

Rate Effects in Resistive Plate Chambers

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Overview

Exact solutions for electric fields of charges in RPCs

Monte Carlo simulations

DC current model

RPC with a gas gap of thickness b and resistive plate of thickness a and volume resistivity $\rho = 1/\sigma$



A current I_0 on the surface causes a voltage drop of $\Delta V = a \cdot \rho \cdot I_0$ across the gas gap.

An avalanche charge Q (pC) at rate R (Hz/cm²) gives a current of $I_0 = R \cdot Q$ (A/cm²).

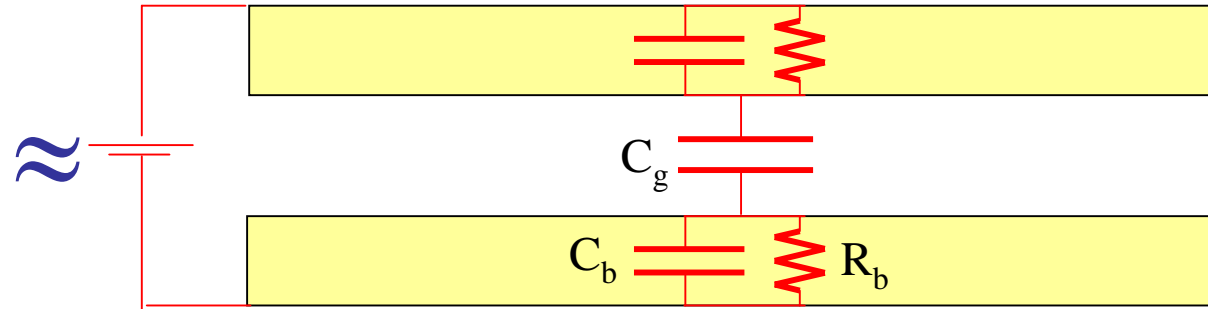
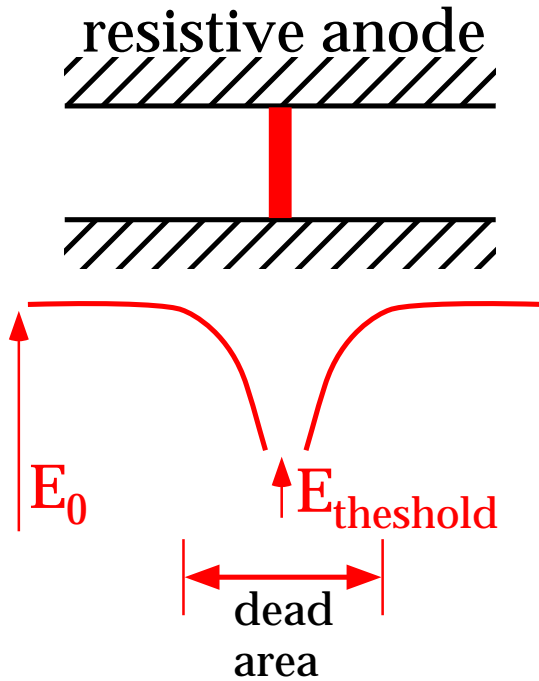
The resistive plate represents a resistance of $a \cdot \rho$ (Ω cm²) between gas gap and metal.

The voltage drop is therefore $\Delta V = \rho \cdot a \cdot I_0 = \rho \cdot a \cdot R \cdot Q$ and the electric field drops by

$$\Delta E_{\text{gap}} = -\rho \cdot a / b \cdot R \cdot Q$$

Single cell model

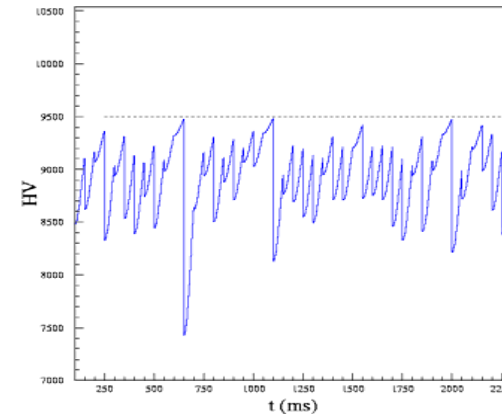
M. Abbrescia, RPC2003



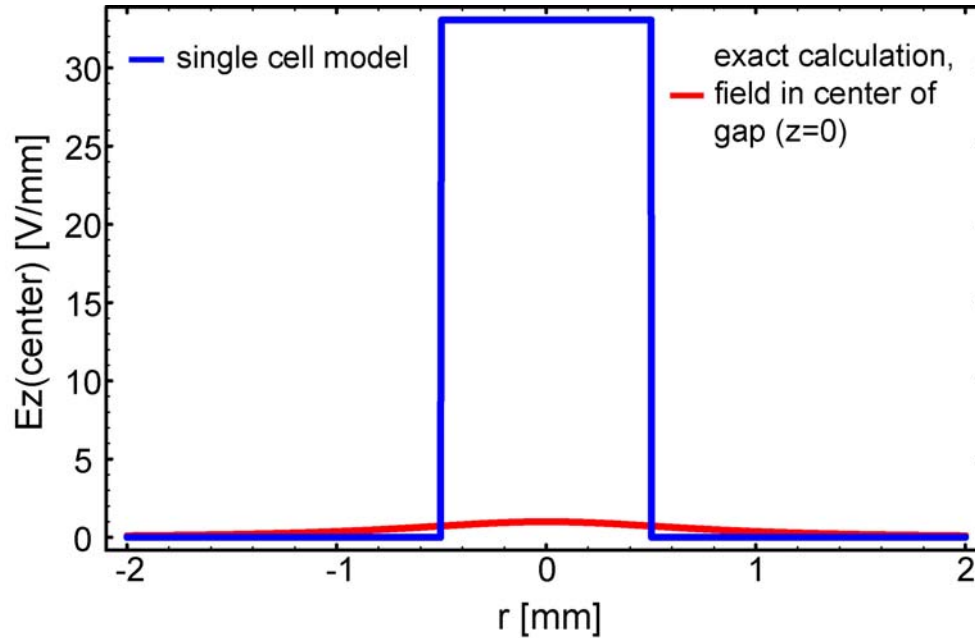
$$\tau = 2R_b(2C_b + C_g) = 2\rho_b \varepsilon_0 \left(2\varepsilon_r + \frac{b}{g} \right)$$

Assumption:

The voltage drop due to a deposited charge q on the plate surface is given by the voltage q/C which is constant across the cell and decays with the single time constant τ .



Comparison of the exact model to the single cell model



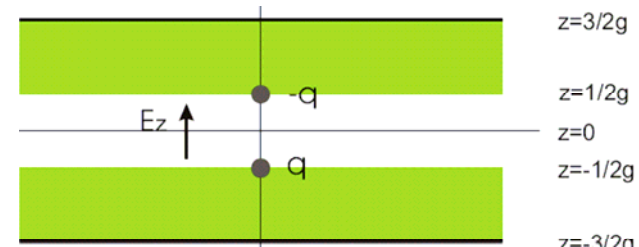
Parameters for this comparison:

- trigger RPC
- $\epsilon_1 = 10 \epsilon_0$
- $g = 2 \text{ mm}$
- $\rho = 10^{10} \Omega\text{cm}$
- $q = 50\text{pC}$

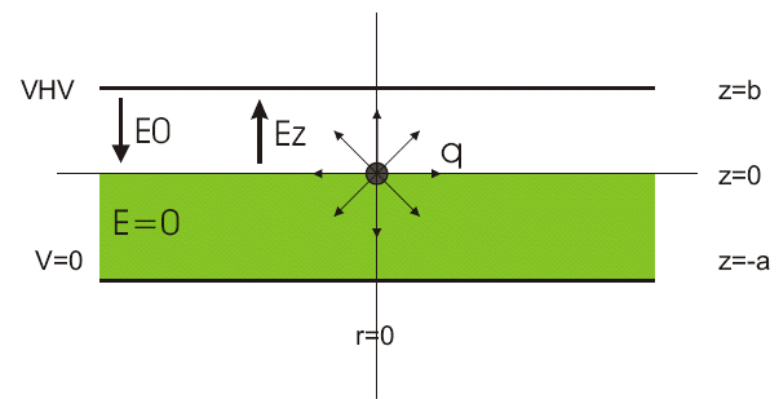
The electric field drop in the single cell model:

$$E = U/g = q/Cg = q/\epsilon_0 A$$

$$A \approx 1 \text{ mm}^2$$



Exact calculation



Without particles traversing the RPC the field in the gas gap is V_{HV}/b and the field in the resistive plate is zero.

The charge sitting on the surface of the resistive plate decreases the field in the gas gap and causes an electric field in the resistive plate.

The electric field in the resistive plate will cause charges to flow in the resistive material which 'destroy' the point charge.

This causes a time dependent electric field $E(x,y,z,t)$ in the gas gap which adds to the externally applied field E_0 .

The electric field in the gas gap due to high rate is then simply given by superimposing this solution for the individual charges.

Quasistatic approximation of Maxwell's equations

Knowing the electrostatic solution for a material with permittivity ϵ , the dynamic solution for a material with permittivity ϵ and conductivity σ is obtained by replacing ϵ with $\epsilon + \sigma /s$ and performing the inverse Laplace transform.

Point charges in media with conductivity

Point Charge in infinite medium of permittivity ϵ_1

$$\phi(r) = \frac{1}{4\pi\epsilon_1} \frac{q}{r}$$



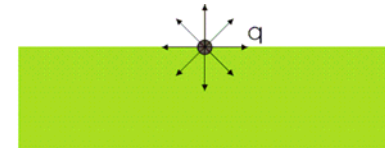
Point Charge placed in an infinite medium with permittivity ϵ_1 and conductivity σ at $t=0$: $q(t) = q \cdot \Theta(t) \rightarrow q(s) = q/s$

$$\phi(r, s) = \frac{1}{4\pi(\epsilon_1 + \sigma/s)} \frac{q/s}{r} \quad \rightarrow \quad \phi(r, t) = \frac{1}{4\pi\epsilon_1} \frac{q}{r} e^{-\frac{t}{\tau}} \quad \tau = \frac{\epsilon_1}{\sigma}$$

Charge is destroyed with characteristic time constant ϵ_1/σ .

Point charge on the boundary of an infinite halfspace with permittivity ϵ_1

$$\phi(r) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \frac{q}{r}$$



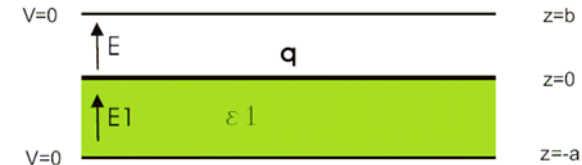
Point Charge placed on the boundary of an infinite halfspace with permittivity ϵ_1 and conductivity σ at $t=0$.

$$\phi(r, s) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1 + \sigma/s)} \frac{q/s}{r} \quad \rightarrow \quad \phi(r, t) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \frac{q}{r} e^{-\frac{t}{\tau}} \quad \tau = \frac{\epsilon_0 + \epsilon_1}{\sigma}$$

Charge sheet in an RPC

Charge sheet with charge density q on the boundary between two media with permittivity ϵ_1 and ϵ_0 and a grounded plate at $z=-a$ and $z=b$. From the conditions $aE_1 + bE=0$ and $-\epsilon_1 E_1 + \epsilon_0 E=q$ we find

$$E = \frac{aq}{a\epsilon_0 + b\epsilon_1}$$



Charge Sheet with charge density q placed in the RPC with resistive plate of permittivity ϵ_1 and conductivity σ : $q(t) = q \cdot \Theta(t) \rightarrow q(s) = q/s$

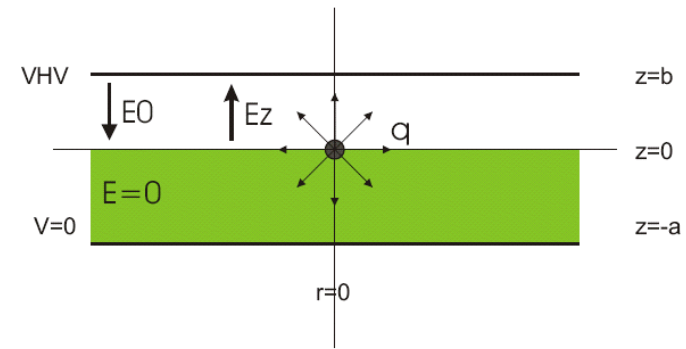
$$E(s) = \frac{aq/s}{a\epsilon_0 + b(\epsilon_1 + \sigma/s)} \rightarrow E(t) = \frac{aq}{a\epsilon_0 + b\epsilon_1} e^{-\frac{t}{\tau}} \quad \tau = \frac{a\epsilon_0 + b\epsilon_1}{b\sigma}$$

Current I_0 on the surface i.e. $q(t) = I_0 \cdot t \rightarrow q(s) = I_0/s^2$

$$E(s) = \frac{aI_0/s^2}{a\epsilon_0 + b(\epsilon_1 + \sigma/s)} \rightarrow E(t) = \frac{aI_0}{b\sigma} (1 - e^{-\frac{t}{\tau}}) \quad t \rightarrow \infty \rightarrow E = \frac{aI_0}{b\sigma}$$

With $I_0 = q \cdot R$ and $\sigma = 1/\rho$ this becomes (of course) equal to the DC model from before.

Point charge in RPC



Point charge in geometry with ϵ_0 and ϵ_1

$$E_z(r, z) = \frac{q}{2\pi(\epsilon_0 + \epsilon_1)} \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} + \frac{q}{2\pi} \int_0^\infty J_0(kr) [f_1(k, z) - f_2(k, z)] dk$$

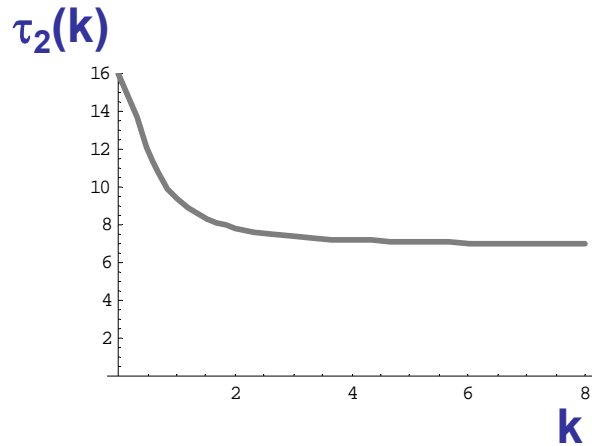
$$f_1(k, z) = \frac{k \cosh[k(b - z)] \sinh(ka)}{\epsilon_0 \cosh(kb) \sinh(ka) + \epsilon_1 \cosh(ka) \sinh(kb)} \quad f_2(k, z) = \frac{ke^{-kz}}{\epsilon_0 + \epsilon_1}$$

Point charge placed at position $r=0, z=0$ at time $t=0$, permittivity ϵ_1 , conductivity σ

$$E_z(r, z, t) = \frac{q}{2\pi(\epsilon_0 + \epsilon_1)} \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} e^{-t/\tau_1} + \frac{q}{2\pi} \int_0^\infty J_0(kr) [f_1(k, z) e^{-t/\tau_2(k)} - f_2(k, z) e^{-t/\tau_1}] dk$$

$$\tau_1 = \frac{\epsilon_0 + \epsilon_1}{\sigma} \quad \tau_2(k) = \frac{\epsilon_0 \cosh(kb) \sinh(ka) + \epsilon_1 \cosh(ka) \sinh(kb)}{\sigma \cosh(ka) \sinh(kb)}$$

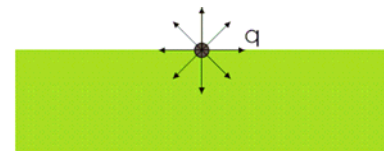
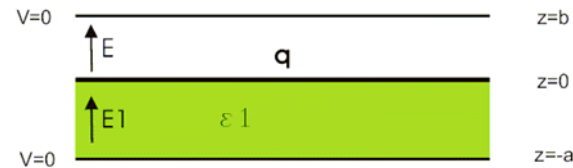
Point charge in RPC



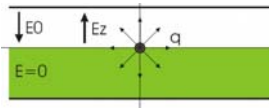
$$\tau_2(k) = \frac{\varepsilon_0 \cosh(kb) \sinh(ka) + \varepsilon_1 \cosh(ka) \sinh(kb)}{\sigma \cosh(ka) \sinh(kb)}$$

$$\lim_{k \rightarrow 0} \tau_2(k) = \frac{a\varepsilon_0 + b\varepsilon_1}{b\sigma}$$

$$\lim_{k \rightarrow \infty} \tau_2(k) = \frac{\varepsilon_0 + \varepsilon_1}{\sigma}$$



Charge decays with a continuous distribution of time constants between τ (charge sheet in RPC) and τ_1 (point charge at infinite half space).



Method for Monte Carlo Simulations

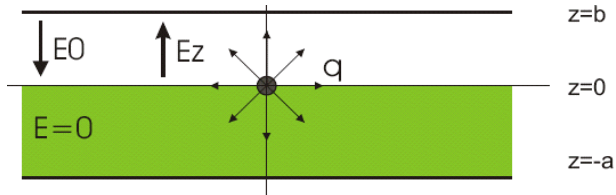
A single gap RPC of area $A = 3 \times 3 \text{ cm}^2$ is simulated.

For each time step (Δt) a new number of charges ($\Delta t \cdot R \cdot A$) is distributed randomly on the surface of the resistive plate.

The z-component of the electric field of all charges in the resistive plates is calculated at always the same position (center of RPC area, center of gap or close to electrodes) at all time steps and added to the applied field: $E_{\text{tot}} = E_0 + \sum E_z(r, z, t)$.

All charges are kept until their field contribution has fallen below 10^{-26} V/cm (up to 60s for Timing RPC).

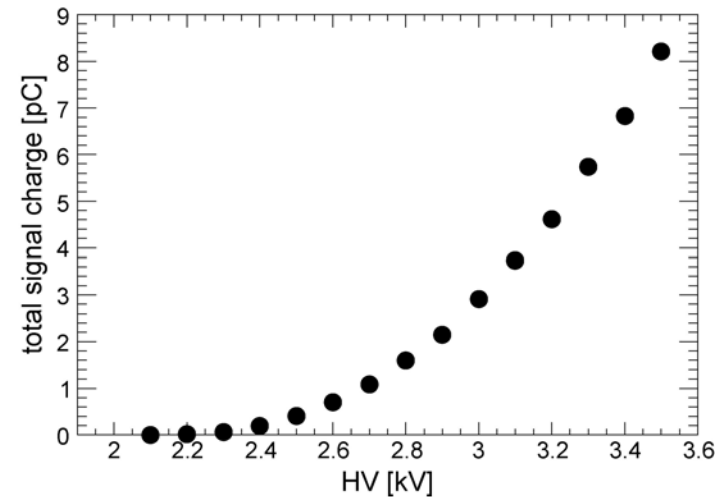
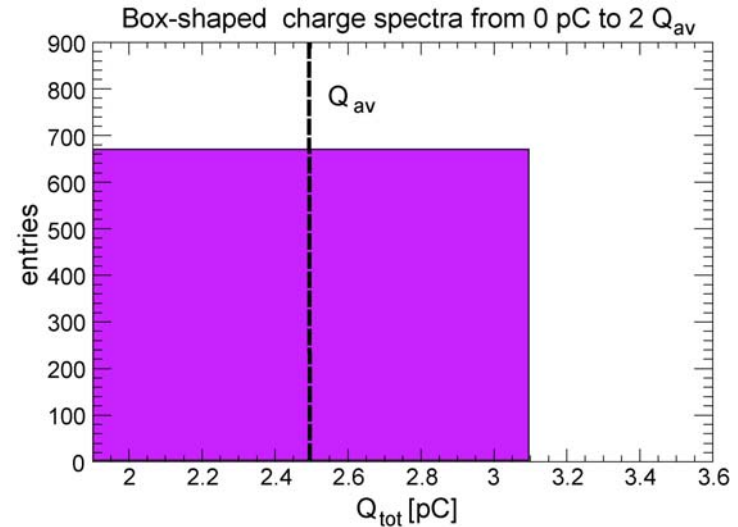
Input Parameters for Timing RPC

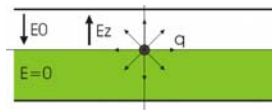


- $HV = 3kV \Rightarrow E_0 = 100 \text{ kV / cm}$
- $\epsilon_1 = 8 \epsilon_0$
- $a = 3 \text{ mm}$
- $b = 0.3 \text{ mm}$
- $\rho = 10^{12} \Omega\text{cm}$

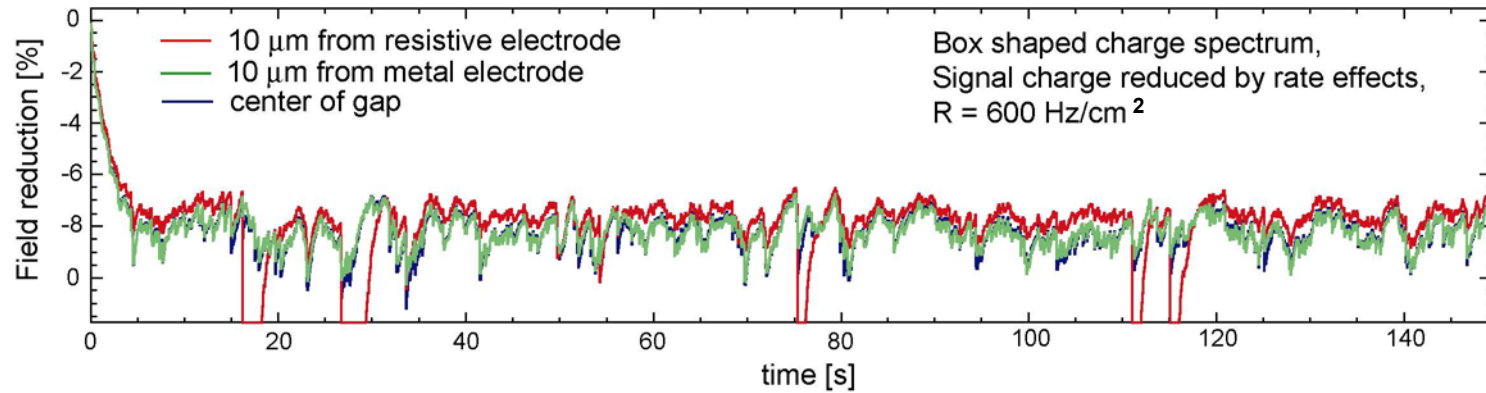
We use box-shaped charge spectra from 0 pC to 2 times the average total signal charge.

For the average total signal charge as a function of the HV we use simulated data.

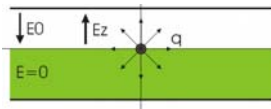




Monte Carlo for Timing RPCs

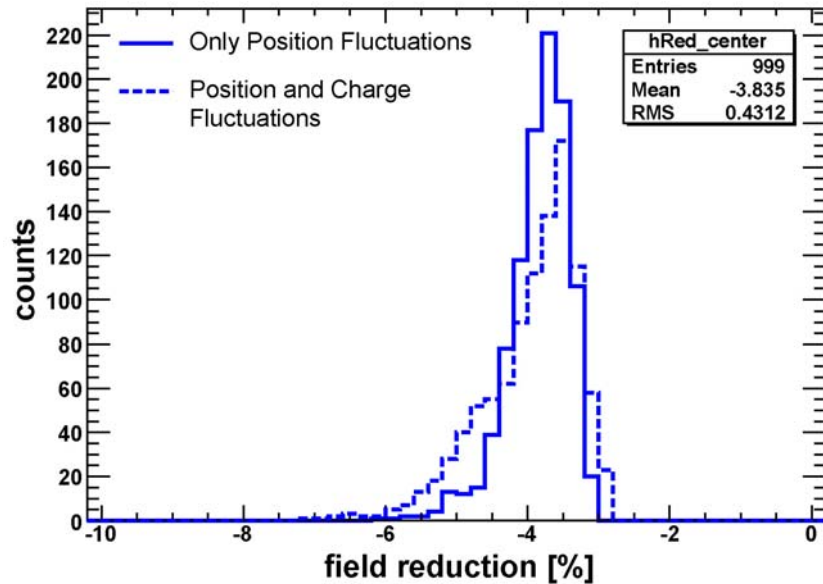
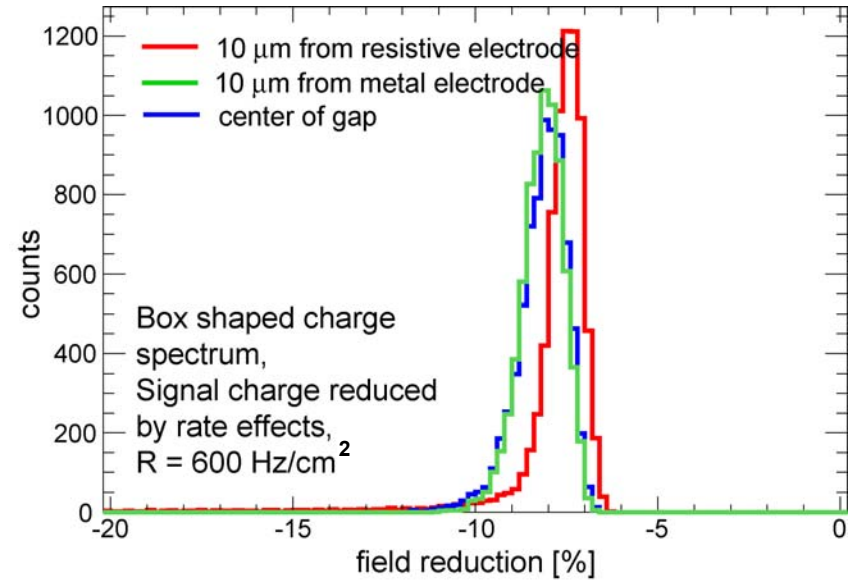
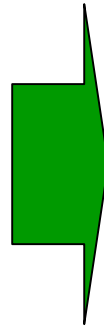


Fluctuations of the electric field at three different z-positions in the gap.

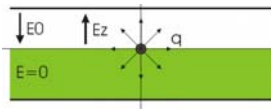


Monte Carlo for Timing RPCs: Results (1)

The field fluctuations at the three different z-positions in the gas gap. The mean values are the same everywhere. Close to the resistive plate the r.m.s. is the largest.

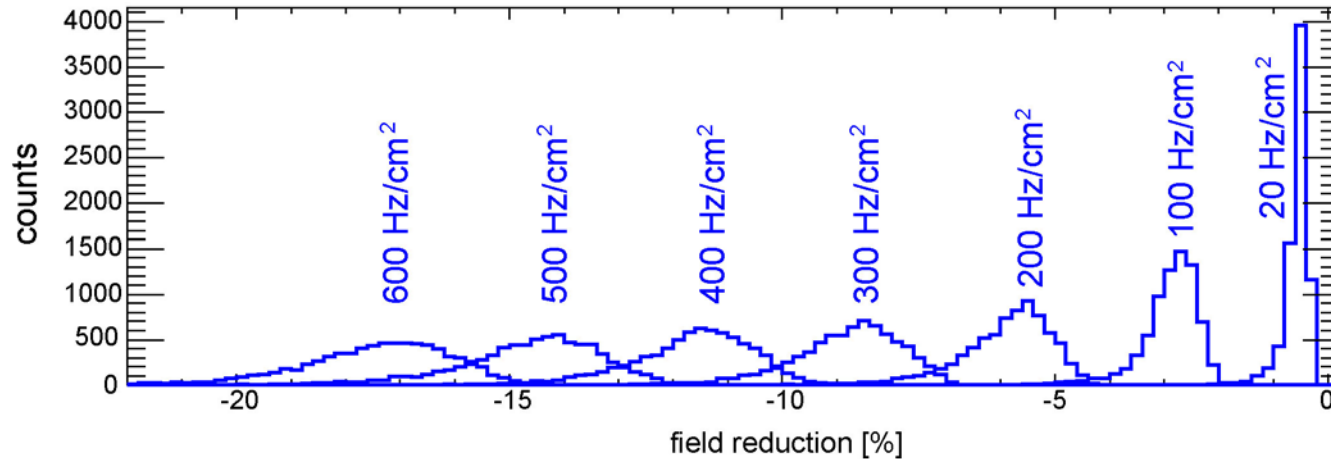


Position and charge fluctuations contribute to the field variations. The average field reduction is the same in both cases.

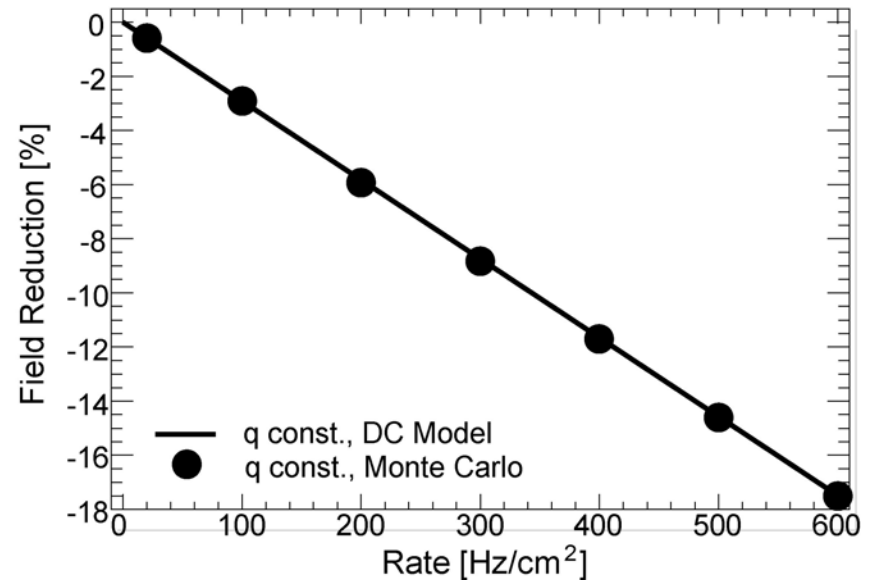
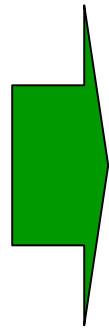


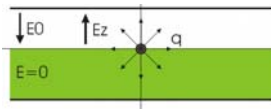
Monte Carlo for Timing RPCs: Results (2)

Here the total avalanche charge is kept constant as a function of rate:



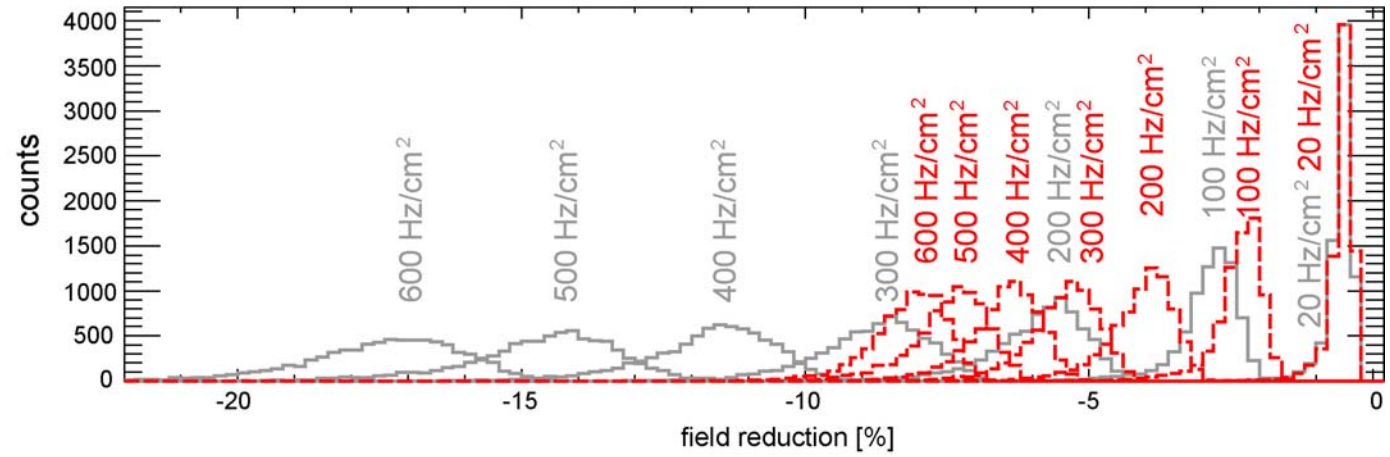
The average field reduction in the gap center is exactly the same as the one calculated from the DC model.



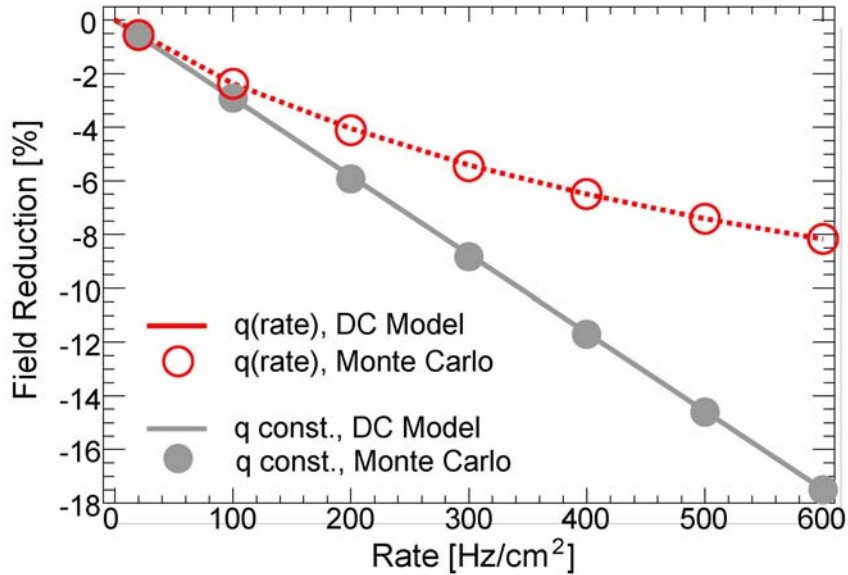
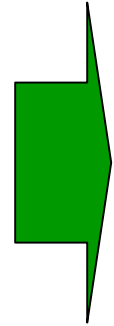


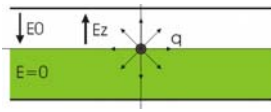
Monte Carlo for Timing RPCs: Results (3)

Total avalanche charge changes with the electric field variations with rate:



The average field reduction in the gap center is exactly the same as the one calculated from the DC model.



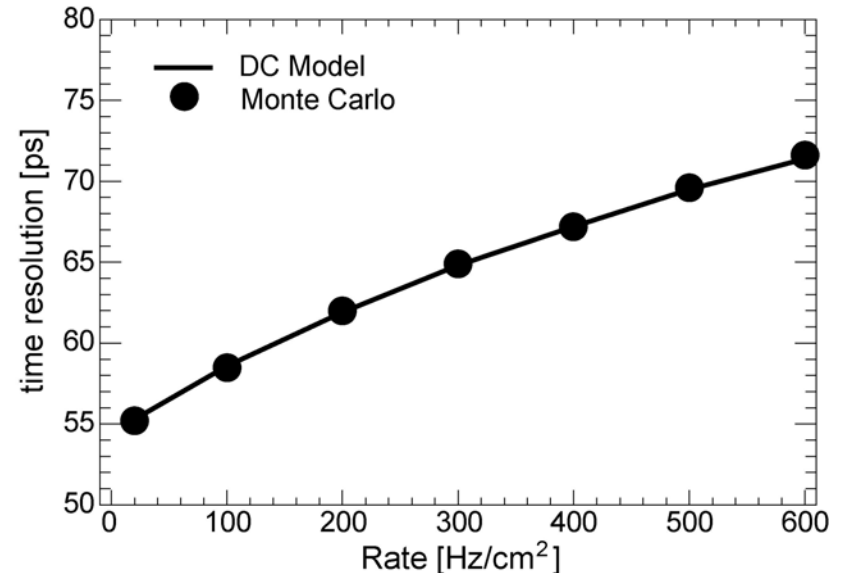
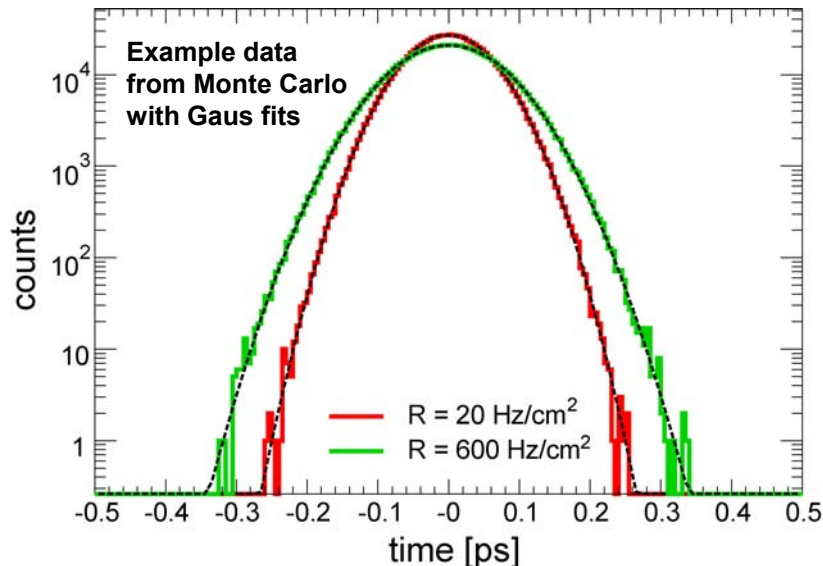


Monte Carlo for Timing RPCs: Results (4)

The time resolution for a given electric field can be calculated at all time steps during a simulation using the analytic formula:

$$\sigma_t = 1.28 / (v_D(\alpha-\eta))$$

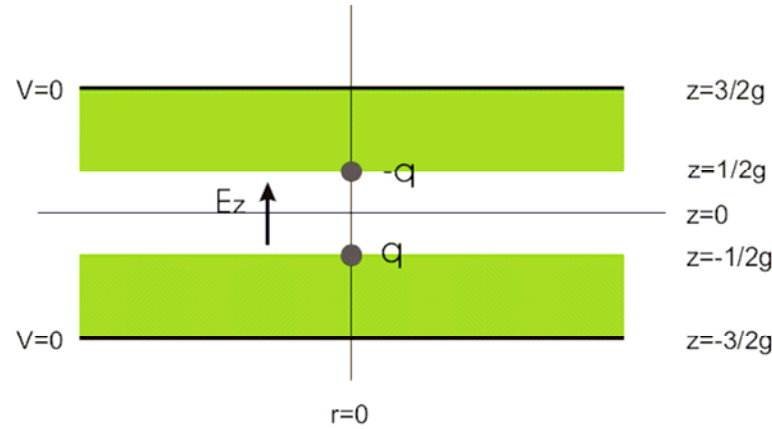
Comparison of timing from DC Modell and from Monte Carlo including the fluctuations:



⇒ According to this simulation the field fluctuations have no influence on the time resolution.

Exact 'cured' solution for point charge in RPC with two plates

Exact solution for the time dependent electric field E_z in the gas gap for an RPC with gap g and two resistive plates g for charge q at $-g/2$ and $-q$ at $g/2$.



$$E_z(r, z, t) = \frac{q}{2\pi(\varepsilon_0 + \varepsilon_1)} \left(\frac{\frac{g}{2} - z}{((r^2 + (\frac{g}{2} - z)^2)^{\frac{3}{2}}} + \frac{\frac{g}{2} + z}{((r^2 + (\frac{g}{2} + z)^2)^{\frac{3}{2}}} \right) e^{-t/\tau_1} + \frac{q}{2\pi} \int_0^\infty J_0(kr) \left[f_1(k, z) e^{-t/\tau_2(k)} - f_2(k, z) e^{-t/\tau_1} \right] dk$$

$$f_1(k, z) = \frac{2k \cosh(\frac{gk}{2}) \cosh(kz)}{\varepsilon_1 + (\varepsilon_1 + \varepsilon_2) \cosh(kg)} \quad f_2(k, z) = \frac{2k e^{-kg/2} \cosh(kz)}{\varepsilon_0 + \varepsilon_1} \quad \tau_1 = \frac{\varepsilon_0 + \varepsilon_1}{\sigma} \quad \tau_2(k) = \frac{\varepsilon_0 + \varepsilon_1}{\sigma} + \frac{\varepsilon_0}{\sigma \cosh(kg)}$$

Summary / Conclusions

We calculate rate effects in RPCs by using the exact time dependend solutions for the electric field of a point charge on the resistive plate of an RPC.

The charges decay with a continuous distribution of time constants. The two limiting cases are a continuous charge sheet (DC Model) and a point charge at an infinite half space.

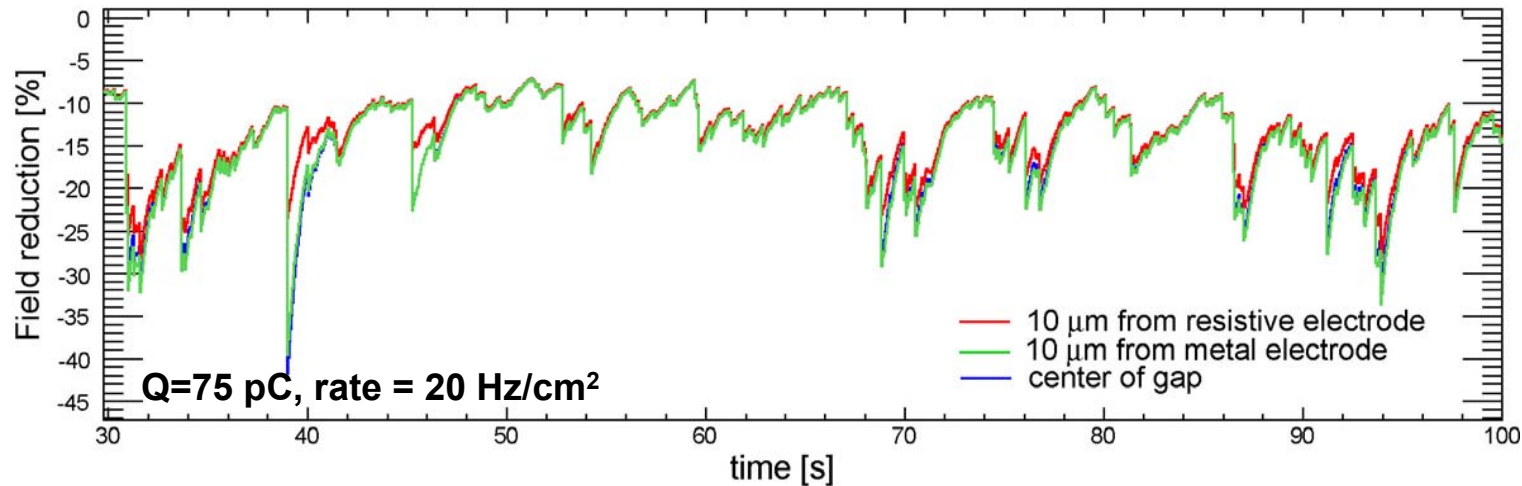
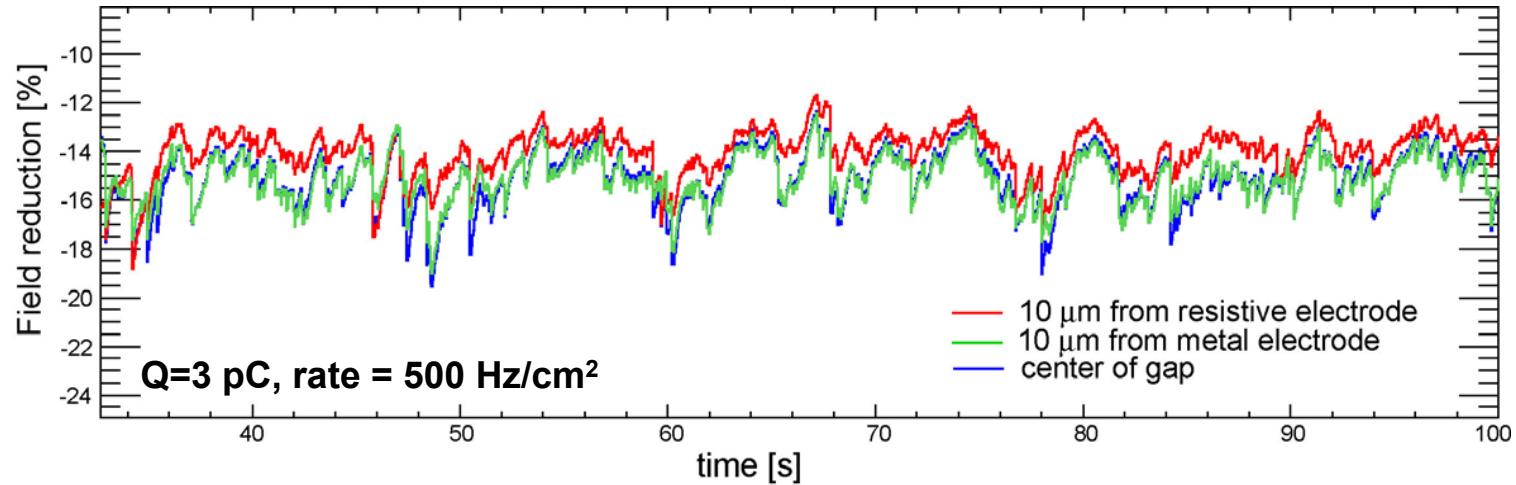
We present a Monte Carlo simulation for single gap Timing RPCs with one resistive plate.

The electric field fluctuates due to the particle flux around a mean value which is equal to the value derived with the DC Model.

The simulation suggests that these field fluctuations have no influence on the time resolution for a single gap of the investigated geometry.

More on the field fluctuations (1)

Field fluctuations for constant charge per time (Q*R):



More on the field fluctuations (2)

The mean field reduction (mean value from histograms) is the same for both cases, as expected.

In the case of large charges and low rate an influence on the timing is visible.

