Rate Effects in Resistive Plate Chambers

Christian Lippmann(GSI), Werner Riegler (CERN)

and Alexander Kalweit (TU Darmstadt)

Overview

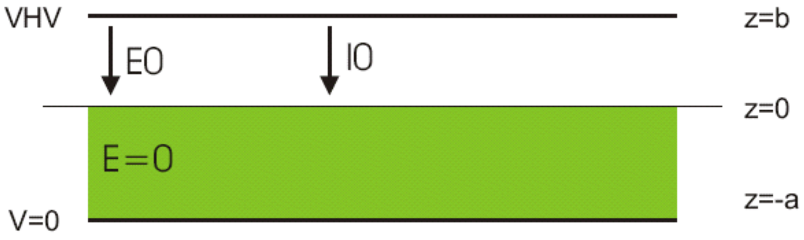
Exact solutions for electric fields of charges in RPCs

Monte Carlo simulations

Analytic expressions for rate effects

DC current model

RPC with a gas gap of thickness b and resistive plate of thickness a and volume resistivity $\rho = 1/\sigma$



$E_0 = V_{HV}/b$

A current I_0 on the surface causes a voltage drop of $\Delta V = a*\rho*I_0$ across the gas gap.

An avalanche charge Q (pC) at rate R (Hz/cm^2) gives a current of I_0=R*Q (A/cm^2).

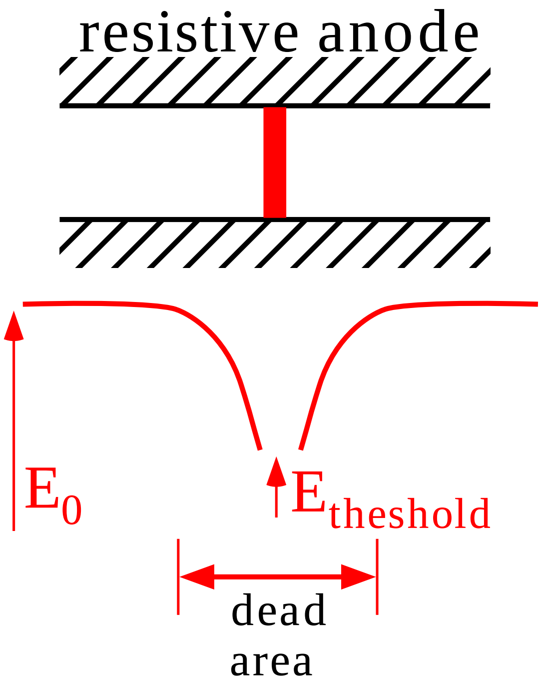
The resistive plate represents a resistance of $a*\rho$ (Ω cm^2) between gas gap and metal.

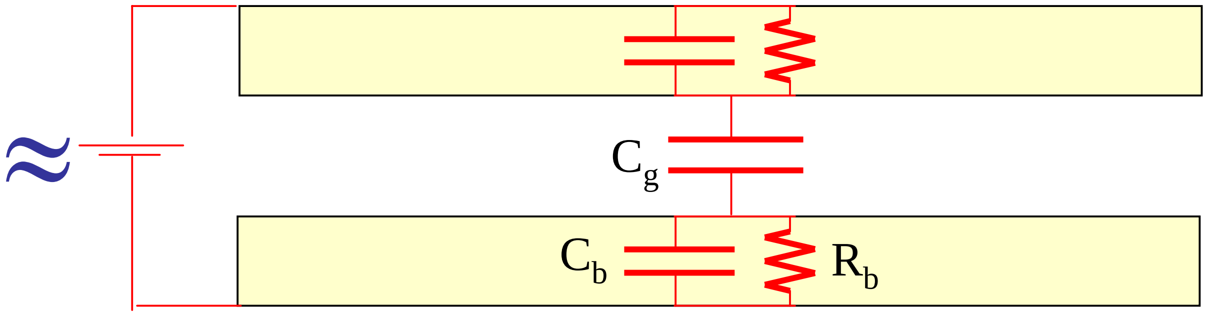
The voltage drop is therefore $\Delta V = \rho*a*I_0 = \rho*a*R*Q$ and the electric field drops by

$$\Delta E_{gap} = -\rho*a/b*R*Q$$

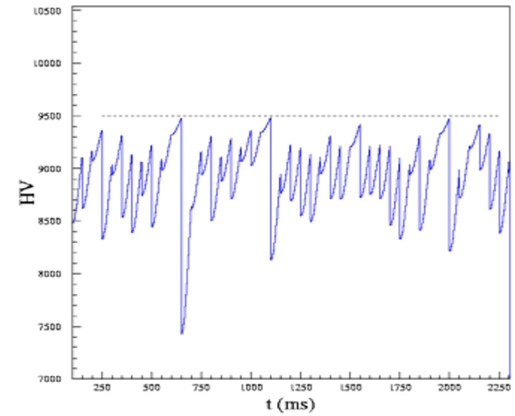
Single cell model

M. Abbrescia, RPC2003





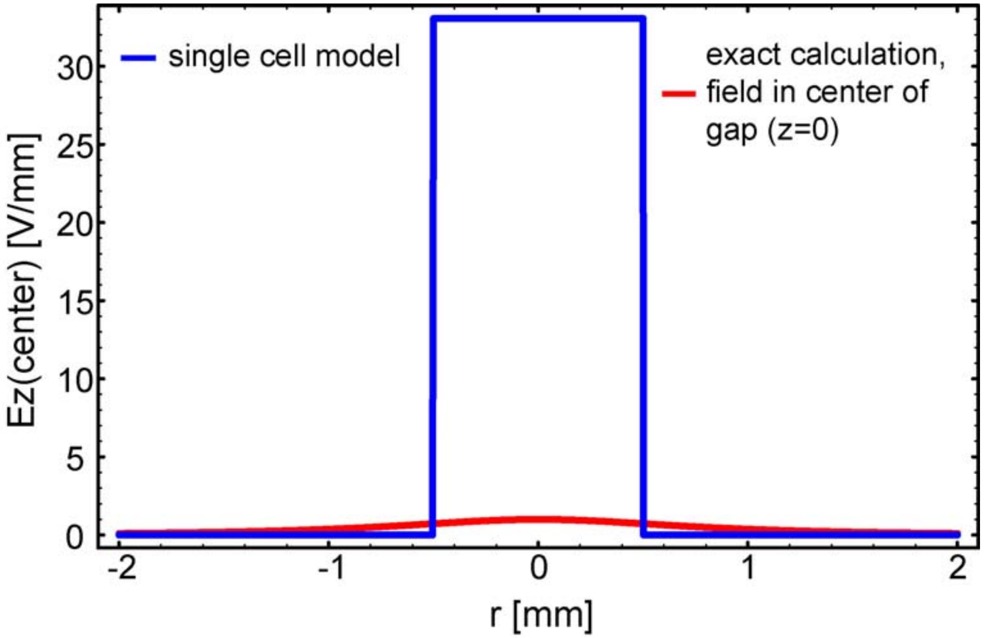
$$\tau = 2R_b\left(2C_b + C_g\right) = 2\rho_b\varepsilon_0\left(2\varepsilon_r + \frac{b}{g}\right)$$



Assumption:

The voltage drop due to a deposited charge q on the plate surface is given by the voltage q/C which is constant across the cell and decays with the single time constant τ.

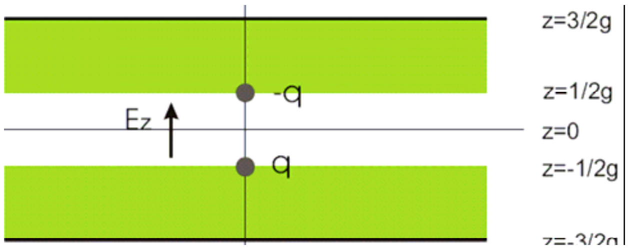
Comparison of the exact model to the single cell model



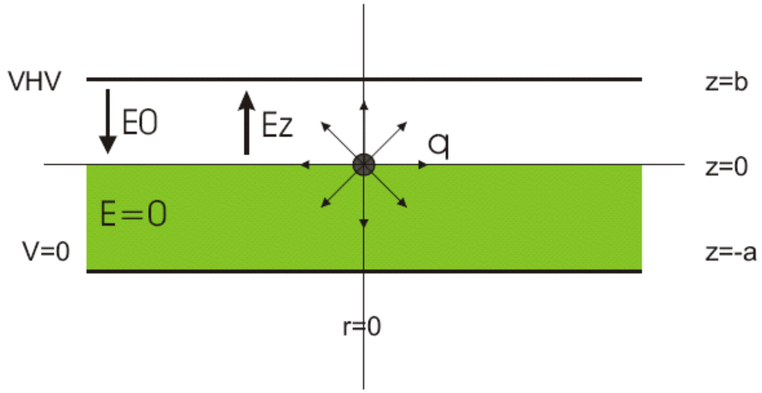
Parameters for this comparison:

- **trigger RPC**
- $\varepsilon_1 = 10\ \varepsilon_0$
- **g = 2 mm**
- $\rho = 10^{10}\ \Omega cm$
- **q = 50pC**

The electric field drop in the single cell model:
$E = U/g = q/Cg = q/\varepsilon_0 A$
$A \approx 1\ mm^2$



Exact calculation



Without particles traversing the RPC the field in the gas gap is V_{HV}/b and the field in the resistive plate is zero.

The charge sitting on the surface of the resistive plate decreases the field in the gas gap and causes an electric field in the resistive plate.

The electric field in the resistive plate will cause charges to flow in the resistive material which 'destroy' the point charge.

This causes a time dependent electric field E(x,y,z,t) in the gas gap which adds to the externally applied field E_0.

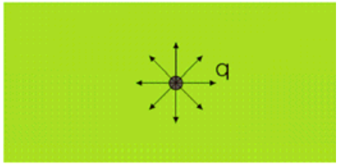
The electric field in the gas gap due to high rate is then simply given by superimposing this solution for the individual charges.

Quasistatic approximation of Maxwell's equations

Knowing the *electrostatic solution* for a material with permittivity ε, the *dynamic solution* for a material with permittivity ε and conductivity σ is obtained by replacing ε with ε + σ /s and performing the inverse Laplace transform.

Point charges in media with conductivity

Point Charge in infinite medium of permittivity ε$_1$



$$\phi(r) = \frac{1}{4\pi\varepsilon_1}\frac{q}{r}$$

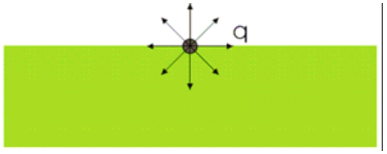
Point Charge placed in an infinite medium with permittivity ε$_1$ and conductivity σ at t=0: q(t) = q*Θ(t) → q(s)=q/s

$$\phi(r,s) = \frac{1}{4\pi(\varepsilon_1 + \sigma/s)}\frac{q/s}{r} \quad \rightarrow \quad \phi(r,t) = \frac{1}{4\pi\varepsilon_1}\frac{q}{r}e^{-\frac{t}{\tau}} \quad \tau = \frac{\varepsilon_1}{\sigma}$$

Charge is destroyed with characteristic time constant ε$_1$/σ.

Point charge on the boundary of an infinite halfspace with permittivity ε$_1$

$$\phi(r) = \frac{1}{2\pi(\varepsilon_0 + \varepsilon_1)}\frac{q}{r}$$



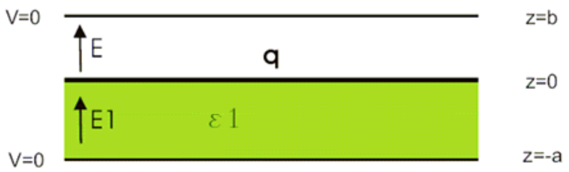
Point Charge placed on the boundary of an infinite halfspace with permittivity ε$_1$ and conductivity σ at t=0.

$$\phi(r,s) = \frac{1}{2\pi(\varepsilon_0 + \varepsilon_1 + \sigma/s)}\frac{q/s}{r} \quad \rightarrow \quad \phi(r,t) = \frac{1}{2\pi(\varepsilon_0 + \varepsilon_1)}\frac{q}{r}e^{-\frac{t}{\tau}} \quad \tau = \frac{\varepsilon_0 + \varepsilon_1}{\sigma}$$

Charge sheet in an RPC

Charge sheet with charge density q on the boundary between two media with permittivity ε_1 and ε_0 and a grounded plate at z=-a and z=b. From the conditions $aE_1 + bE=0$ and $-\varepsilon_1 E_1 + \varepsilon_0 E = q$ we find

$$E = \frac{aq}{a\varepsilon_0 + b\varepsilon_1}$$



Charge Sheet with charge density q placed in the RPC with resistive plate of permittivity ε_1 and conductivity σ: $q(t) = q^*\Theta(t)$ → $q(s)=q/s$

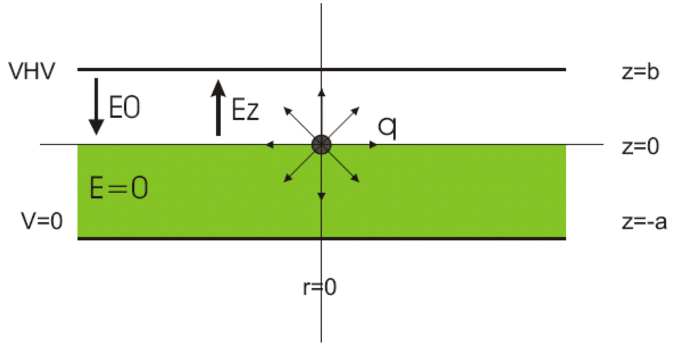
$$E(s) = \frac{aq/s}{a\varepsilon_0 + b(\varepsilon_1 + \sigma/s)} \quad \rightarrow \quad E(t) = \frac{aq}{a\varepsilon_0 + b\varepsilon_1} e^{-\frac{t}{\tau}} \qquad \tau = \frac{a\varepsilon_0 + b\varepsilon_1}{b\sigma}$$

Current I_0 on the surface i.e. $q(t) = I_0{}^*t$ → $q(s)=I_0/s^2$

$$E(s) = \frac{aI_0/s^2}{a\varepsilon_0 + b(\varepsilon_1 + \sigma/s)} \quad \rightarrow \quad E(t) = \frac{aI_0}{b\sigma}\left(1 - e^{-\frac{t}{\tau}}\right) \quad \stackrel{t\to\infty}{\rightarrow} \quad E = \frac{aI_0}{b\sigma}$$

With $I_0 = q^*R$ and $\sigma = 1/\rho$ this becomes (of course) equal to the DC model from before.

Point charge in RPC



Point charge in geometry with ε_0 and ε_1

$$E_z(r,z) = \frac{q}{2\pi(\varepsilon_0+\varepsilon_1)}\frac{z}{(r^2+z^2)^{\frac{3}{2}}} + \frac{q}{2\pi}\int_0^\infty J_0(kr)\left[f_1(k,z) - f_2(k,z)\right] dk$$

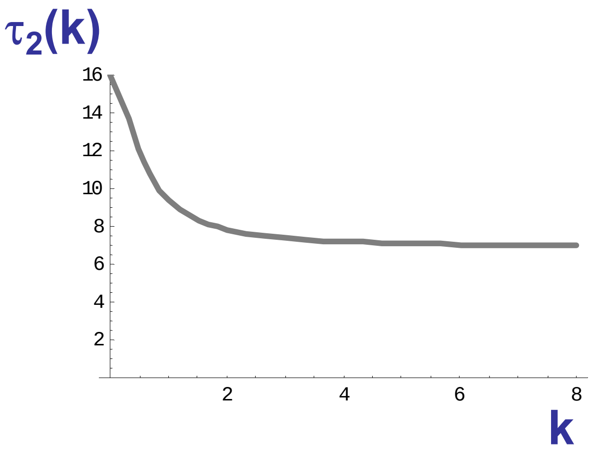
$$f_1(k,z) = \frac{k\cosh[k(b-z)]\sinh(ka)}{\varepsilon_0\cosh(kb)\sinh(ka)+\varepsilon_1\cosh(ka)\sinh(kb)} \qquad f_2(k,z) = \frac{ke^{-kz}}{\varepsilon_0+\varepsilon_1}$$

Point charge placed at position r=0, z=0 at time t=0, permittivity ε_1, conductivity σ

$$E_z(r,z,t) = \frac{q}{2\pi(\varepsilon_0+\varepsilon_1)}\frac{z}{(r^2+z^2)^{\frac{3}{2}}}e^{-t/\tau_1} + \frac{q}{2\pi}\int_0^\infty J_0(kr)\left[f_1(k,z)\,e^{-t/\tau_2(k)} - f_2(k,z)\,e^{-t/\tau_1}\right] dk$$

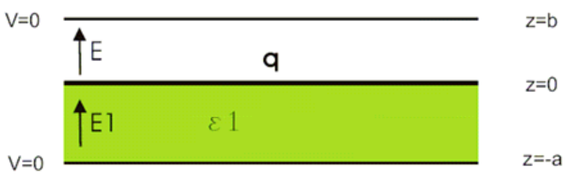
$$\tau_1 = \frac{\varepsilon_0+\varepsilon_1}{\sigma} \qquad \tau_2(k) = \frac{\varepsilon_0\cosh(kb)\sinh(ka)+\varepsilon_1\cosh(ka)\sinh(kb)}{\sigma\,\cosh(ka)\sinh(kb)}$$

Point charge in RPC

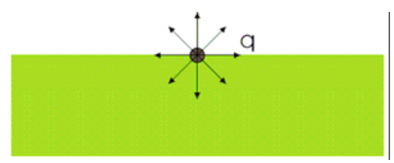


$$\tau_2(k) = \frac{\varepsilon_0 \cosh(kb)\sinh(ka) + \varepsilon_1 \cosh(ka)\sinh(kb)}{\sigma \, \cosh(ka)\sinh(kb)}$$

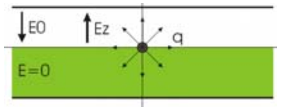
$$\lim_{k \to 0} \tau_2(k) = \frac{a\varepsilon_0 + b\varepsilon_1}{b\sigma}$$



$$\lim_{k \to \infty} \tau_2(k) = \frac{\varepsilon_0 + \varepsilon_1}{\sigma}$$



Charge decays with a continuous distribution of time constants between τ (charge sheet in RPC) and τ_1 (point charge at infinite half space).



Method for Monte Carlo Simulations

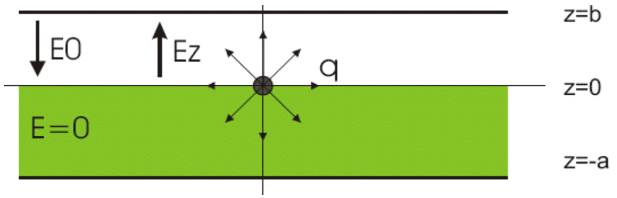
A <u>single gap</u> RPC of area A = 3*3 cm^2 is simulated.

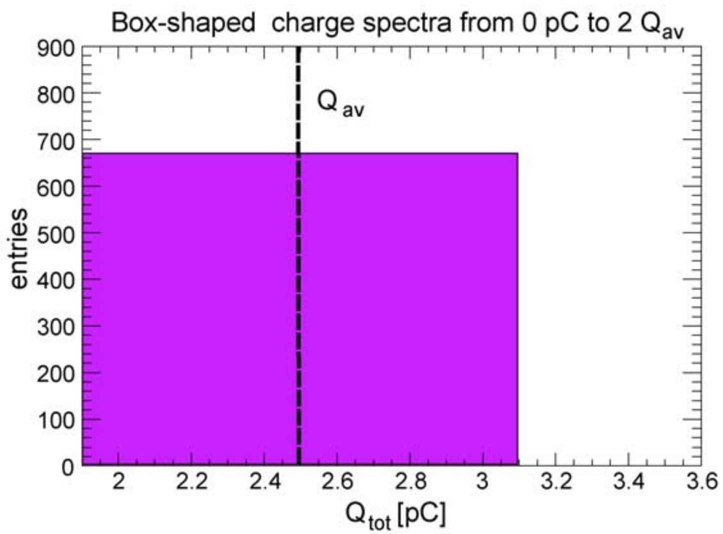
For each time step (Δt) a new number of charges (Δt*R*A) is distributed randomly on the surface of the resistive plate.

The z-component of the electric field of all charges in the resistive plates is calculated at always the same position (center of RPC area, center of gap or close to electrodes) at all time steps and added to the applied field: $E_{tot} = E_0 + \sum E_z(r,z,t)$.

All charges are kept in memory until their field contribution has fallen below 10^{-26} V/cm (up to 60s for Timing RPC).

Input Parameters for Timing RPC

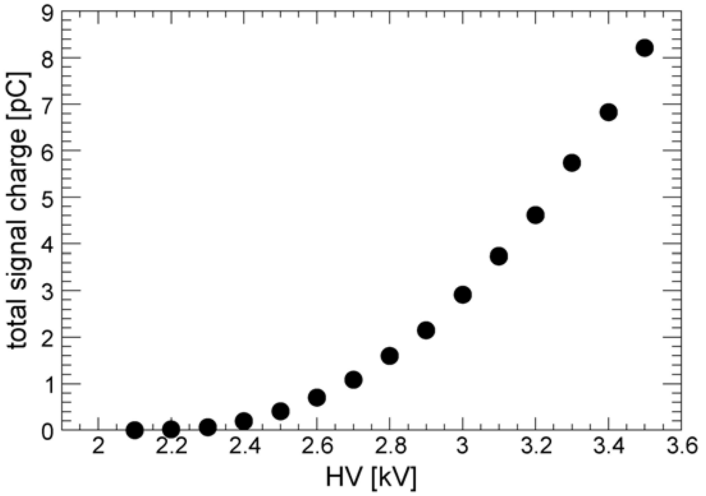


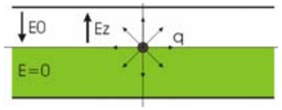


- HV = 3kV ⇒ E_0 = 100 kV / cm
- $\varepsilon_1 = 8\ \varepsilon_0$
- a = 3 mm
- b = 0.3 mm
- $\rho = 10^{12}\ \Omega$cm

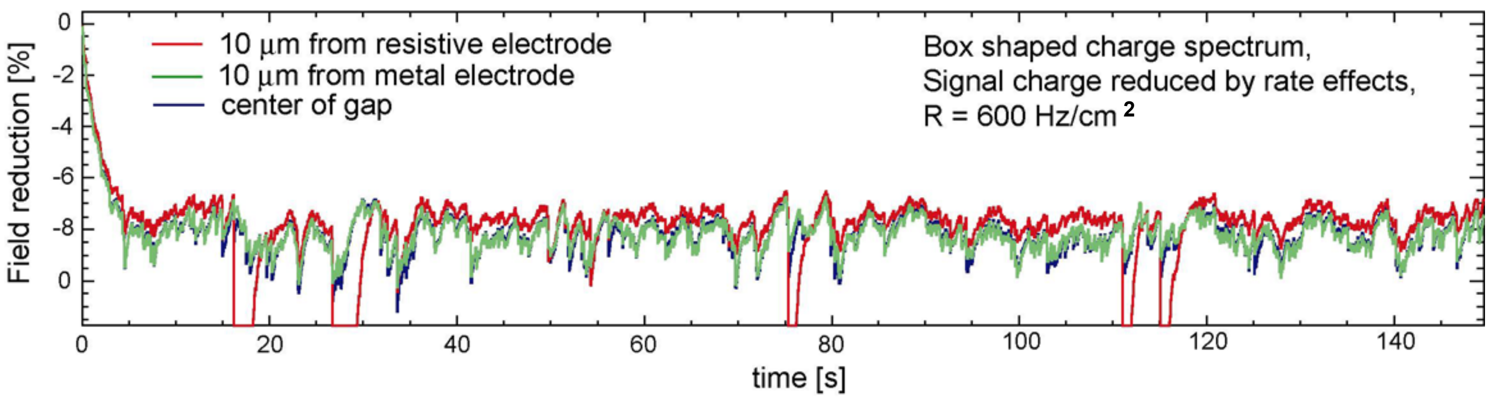
We use box-shaped charge spectra from 0 pC to 2 times the average total signal charge.

For the average total signal charge as a function of the HV we use simulated data.

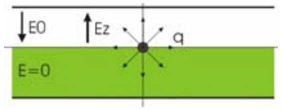




Monte Carlo for Timing RPCs

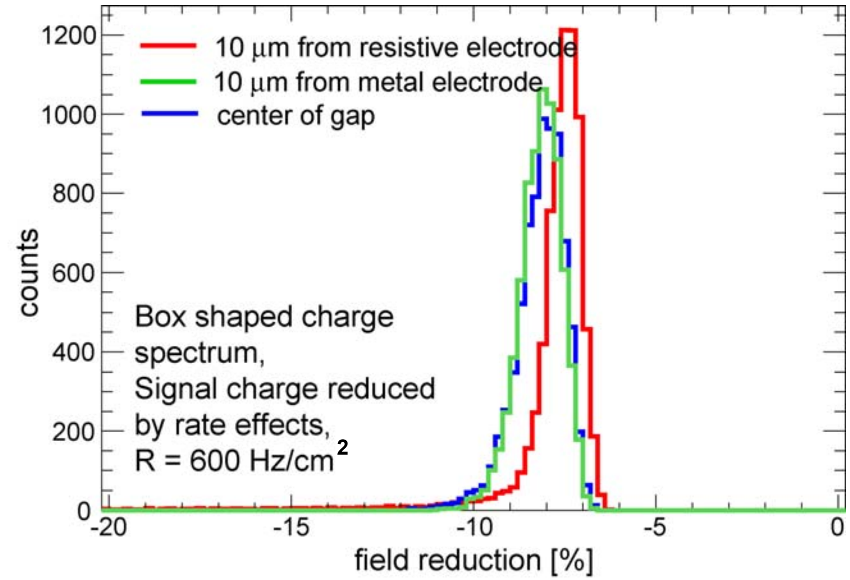


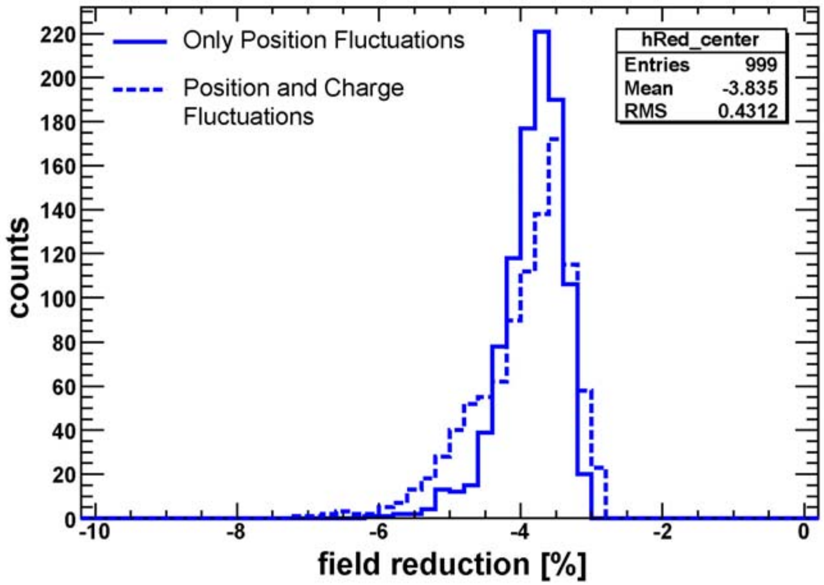
Fluctuations of the electric field at three different z-positions in the gap.



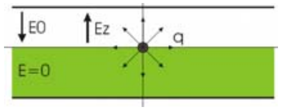
Monte Carlo for Timing RPCs: Results (1)

The field fluctuations at the three different z-positions in the gas gap. The mean values are the same everywhere. Close to the resistive plate the r.m.s. is the largest.



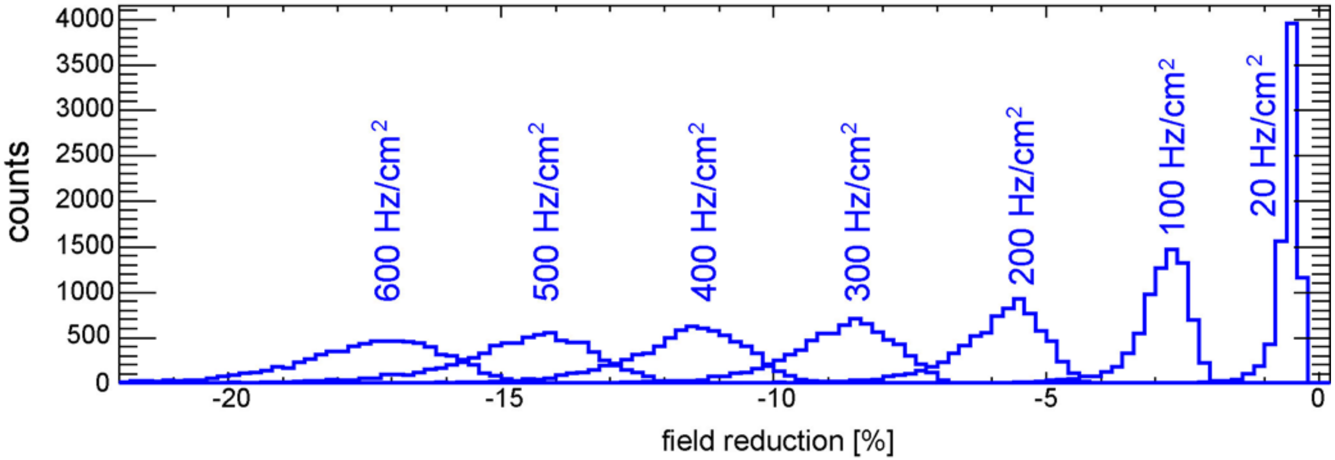


Position and charge fluctuations contribute to the field variations. The average field reduction is the same in both cases.

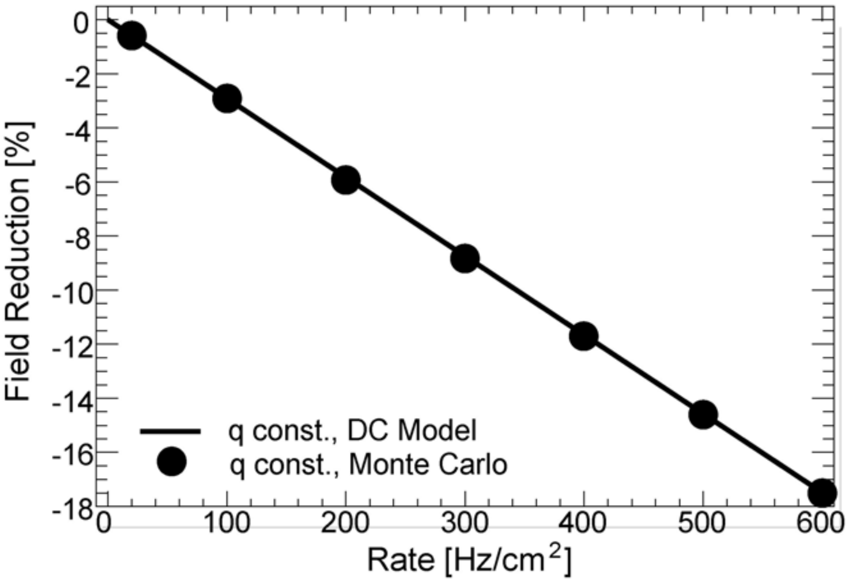


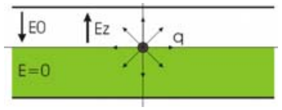
Monte Carlo for Timing RPCs: Results (2)

Here the total avalanche charge is kept constant for all rates:



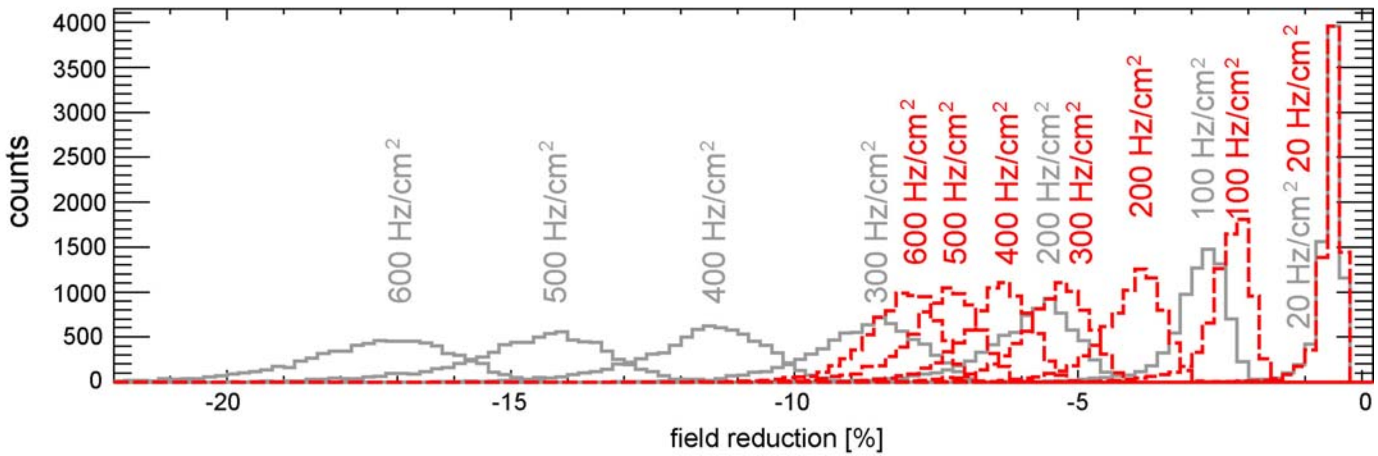
The average field reduction in the gap center is exactly the same as the one calculated from the DC model.



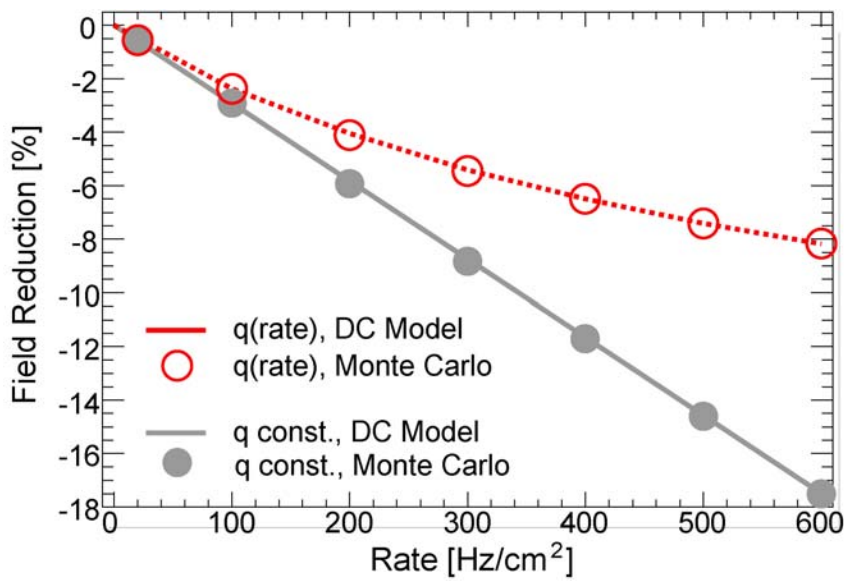


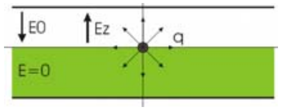
Monte Carlo for Timing RPCs: Results (3)

Total avalanche charge decreases as the electric field decreases with rate:



The average field reduction in the gap center is exactly the same as the one calculated from the DC model.



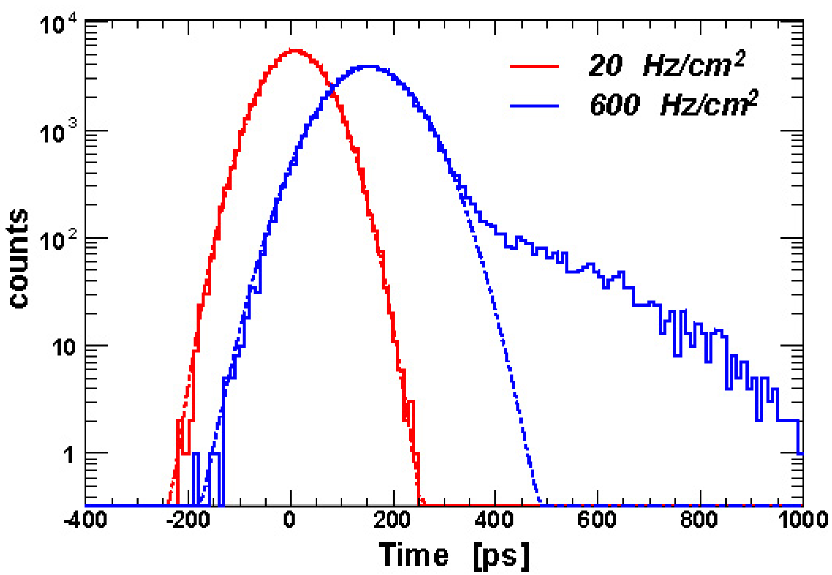


Monte Carlo for Timing RPCs: Results (4)

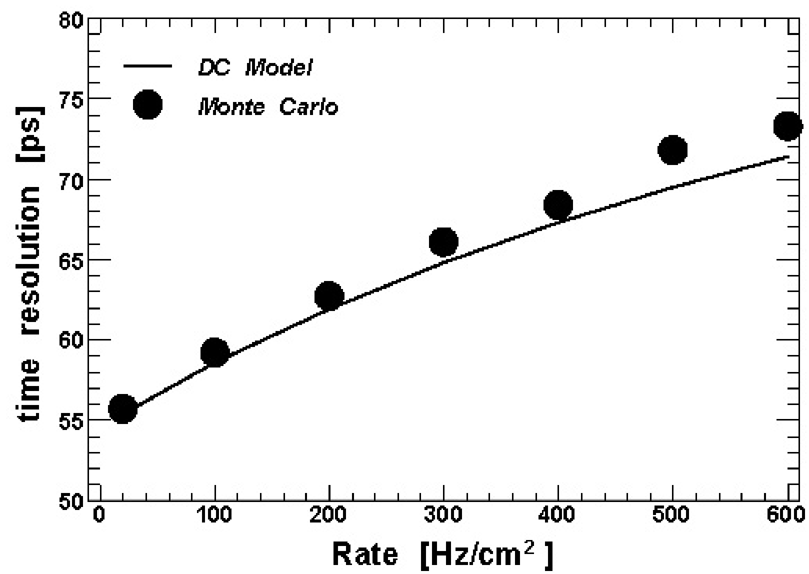
The (local) *threshold crossing time* and the *time resolution* for a given electric field can be calculated at all time steps during a simulation using the analytic formulas:

$$t_0 = \ln(Q_{thr}) / (v_D(\alpha-\eta)) \quad \text{and} \quad \sigma_t = 1.28 / (v_D(\alpha-\eta)) .$$

Monte Carlo:



Comparison to DC Modell:



⇒ **According to this simulation the field <u>fluctuations</u> have a small influence on the time resolution.**

Analytic Formulas for Mean and Standard Deviation (1)

In RPC2005 Gonzalez-Diaz et al. proposed to use *Campbell's theorem* in order to arrive at an analytic expression for the average field drop and the variation.

The theorem states that for a signal $E(t) = \sum_n a_n f(t - t_n)$

where a_n is from a random amplitude distribution and t_n are random (exponentially distributed) times with an average frequency ν, the average and standard deviation of $E(t)$ are

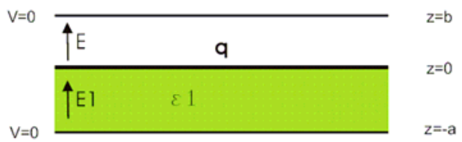
$$\overline{E} = \nu \overline{a} \int f(t) dt \qquad \text{and} \qquad (\Delta E)^2 = \nu \overline{a^2} \int f(t)^2 dt \, .$$

Analytic Formulas for Mean and Standard Deviation (2)

The theorem cannot be applied to our exact solution, since the signal shape is not constant.

We can however approximate the situation by assuming

$$E_z(r,z,t) \approx q E_z^{stat}(r,z)\, e^{-t/\tau_2}$$



This gives an electric field at time *t* of

$$E(t) = \sum_n q_n\, E_z^{stat}(r_n, z)\, e^{-\frac{t-t_n}{\tau_2}}$$

where q_n, r_n and t_n are the charge, position and time of event *n*.

Analytic Formulas for Mean and Standard Deviation (3)

Applying Campbell's theorem with a flux Φ of particles ($\nu = r{*}2\pi{*}\Phi$) we find the average field in the gap of

$$\begin{aligned} \overline{E} &= \nu\, \overline{q}\, \overline{E_z^{stat}} \int_0^\infty e^{-\frac{t}{\tau_2}}\, dt \\ &= \Phi\, \overline{q} \int_0^\infty 2\, r\, \pi\; E_z^{stat}(r,z)\, dr\, \tau_2 \\ &= \rho\, \frac{a}{b}\, \Phi \overline{q}\ . \end{aligned}$$

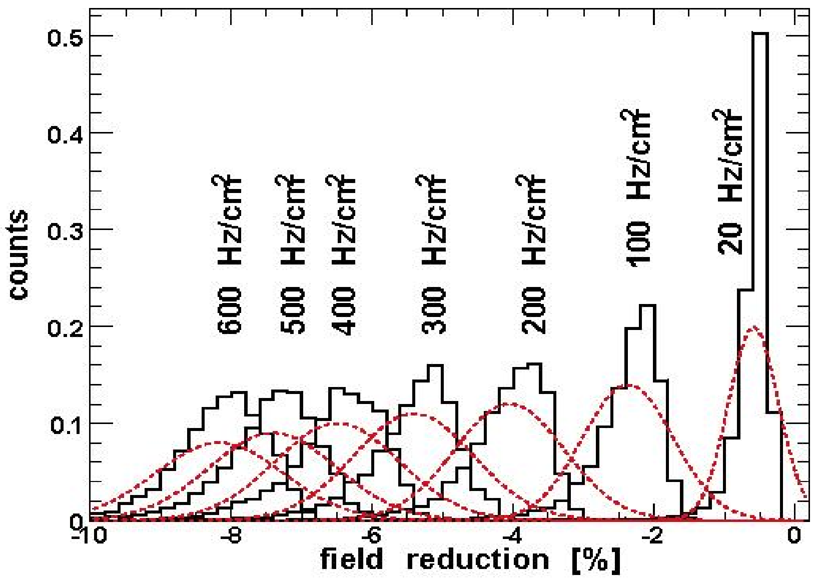
This shows that the choice of τ_2 for the overall time constant guarantees that the model is giving the correct average DC drop. For the relative variance we get

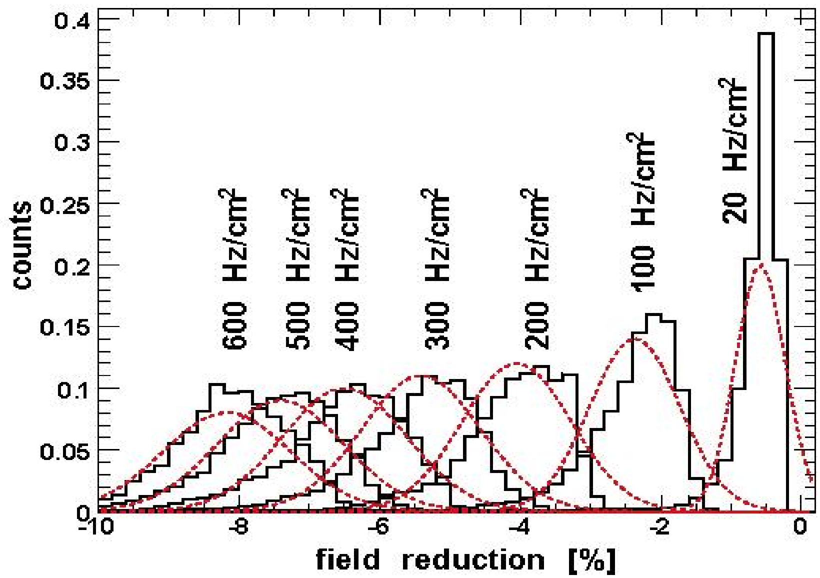
$$\frac{\Delta E(z)}{\overline{E}} = \frac{1}{\sqrt{2\Phi\tau A(z)}} \sqrt{1 + \frac{(\Delta q)^2}{\overline{q}^2}}$$

with an *effective area* of

$$A(z) = \frac{\left[\int_0^\infty 2r\pi E_z^{stat}(r,z) dr\right]^2}{\int_0^\infty 2r\pi E_z^{stat}(r,z)^2 dr}$$

Comparison to Monte Carlo





Monte Carlo with exact solution leads to lower standard deviations of the field values than analytic calculation.

Monte Carlo with

$$E(t) = \sum_n q_n E_z^{stat}(r_n, z)\, e^{-\frac{t-t_n}{\tau_2}}$$

leads to good agreement of the field values with the analytic calculation.

The continuous time constant makes a difference!!

Summary / Conclusions

We calculate rate effects in RPCs by using the *exact* time dependend solutions for the electric field of a point charge on the resistive plate of an RPC.

The charges decay with a *continuous distribution of time constants*. The two limiting cases are a continuous charge sheet (DC Model) and a point charge at an infinite half space.

We present a Monte Carlo simulation for single gap Timing RPCs with one resistive plate.

The electric field fluctuates due to the particle flux around a mean value which is equal to the value derived with the DC Model.

The simulation suggests that these field fluctuations have little influence on the time resolution for a single gap of the investigated geometry.

An analytic calculation using Campbell's theorem (Gonzalez-Diaz et al., RPC2005) can be used to approximate rate effects.