Rate effects in resistive plate chambers

C. Lippmann [a], W. Riegler [b] and A. Kalweit [c]

[a] *GSI, Darmstadt, Germany*
[b] *PH-Division, CERN, Geneva, Switzerland*
[c] *Technische Universität Darmstadt, Germany*

Abstract

The resistive plates in RPCs cause a drop of the electric field in the gas gap at high particle rates or large gas gain, which affects efficiency and time resolution. This effect is typically estimated by assuming the particle flux to be a DC current that causes a voltage drop when it passes through the resistive plate. In an improved model by Abbrescia (Nucl. Instr. Meth. A 533 (2004) 7), the fluctuation of the field in the gas gap is modelled by assuming that the avalanche partially discharges a small capacitor which gets recharged with a time constant characteristic for the given RPC. In our approach, the effect is calculated by using the exact analytic solution for the time dependent electric field of a point charge sitting on the surface of a resistive plate in an RPC. This is—by definition—the best possible approximation to reality. The solution is obtained using the quasi-static approximation of Maxwell's equations. The formulas are presented as integral representations with 'cured' integrands, which allow easy numerical evaluation for Monte Carlo simulations. The solutions show that the charges in RPCs are 'destroyed' with a continuous distribution of time constants which are related in a very intuitive way to some limiting cases. Using these formulas we present a Monte Carlo simulation of rate effects, proving the applicability of this approach. Finally, we compare the Monte Carlo results to analytical calculations, similar to the ones proposed by Gonzalez-Diaz et al. (see proceedings of this conference).

1. Introduction

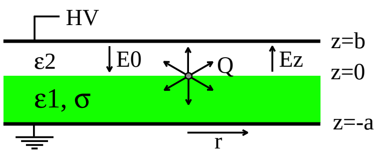

Fig. 1. RPC geometry. We use a resistive layer of thickness $a = 3\,\text{mm}$, a gap width of $b = 0.3\,\text{mm}$, permittivities $\varepsilon_1 = 8\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$ and resistivity $\rho = 1/\sigma = 10^{12}\,\Omega\text{cm}$. A point charge Q is situated at $r = z = 0$.

In this report we investigate the geometry from Fig. 1. Before the passage of a particle the electric field in the gas gap is given by $E_0 = V_{HV}/b$ and the electric field in the resistive plate (with resistivity ρ) is zero. When a charge Q is put on the surface of the electrode, the resulting electric field in the resistive plate will cause charges to flow and 'destroy' the charge, which causes a time dependent electric field in the gas gap. Using the quasi-static approximation of Maxwell's equations we obtain this solution from a static solution [1]. Assuming a point charge Q at position $r = 0, z = 0$ on the boundary of two infinite half-spaces of dielectric permittivities ε_1 and ε_2, the z-component of the electric field in the half space at $z > 0$ is given by [2]

$$E_z(r,z) = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\,\frac{z}{(r^2+z^2)^{\frac{3}{2}}} \tag{1}$$

In case the half-space $z < 0$ has permittivity ε_1 and conductivity $\sigma = 1/\rho$ and we put the charge in place at $t = 0$ we have to replace ε_1 by $\varepsilon_1 + \sigma/s$ and Q by Q/s and perform the inverse Laplace transform, giving

$$E_z(r,z,t) = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\,\frac{z}{(r^2+z^2)^{\frac{3}{2}}}\,e^{-\frac{t}{\tau_1}}\ , \tag{2a}$$

$$\text{with}\quad \tau_1 = \frac{\varepsilon_1+\varepsilon_2}{\sigma}\ . \tag{2b}$$

The charge and the electric field are therefore 'destroyed' with the time constant τ_1 which is about 0.8 s for our parameters. In another example we place a charge sheet with density q at position $z = 0$ in the RPC from Fig. 1. This causes a static electric field of

$$E_z = \frac{aq}{a\varepsilon_1 + b\varepsilon_2}\ . \tag{3}$$

Giving the plate at $z < 0$ a conductivity σ and placing the charge sheet at $t = 0$ we replace ε_1 by $\varepsilon_1 + \sigma/s$ and q by q/s and perform the inverse Laplace transform, giving

$$E_z(t) = \frac{a\,q}{a\varepsilon_1 + b\varepsilon_2}\,e^{-\frac{t}{\tau_2}}\ , \tag{4a}$$

$$\text{with}\qquad \tau_2 = \frac{a\varepsilon_1 + b\varepsilon_2}{b\sigma}\ , \tag{4b}$$

where τ_2 is about 1.6 s for our parameters. Assuming now that a constant DC current I_0 feeds the charge sheet (i.e. $q(t) = I_0 t$, $q(s) = I_0/s^2$) we obtain after the inverse Laplace transform instead of Eq. 4a

$$E_z(t) = \rho\,\frac{a}{b}\,I_0\,(1 - e^{-\frac{t}{\tau_2}})\ . \tag{5}$$

For $t \to \infty$ and after replacing I_0 with $Q\,\Phi$, where Φ is the particle flux per area and Q is the avalanche charge, we find that the DC current leads to a static electric field of

$$E_z = \rho\,\frac{a}{b}\,\Phi\,Q\ . \tag{6}$$

2. Solution for the point charge

The static solution for E_z, $z > 0$ for a point charge Q at $r = 0, z = 0$ (See Fig. 1) is obtained from Eqs. 56 and 61 (N_{21}) in [3] by replacing $q \to a, g \to b, p \to b$ and taking the derivative with respect to z, which gives

$$\begin{aligned} E_z^{stat}(r,z) &= \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\,\frac{z}{(r^2+z^2)^{\frac{3}{2}}} \\ &+ \frac{Q}{2\pi}\int_0^\infty J_0(kr)\,[f_1(k,z) - f_2(k,z)]\,dk \end{aligned} \tag{7}$$

with $f_1(k,z) =$

$$\frac{k\cosh[k(b-z)]\sinh(ka)}{\varepsilon_2\cosh(kb)\sinh(ka) + \varepsilon_1\cosh(ka)\sinh(kb)}$$

$$\text{and}\quad f_2(k,z) = \frac{ke^{-kz}}{\varepsilon_1+\varepsilon_2}\ .$$

For $k \to \infty$ the integrand decays as $e^{-2bk} - e^{-2ak}$, so for numerical evaluation the integration limit ∞ can be replaced by $10\frac{a+b}{a\,b}$. Giving the resistive plate a conductivity σ and placing the charge Q at $t = 0$ we get the solution

$$\begin{aligned} E_z^{dyn}(r,z,t) &= \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\,\frac{z}{(r^2+z^2)^{\frac{3}{2}}}\,e^{\frac{-t}{\tau_1}} \\ &+ \frac{Q}{2\pi}\int_0^\infty [\,J_0(kr)\,f_1(k,z)\,e^{\frac{-t}{\tau(k)}} \\ &\quad - J_0(kr)\,f_2(k,z)\,e^{\frac{-t}{\tau_1}}\,]\,dk \end{aligned} \tag{8}$$

with

$$\tau(k) = \frac{\varepsilon_2\cosh(kb)\sinh(ka) + \varepsilon_1\cosh(ka)\sinh(kb)}{\sigma\,\cosh(ka)\sinh(kb)} \tag{9}$$

and τ_1 from Eq. 2b. The solution consists of a term giving the point charge at two infinite half-spaces (See Eq. 2a), decaying with a single time constant τ_1 and a term due to the presence of the grounded plates which decays with a distribution of time constants $\tau(k)$. It is easily shown that $\tau(\infty) = \tau_1$, $\tau(0) = \tau_2$ and $\tau_1 < \tau(k) < \tau_2$. The solution is tabulated and used for a Monte Carlo simulation (See section 3).

In order to calculate the spread of the charge in the resistive layer we need also the solution for E_z, $z < 0$ (not given here) and calculate the time dependent charge density $\rho(r,z,t)$ by using the relation $\boldsymbol{\nabla}(\varepsilon\boldsymbol{E}) = \rho$, which gives

$$\begin{aligned} \rho(r,t) &= \delta(z)\,Q\,\delta(r)\,e^{-\frac{t}{\tau_1}} \\ &+ \delta(z)\,\frac{Q}{2\pi}\int_0^\infty J_0(kr)\,k\,(e^{-\frac{t}{\tau(k)}} - e^{-\frac{t}{\tau_1}}) \end{aligned} \tag{10}$$

At $t = 0$ we have the charge Q at the surface of the resistive plate which decays according to τ_1 for

$t > 0$. In addition the charge spreads on the surface according to the second term.

3. Monte Carlo simulation

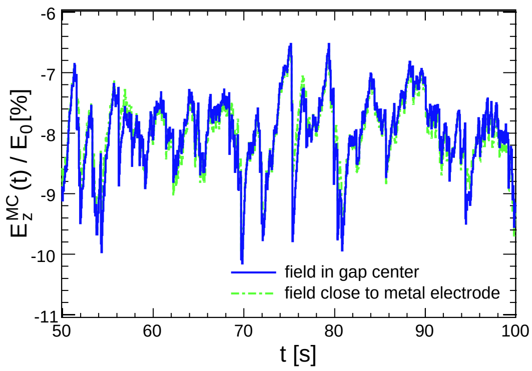

Fig. 2. Example field fluctuations at two positions in the gas gap of an RPC as in Fig. 1 at 3 kV applied voltage and at $\Phi = 600\,\mathrm{Hz/cm^2}$.

In a Monte Carlo simulation we can calculate at all times t at a certain position in the gas gap ($r = 0$, z_0) the sum of the z-components of the electric fields of all charges on the surfaces of the resistive plates:

$$E_z^{MC}(t) = \sum_n Q_n E_z^{dyn}(r_n, z_0, t - t_n) \ . \qquad (11)$$

Here the Q_n are given by the total signal charges of the avalanches that created the charges at position r_n and at time t_n and E_z^{dyn} is given by Eqn. 8. We simulate a single gap RPC of area $A = 3 \times 3\,\mathrm{cm}^2$. For each time step a new number of charges ($t\,\Phi\,A$) is distributed randomly on that area. $E_z^{MC}(t)$ is calculated at always the same position in the center of the RPC area. It has been verified that a further increase of the simulated detector area, which considerably increases computing time, does not improve precision. All charges are kept in memory until their field contributions are negligible, which can take up to 60 s for our parameters. To obtain the avalanche charge Q_n we use as an approximation box-shaped charge spectra from 0 to $2\,Q_{av}$, where Q_{av} is the average total signal charge. Q_{av} however also depends on the rate itself and must be obtained using an iterative approach. As starting values for the different applied voltages we use the simulated values from Fig. 11a in Ref. [4].

An example of the field fluctuations is shown in Fig. 2. Histograms of the field values at the center of the gap at different particle fluxes are shown in Fig. 3a. Position and charge fluctuations both contribute to the field fluctuations. Fig. 3b shows that the average field reduction is exactly the same as the one calculated from the DC model (Eq. 6).

4. Average and standard deviation

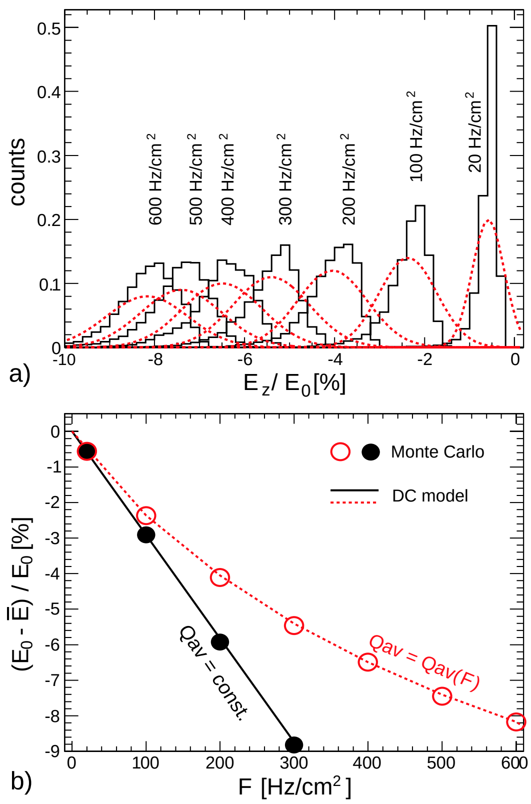

Fig. 3. a) Histograms of the field fluctuations at different particle fluxes for a single gap timing RPC as in Fig. 1 at 3 kV applied high voltage. The lines are Gaus functions with mean and sigma values calculated as described in section 4. b) Average field reductions taken from the histograms (open symbols) compared to calculations using Eq. 6 (broken line). The filled symbols and straight line correspond to the case where the avalanche charge does not depend on the flux.

In [5] it was prosed to use Campbell's theorem in order to arrive at an analytic expression for the average field drop and the variation. The theorem states that for a signal $E(t) = \sum_n a_n f(t - t_n)$, where a_n is from a random amplitude distribution and t_n are random (exponentially distributed) times with an average frequency ν, the average and standard deviation of $E(t)$ are $\overline{E} = \nu\overline{a}\int f(t)dt$ and $(\Delta E)^2 = \nu\overline{a^2}\int f(t)^2 dt$. The theorem cannot be applied to the exact solution (Eq. 7) since the signal shape is not constant. We can however approximate the situation by assuming $E_z^{dyn} \approx Q\,E_z^{stat0}\,e^{-t/\tau_2}$ with $E_z^{stat0} = E_z^{stat}/Q$. This gives an electric field at time t of

$$E(t) \;=\; \sum_n Q_n\, E_z^{stat0}(r_n, z)\, e^{-\frac{t-t_n}{\tau_2}} \tag{12}$$

where Q_n, r_n and t_n are the charge, position and time of event n. Applying Campbell's theorem with a flux Φ of particles ($\nu = r^2\pi\Phi$) we find the average field drop in the gap:

$$\begin{aligned}\overline{E} \;&=\; \nu\,\overline{Q}\,\overline{E_z^{stat0}}\int_0^\infty e^{-\frac{t}{\tau_2}}\,dt \\ &=\; \Phi\,\overline{Q}\int_0^\infty 2\,r\,\pi\;E_z^{stat0}(r,z)\,dr\,\tau_2 \\ &=\; \rho\,\frac{a}{b}\,\Phi\overline{Q}\;.\end{aligned} \tag{13}$$

This shows that the choice of τ_2 for the overall time constant guarantees that the model gives the correct average drop (compare to Eq. 6). For the variance of the field fluctuation we get

$$\begin{aligned}\Delta E(z)^2 \;&=\; \nu\;\overline{Q^2}\;\overline{(E_z^{stat0})^2}\int_0^\infty e^{-\frac{2t}{\tau_2}}\,dt \\ &=\; \Phi\,\overline{Q^2}\,\frac{\tau_2}{2}\int_0^\infty 2r\pi[E_z^{stat0}(r,z)]^2 dr\end{aligned} \tag{14}$$

so the relative variation can be written as

$$\frac{\Delta E(z)}{\overline{E}} \;=\; \frac{1}{\sqrt{2\Phi\tau_2 A(z)}}\sqrt{1+\frac{(\Delta Q)^2}{\overline{Q}^2}} \tag{15}$$

with an effective area of

$$A(z) \;=\; \frac{[\int_0^\infty 2r\pi E_z^{stat0}(r,z)dr]^2}{\int_0^\infty 2r\pi E_z^{stat0}(r,z)^2 dr}\;. \tag{16}$$

For our parameters we obtain in the center of the gap an effective area of $A(0.15\,\mathrm{mm}) \;=\; 5.67\,\mathrm{mm}^2$ and for our box-shaped charge spectrum we find $(\Delta Q)^2/\overline{Q}^2 = 1/3$. In Fig. 3a we include Gaus functions around mean values from Eq. 13 and with sigmas calculated from Eq. 15 as $\frac{\Delta E(z)}{E_0}$. This sigma value is always larger than the r.m.s. from the Monte Carlo histograms, as is shown in the following table:

Φ [Hz/cm^2]	20	100	200	300	400	500	600
$\frac{\Delta E(z)}{E_0}$ (MC) [%]	0.33	0.52	0.58	0.66	0.67	0.70	0.66
$\frac{\Delta E(z)}{E_0}$ (Analytic) [%]	0.35	0.64	0.78	0.85	0.88	0.90	0.91

The difference is explained by the complex distribution of time constants in the exact solution (Eq. 8), which does not enter in Eq. 15. Still, using the analytic expression from Eq. 15 we can get a good order of magnitude estimate for the field fluctuations.

5. Conclusion

We have calculated the field drop and field fluctuations in a single gap RPC by using the analytic expression for the field of a point charge sitting on the surface of a resistive plate. The charge decays with different time constants distributed between the case of a point charge on the boundary of two infinite half spaces and the case of a charge sheet in the investigated RPC geometry. Approximating the solution and using Campbell's theorem, an approximate analytic expression for the field fluctuation was given.

References

[1] T. Heubrandtner and B. Schnizer, The quasi-static electromagnetic approximation for weakly conducting media, Nucl. Instr. Meth. Phys. Res. A 478 (2002) 444.

[2] J.D. Jackson, *Classic Electrodynamics*, Wiley, New York (1975).

[3] Th. Heubrandtner et al., CERN-OPEN 2001-074.

[4] C. Lippmann and W. Riegler, Nucl. Instr. Meth. Phys. Res. A 517 (2004) 54.

[5] D. Gonzalez-Diaz et al., Proceedings of this conference