept at

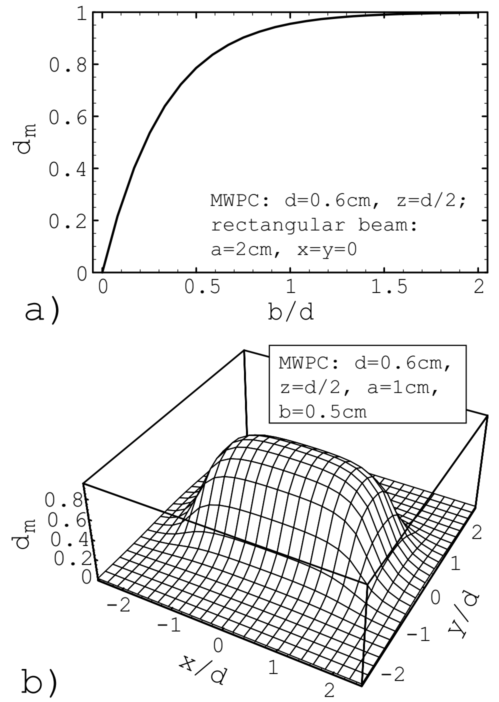

Fig. 2. Value of the modifying factor d_m for a given chamber geometry as a function of the size of the irradiated area in its center (a) and as a function of position for a given size of the space charge density (b).

$2a = 4\,\text{cm}$ and the other one is varied from 0 to $2d$. In Fig. 2b we show the distribution of d_m in a detector of that particular geometry and for a beam with size $2a \times 2b = 2\,\text{cm} \times 1\,\text{cm}$.

2.2 Circular Irradiation

For a beam of circular shape the voltage drop can be calculated using the Green's function for the parallel plate geometry in cylinder coordinates:

$$
\begin{aligned}
&G(r, \phi, z, r', \phi', z') \\
&\quad = \frac{1}{\pi} \sum_{m=0}^{\infty} \int_0^{\infty} dk \, \frac{J_m(kr)\, J_m(kr')}{1 + \delta_{m0}} \\
&\quad \times \frac{\sinh(kz_<)\, \sinh(k(d - z_>))}{\sinh(kd)} \\
&\quad \times \cos(m(\phi - \phi')) \,,
\end{aligned} \tag{9}
$$

where J_m are the Bessel functions of the first kind and δ_{ij} is the Kronecker delta.

2.2.1 Constant Circular Space Charge Density

The voltage drop for a constant space charge density ρ_0 in a circular volume of radius R in the MWPC can be calculated by inserting Eq. (9) into Eq. (2) and performing the integrations over $0 < r' < R$, $0 < z' < d$ and $0 < \phi' < 2\pi$, which yields [7]

$$
\begin{aligned}
&\Delta V(r, z) \\
&\quad = \frac{\rho_0\, R}{\varepsilon_0} \int_0^{\infty} dk \; b_{\text{MWPC}}(k, z) \\
&\quad \times \; \frac{J_1(kR)\, J_0(kr)}{k^2} \,.
\end{aligned} \tag{10}
$$

The variable $b_{\text{MWPC}}(k, z)$ is given by the same expression (Eq. 5) as for the rectangular case. Since this variable approaches 1 for large k, and since the Bessel functions decrease like $1/\sqrt{k}$, the integrand in Eq. (10) decreases like $1/k^3$. This helps in carefully choosing an upper integration limit for numerical integration.

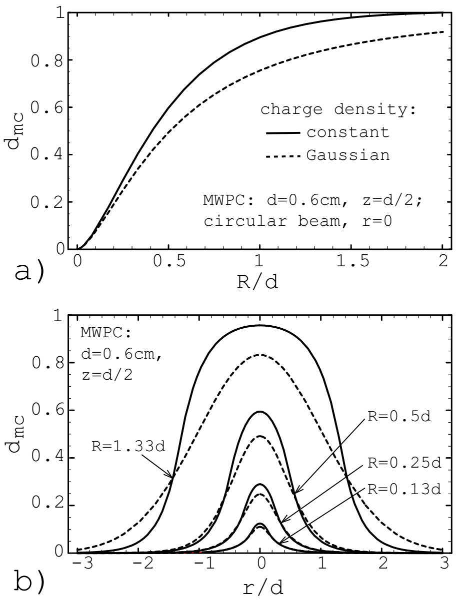

Fig. 3. Value of d_{mc} for a circular beam and uniform and gaussian charge densities for a selected MWPC detector geometry: a) In the center of the charge densities as a function of their radial size; b) as a function of the radial distance for charge densities with several radii.

For $R \to \infty$, Eq. (10) yields the expression for uniform irradiation of the entire detector area, Eq. (6). The value

$$d_{mc} = \frac{\Delta V(r,z)}{\Delta V_0(z)} \tag{11}$$

again relates the voltage drop for localized irradiation to the uniform case (modifying factor). This value is shown in Fig. 3a for a specific chamber geometry ($d = 0.6\,\mathrm{cm}$). In Fig. 3b we also show the distributions of the voltage drop for different radii of space charge densities in a detector for that particular geometry.

2.2.2 *Gaussian Circular Space Charge Density*

Due to Gaussian beam profiles and/or electron diffusion the assumption of a uniform space charge density might not be valid. Thus we will in this section consider the voltage drop for a space charge of Gaussian density in radial direction:

$$\rho(r') = \rho_0 \, e^{-\frac{r'^2}{R^2}} \; . \tag{12}$$

The total charge contained is assumed to be equal to that of a circular disc (section 2.2.1) of radius R: $Q = 2\pi R^2 d \rho_0$. The charge density in the center is set to the same value as for the disc: $\rho(r'=0) = \rho_0$. In z-direction the charge density can be assumed constant to first order. The voltage drop can now be calculated by inserting Eqs. (9) and (12) into Eq. (2) and performing the integrations over r', ϕ' and z', which yields

$$\begin{aligned} \Delta V(r,z) & \\ &= \frac{\rho_0 \, R^2}{2\,\varepsilon_0} \int\limits_0^\infty dk \; b_{\mathrm{MWPC}}(k,z) \\ &\quad \times \; \frac{J_0(kr)}{k} \, e^{-\frac{k^2 R^2}{4}} \; . \end{aligned} \tag{13}$$

Because the integrand decreases exponentially with k, for numerical integration we may choose the upper integration limit to satisfy $k^2R^2/4 \gtrsim 10$. Voltage drops in the center of Gaussian charge distributions are plotted together with the uniform case in Fig. 3. In general the distributions are wider and the voltage drop is less strong in the center, as compared to that of the uniform space charge.

3 Formulas for Drift Tube

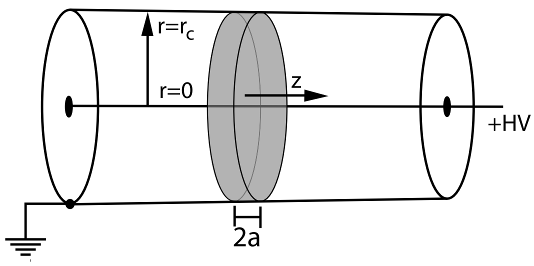

Fig. 4. Drift tube with space charge over a length of $2a$.

Fig. 4 shows a schematic view of a drift tube. In absence of the wires it is simply a grounded cylinder of radius r_c. The Green's function for this geometry is given by

$$G(r,\phi,z,r',\phi',z') = \frac{1}{r_c\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{J_m(j_{mn}\,r'/r_c)}{(1+\delta_{m0})\,j_{mn}} \times \frac{J_m(j_{mn}\,r/r_c)}{(J_{m+1}(j_{mn}))^2} \times e^{-\frac{j_{mn}}{r_c}|z-z'|} \times \cos m(\phi-\phi')\ , \quad (14)$$

where j_{mn} is the n^{th} zero of J_m, i.e. $J_m(j_{mn})=0$.

3.1 Constant Space Charge Density

For a constant space charge density ρ_0 in the drift tube along a section of length $2a$, the voltage drop can be calculated by inserting Eq. (14) into Eq. (2) and performing the integrations over $-a < z' < a$, $0 < r' < r_c$ and $0 < \phi' < 2\pi$, which yields

$$\Delta V(r,z) = \frac{\rho_0\,r_c^2}{\varepsilon_0}\sum_{n=1}^{\infty}\frac{J_0(j_{0n}\,r/r_c)}{j_{0n}^3\,J_1(j_{0n})} \times b_{\mathrm{DT}}(n,z)\ , \quad (15)$$

with [8]

$$b_{\mathrm{DT}}(n,z) = \begin{cases} \text{if } |z|<a: \\ 2-e^{-\frac{j_{0n}}{r_c}(a+z)}-e^{\frac{j_{0n}}{r_c}(z-a)} \\ \text{if } z\geq a: \\ e^{\frac{j_{0n}}{r_c}(a-z)}-e^{\frac{-j_{0n}}{r_c}(a+z)} \\ \text{if } z\leq -a: \\ -e^{\frac{j_{0n}}{r_c}(z-a)}+e^{\frac{j_{0n}}{r_c}(a+z)}\ . \end{cases} \quad (16)$$

Since the wire of a Drift Tube is sitting in the center at $r=0$, Eq. (15) can be simplified to [9]

$$\Delta V(z) = \frac{\rho_0\,r_c^2}{\varepsilon_0}\sum_{n=1}^{\infty}\frac{b_{\mathrm{DT}}(n,z)}{j_{0n}^3\,J_1(j_{0n})}\ . \quad (17)$$

For $a\to\infty$, meaning for uniform irradiation over the full length of the tube, Eq. (17) becomes

$$\Delta V_0 = \frac{\rho_0}{4\varepsilon_0}\,r_c^2 \quad (18)$$

with the modifying factor

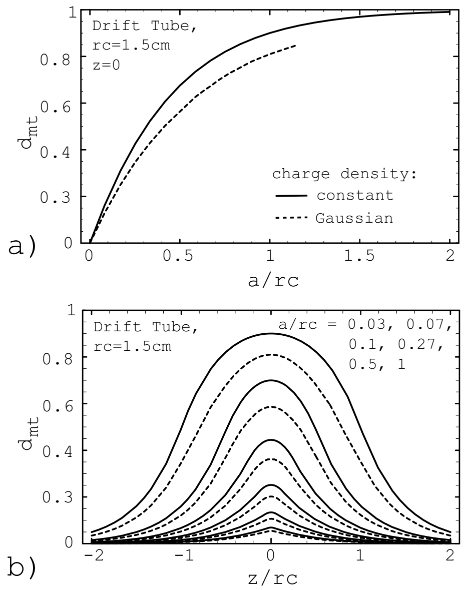

Fig. 5. a) Value of d_{mt} for a uniform space charge density along a section of length $2a$ and for a Gaussian charge distribution along z. b) Distributions of the voltage drop due to localized space charge densities of several widths for the two cases. We chose a tube with $r_c = 1.5\,\mathrm{cm}$.

$$d_{mt} = \frac{\Delta V(z)}{\Delta V_0} \ . \tag{19}$$

3.2 Gaussian Space Charge Density

We now assume a space charge of Gaussian density in z-direction

$$\rho(z') = \rho_0\, e^{-\frac{\pi\, z'^2}{4\, a^2}} \ , \tag{20}$$

where the total charge contained is equal to that of a uniform space charge along a section of length $2a$ (section 3.1): $Q = 2a\pi r_c^2 \rho_0$. The charge density in the center of the Gaussian charge distribution is set to $\rho(z' = 0) = \rho_0$. In r-direction the charge density can be assumed constant to first order. The voltage drop can be calculated by inserting Eqs. (14) and (20) into Eq. (2) and performing the integrations over $-a < z' < a$, $0 < r' < r_c$ and $0 < \phi' < 2\pi$. The result is equal to Eqs. (15) and (17), but the term depending on z is now given by

$$b_{\mathrm{DT}}(n, z) \ = \begin{cases} \text{if } |z| < a : \\ d_+(n,z)\ [f_+(n) - f_1(n,z)] \\ \ +d_-(n,z)\ [f_+(n) + f_2(n,z)], \\ \text{if } z \geq a : \\ d_-(n,z)\ [f_-(n) + f_+(n,z)], \\ \text{if } z \leq -a : \\ d_+(n,z)\ [f_-(n) + f_+(n,z)]. \end{cases} \tag{21}$$

with

$$d_\pm(n,z) = \ a\, e^{\frac{j_{0n}\left(j_{0n} a^2 \pm \pi r_c z\right)}{\pi r_c^2}} \ , \tag{22}$$

$$f_\pm(n) = \ f_e\left(\frac{\pi r_c \pm 2 a j_{0n}}{2\sqrt{\pi} r_c}\right) \ , \tag{23}$$

$$f_1(n,z) = f_e\left(\frac{a j_{0n}}{\sqrt{\pi} r_c} + \frac{\sqrt{\pi} z}{2a}\right) \ , \tag{24}$$

$$f_2(n,z) = \ f_e\left(\frac{\pi z - \frac{2a^2 j_{0n}}{r_c}}{2a\sqrt{\pi}}\right) \ , \tag{25}$$

where f_e is the error function

$$f_e(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \ . \tag{26}$$

The expression converges well in the region $0 \leq a \lesssim r_c$.

Fig. 5 shows the value of $\Delta V(z)/\Delta V_0$

for a localized space charge density and for a selected tube geometry ($r_c = 1.5\,\mathrm{cm}$) for the two cases from section 3.1 and from this section. As the area of irradiation increases, the voltage drop in the center of the irradiated area approaches ΔV_0 from Eq. (18).

4 Conclusions

The presented formulas allow straight forward calculation of the effective voltage drop on the amplification wires of MWPCs and Drift Tubes due to localised space charge. To first order the space charge density can be assumed constant in the uniformly irradiated case. As compared to the case where the whole area of an MWPC is irradiated, the voltage drop in the center of a localized irradiation area with radius comparable to the detector thickness is lower by a factor of about 0.9. The same is true for the irradiation of a section of a Drift Tube of length comparable to the tube diameter. More complex cases, where non-uniform, Gaussian space charge distributions are assumed, can be calculated as well.

Knowing the voltage drop, one can easily calculate the gain drop due to a high particle flux in the detector, if the gas parameters (primary ionisation, ion mobility and gas gain) are known.

References

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[4] Th. Heubrandtner, B. Schnizer, C. Lippmann and W. Riegler, Nucl. Instr. Meth. Phys. Res. A 489 (2002), 439-443.

[5] In mathematica 5.2 this reads: dV[x_,y_,z_] := 4*rho/Pi^2/eps* NIntegrate[(Sin[k1*a]*Sin[k2*b]* Cos[k1*x]*Cos[k2*y])/(k1*k2* (k1^2+k2^2))*b[Sqrt[k1^2+k2^2],z], {k1,0,limit},{k2,0,limit}]

[6] In mathematica 5.2 this reads: b[k_,z_] := 1-Cosh[k*z]+Sinh[k*z]* Tanh[k*d/2]

[7] In mathematica 5.2 this reads: dV[r_,z_] := rho/eps*NIntegrate[R* BesselJ[1,k*R]*BesselJ[0,k*r]/k^2* b[k,z],{k,0,limit}]

[8] In mathematica 5.2 this reads: b[n_,z_] := If[Abs[z]<a,2-Exp[-bzj[0, n]/R*(a+z)]-Exp[-bzj[0,n]/R*(a-z)], If[z>=a,Exp[-bzj[0,n]/R*(z-a)]-Exp[-bzj[0,n]/R*(z+a)],-Exp[-bzj[0, n]/R*(a-z)]+Exp[bzj[0,n]/R*(z+a)]]]

[9] In mathematica 5.2 this reads: dV[z_] := rho/eps*R^2*Sum[1/((bzj[0,n])^3* BesselJ[1,bzj[0,n]])*b[n,z],{n,1,limit}] using the *BesselZeros* package and bzj[m_,n_]:=BesselJZeros[m,{n,n}][[1]]