Momentum Resolution of the E877 Spectrometer - Calculation

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1 Motivation

The finite momentum resolution of our spectrometer smears out the peak in the two-pion correlation function. As a consequence, the extracted source size is too small. This effect is of the order of 5% for R_{out} but probably much more significant for the out-long cross term. Understanding the momentum resolution is necessary to understand this distortion of the correlation function.

2 Introduction

The momentum resolution results from the finite position resolution of the detectors and from multiple scattering. It can be completely described by a 3×3 covariance matrix. The 3 dimensions can be for example (p, θ, ϕ) or (p_x, p_y, p_z), not necessarily in the LAB frame. This matrix is symmetric and thus has six independent elements. The non-diagonal elements are non negligible. They account for the fact that the individual momentum components are affected in a correlated way. For example, an error in the determination of the horizontal position at DC2 will cause a simultanous error of the reconstructed p and θ.

Following effects contribute to the momentum resolution (see Fig. 1):

1. Multiple scattering in target (MS_0)
2. Multiple scattering between target and DC2 (MS_1).
3. Multiple scattering in DC2 (MS_2)
4. Multiple scattering between DC2 and DC3 (MS_3)
5. DC2 and DC3 resolutions

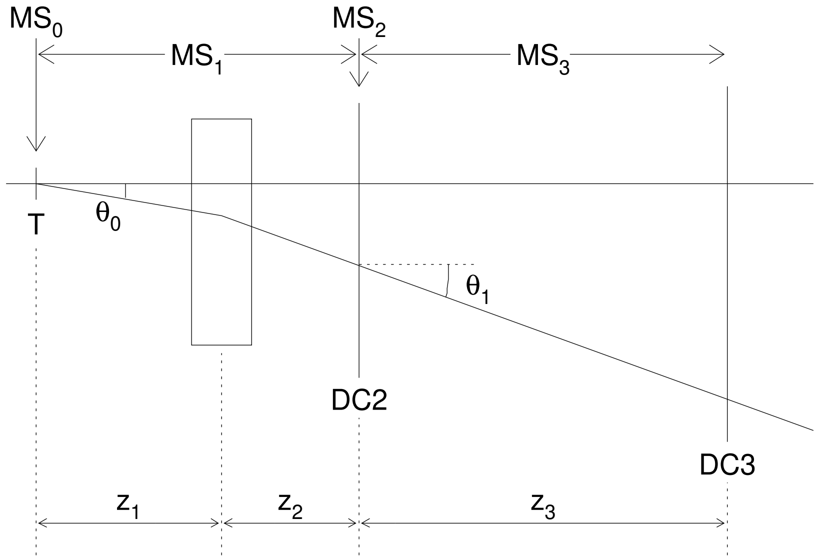

Figure 1. E877 spectrometer. Multiple scattering occurs in target (MS_0), between target and DC2 (MS_1), in DC2 (MS_2) and between DC2 and DC3 (MS_3). Finite x and y position resolution of DC2 and x of DC3 also contribute to momentum resolution.

I used two different approaches to calculate the momentum resolution: analytic and Monte Carlo. The analytic calculation had as a result analytic formulas for the six elements of the covariance matrix. They can be used to study the dependence of the momentum resolution on various parameters of the setup. The Monte Carlo routine for a given original momentum returns the reconstructed momentum vector. It can be used in simulations instead of GEANT if the momentum resolution is the only thing we are interested in. It is much faster and simpler to use than GEANT. If I call this routine many times with the same input parameters I will obtain some distribution of the reconstructed momentum. The width of this distribution should be equal to the prediction of the analytic formula if the two methods are consistent. The next sections describe the two methods and show the comparison.

3 Analytic calculation

This calculation was done with the help of the symbolic computation package MAPLE. Its basis are subsequent transformations from one set of variables to the other. The covariance matrix V is transformed along with the variables according to the formula

17.37 from CERN Booklet 1994:

$$V_{i,j}^{y} = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial y_i}{\partial x_m} \frac{\partial y_j}{\partial x_n} V_{m,n}^{x} \tag{1}$$

where x (y) is the old (new) variable set and V^x (V^y) is the old (new) covariance. The first set of variables contains the original momentum components. The last is the reconstructed momentum. The first covariance has nine zeros as elements - this corresponds to a fixed initial momentum. The last covariance describes the reconstructed momentum spread - this is the momentum resolution I am looking for. The intermediate sets of variables were chosen such that the detector resolution and the multiple scattering can be easily added to the covariance.

Following is the list of subsequent sets of variables. x and y always denote the horizontal and vertical position respectively. θ (ϕ) is the horizontal (vertical) angle. The small angle approximation is used: $\theta = p_x/p_z$ and $\phi = p_y/p_z$.

- $\vec{a} = (p, \theta, \phi)$ original momentum
  All elements of the covariance matrix V^a are originally 0. Now the multiple scattering in target can be introduced by setting

  $$V_{2,2}^{a} = V_{3,3}^{a} = \left(\frac{0.0136}{\beta p} \cdot MS_0 \right)^2 \tag{2}$$

  where MS_0 describes the thickness of the medium the particle must traverse (see Formula 10.8 in CERN Booklet 1994):

  $$MS_0 = \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \log \left(\frac{x}{X_0} \right) \right) \tag{3}$$

- $\vec{b} = (x, \theta, y)$ at DC2
  The propagation through the magnet (and thus the transformation from $\vec{a}$ to $\vec{b}$) is done according to the single bend and the small angles approximations:

  $$p = \frac{0.3BL}{\theta_1 - \theta_0} \tag{4}$$

  where B and L are the field and the effective length, p is the momentum and θ_0 and θ_1 are the horizontal angles before and after the magnet respectively. Consequently, $\vec{b}$ is related to $\vec{a}$ in the following way:

  $$\begin{aligned} b_1 &= \frac{0.3BLz_2}{a_1} + (z_1 + z_2)\, a_2 \\ b_2 &= \frac{0.3BL}{a_1} + a_2 \\ b_3 &= (z_1 + z_2)\, a_3 \end{aligned}$$

The covariance V^b is calculated from V^a according to Eq.1. Then the multiple scattering between target and DC2 is added to it:

$$\begin{aligned}
V^b_{1,1} &\rightarrow V^b_{1,1} + \left(\frac{0.0136}{\beta p} \cdot MS_1\right)^2 * \frac{1}{3} * (z_1 + z_2)^2 \\
V^b_{2,2} &\rightarrow V^b_{2,2} + \left(\frac{0.0136}{\beta p} \cdot MS_1\right)^2 \\
V^b_{1,2} &\rightarrow V^b_{1,2} + \left(\frac{0.0136}{\beta p} \cdot MS_1\right)^2 * \frac{1}{2} * (z_1 + z_2) \\
V^b_{3,3} &\rightarrow V^b_{3,3} + \left(\frac{0.0136}{\beta p} \cdot MS_1\right)^2 * \frac{1}{3} * (z_1 + z_2)^2
\end{aligned}$$

Subsequently the multiple scattering in DC2 is added to the diagonal element related to the horizontal angle:

$$V^b_{2,2} \rightarrow V^b_{2,2} + \left(\frac{0.0136}{\beta p} \cdot MS_2\right)^2$$

- $\vec{c} = (x \text{ at DC2}, x \text{ at DC3}, y \text{ at DC2})$

$$\begin{aligned}
c_1 &= b_1 \\
c_2 &= b_1 + z_3 b_2 \\
c_3 &= b_3
\end{aligned}$$

After V^b is transformed to V^c, the multiple scattering between DC2 and DC3 and the detector resolution are being added:

$$\begin{aligned}
V^c_{2,2} &\rightarrow V^c_{2,2} + \left(\frac{0.0136}{\beta p} \cdot MS_3\right)^2 * \frac{1}{3} * {z_3}^2 \\
V^c_{1,1} &\rightarrow V^c_{1,1} + {\Delta x_{DC2}}^2 \\
V^c_{2,2} &\rightarrow V^c_{2,2} + {\Delta x_{DC3}}^2 \\
V^c_{3,3} &\rightarrow V^c_{3,3} + {\Delta y_{DC2}}^2
\end{aligned}$$

At this point the particle propagation through the spectrometer is completed and the remaining transformations describe the momentum reconstruction.

- $\vec{d} = (p, \theta, \phi)$ reconstructed

$$\begin{aligned}
d_1 &= \frac{0.3\ B\ L\, z_1 z_3}{(z_1 + z_2)\, c_2 - (z_1 + z_2 + z_3)\, c_1} \\
d_2 &= \frac{(z_2 + z_3)\, c_1 - z_2\, c_2}{z_1 z_3} \\
d_3 &= \frac{c_3}{z_1 + z_2}
\end{aligned}$$

The covariance V^d, calculated according to Eq.1 from V^c, looks simple enough to write it here explicitly (after substituting values for z_1, z_2 and z_3):

$$
\begin{aligned}
V^d_{1,1} &= \frac{4.06 \cdot 10^{-4} {\Delta x_{DC2}}^2 + 8.84 \cdot 10^{-5} {\Delta x_{DC3}}^2}{(BL)^2} p^4 + \\
&\quad \frac{2.07 \cdot 10^{-3} {MS_1}^2 + 6.20 \cdot 10^{-3} {MS_2}^2 + 2.07 \cdot 10^{-3} {MS_3}^2}{(BL)^2} \frac{p^2}{\beta^2}, \\
V^d_{2,2} &= 1.95 \cdot 10^{-5} {\Delta x_{DC2}}^2 + 1.43 \cdot 10^{-6} {\Delta x_{DC3}}^2 + \\
&\quad (1.85 \cdot 10^{-4} {MS_0}^2 + 4.97 \cdot 10^{-5} {MS_1}^2 + 10^{-4} {MS_2}^2 + 3.35 \cdot 10^{-5} {MS_3}^2)\ p^{-2} \beta^{-2}, \\
V^d_{3,3} &= 3.44 \cdot 10^{-6} {\Delta y_{DC2}}^2 + (1.85 \cdot 10^{-4} {MS_0}^2 + 6.17 \cdot 10^{-5} {MS_1}^2)\ p^{-2} \beta^{-2}, \\
V^d_{1,2} &= \frac{8.91 \cdot 10^{-5} {\Delta x_{DC2}}^2 + 1.13 \cdot 10^{-5} {\Delta x_{DC3}}^2}{BL}\ p^2 + \\
&\quad \frac{8.46 \cdot 10^{-5} {MS_1}^2 + 7.89 \cdot 10^{-4} {MS_2}^2 + 2.63 \cdot 10^{-4} {MS_3}^2}{BL}\ \beta^{-2}.
\end{aligned}
$$

where p is in GeV/c, angles in radians, positions in cm and BL is in Tm. The vertical resolution is not correlated with anything: $V^d_{1,3} = V^d_{2,3} = 0$. The usually quoted Δp and $\Delta\theta$ and their correlation coefficient can be easily obtained from V^d:

$$
\Delta p = \sqrt{V^d_{1,1}} \qquad \Delta\theta = \sqrt{V^d_{2,2}} \qquad \rho_{p\theta} = \frac{V^d_{1,2}}{\sqrt{V^d_{1,1} V^d_{2,2}}} \tag{5}
$$

Two more variable sets may be useful:

- $\vec{e} = (p_x, p_y, p_z)$ in LAB

$$
\begin{aligned}
e_1 &= d_1 d_2 \\
e_2 &= d_1 d_3 \\
e_3 &= d_1 \sqrt{1 - {d_2}^2 - {d_3}^2}
\end{aligned}
$$

and finally

- $\vec{f} = (p_x, p_y, p_z)$ in a coordinate frame moving with rapidity y_F

$$
\begin{aligned}
f_1 &= e_1 \\
f_2 &= e_2 \\
f_3 &= \cosh(y_F)\ (e_3 - \tanh(y_F) \sqrt{mass^2 + {e_1}^2 + {e_2}^2 + {e_3}^2}\ )
\end{aligned}
$$

The formulas describing V_f are somewhat lengthy. I used the MAPLE's ability of producing FORTRAN output to create six FORTRAN functions which represent the six elements of V^f.

4 How YOU can use the analytic formula

This note is accompanied by six files containing FORTRAN routines: `vf11.f`, `vf22.f`, `vf33.f`, `vf12.f`, `vf13.f` and `vf23.f`, and by 3 files containing parameters: `params.93`, `params.94` and `params.95`. The parameters (determined to my best knowledge but with limited warranty) are also summarized in a table at the end of this note. Suppose you want to know what was the px-resolution in the 1994 run if the momentum is calculated in the midrapidity frame. First you should copy `params.94` to `params.inc`. Then you should edit `params.inc` and change the parameter `drap`, which denotes the coordinate frame rapidity, to 1.57. Then you can (for example) use PAW to display the px-resolution as a function of momentum for $\theta = -0.1$ and $\phi = 0.0$:

```
PAW > fun1 10 vf11.f(x,-0.1,0.0)**0.5 100 0 10
```

5 Monte Carlo calculation

The FORTRAN subroutine `propagate` calculates the reconstructed momentum $\vec{p}_{rec}$ for a given original momentum $\vec{p}$ (in GeV/c) and $mass$ (in GeV/c^2).

******

```
      subroutine propagate(mass,px,py,pz,px_rec,py_rec,pz_rec)
      implicit none
      real mass
      real px,py,pz
      real px_rec,py_rec,pz_rec

      real p,theta0,theta1,phi
      real p_rec,theta0_rec,theta1_rec,phi_rec
      real x1,x2,x3,y2
      real x1_rec,x2_rec,x3_rec,y2_rec
      real z1,z2,z3
      real beta,pxz,pxz_rec
      real rangauss,ran1,ran2
      real DT
      real L,B
      real DC2xres,DC3xres,DC2yres
      real MS0,MS1,MS2,MS3

      z1=310.2
      z2=228.6
      z3=615.8
```

```
      L = 1.045
      B = 0.262
      DC2xres = 0.008
      DC2yres = 0.1
      DC3xres = 0.008
      MS0 = 0.40
      MS1 = 0.024
      MS2 = 0.070
      MS3 = 0.032

      theta0=px/pz
      phi=py/pz
      pxz=(px**2+pz**2)**0.5
      p=(pxz**2+py**2)**0.5
      beta = 1.0/(mass**2/p**2+1)**0.5

c multiple scattering in target
      theta0=theta0+rangauss()*DT(beta,p,1,MS0)
      phi=phi+rangauss()*DT(beta,p,1,MS0)
c propagation
      x1=theta0*z1
      theta1=theta0+0.3*B*L/pxz
      x2=x1+theta1*z2
      y2=(z1+z2)*phi
c multiple scattering before DC2
      ran1=rangauss()
      ran2=rangauss()
      x2=x2+(z1+z2)*DT(beta,p,1,MS1)*(ran1/12**0.5+ran2/2.0)
      theta1=theta1+ran2*DT(beta,p,1,MS1)
      y2=y2+(z1+z2)*rangauss()*DT(beta,p,1,MS1)
c multiple scattering in DC2
      theta1=theta1+rangauss()*DT(beta,p,1,MS2)
c propagation
      x3=x2+theta1*z3
c multiple scattering before DC3
      x3=x3+rangauss()*z3*DT(beta,p,1,MS3)/3**0.5
c detector resolution
      x2_rec=x2+rangauss()*DC2xres
      x3_rec=x3+rangauss()*DC3xres
      y2_rec=y2+rangauss()*DC2yres
c reconstruction
      x1_rec=x2_rec-(x3_rec-x2_rec)/z3*z2
      theta0_rec=x1_rec/z1
      theta1_rec=(x3_rec-x2_rec)/z3
      pxz_rec=0.3*B*L/(theta1_rec-theta0_rec)
```

```
      phi_rec=y2_rec/(z1+z2)

      px_rec=pxz_rec*theta0_rec
      pz_rec=pxz_rec*(1-theta0_rec**2)**0.5
      py_rec=pz_rec*phi_rec
      return
      end
******************************************************************************
      real function DT(beta,p,z,thick)
      implicit none
      real beta,p,thick
      integer z
      DT=0.0136/beta/p*z*thick
      return
      end
******************************************************************************
* Random numbers with Gaussian distribution, mean=0 sigma=1
      real function rangauss()
      implicit none
      integer l
      real xdummy
      real u1,u2,v1,v2,s,z1,z2,rndm

 10   u1=rndm(l)
      u2=rndm(l)
      v1=2*u1-1
      v2=2*u2-1
      s=v1**2+v2**2
      if (s.gt.1.0) goto 10
      z1=v1*(-2*log(s)/s)**0.5
c     z2=v2*(-2*log(s)/s)**0.5
      rangauss=z1
      return
      end

******
```

6 How YOU can use the Monte Carlo routine

You can simulate the momentum resolution in your Monte Carlo program by calling the subroutine `propagate` for each track, and then substituting original momentum $\vec{p}$ by the reconstructed $\vec{p}_{rec}$. The parameters inside `propagate` must be edited depending on which year's data you want to simulate (see table in the Appendix).

7 Comparison of analytic and Monte Carlo calculations with GEANT

The momentum resolution of course can also be extracted from the GEANT simulation. Multidimensional histograms containing the difference between the original and the reconstructed momentum must be filled and subsequently fitted by Gaussian with cross-term. The results of such an analysis agree well with the results obtained using two previously described methods (Fig. 2).

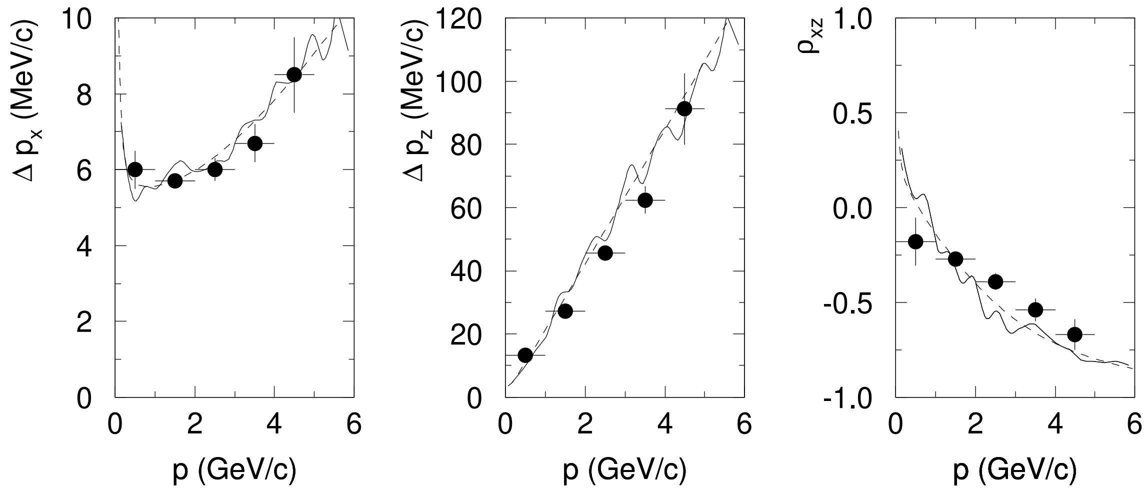

Figure 2. Comparison of different methods to calculate the momentum resolution. Presented are Δp_x, Δp_z and their correlation ρ_{xz} as a function of momentum for pions. The solid and dashed lines represent Monte Carlo and analytic calculation, respectively, done for emission angles $\theta = -0.075$ and $\phi = 0$. The black points are from Gaussian fit to GEANT data with a cut on the horizontal emission angle $-0.10 < \theta < -0.05$.

8 Appendix - Parameter values

The contents of files `params.93`, `params.94`, `params.95` and `params.kwon` is summarized in this table:

	L [m]	B [T]	Δx_{DC2}	Δy_{DC2}	Δx_{DC3}	MS_0	MS_1	MS_2	MS_3
1992 Jimmy	1.045	0.262	0.012?		0.012?		0.160		
1993 Kwon	1.045	0.262	0.008	0.1 or 0.2	0.008	0.40	0.024	0.070	0.032
1993	1.045	0.262	0.030	0.2	0.060	0.27	0.080	0.083	0.044
1994	1.045	0.335	0.030	0.2	0.060	0.27	0.082	0.083	0.044
1995	1.045	0.335	0.030	0.2	0.060	0.27	0.091	0.083	0.044

'1993 Kwon' denotes the GEANT simulation done by Kwon. In the GEANT setup definition wire chambers do not have windows and wires. That's why the MS_2 and MS_3 values there are somewhat different from the other parameter sets which are my best guess based on what the detectors builders told me. Also the detector resolution used in GEANT might be too optimistic.

The value of MS_0 is based on assumption that the interaction took place in the center of the target volume (in z direction). The actual values will be distributed between 0 and approximately $sqrt2\ MS_0$.

The B-z-θ sign convention is following:
- spectrometer acceptance covers mainly negative horizontal angles ($-0.15 < \theta < 0.01$)
- use positive value of B to describe positive particles in A-field or negative particles in B-field
- use negative value of B to describe negative particles in A-field or positive particles in B-field