Normalization of the HBT correlation function

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In the literature there exist two definitions of the correlation function which differ by the normalization. Starting from these two different definitions, authors arrive at the same relation between the correlation function and the particle emission (source) function. This means that some of the derivations are wrong. The first definition [1] is

$$C_1(\mathbf{p}_1,\mathbf{p}_2) = \frac{\sigma \dfrac{d^6\sigma}{d^3p_1 d^3p_2}}{\dfrac{d^3\sigma}{d^3p_1}\dfrac{d^3\sigma}{d^3p_2}} = \frac{\dfrac{d^6 n}{d^3p_1 d^3p_2}}{\dfrac{d^3 n}{d^3p_1}\dfrac{d^3 n}{d^3p_2}}\ , \tag{1}$$

where $d^3\sigma/d^3p$ and $d^6\sigma/d^3p_1/d^3p_2$ are the single-particle and the two-particle cross sections, respectively, and σ is the total (inelastic) cross section, while d^3n/d^3p and $d^6n/d^3p_1/d^3p_2$ are the particle and the pair yields per event, respectively. The relations between the cross sections and yields are $d^3n/d^3p = 1/\sigma\ d^3\sigma/d^3p$, $d^6n/d^3p_1/d^3p_2 = 1/\sigma\ d^6\sigma/d^3p_1/d^3p_2$.

The second definition [2, 3] differs from the first one by the normalization factor:

$$C_2(\mathbf{p}_1,\mathbf{p}_2) = \frac{\langle n\rangle^2}{\langle n(n-1)\rangle} \frac{\dfrac{d^6 n}{d^3p_1 d^3p_2}}{\dfrac{d^3 n}{d^3p_1}\dfrac{d^3 n}{d^3p_2}} \tag{2}$$

with $\langle n\rangle$ and $\langle n(n-1)\rangle$ being the mean number of particles and mean number of particle pairs per event, respectively.

The third definition [4] is

$$C_3(\mathbf{p}_1,\mathbf{p}_2) = \frac{P(\mathbf{p}_1,\mathbf{p}_2)}{P(\mathbf{p}_1)P(\mathbf{p}_2)}, \tag{3}$$

where $P(\mathbf{p}_1)$ and $P(\mathbf{p}_1,\mathbf{p}_2)$ are called probabilities of observing a single particle with momentum $\mathbf{p}_1$ and a pair of particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively. The word 'probability' can be understood in two ways: 1) $P(\mathbf{p}_1)d\mathbf{p}_1$ is the mean multiplicity of particles with momentum between $\mathbf{p}_1$ and $\mathbf{p}_1 + d\mathbf{p}_1$ per event, in this case $P(\mathbf{p}_1) = d^3n/d^3p_1$, or 2) $P(\mathbf{p}_1)$ is the probability density for a given particle to have momentum $\mathbf{p}_1$. In the last case $P(\mathbf{p}_1) = 1/\langle n\rangle\ d^3n/d^3p_1$. Thus, depending on the interpretation of $P(\mathbf{p}_1)$ and $P(\mathbf{p}_1,\mathbf{p}_2)$, the definition (3) becomes equivalent to (1) or (2). In the rest of this note we consider the difference between (1) and (2).

Using these two different definions, different authors arrive at the same expression relating the correlation function to the source function. The reason for this is that, in spite of declaring different normalizations at the beginning, everybody (except, probably, [2] where the normalization is kept through-out the paper) starts his/her calculation from the same expression:

$$C = \frac{\langle a_1^\dagger a_2^\dagger a_2 a_1\rangle}{\langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle}\ . \tag{4}$$

For chaotic boson source this leads to

$$C(\mathbf{p}_1,\mathbf{p}_2) = 1 + \frac{|\langle a_1^\dagger a_2\rangle|^2}{\langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle} = 1 + |f(\mathbf{p}_1,\mathbf{p}_2)|^2 \tag{5}$$

with the function $f(\mathbf{p}_1,\mathbf{p}_2)$ depending on the shape of the boson source and asymptotically approaching zero for $|\mathbf{p}_2 - \mathbf{p}_1| \to \infty$. For the Fermi-Dirac statistics there is a minus sign in front of the second term.

The equivalence of the definitions (1) and (4) can be shown using the expressions for one and two particle invariant distributions:

$$E_1\frac{d^3n}{d^3p_1} = \langle a_1^\dagger a_1\rangle, \qquad E_1E_2\frac{d^6n}{d^3p_1d^3p_2} = \langle a_1^\dagger a_2^\dagger a_2 a_1\rangle\ . \tag{6}$$

It also can be shown that the definition (2) and Eq. (5) are inconsistent. Indeed, combining them we obtain

$$\frac{d^6n}{d^3p_1d^3p_2} = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}\frac{d^3n}{d^3p_1}\frac{d^3n}{d^3p_2}\left(1 + |f(\mathbf{p}_1,\mathbf{p}_2)|^2\right)\ . \tag{7}$$

Let us integrate this equation over $\mathbf{p}_1$ and $\mathbf{p}_2$. Since the integral of $d^6n/d^3p_1/d^3p_2$ is equal to the mean number of pairs per event $\langle n(n-1)\rangle$, and the integral of d^3n/d^3p is the mean multiplicity $\langle n\rangle$, the integration of (7) leads to

$$\int \frac{d^3n}{d^3p_1}\frac{d^3n}{d^3p_2}\ |f(\mathbf{p}_1,\mathbf{p}_2)|^2\ d^3p_1d^3p_2 = 0, \tag{8}$$

which is not possible unless $f(\mathbf{p}_1,\mathbf{p}_2)$ is identically equal to zero. Thus, a derivation starting from (2) and leading to (5) must contain an error.

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