Space Charge Effects and Induced Signals in Resistive Plate Chambers

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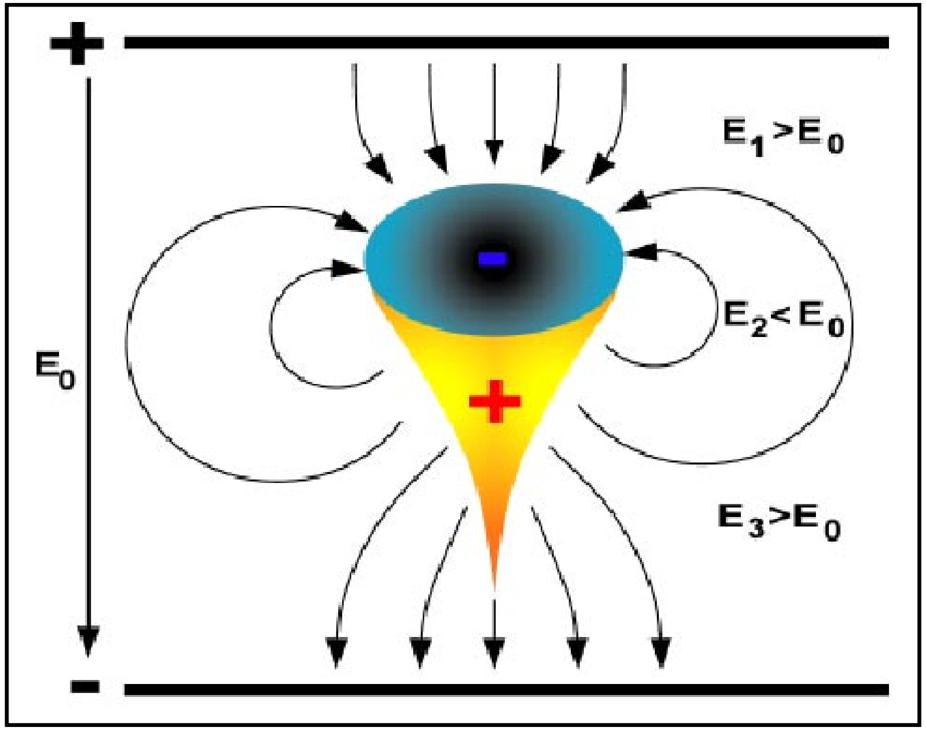
- Overview and Principles
- Avalanche Simulation Program
- Some Results
- Analytic Solutions for Weighting Fields
- Conclusion

Simulation of Avalanches

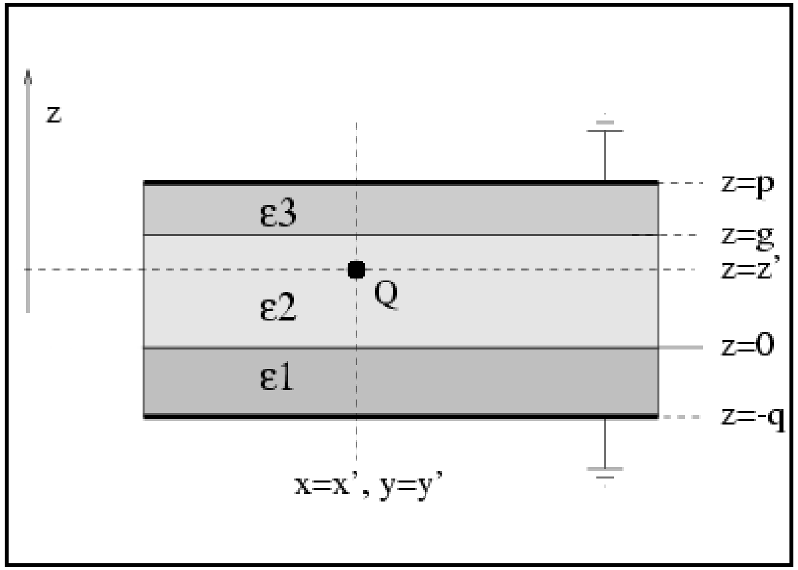
- Monte Carlo C/C++ program using ROOT framework
- Divide gas gap into several steps
- Create primary clusters
  - Mean free path λ; cluster size distribution [HEED]
- Let electrons multiply and drift towards the anode
- Include space charge effect
  - Calculate dynamically the field E of the space charge at each step where electrons are multiplied
  - α(E), η(E) [IMONTE 4.5]
  - v_D(E) [MAGBOLTZ 2]
- Use diffusion
  - D_T and D_L [MAGBOLTZ 2]
- No photons
- Induced charge: $q_{ind} = \sum_{i=0}^{Steps} \int_0^T \frac{\overrightarrow{E_w}}{V_w} \overrightarrow{v_i}(t)\, q_i\, dt$



Geometry

- Cylindrical coordinates
- x, y, z, ρ, φ = coordinates of point of observation
- x', y', z', ρ', φ' = coordinates of charge
- p, g, q define thickness of layers

$$
\begin{aligned}
R^2 &= |\vec{r} - \vec{r}\,'|^2 = \\
&= (x-x')^2 + (y-y')^2 + (z-z')^2 \\
&= \rho^2 - 2\rho\rho' \cos(\phi-\phi') + \rho'^2 + (z-z')^2 \\
&= P^2 + (z-z')^2
\end{aligned}
$$



Static Electric Fields in an Infinite Plane Condenser with Three Homogeneous Layers

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$$\Phi(\rho,\phi,z) \;=\; \frac{Q}{4\pi\varepsilon_2}\Big[\frac{1}{\sqrt{P^2+(z-z')^2}} + \frac{(\varepsilon_2-\varepsilon_3)}{(\varepsilon_2+\varepsilon_3)\sqrt{P^2+(2g-z-z')^2}} - \frac{(\varepsilon_1-\varepsilon_2)}{(\varepsilon_1+\varepsilon_2)\sqrt{P^2+(z+z')^2}}$$

$$+ \frac{1}{(\varepsilon_1+\varepsilon_2)(\varepsilon_2+\varepsilon_3)} \int_0^\infty d\kappa\; J_0(\kappa P)\,\frac{R(\tau,z,z')}{D(\kappa)}\Big], \qquad 0 \le z \le g$$

$$\begin{aligned} D(\kappa) \;=\; & (\varepsilon_1+\varepsilon_2)(\varepsilon_2+\varepsilon_3)\,(1-e^{-2\kappa\,(p+q)}) - (\varepsilon_1-\varepsilon_2)(\varepsilon_2+\varepsilon_3)(e^{-2\kappa\,p}-e^{-2\kappa\,q}) \\ & - (\varepsilon_1+\varepsilon_2)(\varepsilon_2-\varepsilon_3)(e^{-2\kappa\,(p-g)}-e^{-2\kappa\,(q+g)}) + (\varepsilon_1-\varepsilon_2)(\varepsilon_2-\varepsilon_3)(e^{-2\kappa g}-e^{-2\kappa\,(p+q-g)}) \end{aligned}$$

$$\begin{aligned} R(\kappa;z,z') \;=\; & (\varepsilon_1+\varepsilon_2)^2(\varepsilon_2+\varepsilon_3)^2\,\left[e^{\kappa(-2p-2q+z-z')} + e^{\kappa(-2p-2q-z+z')}\right] \\ & -(\varepsilon_1+\varepsilon_2)^2\,(\varepsilon_2-\varepsilon_3)^2\,e^{\kappa(-4g-2q+z+z')} - 4\varepsilon_1\varepsilon_2(\varepsilon_2+\varepsilon_3)^2\,e^{\kappa(-2q-z-z')} \\ & -(\varepsilon_1-\varepsilon_2)^2\,(\varepsilon_2+\varepsilon_3)^2\,e^{\kappa(-2p-z-z')} - \left(\varepsilon_1{}^2-\varepsilon_2{}^2\right)(\varepsilon_2-\varepsilon_3)^2\,e^{\kappa(-4g+z+z')} \\ & + \left(\varepsilon_1{}^2-\varepsilon_2{}^2\right)(\varepsilon_2+\varepsilon_3)^2\,\left[-e^{\kappa(-2p-2q-z-z')} + e^{\kappa(-2p+z-z')} + e^{\kappa(-2p-z+z')}\right] \\ & -4\,\left(\varepsilon_1{}^2-\varepsilon_2{}^2\right)\,\varepsilon_2\varepsilon_3\,e^{\kappa(-2p-2q+z+z')} - 4\,(\varepsilon_1+\varepsilon_2)^2\varepsilon_2\varepsilon_3\,e^{\kappa(-2p+z+z')} \\ & +(\varepsilon_1-\varepsilon_2)^2\left(\varepsilon_2{}^2-\varepsilon_3{}^2\right)\,e^{\kappa(-2g-z-z')} + 4\,\varepsilon_1\varepsilon_2\left(\varepsilon_2{}^2-\varepsilon_3{}^2\right)\,e^{\kappa(2g-2p-2q-z-z')} \\ & +(\varepsilon_1+\varepsilon_2)^2\left(\varepsilon_2{}^2-\varepsilon_3{}^2\right)[-e^{\kappa(-2g-2q+z-z')} - e^{\kappa(-2g-2q-z+z')} + e^{\kappa(-2g-2p-2q+z+z')}] \\ & + \left(\varepsilon_1{}^2-\varepsilon_2{}^2\right)\left(\varepsilon_2{}^2-\varepsilon_3{}^2\right)[e^{\kappa(-2g-2q-z-z')} - e^{\kappa(-2g+z-z')} - e^{\kappa(-2g-z+z')} + e^{\kappa(-2g-2p+z+z')}] \end{aligned}$$

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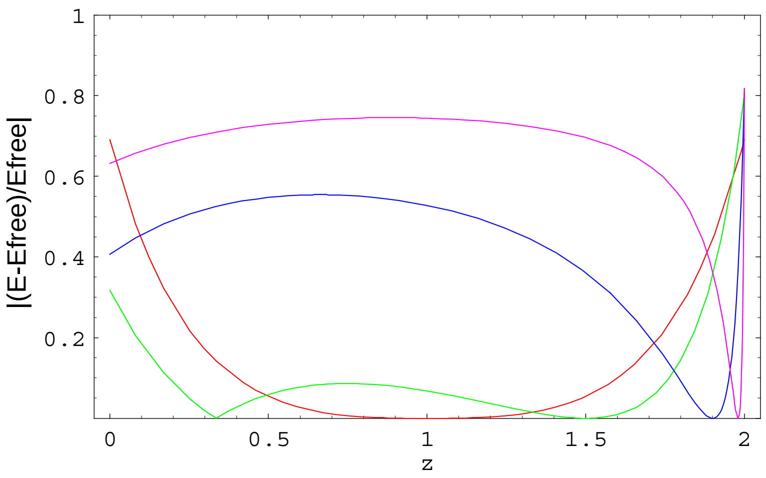
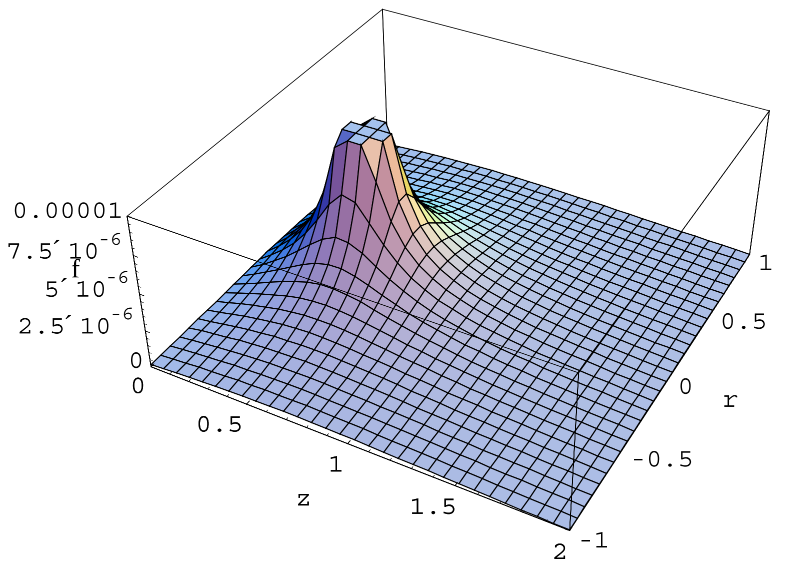
Static Electric Fields in an Infinite Plane Condenser with Three Homogeneous Layers

- Close to the resistive plates, the deviation from the solution of a free point charge becomes important:





The charge distribution in the gap

- **Put charge in gaussian** $\varphi(\rho', \sigma, z')$**:**
- $\sigma = D_T \sqrt{z' - z''}$

  z'' = spot of formation of primary cluster

- $E_z(\rho, \phi, \sigma, z, z') =$

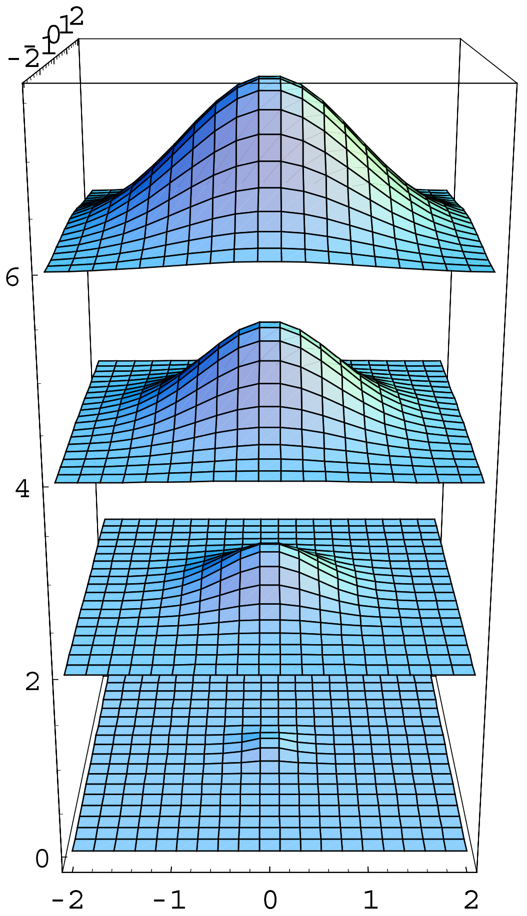
  $$\int_0^{2\pi} \int_0^{\infty} \varphi(\rho', \sigma, z') \frac{-\partial \Phi(\rho, \phi, z, \rho', z')}{\partial z} \rho' \, \partial \rho' \partial \phi'$$

- **Field at spot z:**

$$E_z(z) = E_0 + \int_0^{gapwidth} q(z') \; E_z(\rho, \phi, z, z') \partial z'$$

$$\simeq E_0 + \sum_{i=0}^{Steps} q_i \; E_z(z, z_i', z_i'')$$

$\rho = \phi = 0$ $\qquad$ q = charge at z'



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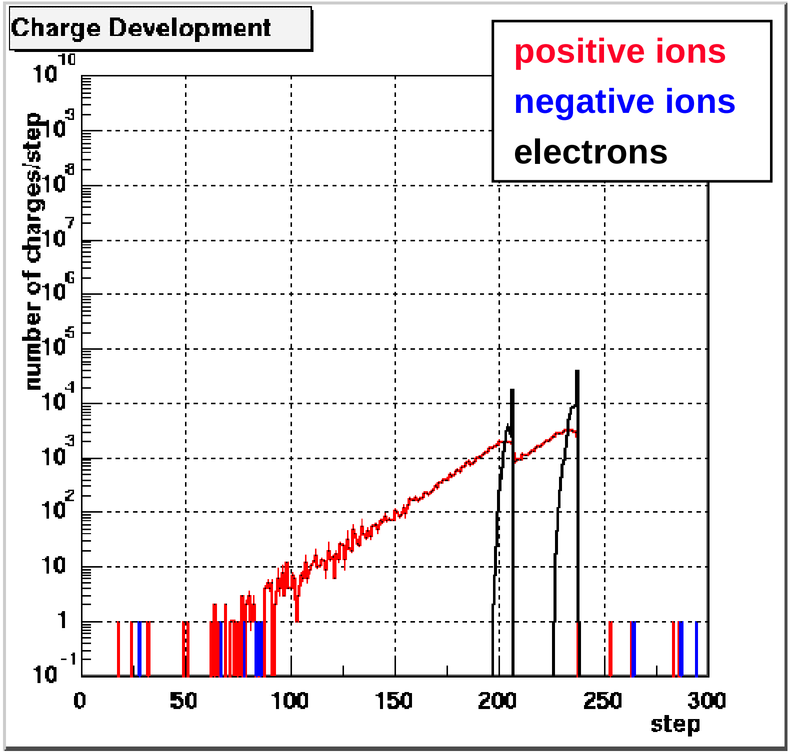
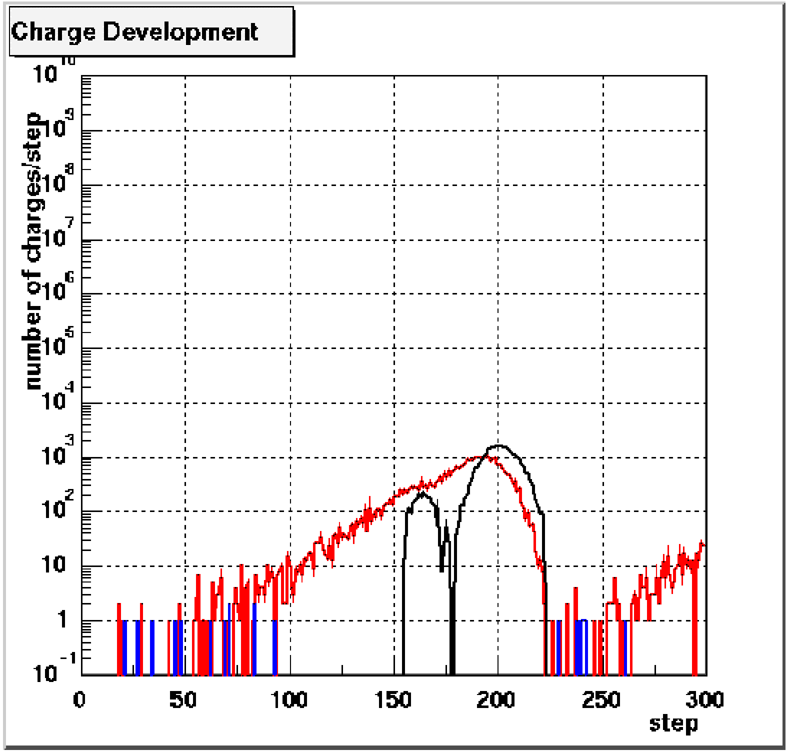
Avalanche Simulation; Snapshots

- No longitudinal diffusion
- Space charge reduces the drift velocity



- Include longitudinal diffusion



Avalanche Simulation; Snapshots

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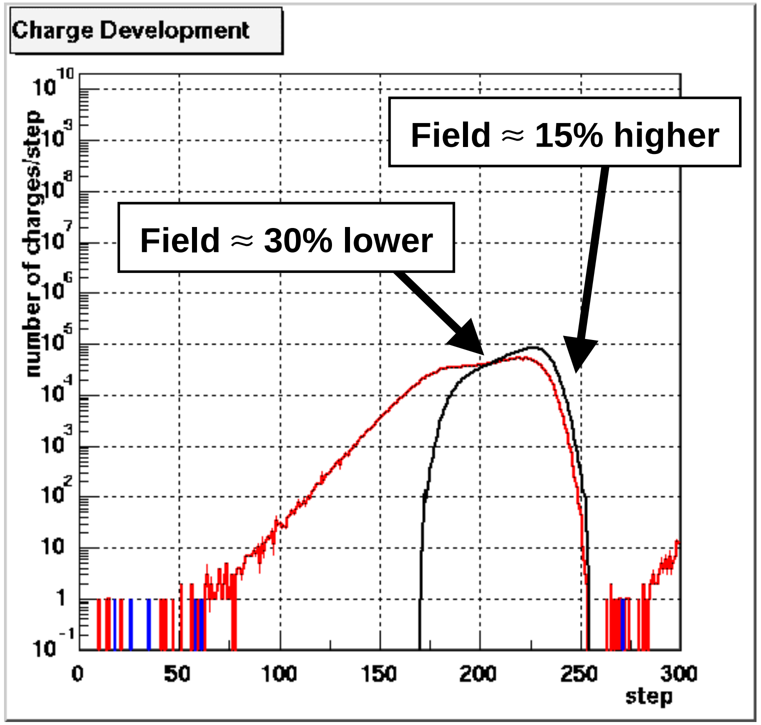
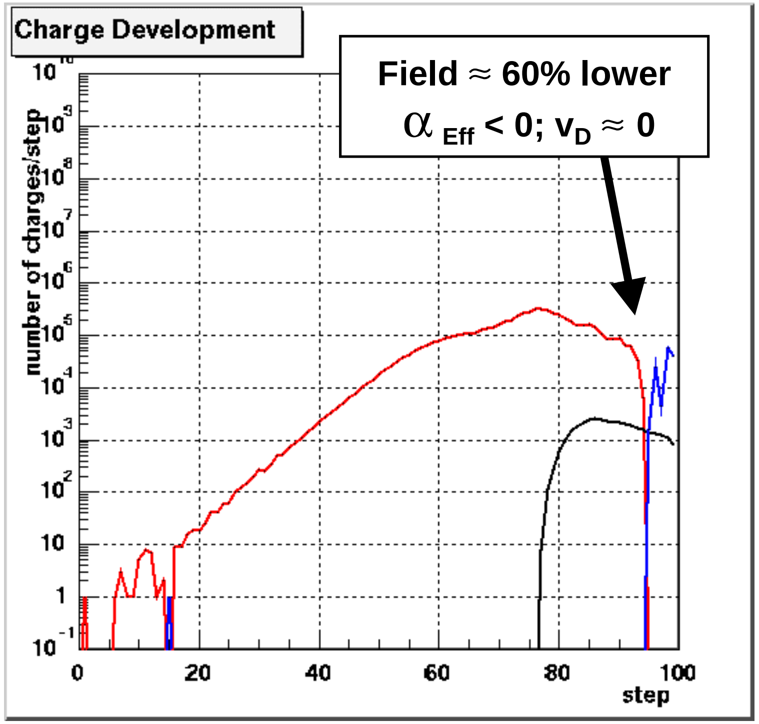
- The space charge reaches sufficient value to hinder growth of avalanche ⇒ more or less linear growth
- Electron cloud is lengthened longitudinally

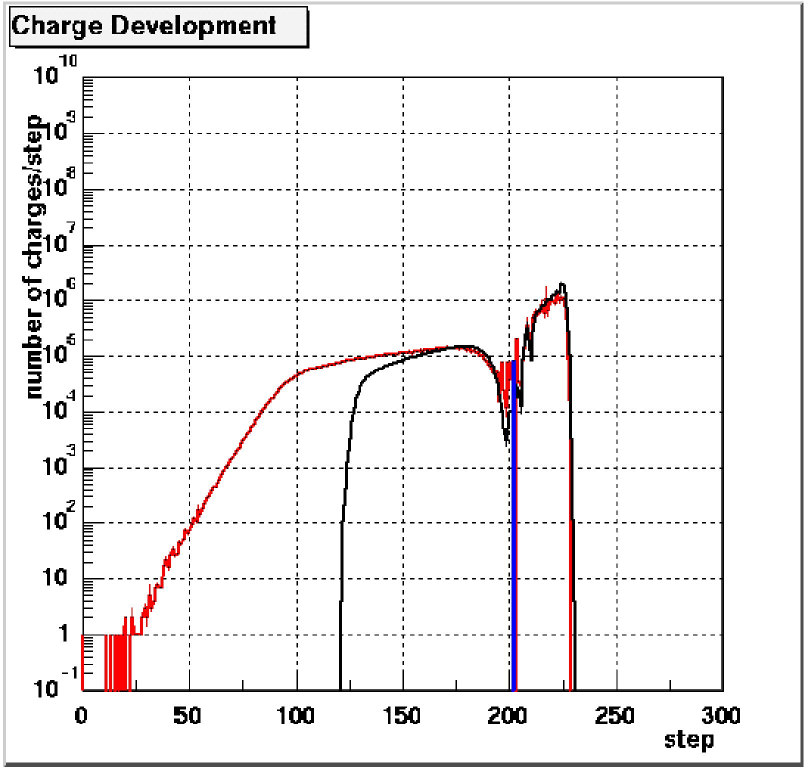


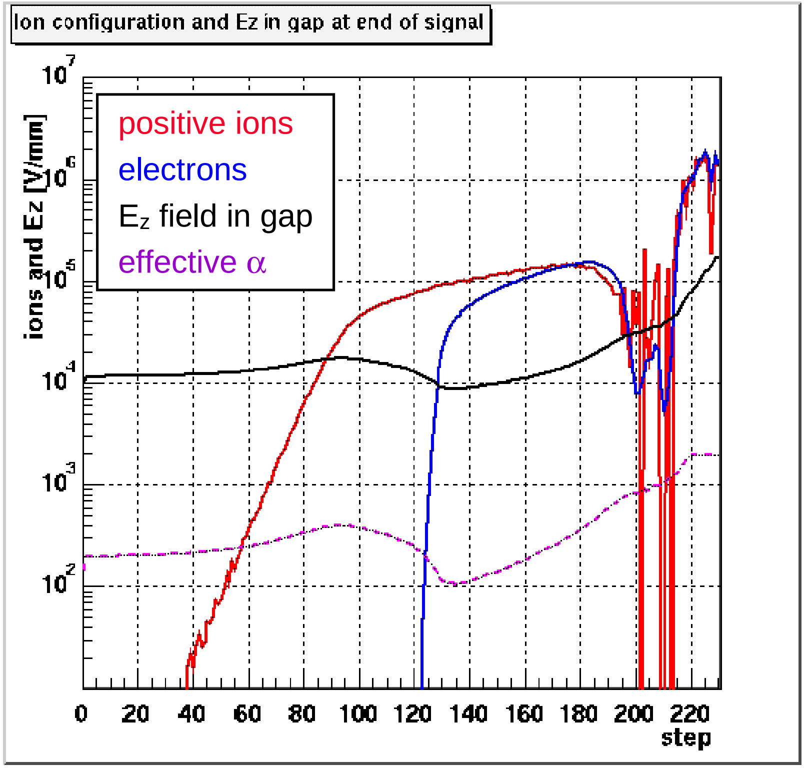
- Electrons that reached anode lower the field right before the anode
- At final stage only attachment; almost no movement
- Note the huge suppression factor



Anode Streamer; Snapshots

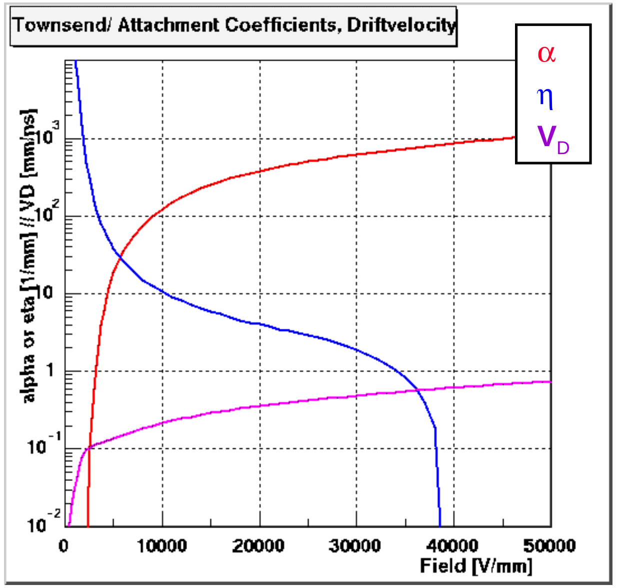
- Some events show exploding electric field:
- after a certain time in saturated mode the field at the head of the avalanche grows extremely fast



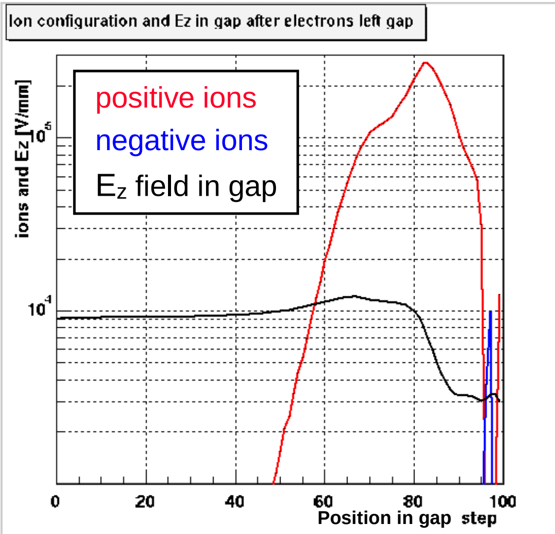


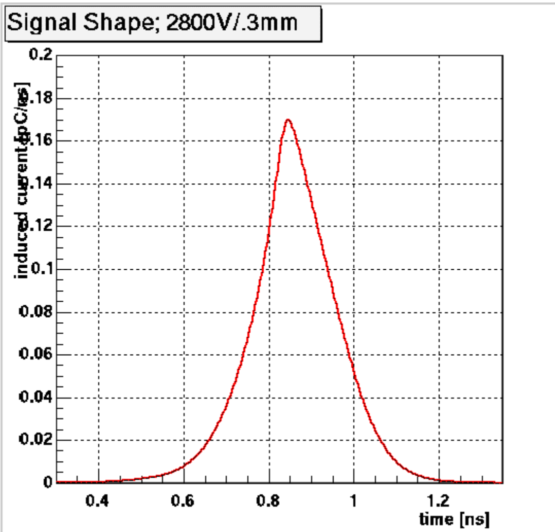
Simulation of Timing RPCs

- 0.3 mm gap; 2x2mm glass (ε = 8)
- $C_2F_4H_2$ / i-C_4H_{10} / SF_6 (85/5/10)
- D_T = 110 [μm/cm] and D_L = 90 [μm/cm]
- Use IMONTE/ MAGBOLTZ for calculation of α, η, v_D:



- top right: electrons just left gap; ions did not move at all (E_0 = 9000 V/mm)
  - Highest field: 1.28 x E_0
  - Lowest field: 0.22 x E_0
- bottom right: Typical signal [pC/ns]





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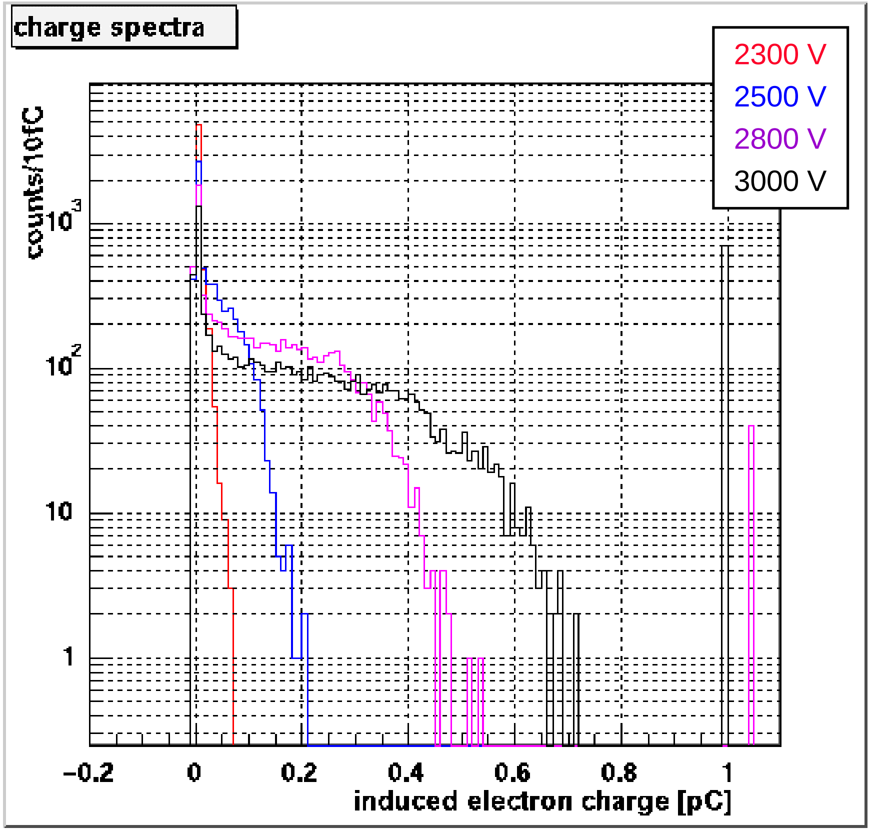
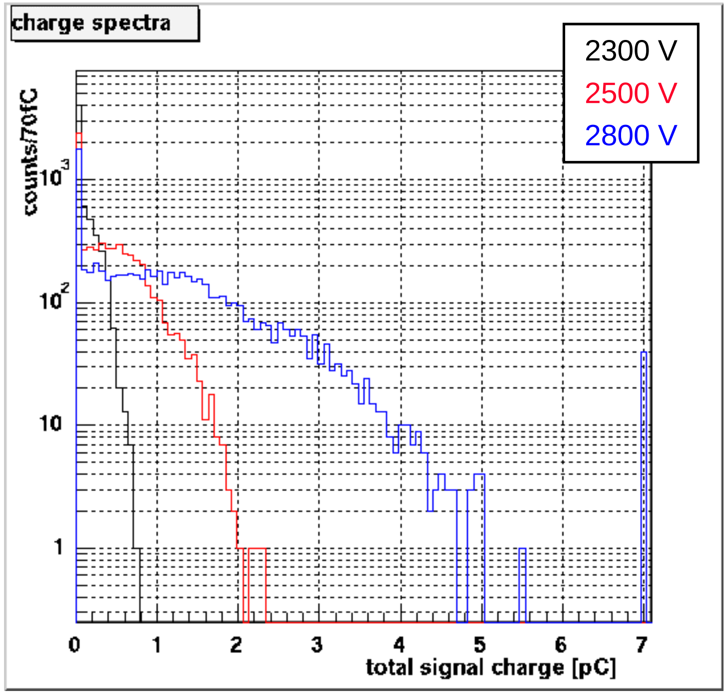
ADC Spectra of Timing RPCs

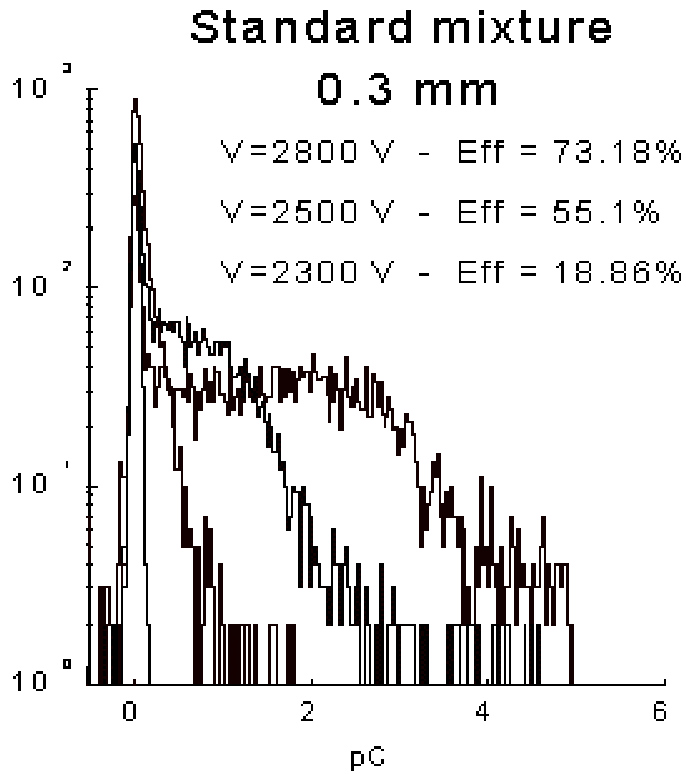
- **Induced fast charge**
- **Efficiencies at 10fC Threshold:**
  2300V: 12.5%
  2500V: 48%
  2800V: 68.3%
  3000V: 75%
  see previous talk [Werner Riegler]
- **peaks at >1pC correspond to Anode Streamer:**
  2800V: 0.5%
  3000V: 10%
- **Without space charge effect the average induced charge at 3000V would be $4{\cdot}10^5$pC**



ADC Spectra of Timing RPCs

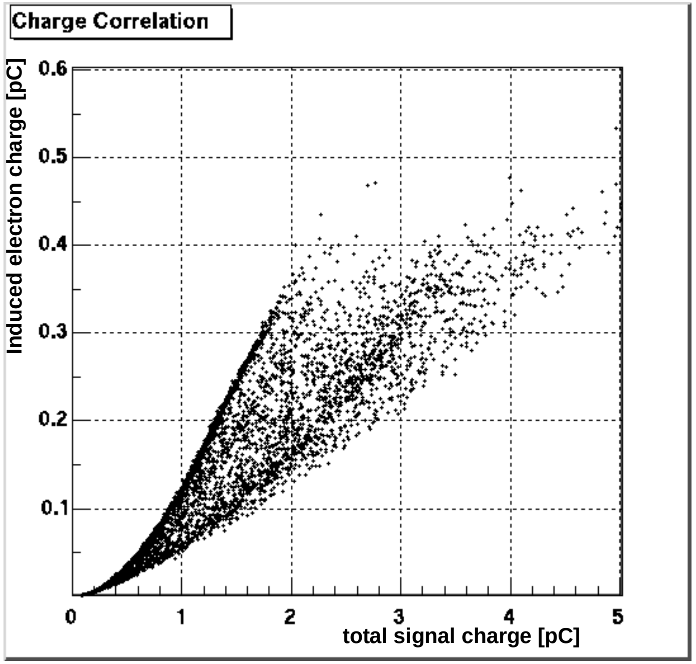
- **Total signal charge and shape of spectra match experimental results very well**
- **taken from** "High Resolution TOF with RPC's", P. Fonte, V. Peskov, LIP-2000-04

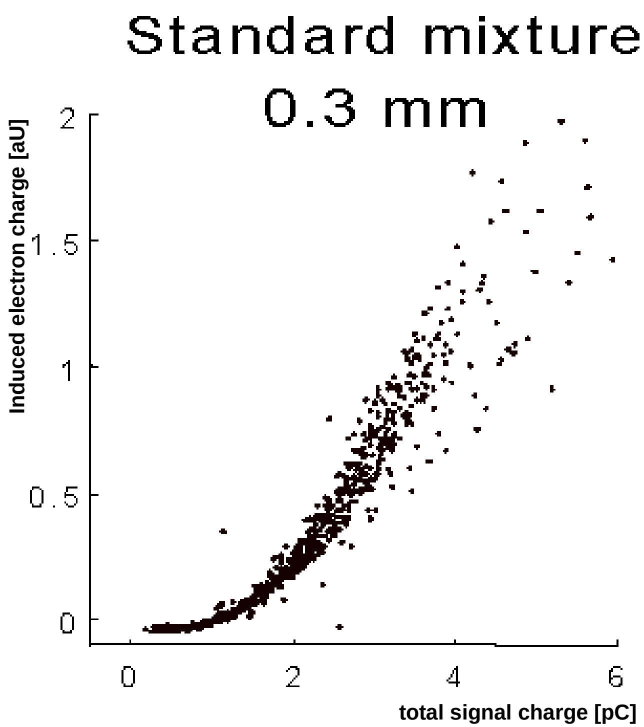




Charge Correlation with Timing RPCs

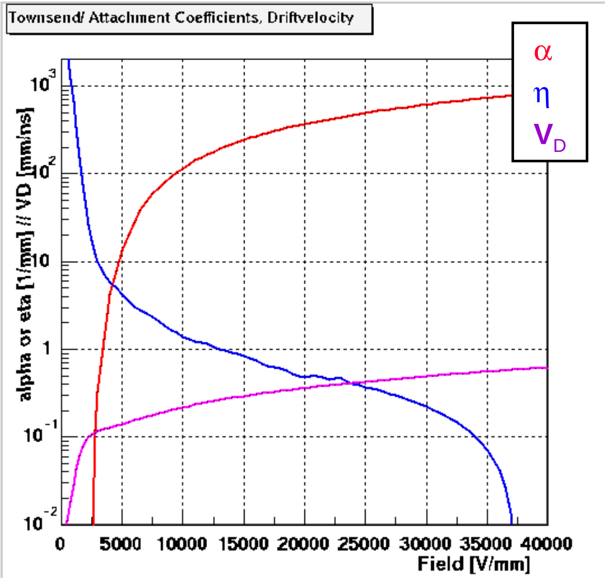
- **Correlation of total signal charge and fast induced charge shows upward curving also observed in experiment**
- **taken from** "High Resolution TOF with RPC's", P. Fonte, V. Peskov, LIP-2000-04



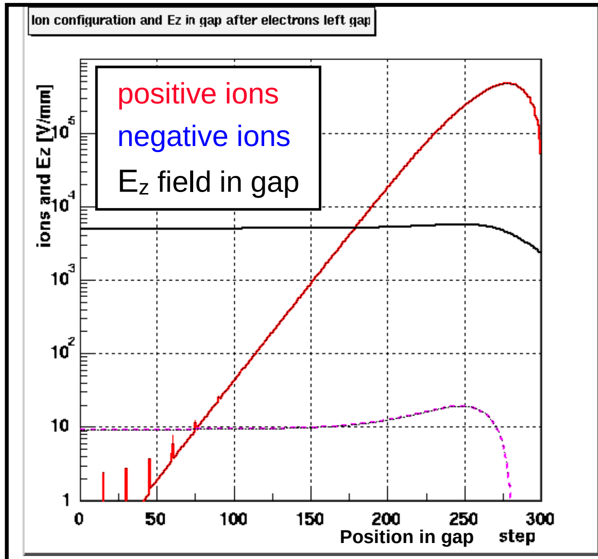


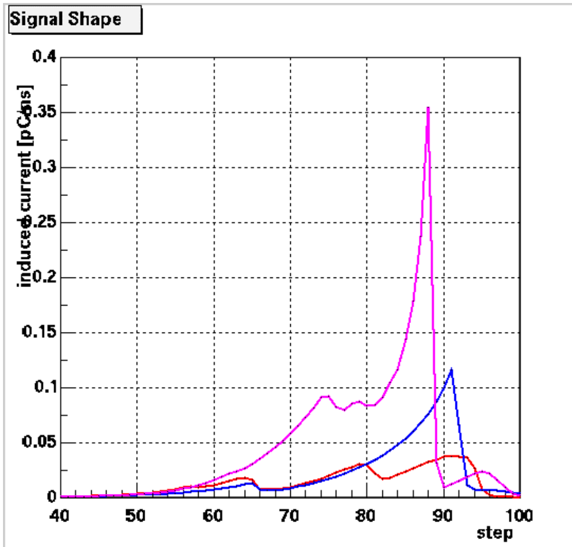
Simulation of Trigger RPCs

- **2 mm gap; 2 x 2mm bakelite (ε = 10), gas: $C_2F_4H_2$ / i-C_4H_{10} / SF_6 (97/2.5/0.5)**
- **Use IMONTE/ MAGBOLTZ for calculation of α, η, v_D :**



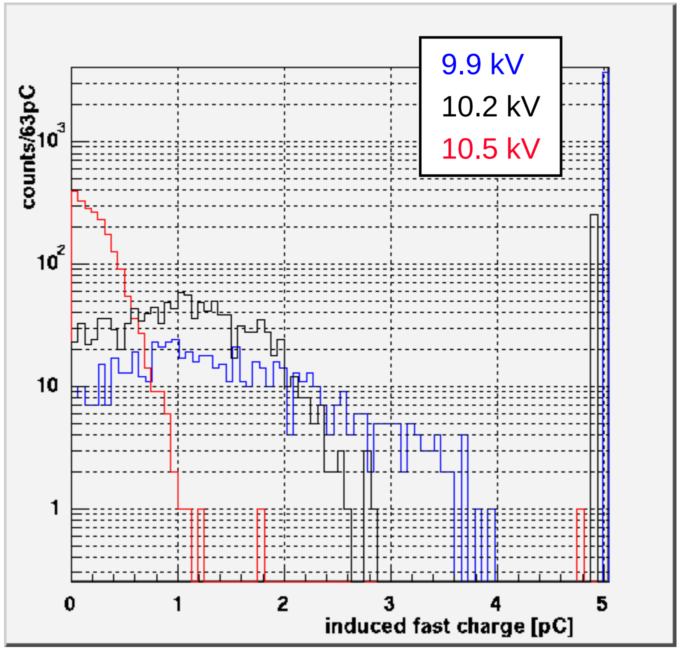
- **top right: electrons just left gap; ions did not move at all (E_0 = 5000 V/mm)**
  - **Electric field deviates by less than 20%.**
- **bottom right: Typical signals [pC/ns]**

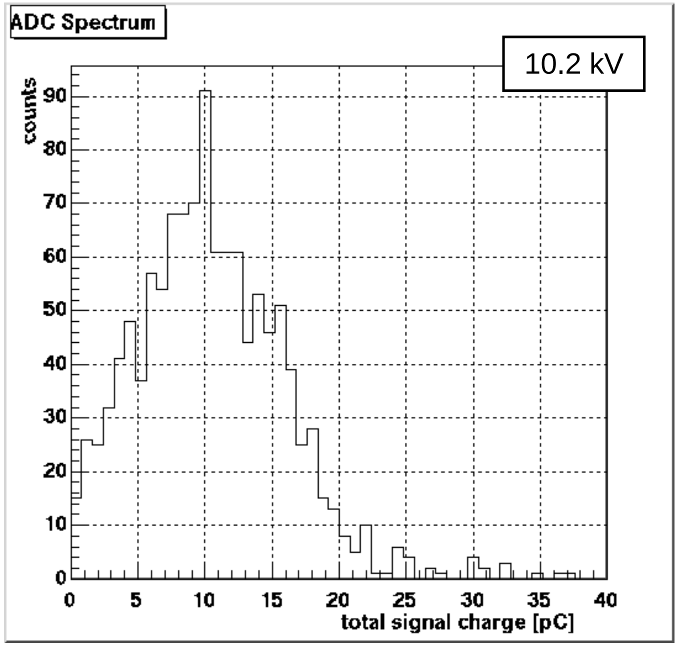




Simulation of Trigger RPCs

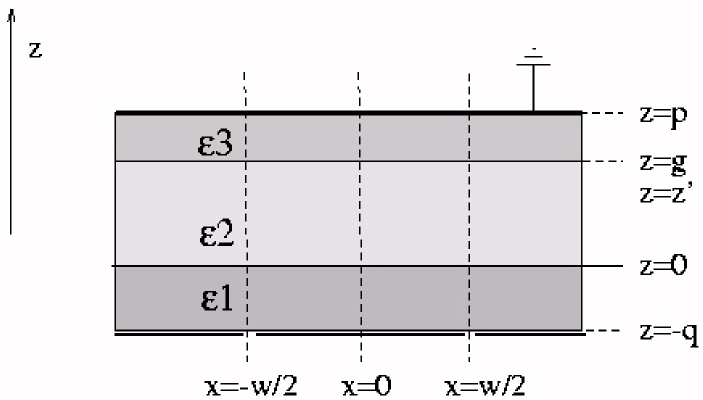
- Fast induced charge spectra
- D_T = 180 [µm/cm] and D_L = 120 [µm/cm] (1.5 times the with MAGBOLTZ simulated values)
- Efficiencies at 100fC Threshold: 9.9kV: 75%; 10.2kV: 97%; 10.5kV: 99%
- Streamer: 9.9kV: 0%; 10.2kV: 18%; 10.5kV: 85%
  not observed in reality
- try to vary the α and η curves





Analytic solutions for Weighting fields

- Analytic expression for the weighting field (z-component) of a strip electrode
- Allows calculation of induced signals and crosstalk in 3 layer RPC geometries
- $D(\kappa)$ has been defined on slide 4



$$E_z(x,z) = V_1\ \varepsilon_1 \frac{2}{\pi}\int_0^{\infty} d\kappa\ \cos(\kappa\ x)\ \sin\left(\kappa\ \frac{w}{2}\right)\ F_2(\kappa,z)$$

with

$$F_2(\kappa,z) = -\frac{2}{D(\kappa)}\Big[(\varepsilon_2+\varepsilon_3)\left(e^{-\kappa(q+z)} + e^{-\kappa(2p+q-z)}\right) - (\varepsilon_2-\varepsilon_3)\left(e^{-\kappa(q+2g-z)} + e^{-\kappa(2p+q-2g+z)}\right)\Big]$$

Conclusions

- An analytic expression for the field of a point charge in a RPC was presented.
- An extensive avalanche simulation program for RPCs was presented as well.
- The efficiencies and the average avalanche charges of 0.3 mm gap timing RPCs can be explained by a huge suppression factor caused by space charge effects.
- The shape of ADC spectra of 2mm gap trigger RPCs can also be explained by space charge effects. A lot of streamers ⇒ need further investigation.
- The analytic solution for the weighting field of a strip electrode was presented.
- The simulation assumes only physical parameters predicted by MAGBOLTZ, IMONTE and HEED and matches the data very well.