Measuring the small

Hanbury Brown-Twiss analysis of nuclear collisions

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- **what is a correlation function**
- **correlations from Bose-Einstein (HBT)**
  - **sensitivity to the source size**
  - **how time enters**
  - **how expansion enters**
  - **how resonances enter (core-halo model)**
- **correlations from Coulomb FSI**
- **correlations from strong FSI**
- **HBT radii and their interpretation**

femtoscopy = measuring fm-sizes via two-particle correlations

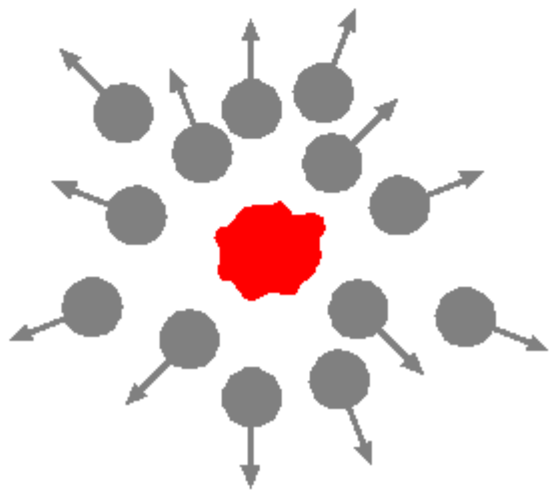
HBT = femtoscopy with identical bosons

relativistic nuclear collision experiments

relativistic nuclear (or hadron) collisions

collide two particles

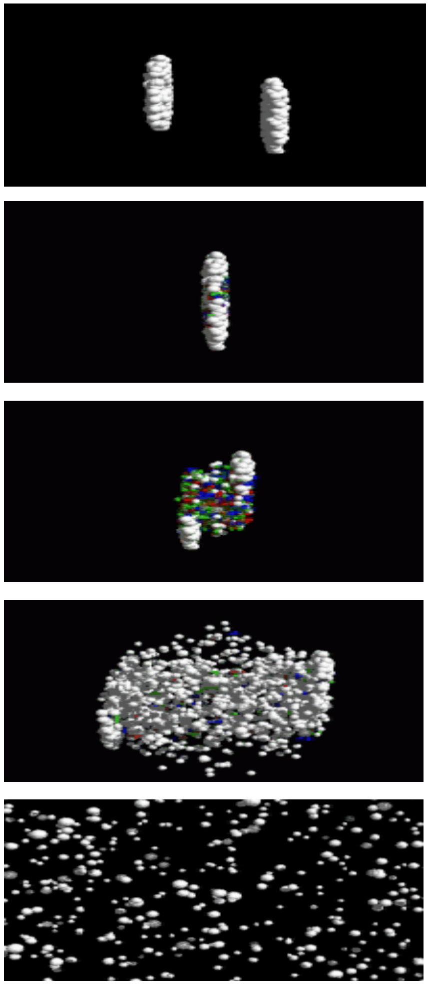
analyze produced particles



collision energy → new particles

relativistic nucleus-nucleus collision

UrQMD 160 GeV Au+Au

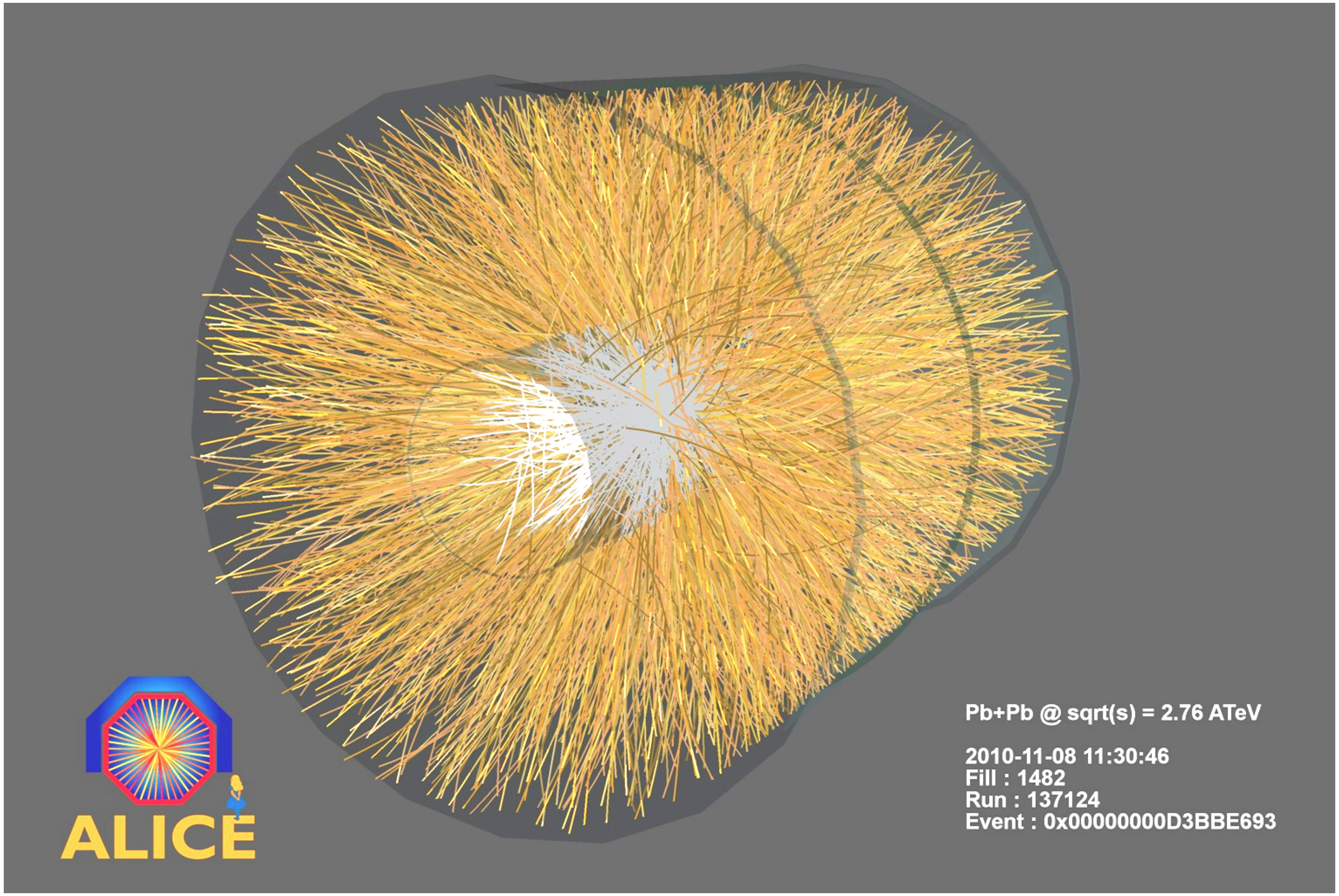


- before collision
- parton collisions
- thermalization
- hadronization
- chemical freezeout (number of particles frozen)
- kinetic freezeout (particle momenta frozen)

collision energies 10 GeV – 10 TeV
particle energies ~ 100 MeV – 1 GeV
sizes ~10 fm

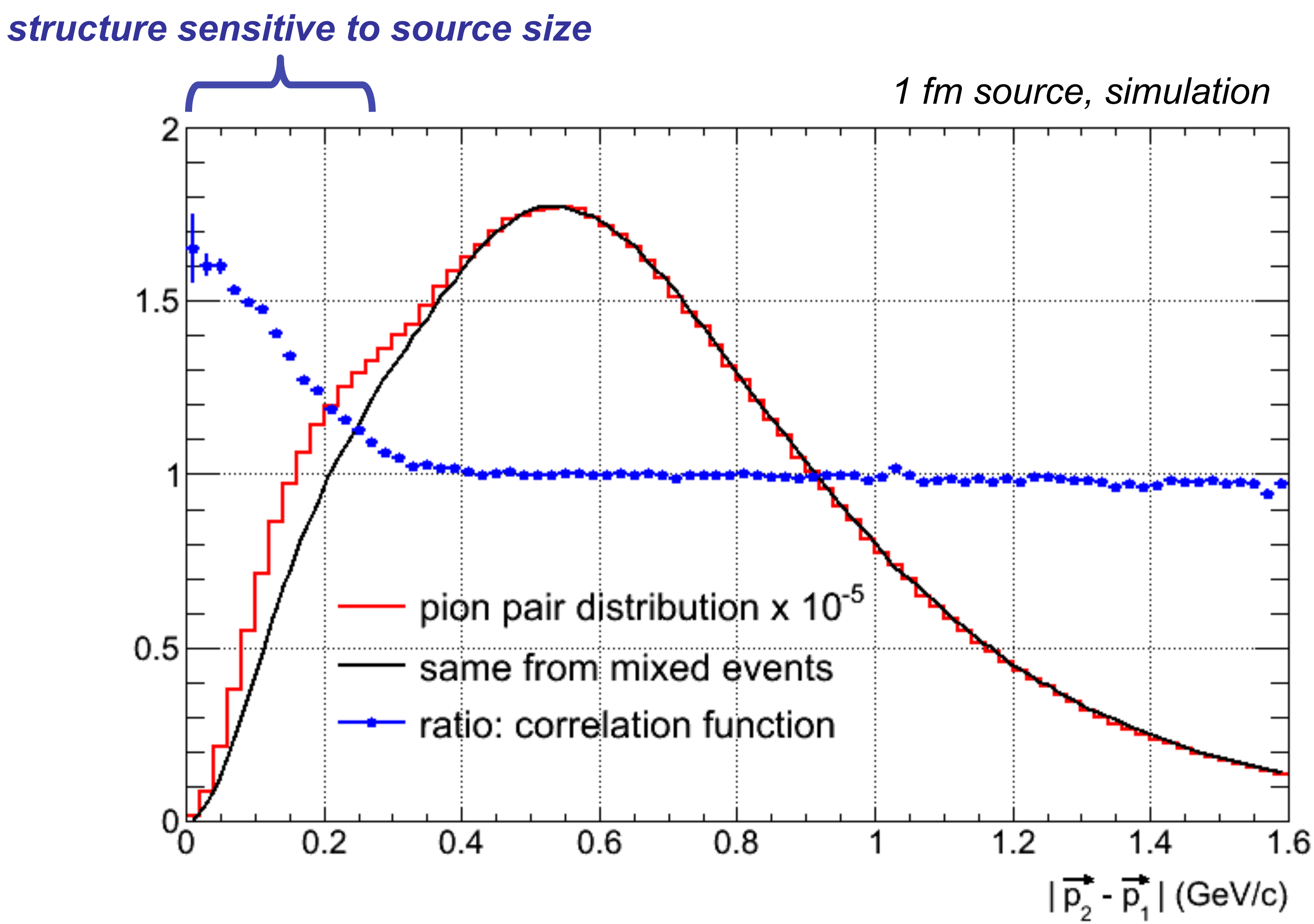
Events and tracks at Larger Hadron Collider

event := data of one Pb-Pb collision
track := reconstructed trajectory of a charged particle



correlation function in experiment

femtoscopy analysis technique



experiment: correlation function = pair distribution / combinatorics

HBT analysis

- select events and tracks, combine tracks into pairs (randomize order!), do event mixing or rotating to get the denominator, both in numerator and denominator remove too-close pairs (two-track resolution cut)
- when doing event mixing, mix only similar events (vertex, mult)
- normalize C to unity outside the peak
- fit the correlation by Bose-Einstein and Coulomb; use finite-size Coulomb; dilute Coulomb by the same factor as Bose-Einstein; use Poissonian maximum-likelihood
- finite momentum resolution broadens and flattens the correlation peak; correct for it
- careful when combining different runs; make sure e.g. that the normalization factor is the same

correlation function in theory

correlation function in theory

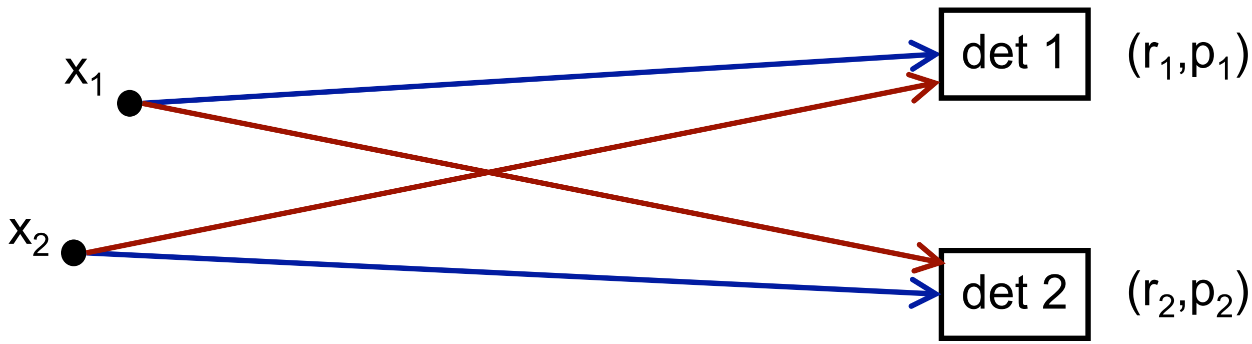
- **two-particle correlation function = relative wave function squared, averaged over the particle source**

$$C_2 = \left\langle \left| \Psi_{12}\left(p_2 - p_1, x_2 - x_1\right)\right|^2 \right\rangle$$

- **dominated by Quantum Statistics (Bose-Einstein, Fermi-Dirac), Coulomb, strong FSI**

Bose-Einstein correlations

emission of identical bosons from 2 fixed points



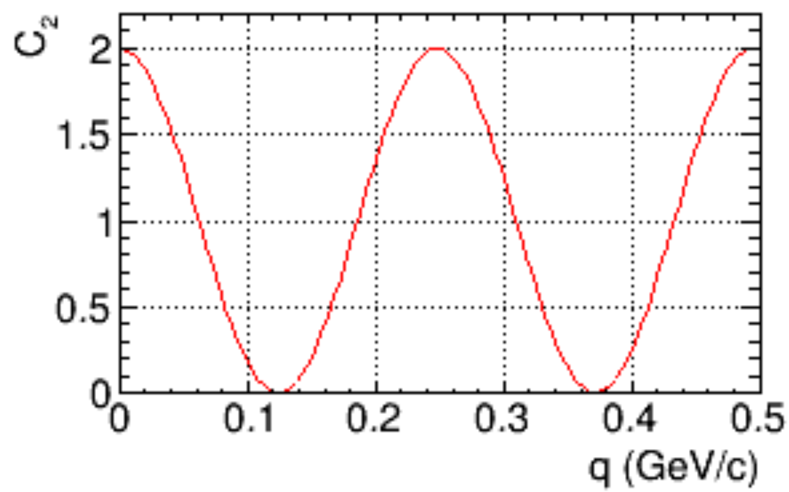
$$\Psi_{12} = \frac{1}{\sqrt{2}}\left(e^{ip_1(r_1-x_1)}e^{ip_2(r_2-x_2)} + e^{ip_1(r_1-x_2)}e^{ip_2(r_2-x_1)}\right)$$

$$|a+b|^2 = |a|^2 + |b|^2 + 2\,\mathrm{Re}\left(ab^*\right)$$

$$\underbrace{|\Psi_{12}|^2}_{\text{gain in number of pairs, aka } C_2} = 1 + \cos\left(\underbrace{(p_2 - p_1)}_{q}(x_2 - x_1)\right)$$

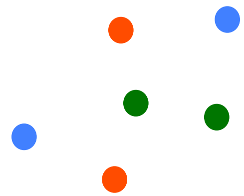
gain in number of pairs, aka C_2

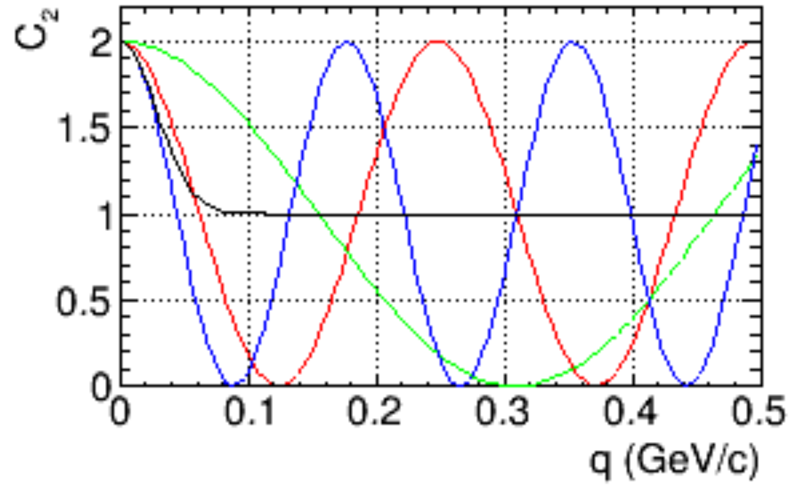
q



emission of identical bosons from a finite-size volume

S(**x**,t,**p**)





black line: Bose-Einstein correlation for finite-size source

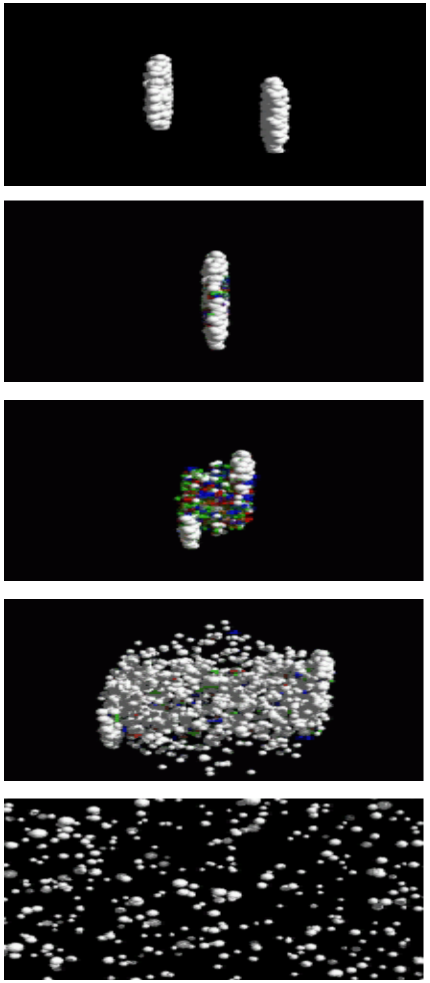
$$C_2 = \left\langle \left| \Psi_{12} \right|^2 \right\rangle = 1 + \left\langle \cos(q(x_2 - x_1)) \right\rangle$$

$$C_2 = 1 + \left| \int S(x, \vec{p}) \mathrm{e}^{-iqx} \, d^4x \right|^2$$

BE correlation function = 1 + (Fourier transform of the source)^2

particle freeze-out – final stage of nuclear collision

UrQMD 160 GeV Au+Au



- before collision
- parton collisions
- thermalization
- hadronization
- chemical freezeout (number of particles frozen)
- kinetic freezeout (particle momenta frozen)

source size = width of the distribution of the points of last interaction of particles

how the time enters

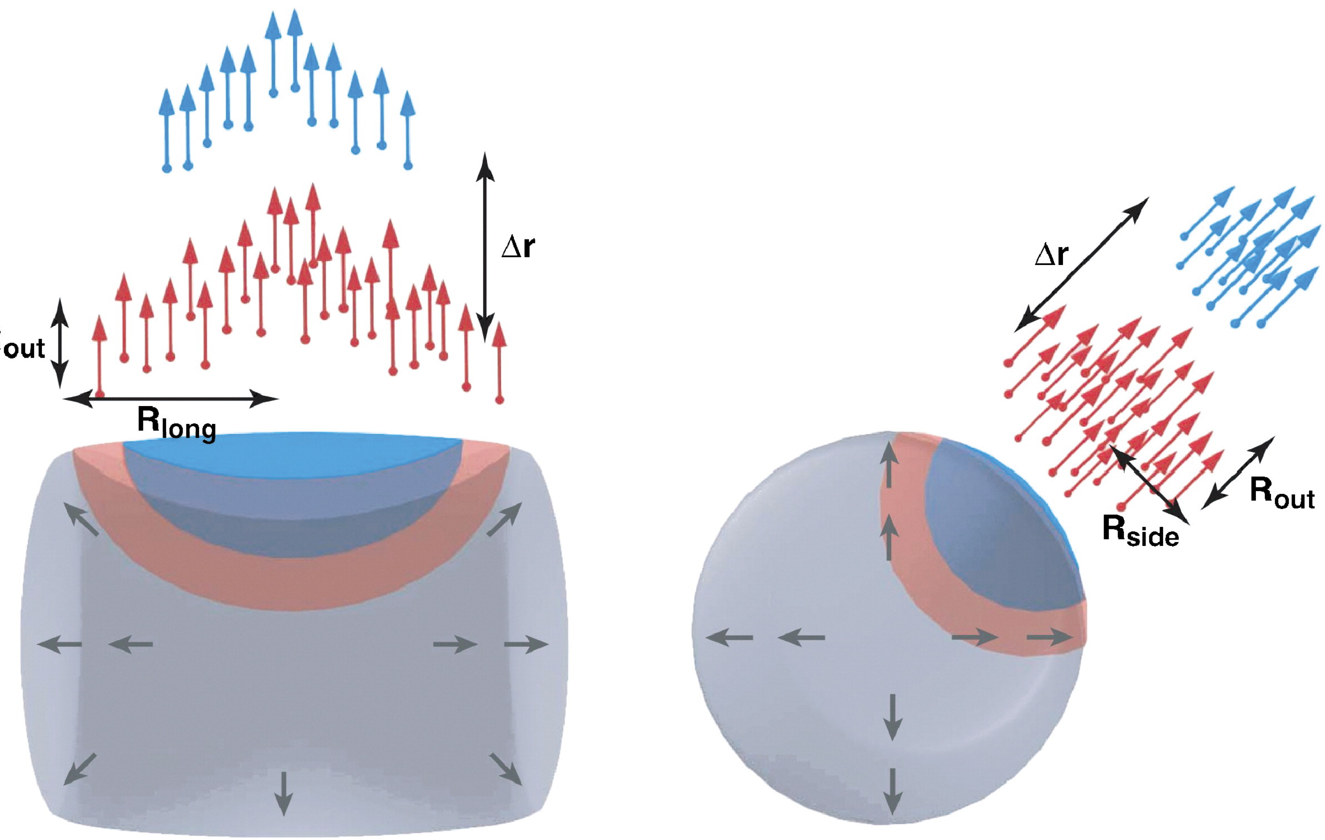
source S(**x**,t,**p**) is 7-dimensional
correlation function $C_2(\mathbf{p_1},\mathbf{p_2})$ is 6-dimensional
So one dimension gets lost. Which one?

What matters is the distance at the freeze-out time of the **second** particle. By that time the first particle has traveled already a bit. You won't be able to distinguish whether the first particle was created far away or simply much earlier. Time adds to the dimension along the pair momentum.

$$
\begin{aligned}
(p_2 - p_1)(x_2 - x_1) &= (E_2 - E_1)(t_2 - t_1) - (\vec{p}_2 - \vec{p}_1)(\vec{x}_2 - \vec{x}_1) \\
&= \vec{\beta}(\vec{p}_2 - \vec{p}_1)(t_2 - t_1) - (\vec{p}_2 - \vec{p}_1)(\vec{x}_2 - \vec{x}_1) \\
&= (\vec{p}_2 - \vec{p}_1)\left\{\vec{\beta}(t_2 - t_1) - (\vec{x}_2 - \vec{x}_1)\right\} \\
&= (\vec{p}_2 - \vec{p}_1)\left\{\vec{\beta}t_2 - \vec{x}_2 - \vec{\beta}t_1 + \vec{x}_1\right\}
\end{aligned}
$$

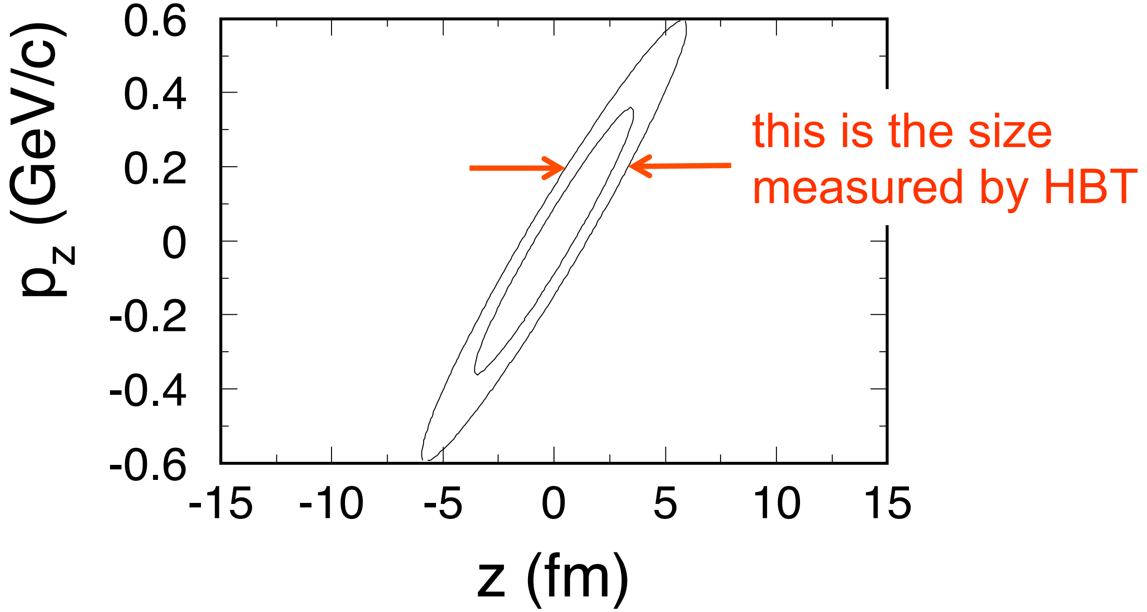
finite duration adds to the source size in the direction of pair momentum

how the expansion enters



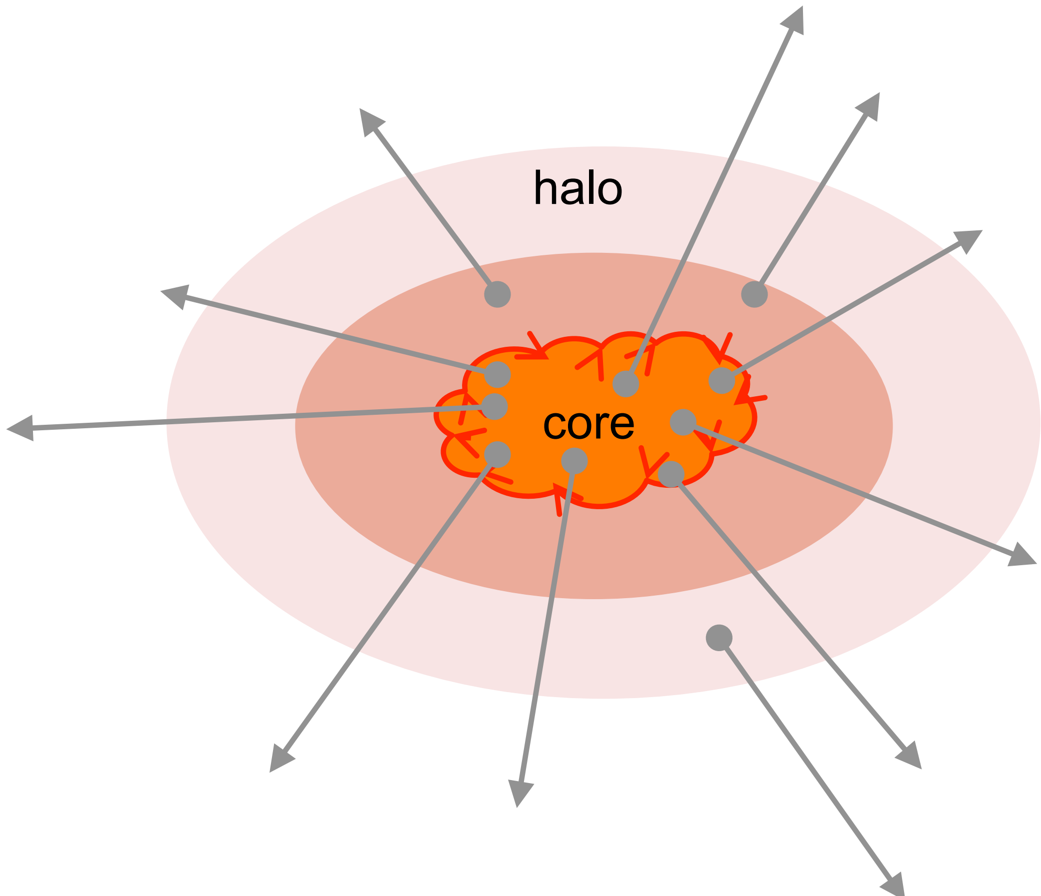
Lisa MA, et al. 2005.
Annu. Rev. Nucl. Part. Sci. 55:357–402

how the expansion enters



HBT technique measures the size of the source of pions with fixed three-momentum ("homogeneity length"). It may be smaller than the total pion source.

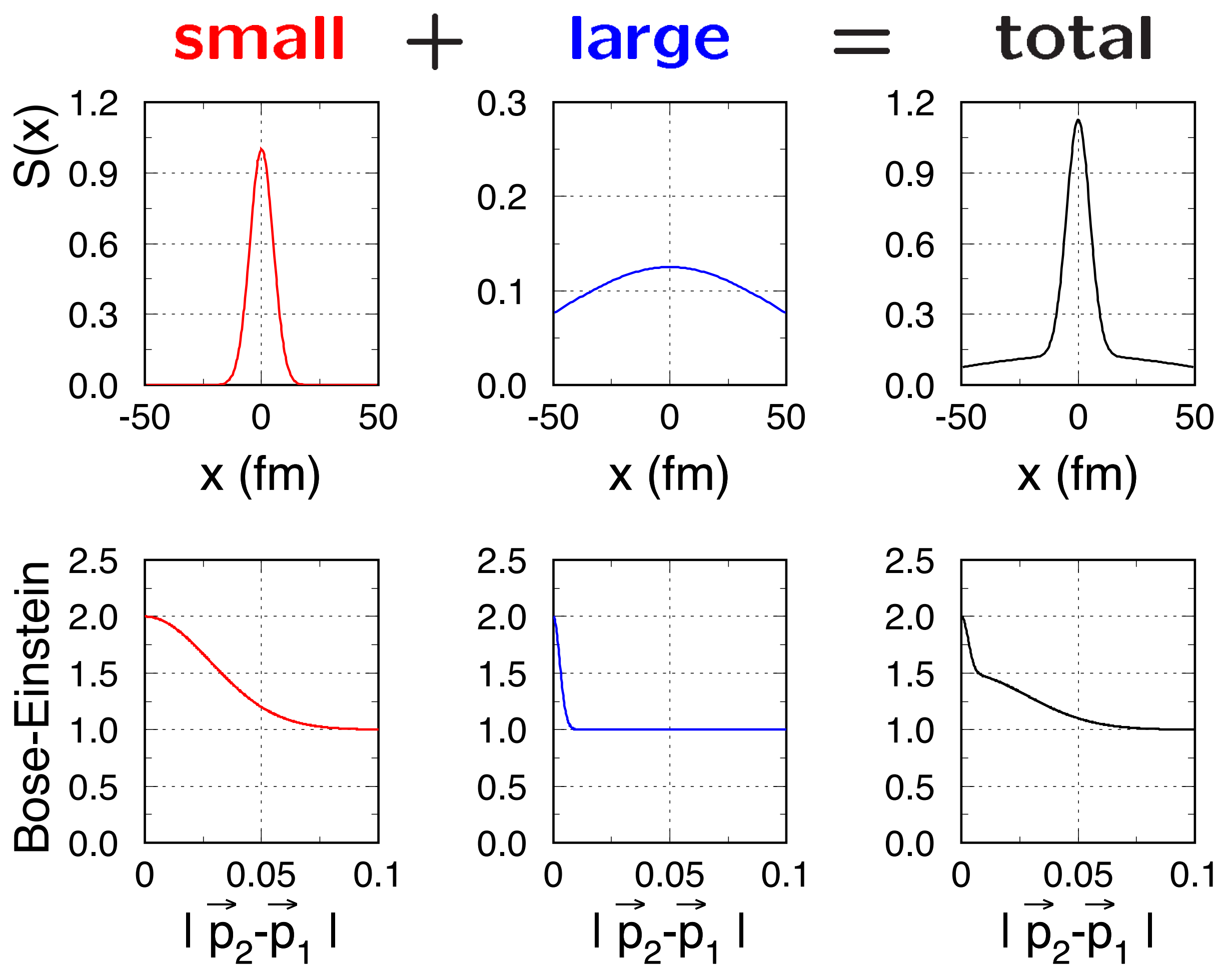
how long-lived resonances enter



core := directly produced pions
halo := pions from decays

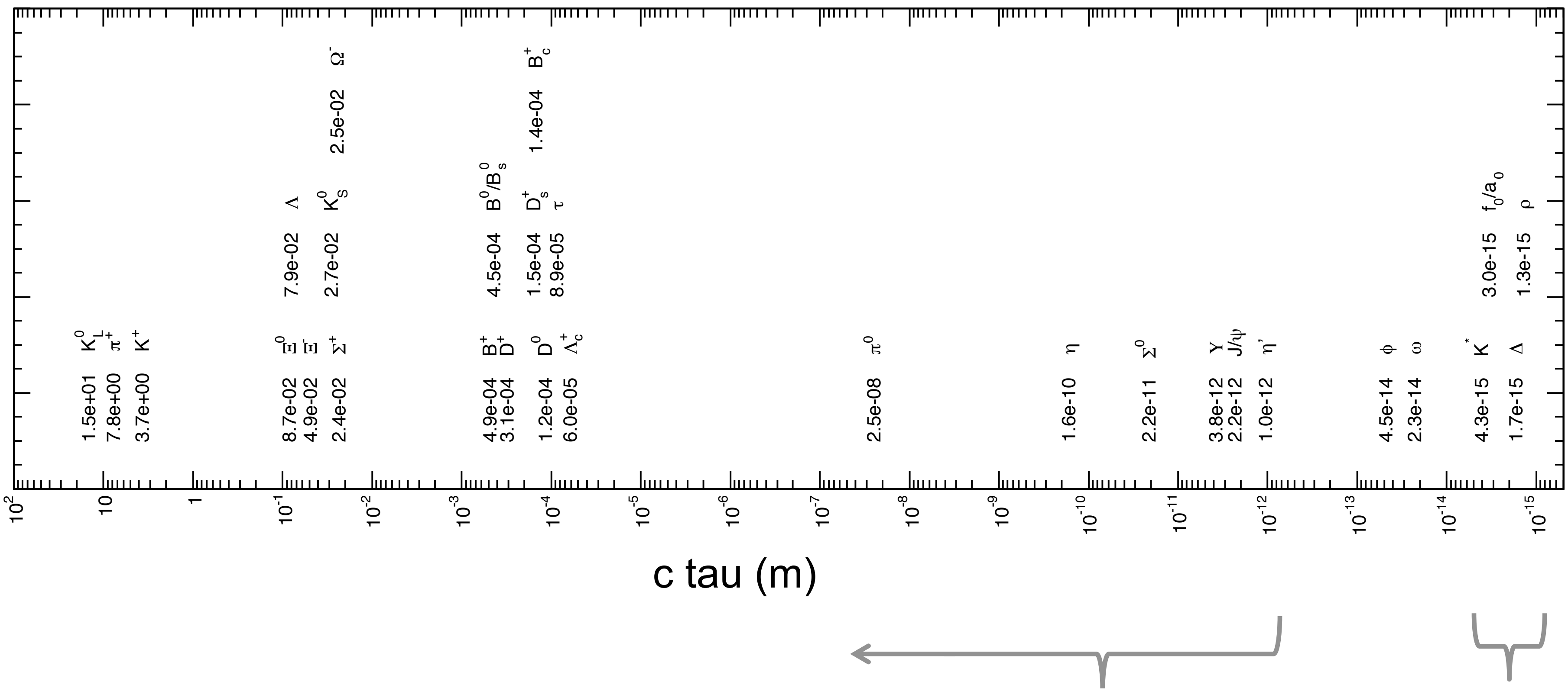
pion pairs from core-core, core-halo, halo-halo

how long-lived resonances enter



Long-lived resonances (cτ > 20 fm) produce pions at large radii → unresolvable spike in C → reduced correlation strength

resonances



long-lived resonances produce halo

short enlarge the core

Coulomb FSI correlations

Quantum-mechanical derivation of Coulomb correlations

- factor modifying production of two charged particles with momentum difference $\Delta p(=Q_{inv})$ and position Δr

  =

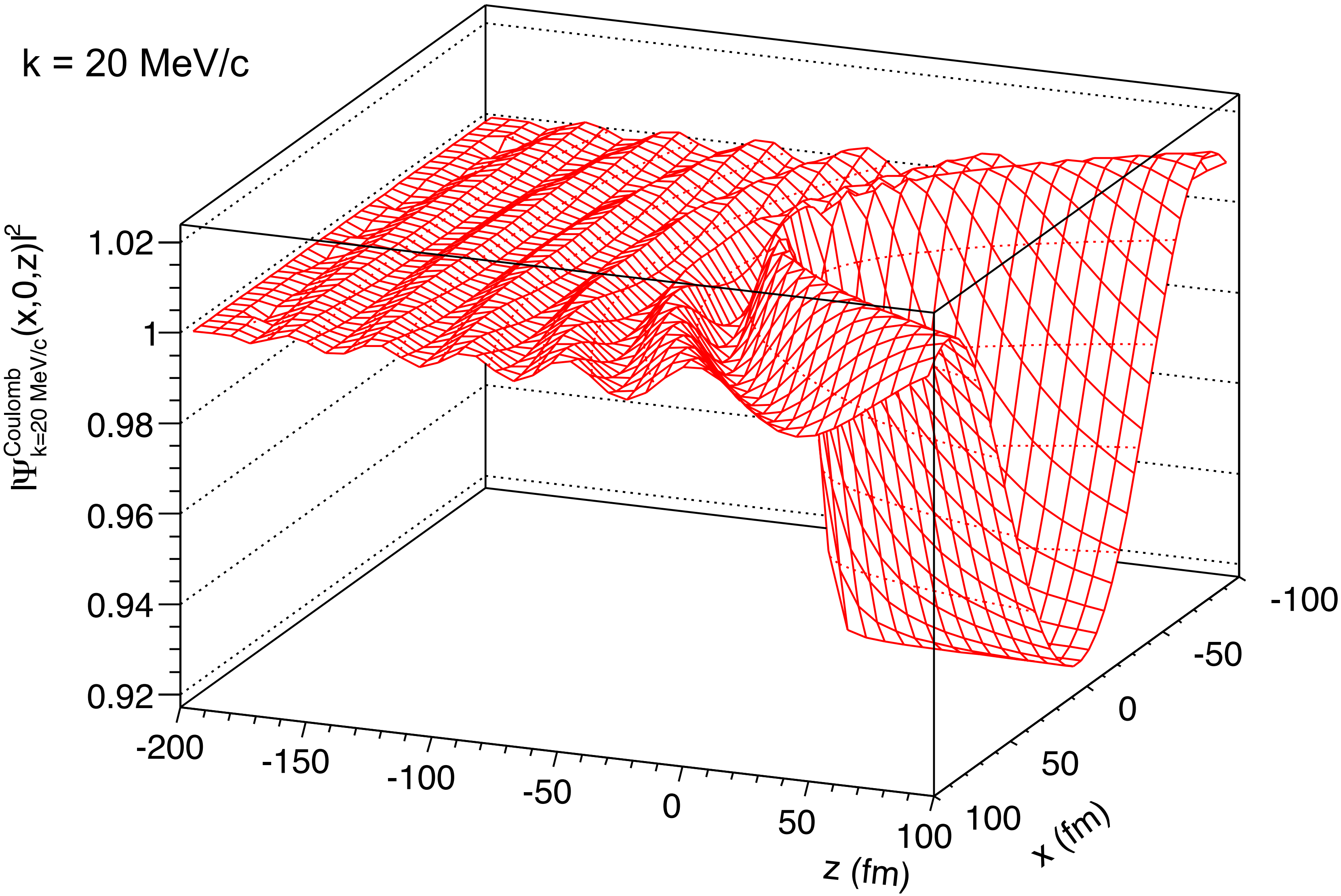
  squared wave-function of scattering of charged particle in Coulomb field $|\psi(Q_{inv}/2, \Delta r)|^2 / |\psi(Q_{inv}/2, \infty)|^2$

- nonrel. Schrödinger equation with Coulomb potential can be solved analytically (e.g. Merzbacher QM, Messiah QM):

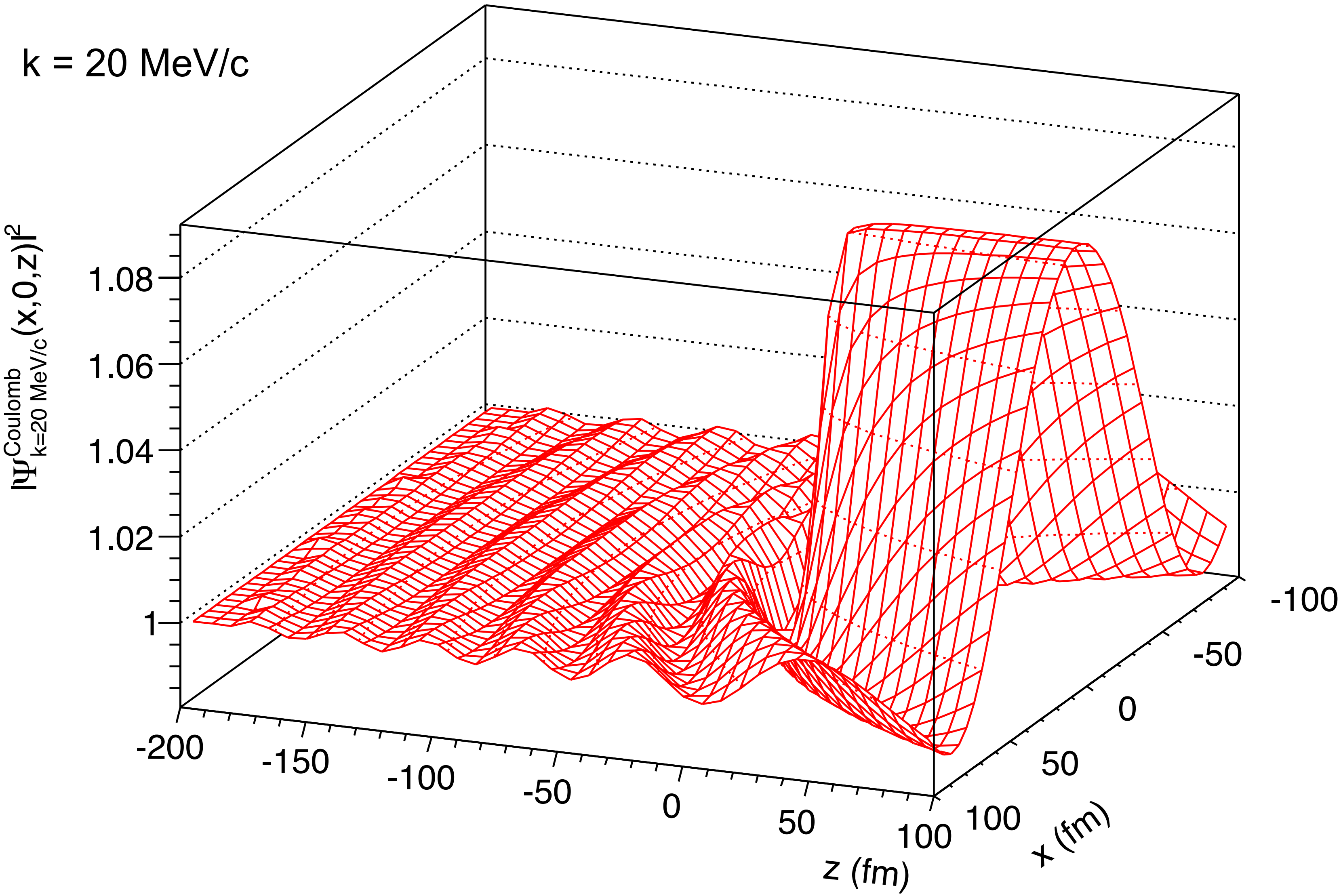
$$H(\mathbf{k}, \mathbf{r}) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \left| F\left(-i\eta; 1; ik(r - \frac{\mathbf{r}\cdot\mathbf{k}}{k})\right)\right|^2$$

where $k=Q_{inv}/2$ is relative momentum, r is relative position, $\eta = Z_1 Z_2 m\alpha/k$ is the relative velocity, m is reduced mass, and F is the confluent hypergeometric function

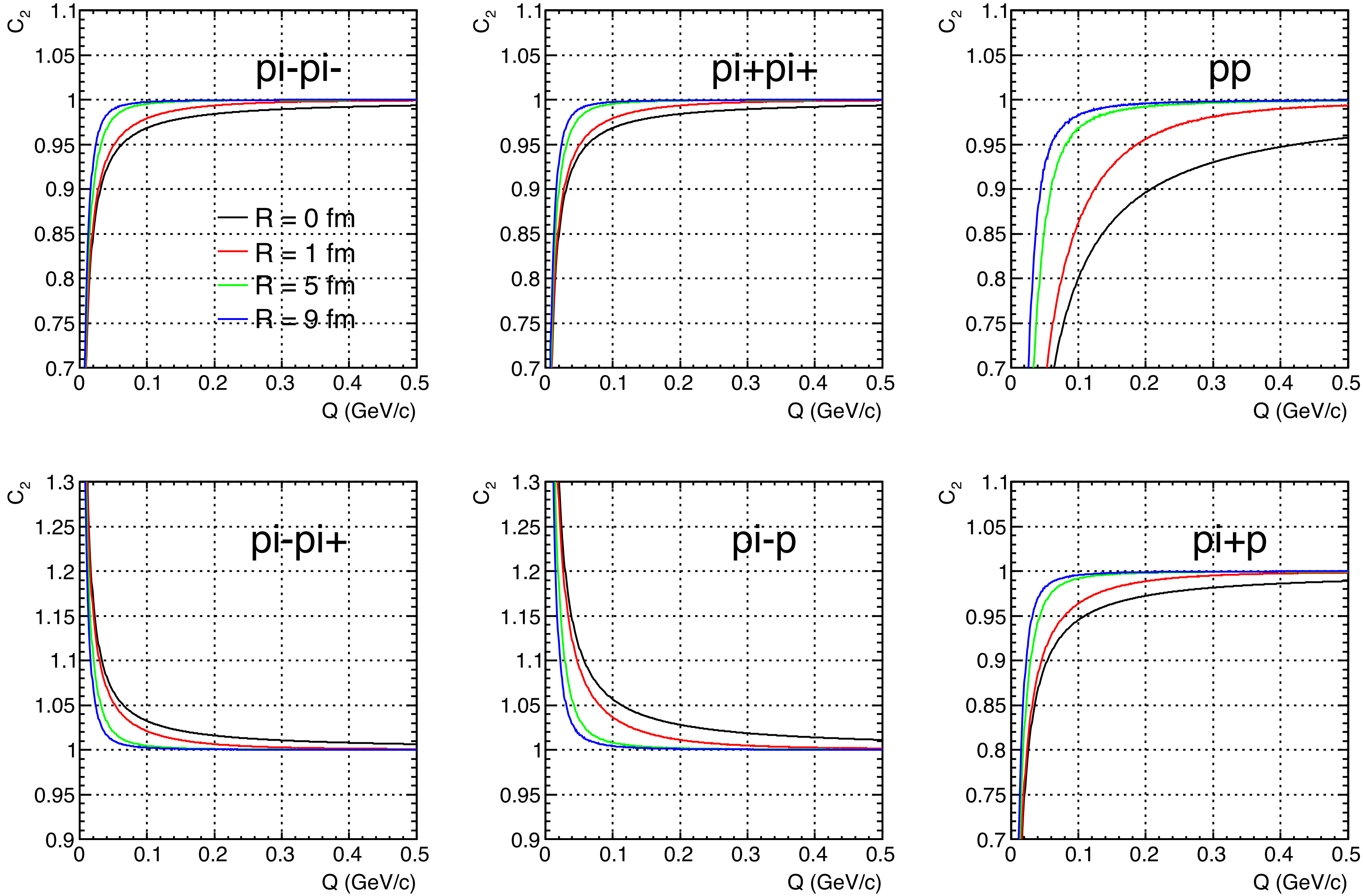
Wave function of charged particle in repulsive Coulomb field, squared, normalized to 1 at z-∞



Wave function of charged particle in attractive Coulomb field, squared, normalized to 1 at -∞



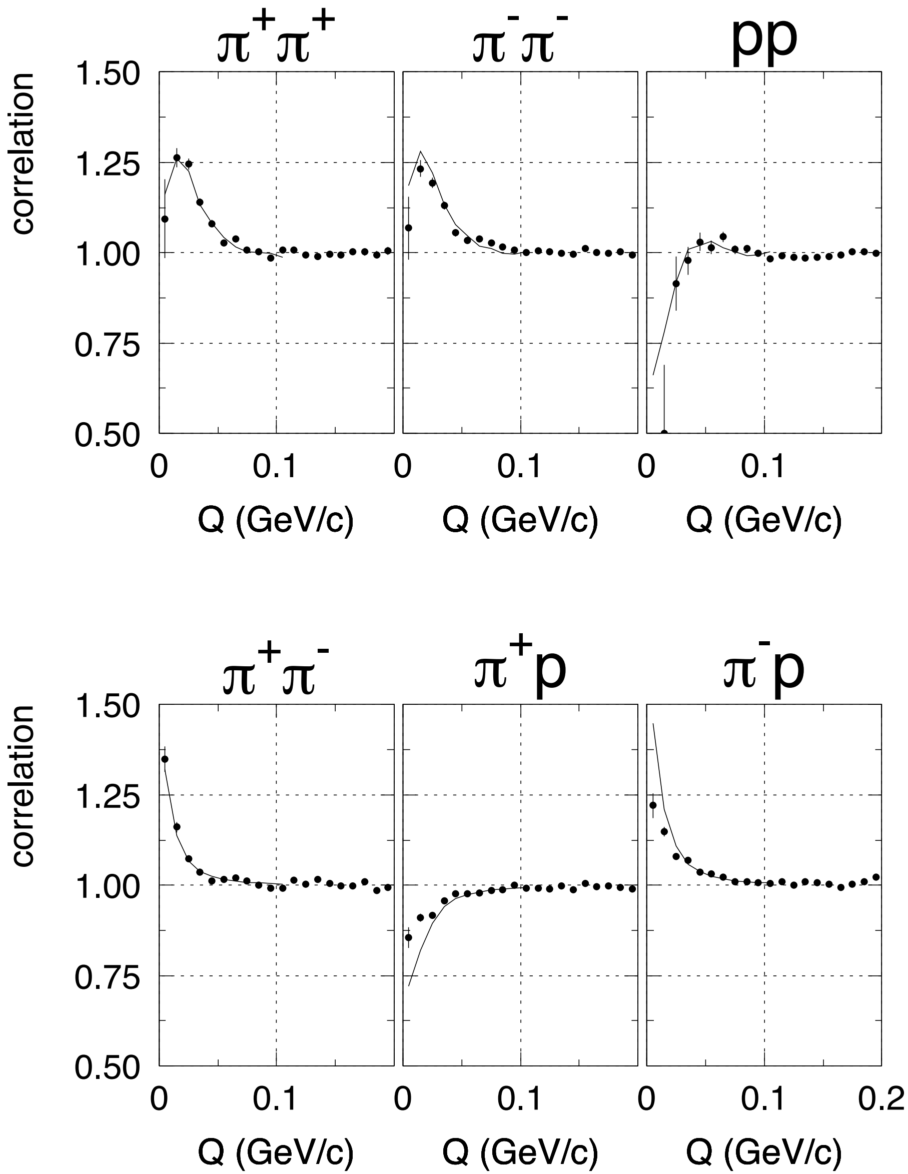
Coulomb correlations



Coulomb correlations

- positive for unlike-sign, negative for like-sign
- note: unlike-sign NOT equal to 1/like-sign
- narrow for pions, wider for heavier particles – relative velocity is what matters
- for point-like source, equal to Gamow factor
- quickly decrease (i.e. approach unity) with increasing source size

Coulomb (and other) correlations

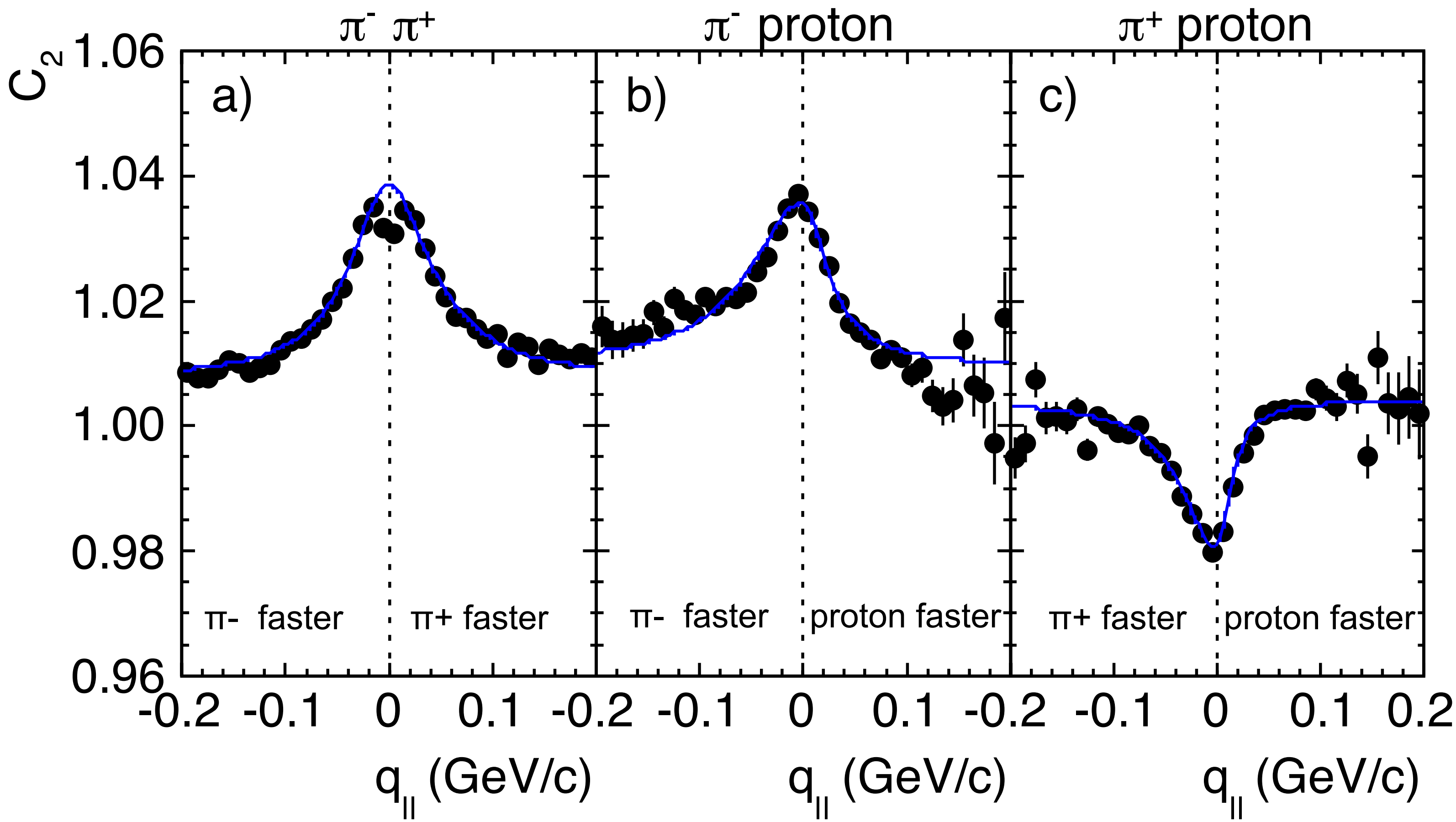


correlation functions from Au+Au at 10.8A GeV (experiment E877)

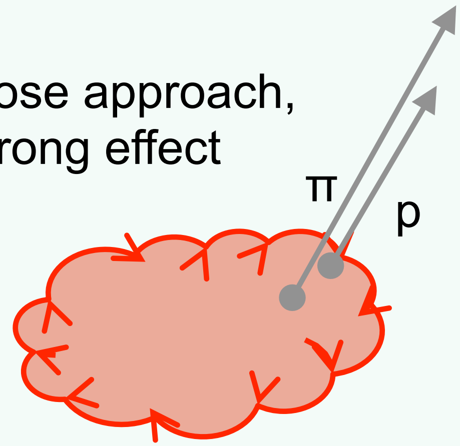
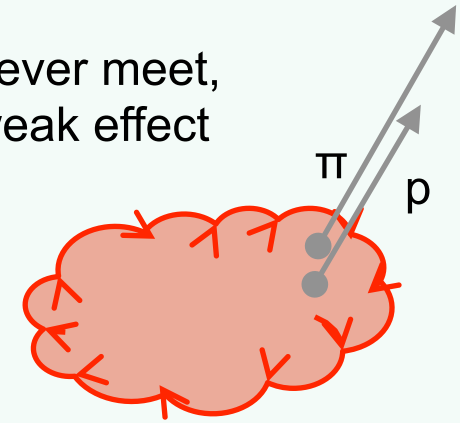
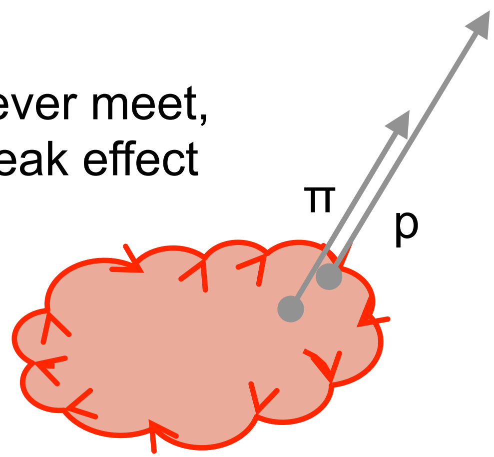
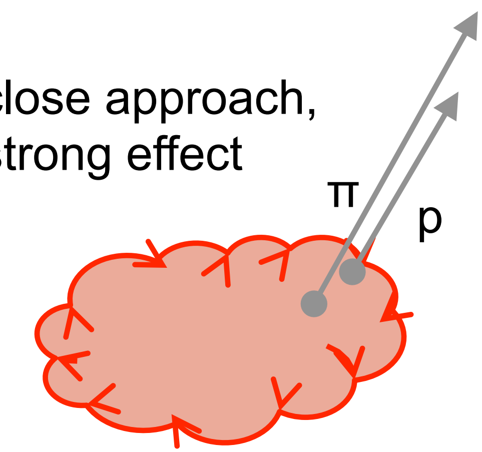
compared to RQMD (line)

An application of Coulomb correlations: deducing displacement between sources of different particle species (Lednicky)

non-identical particle correlations in Pb-Au at 158A GeV/c, CERES/NA45



when pion is faster than proton, effect of interaction is stronger

	proton closer to the edge	pion closer to the edge
pion faster	close approach, strong effect 	never meet, weak effect 
proton faster	never meet, weak effect 	close approach, strong effect 

protons are emitted ~5 fm ahead of pions

Strong FSI correlations

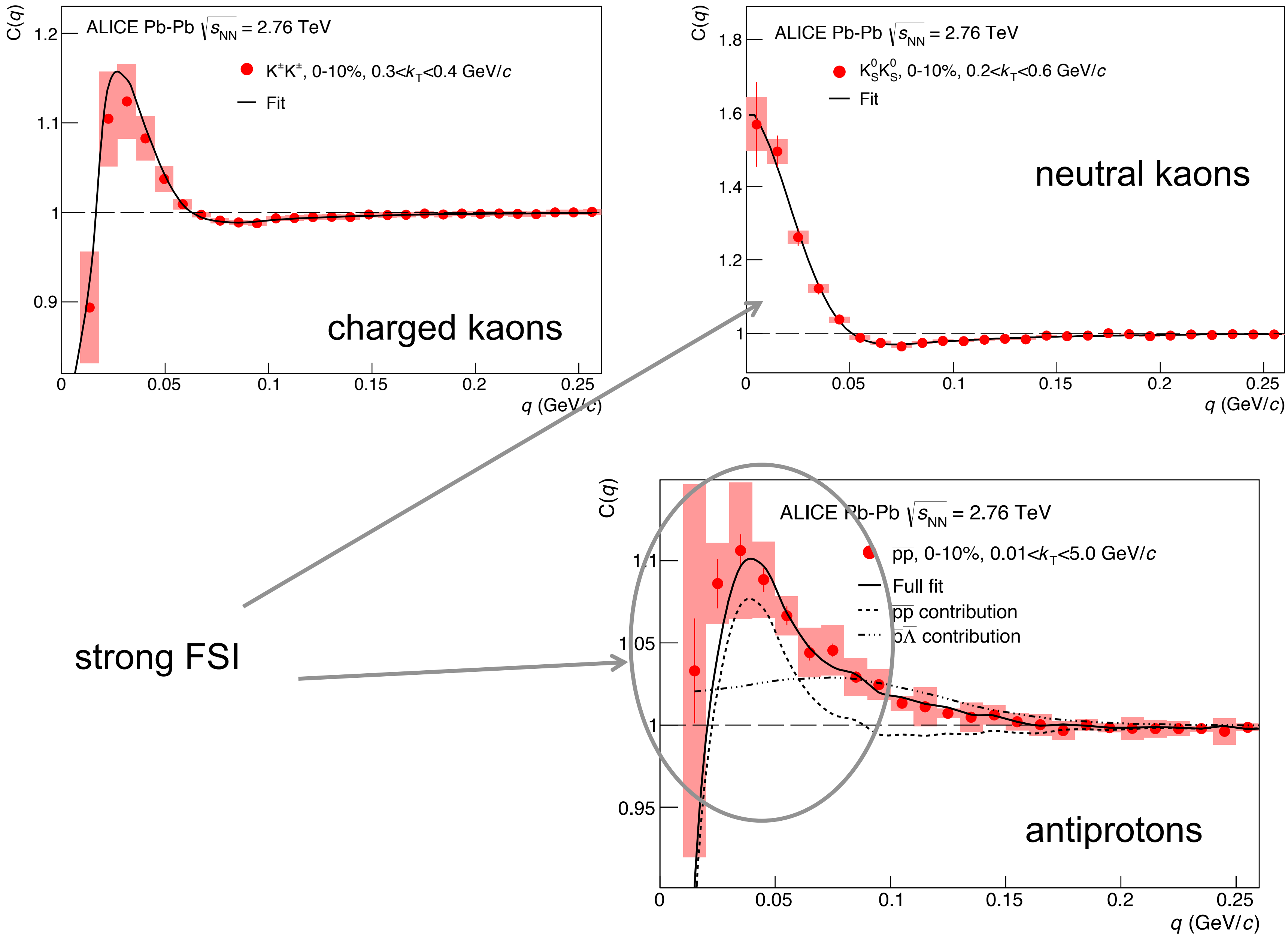
correlation caused by strong FSI

- **small relative momentum, s-wave is sufficient. Example for $K^0_S K^0_S$:**

$$C_{\mathrm{strongFSI}}(q,R) = \frac{1}{2}\left[\left|\frac{f(q)}{R}\right|^2 + \frac{4\Re f(q)}{\sqrt{\pi}R}F_1(qR) - \frac{2\Im f(q)}{R}F_2(qR)\right],$$

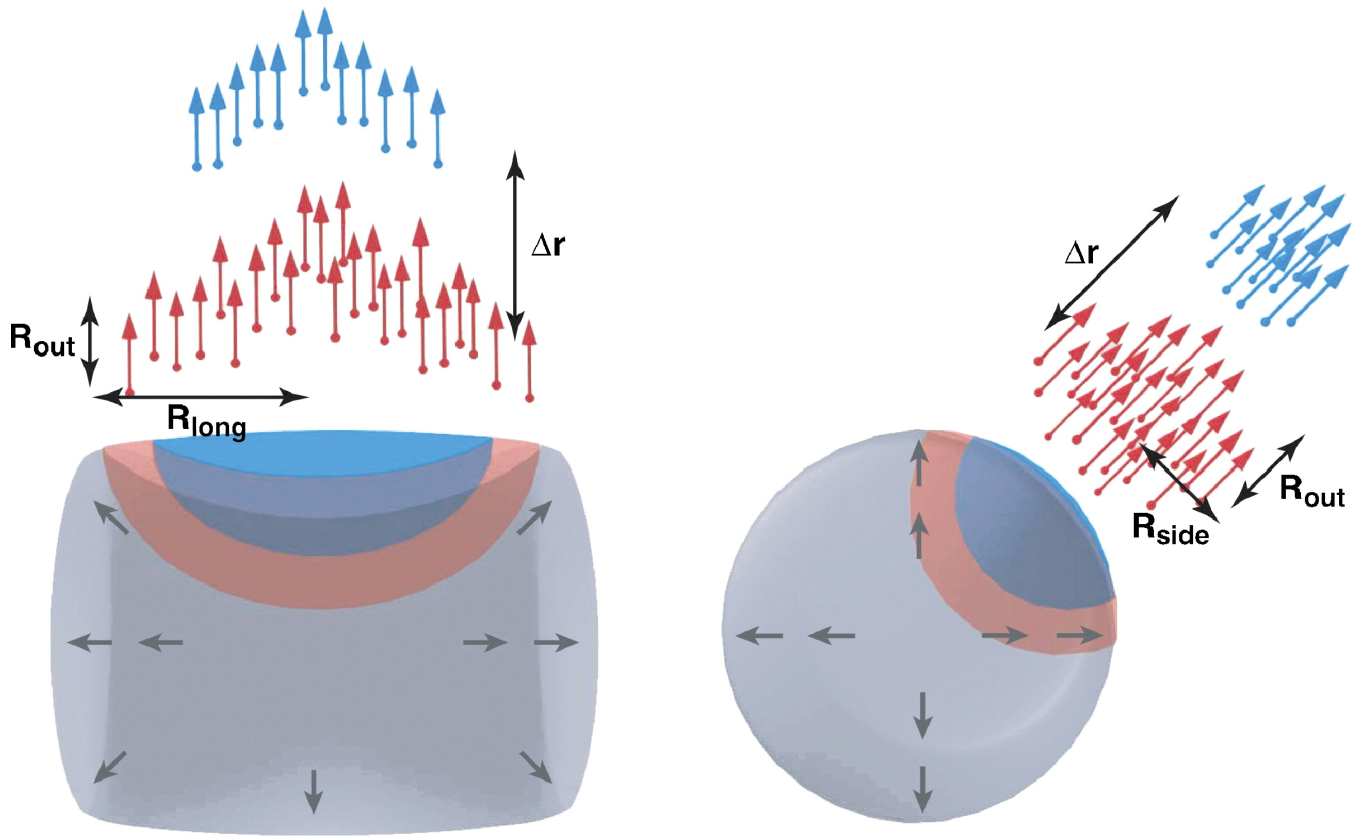
$$F_1(z) = \int_0^z \mathrm{d}x \frac{e^{x^2-z^2}}{z};\ F_2(z) = \frac{1-e^{-z^2}}{z}.$$

- **if the interaction is resonant, then the resonance peak height ~ 1 / source volume**


ALICE Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV
$K^{\pm}K^{\pm}$, 0-10%, 0.3<k_T<0.4 GeV/c
Fit
charged kaons
C(q)
q (GeV/c)
ALICE Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV
$K^0_SK^0_S$, 0-10%, 0.2<k_T<0.6 GeV/c
Fit
neutral kaons
ALICE Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV
$\overline{p}\overline{p}$, 0-10%, 0.01<k_T<5.0 GeV/c
Full fit
$\overline{p}\overline{p}$ contribution
$\overline{p}\overline{\Lambda}$ contribution
antiprotons
strong FSI

HBT radii and their interpretation

definition of out-side-long axes



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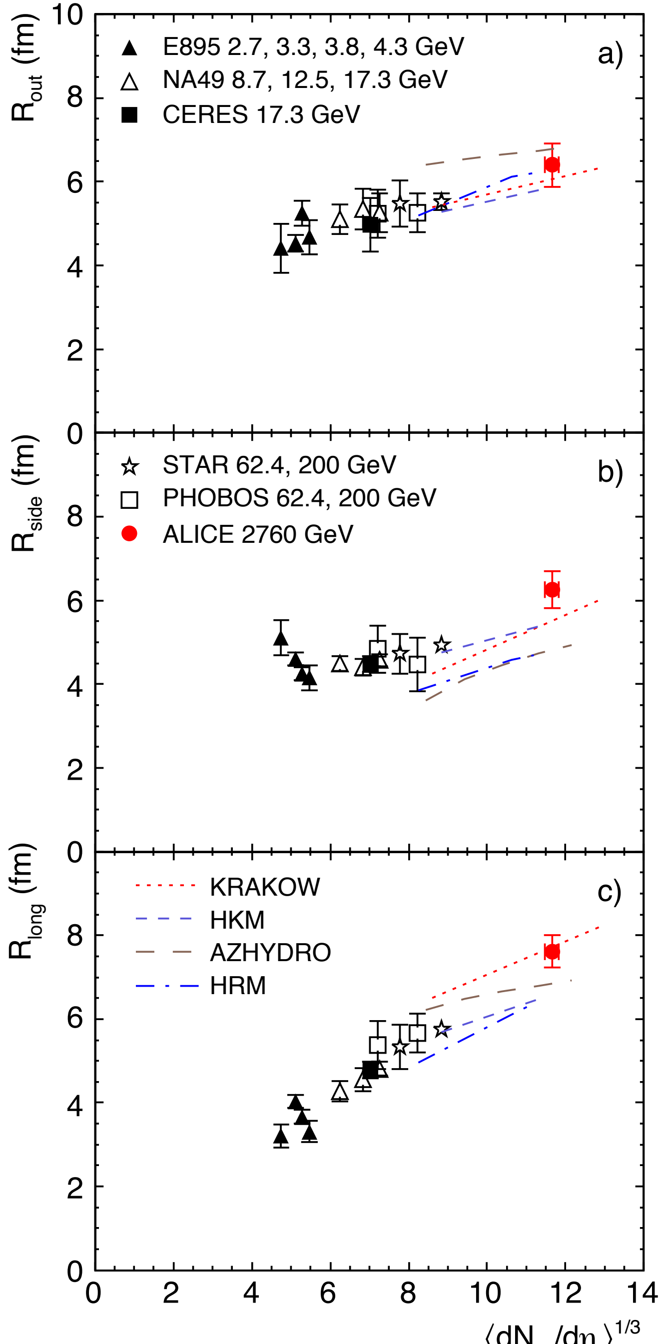
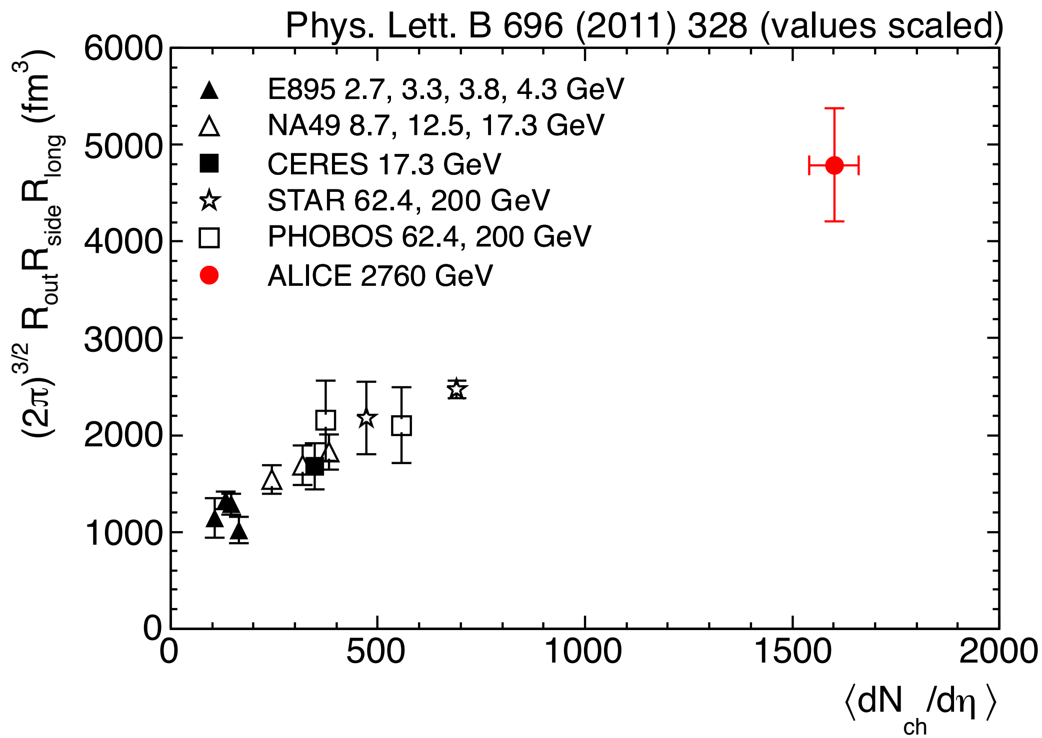
standard way to parametrize the source size in 3-dim: R_{out}, R_{side}, R_{long}

pion HBT

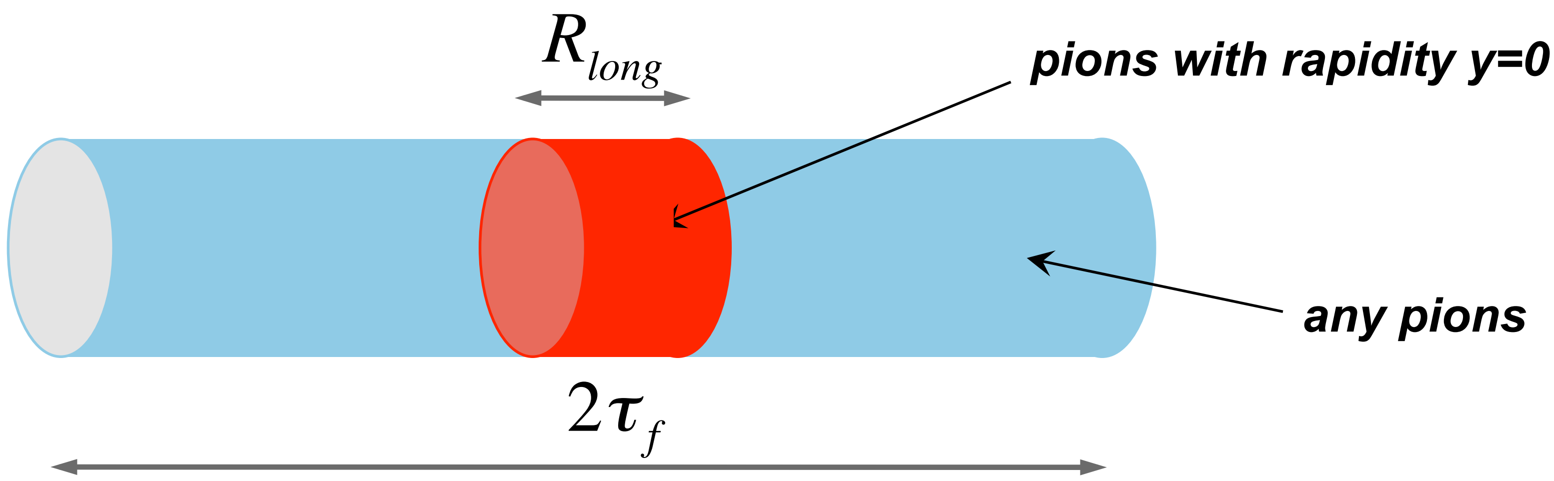




homogeneity volume 2 x larger than at RHIC

growth with energy reasonably well described by models tuned to RHIC data, containing early flow, cross-over, realistic EOS, and resonances

deducing expansion time from R_{long}



Makhlin-Sinyukov

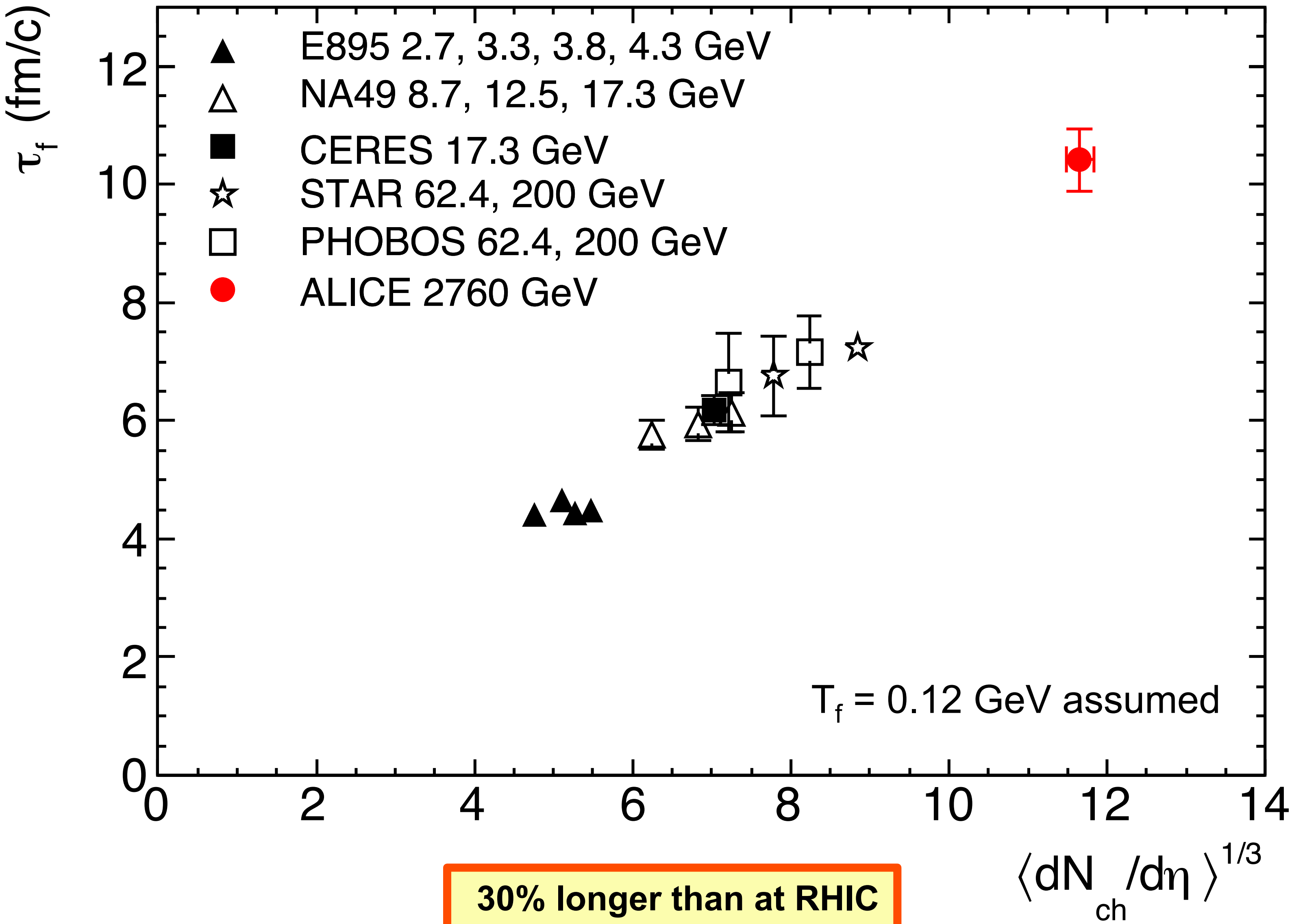
$$R_{long} = \tau_f \sqrt{\frac{T}{m_t}}$$

Herrmann-Bertsch

$$R_{long} = \tau_f \sqrt{\frac{T}{m_t} \frac{K_2(m_t/T)}{K_1(m_t/T)}}$$

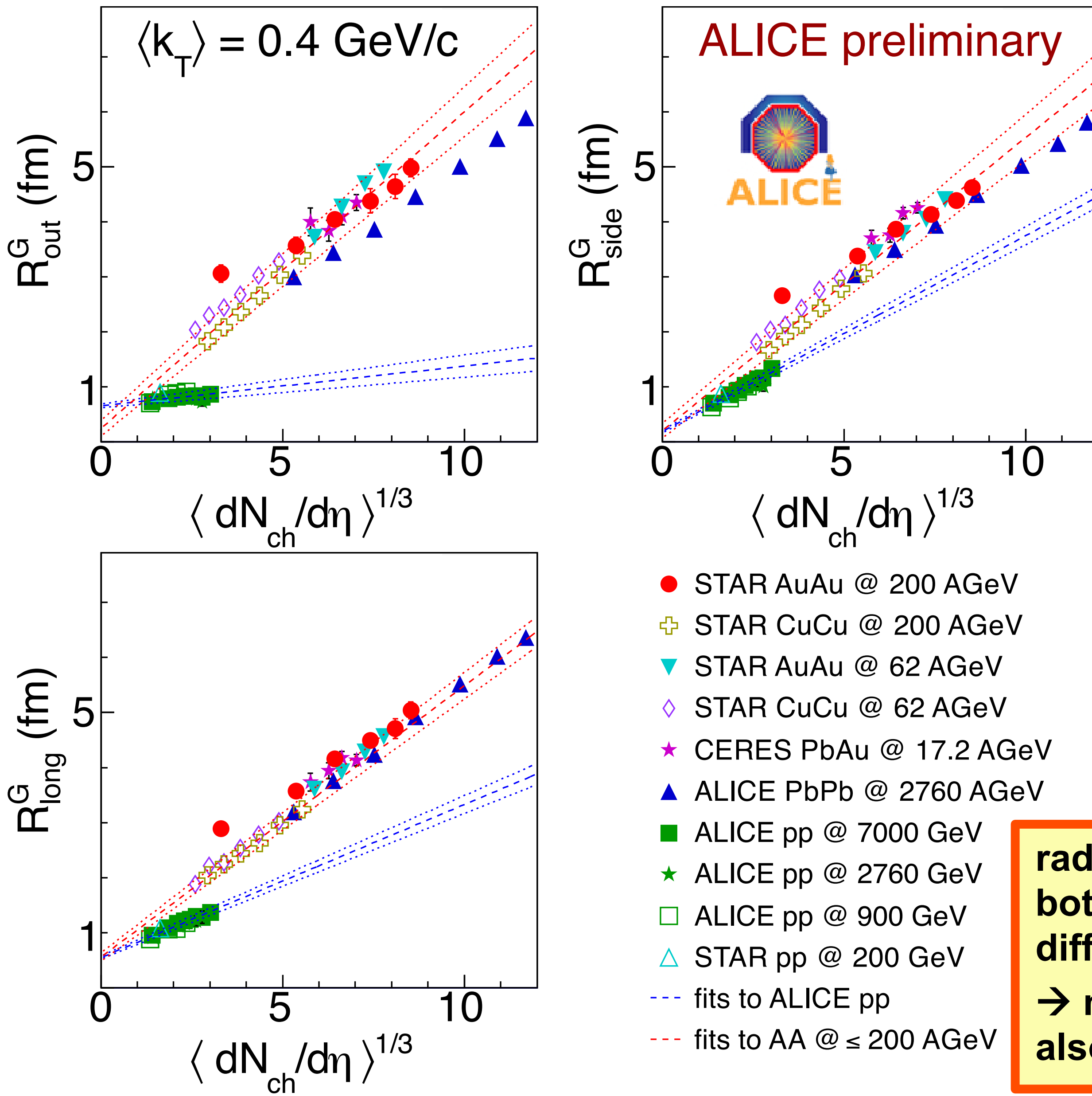
$\tau_f \sim$ ***inverse of the longitudinal Hubble constant***

expansion time in central Au and Pb collisions



30% longer than at RHIC

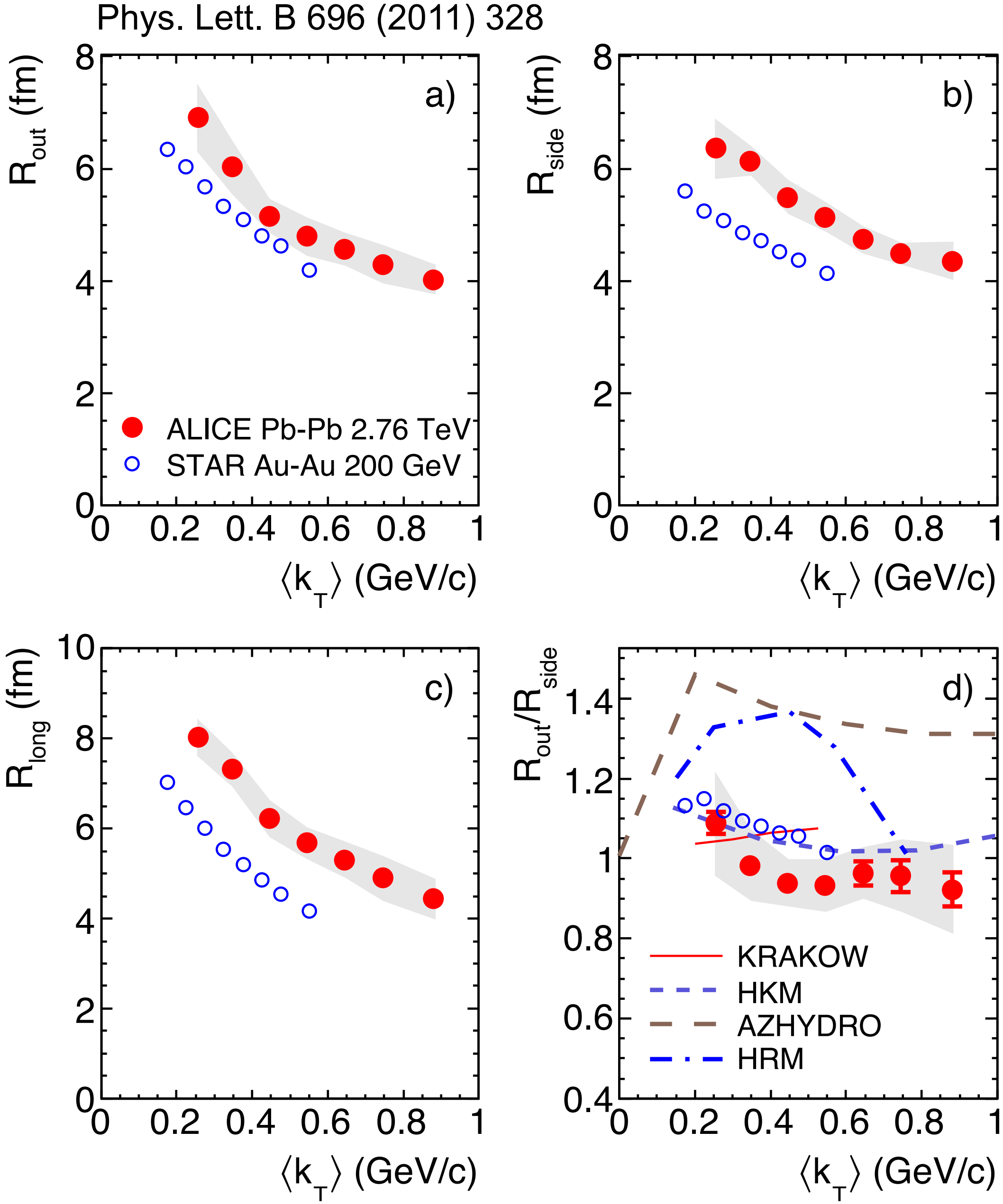
pion HBT



radii increase with multiplicity both in pp and Pb-Pb but with different slopes

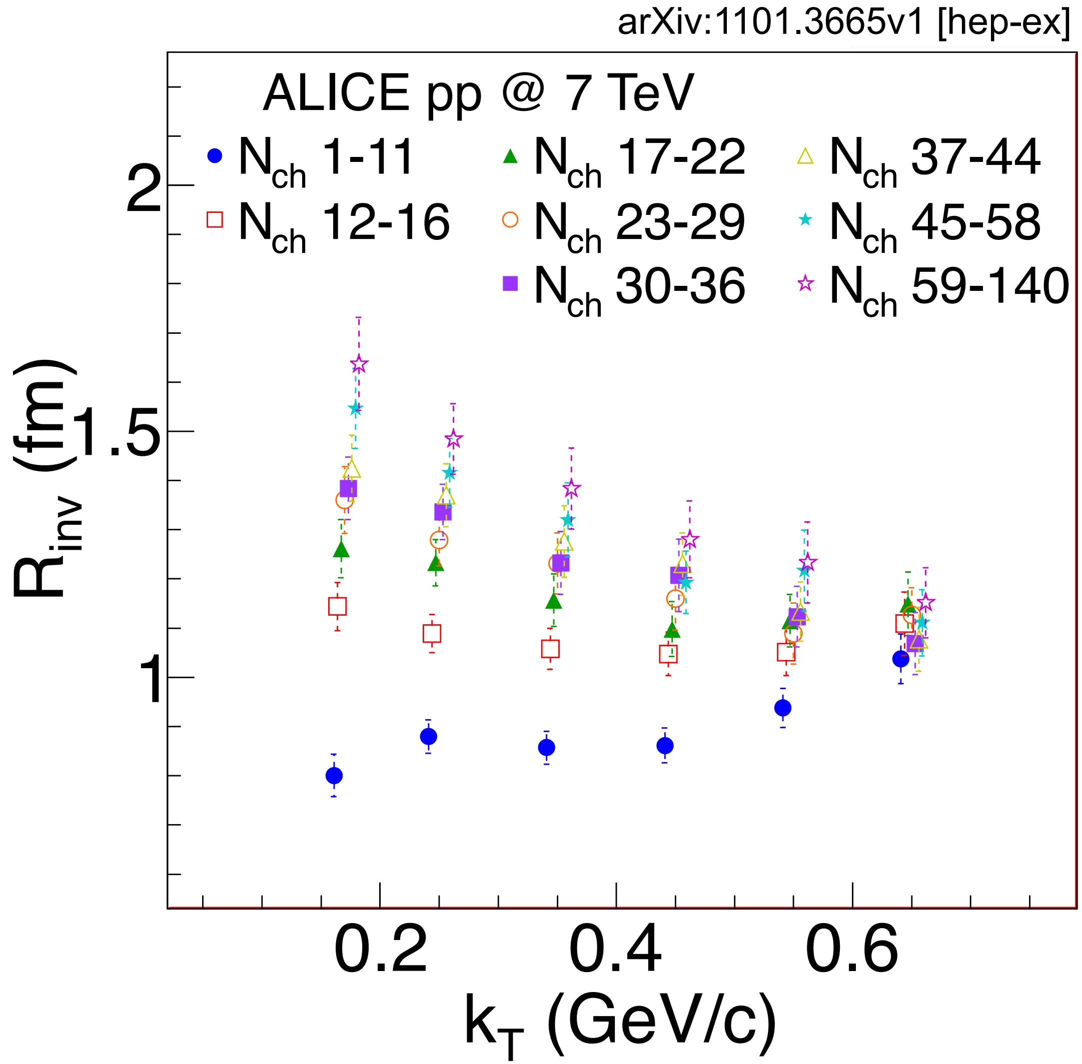
→ not only final multiplicity but also initial geometry matters

pion HBT



k_T dependence – signature of transverse flow

pion HBT



in pp, a similar k_T dependence develops with increasing multiplicity
→ flow in high-multiplicity pp?

briefly discussed   not discussed

- concept of two-particle correlation function
- Bose-Einstein correlations (time axis, expansion, resonances)
- Coulomb correlations (examples, deducing displacement between sources of different particle species)
- strong FSI correlations (examples, deducing parameters of strong interactions between given species)
- pion source size vs multiplicity and freeze-out criterion
- pion source size vs momentum and expansion
- deducing reaction duration
- comparing pp with Pb-Pb and separating initial and final effects
- indication of collective expansion in high-multiplicity pp collisions
- deducing pion phase-space density from the integral of the correlation peak and pion spectra
- deducing source velocity from the out-long cross term (Yano-Koonin-Podgoretsky)
- azimuthal HBT
- source size in pp events with and without jets
- deducing coherence and/or source asymmetry from 3-pion correlations
- imaging technique

common misunderstandings, errors, and misnomers

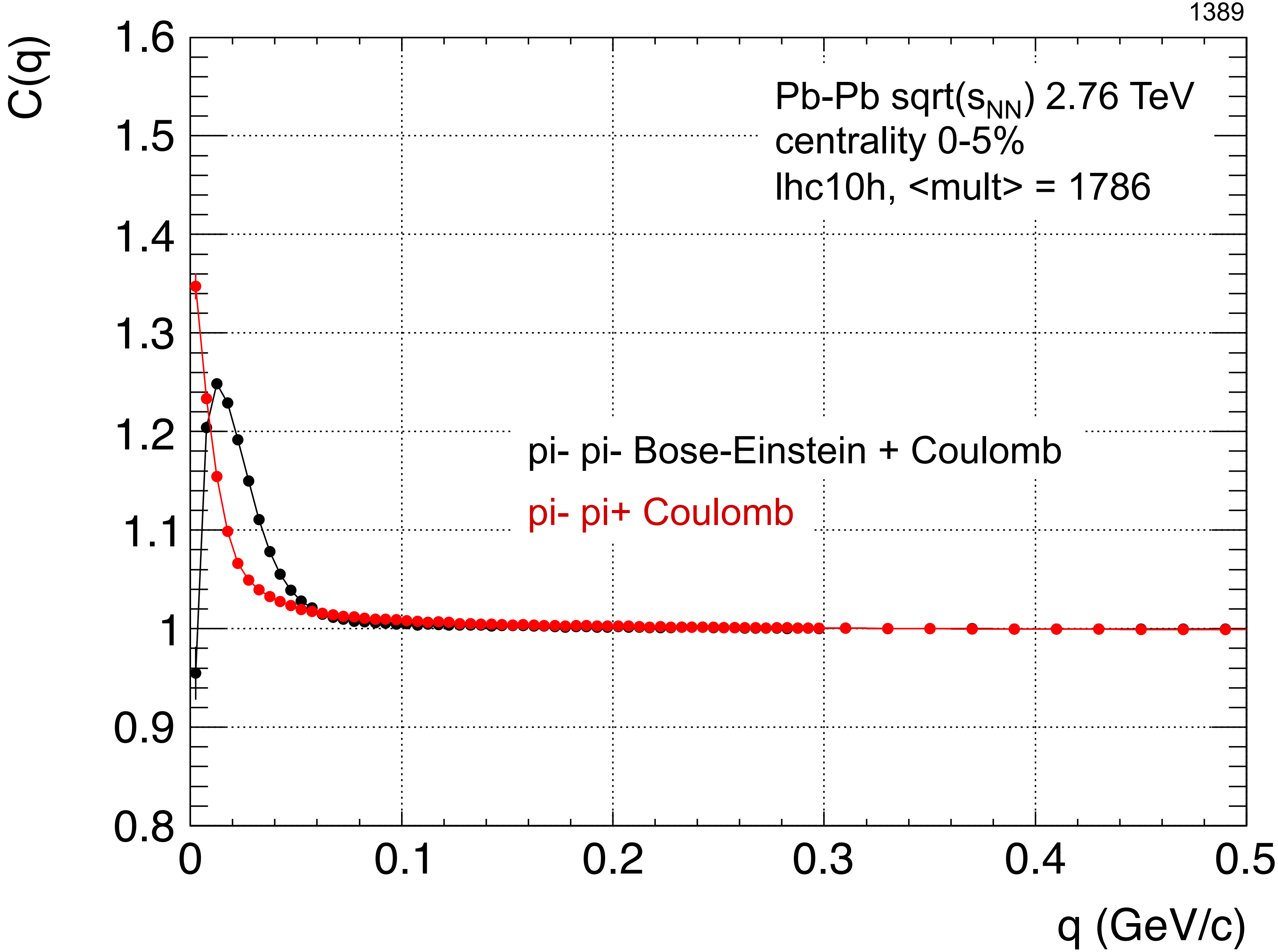
- **our HBT method comes from astronomy**
  truth: Hanbury Brown and Twiss measured stars, Goldhaber measured pion source, Kopylov noticed that these two things are related
- **HBT in astronomy = HBT in high-energy physics**
  truth: they are opposite
  star >> detector, pion source << detector
- **resonances produce correlated pairs and thus destroy HBT correlations**
  truth: resonances produce pions at large radii and thus produce a narrow unresolvable spike in C_2 -- effectively reduce the amplitude of the HBT peak
- **two-track resolution is not a problem in pp because the multiplicities are low**
  truth: two-track resolution effect is independent of multiplicity. But in pp it hurts less because the correlation peak is wide.

common misunderstandings, errors, and misnomers

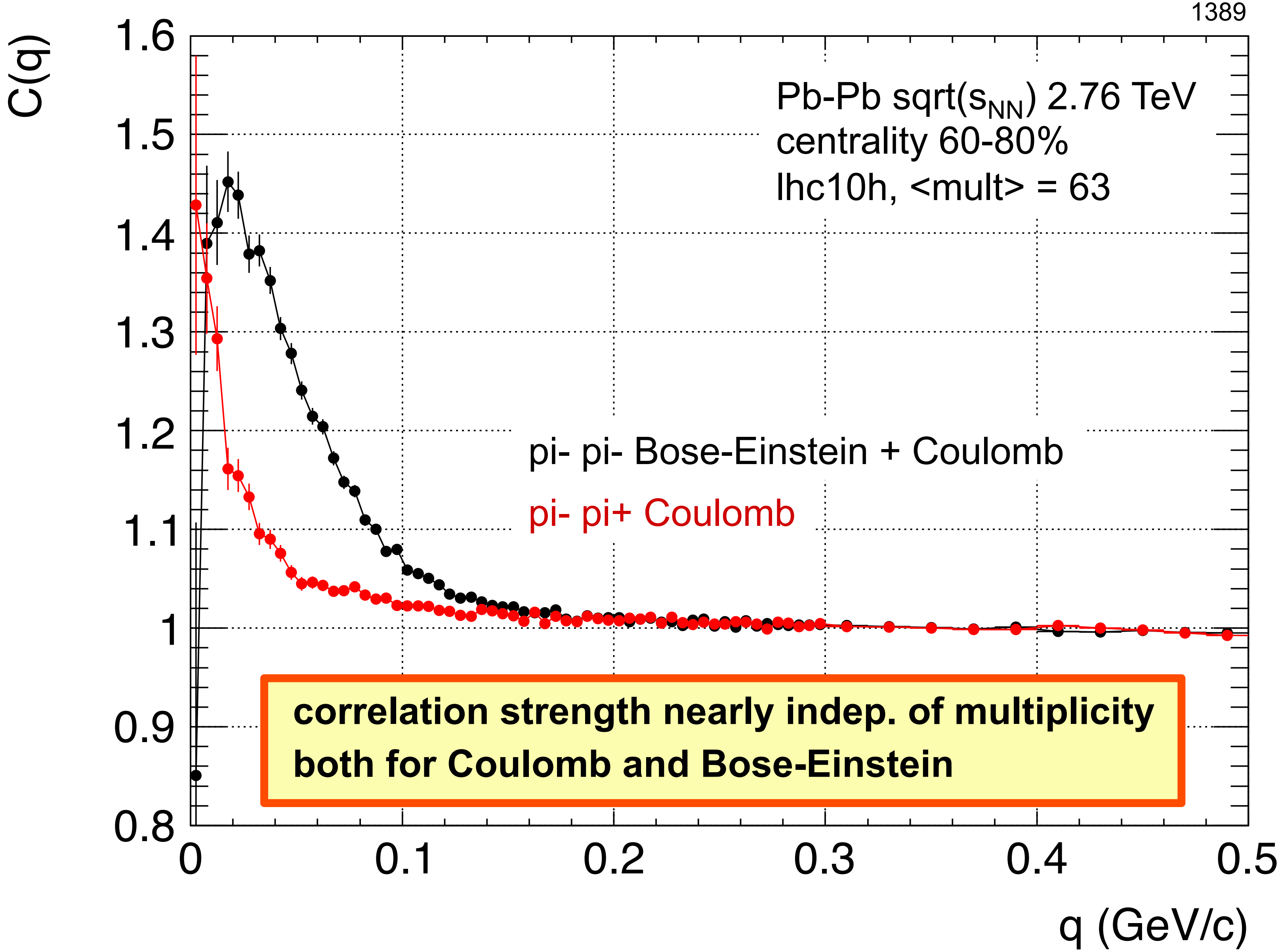
- exclude events with less than 2 pions from event mixing because these events do not contribute to the numerator
  truth: these events must be included, otherwise bias
- normalize numerator and denominator to the same number of pairs
  correct procedure: make number of event-event pairs in mixing equal to the number of events in the numerator; or, even better, normalize such that the flat part is = 1
- combine two correlation functions by weighted average, with weight = $1/\sigma^2$
  correct procedure: weight = number of mixed pairs
- least-square fit of the correlation function
  correct: maximum likelihood with Poisson errors

backup

Bose-Einstein and Coulomb correlations: everybody with everybody

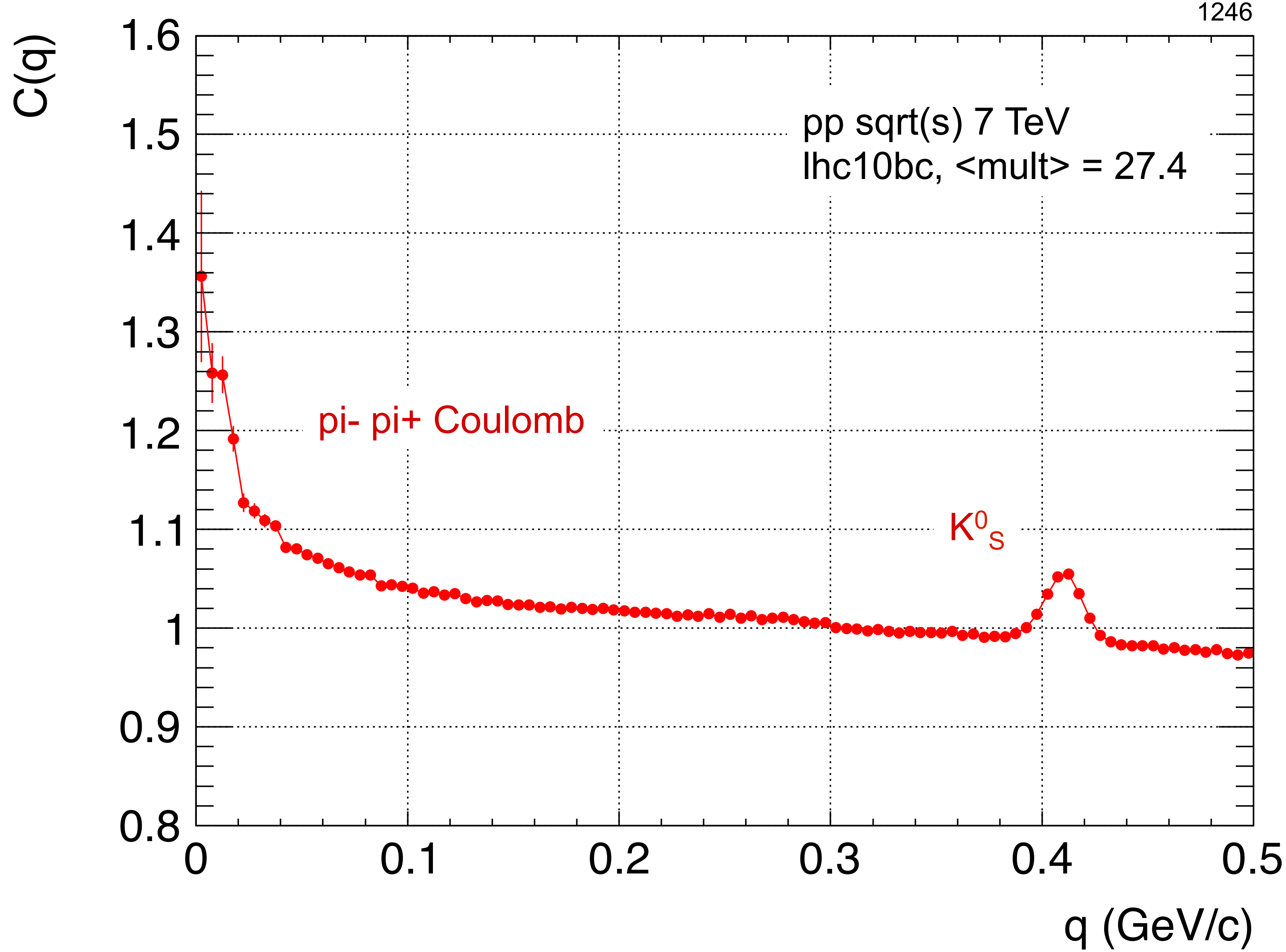


Bose-Einstein and Coulomb correlations: everybody with everybody



correlation strength nearly indep. of multiplicity both for Coulomb and Bose-Einstein

Bose-Einstein and Coulomb correlations: everybody with everybody



Bose-Einstein and Coulomb correlations: everybody with everybody