

# decoding the QCD phase structure with relativistic nuclear collisions

- brief introduction – the LHC era and relativistic nuclear collisions
- the hadron resonance gas and (u,d,s) hadron production
- Dashen-Ma-Bernstein taken seriously
- experimental determination of the QCD phase boundary
- loosely bound objects
- summary and outlook

pbm

Stern-Gerlach Medal presentation (1)

DPG-Frühjahrstagung  
München, 17. - 22. März 2019



UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
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phenomenology results obtained in collaboration with  
Anton Andronic, Krzysztof Redlich, and Johanna Stachel  
arXiv:1710.09425,  
Nature 561 (2018) 321

most of the new data are from the ALICE collaboration  
at the CERN LHC

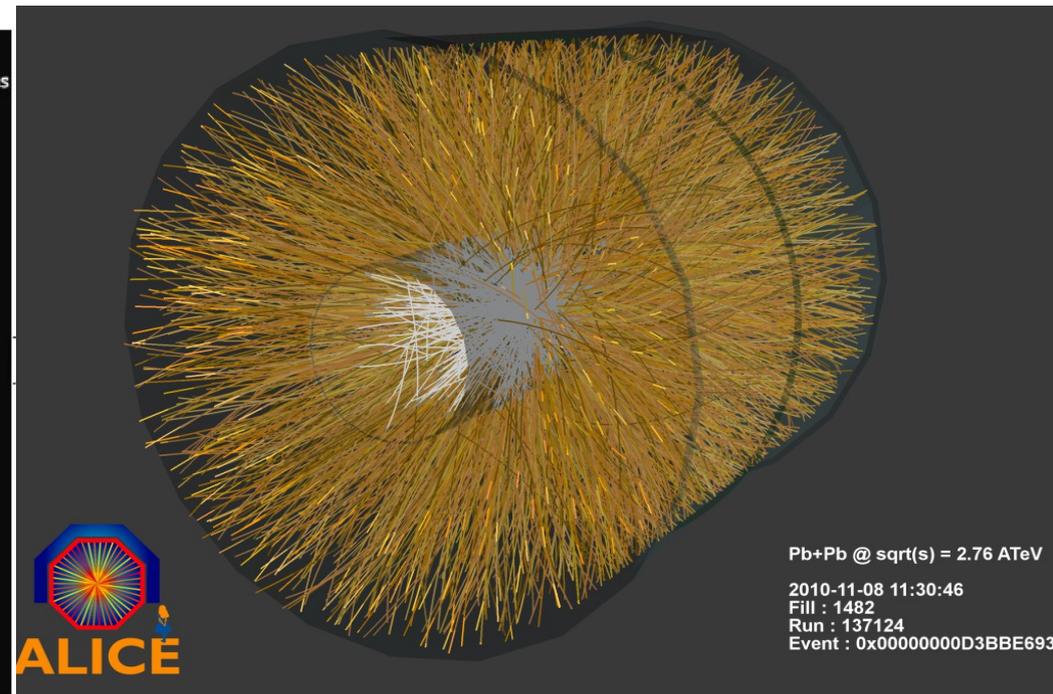
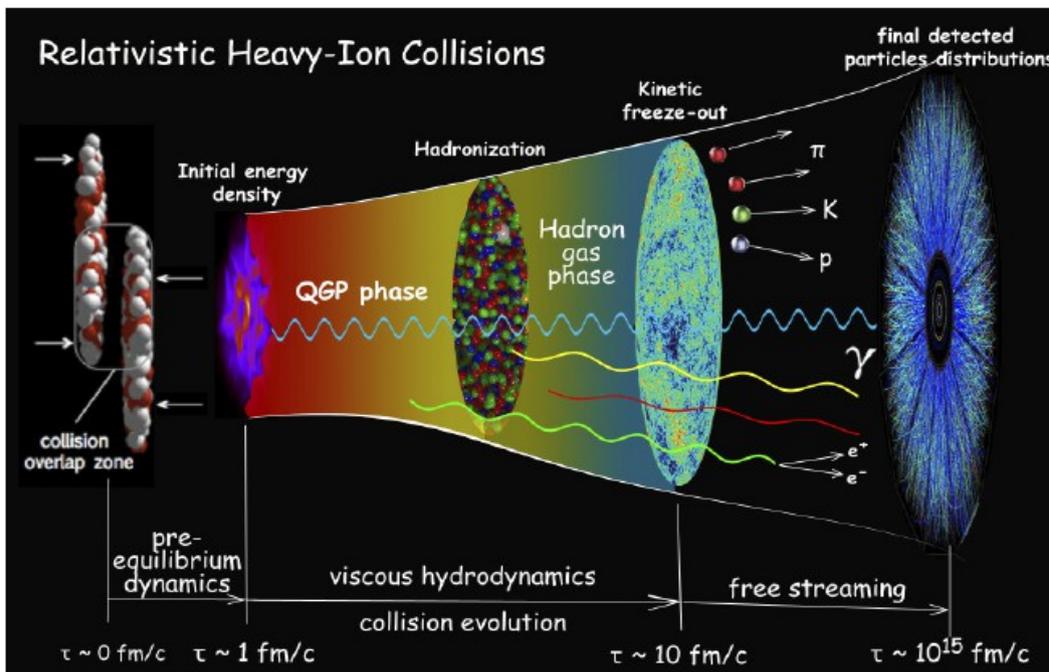
newest results including pion-nucleon phase shifts  
from arXiv:1808.03102



## time line and matter in the early universe

- inflation up to  $10^{-32}$  s
- $10^{-32}$  to  $10^{-12}$  s: cosmic matter consists of **massless** particles and fields quarks, leptons, neutrinos, photons, Z,  $W^\pm$ , H ??? lots of speculations
- $10^{-12}$  s: electroweak phase transition,  $T \approx 100$  GeV
- $10^{-12}$  –  $10^{-5}$  s quark-gluon plasma phase  
particles acquire mass through Higgs mechanism, QGP consists of:  
 $\bar{q}qg\bar{l}l\gamma ZW^\pm H$ , all in equilibrium
- $10^{-5}$  s QCD phase transition,  $T = 155$  MeV
- $10^{-5}$  s to 1 s annihilation phase,  $T(1 \text{ s}) \approx 1$  MeV  
cosmic matter converts into protons, neutrons, leptons, neutrinos, photons
- $t > 1$  s: leptons annihilate and reheat universe, neutrinos decouple, light element production commences

# the Quark-Gluon Plasma formed in nuclear collisions at very high energy



Paul Sorensen and Chun Shen

# PbPb collisions at LHC at $\sqrt{s} = 5.02$ A TeV

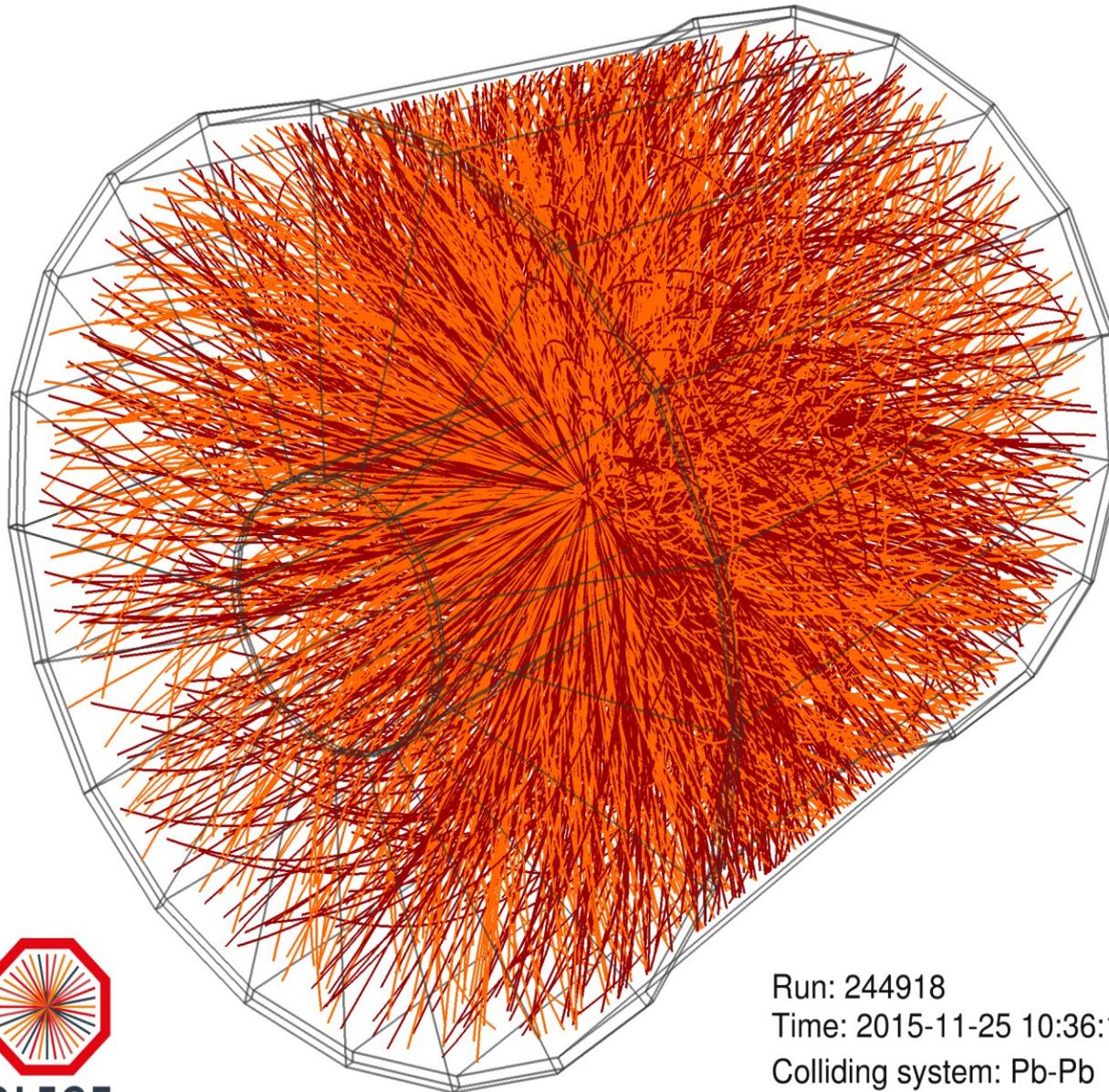
Run1: 3 data taking campaigns  
pp, pPb, Pb—Pb  
> 170 publications

Run2 with 13 TeV pp  
Pb—Pb run 5 TeV/u  
p-Pb Run at 5 and 8 TeV  
> 50 publications

Nov. 2018: PbPb 5 TeV/u

Snapshot taken with the ALICE  
TPC

central Pb-Pb collisions  
more than 32000  
particles produced per collision



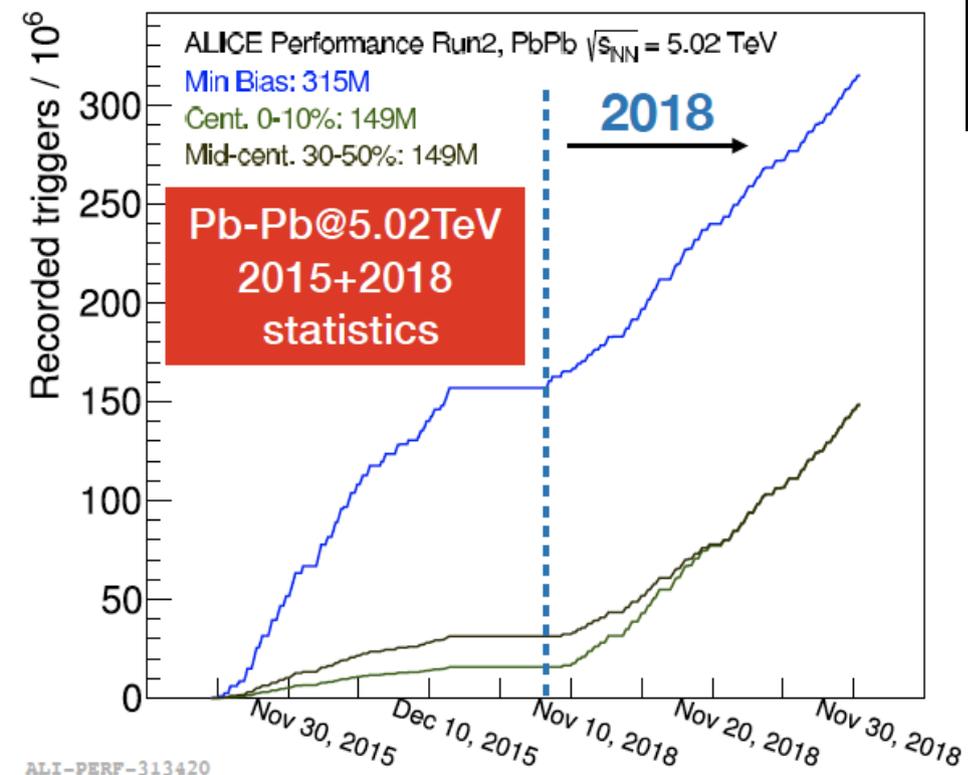
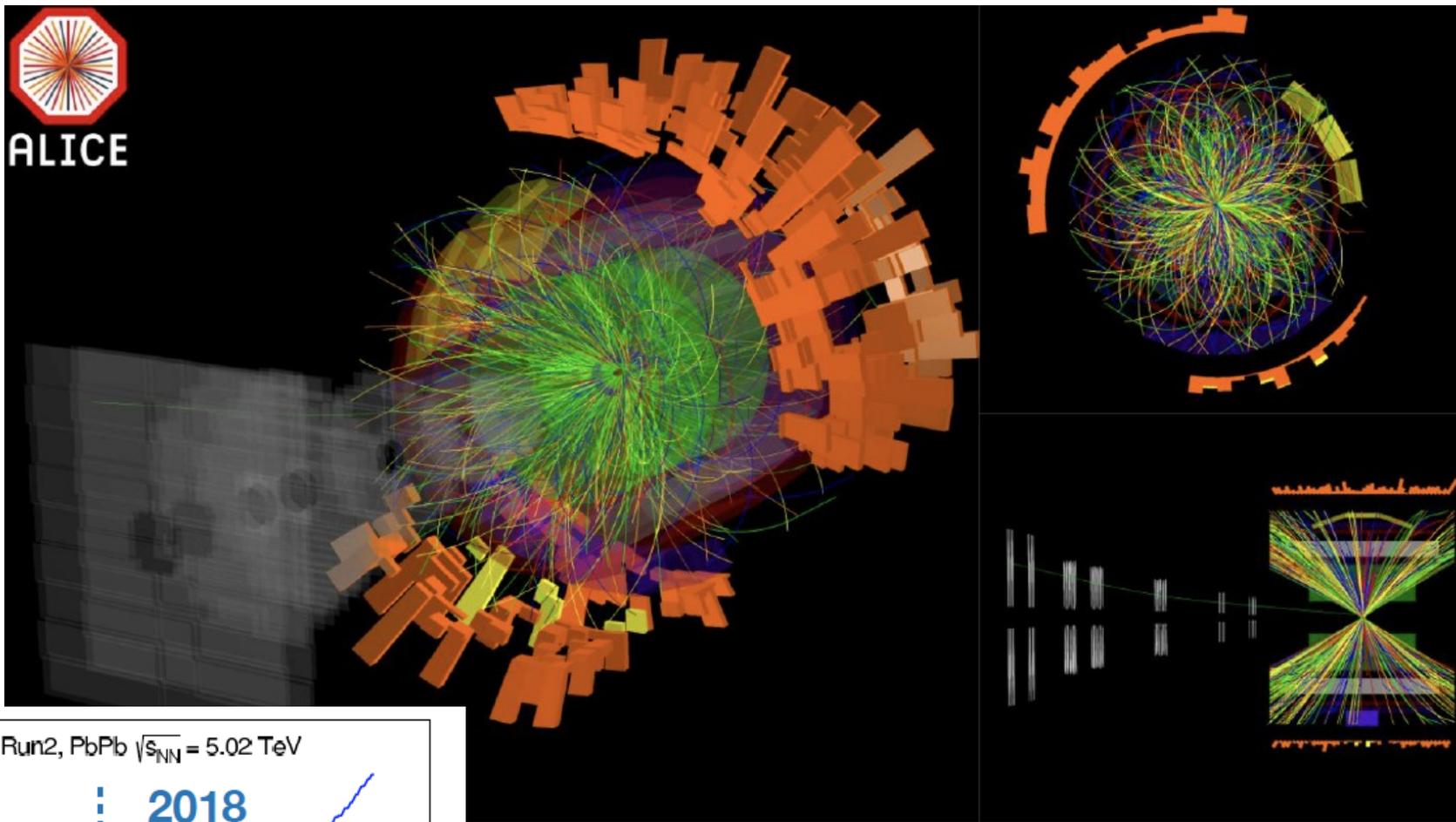
Run: 244918  
Time: 2015-11-25 10:36:18  
Colliding system: Pb-Pb  
Collision energy: 5.02 TeV



inside the TPC field cage, 2004

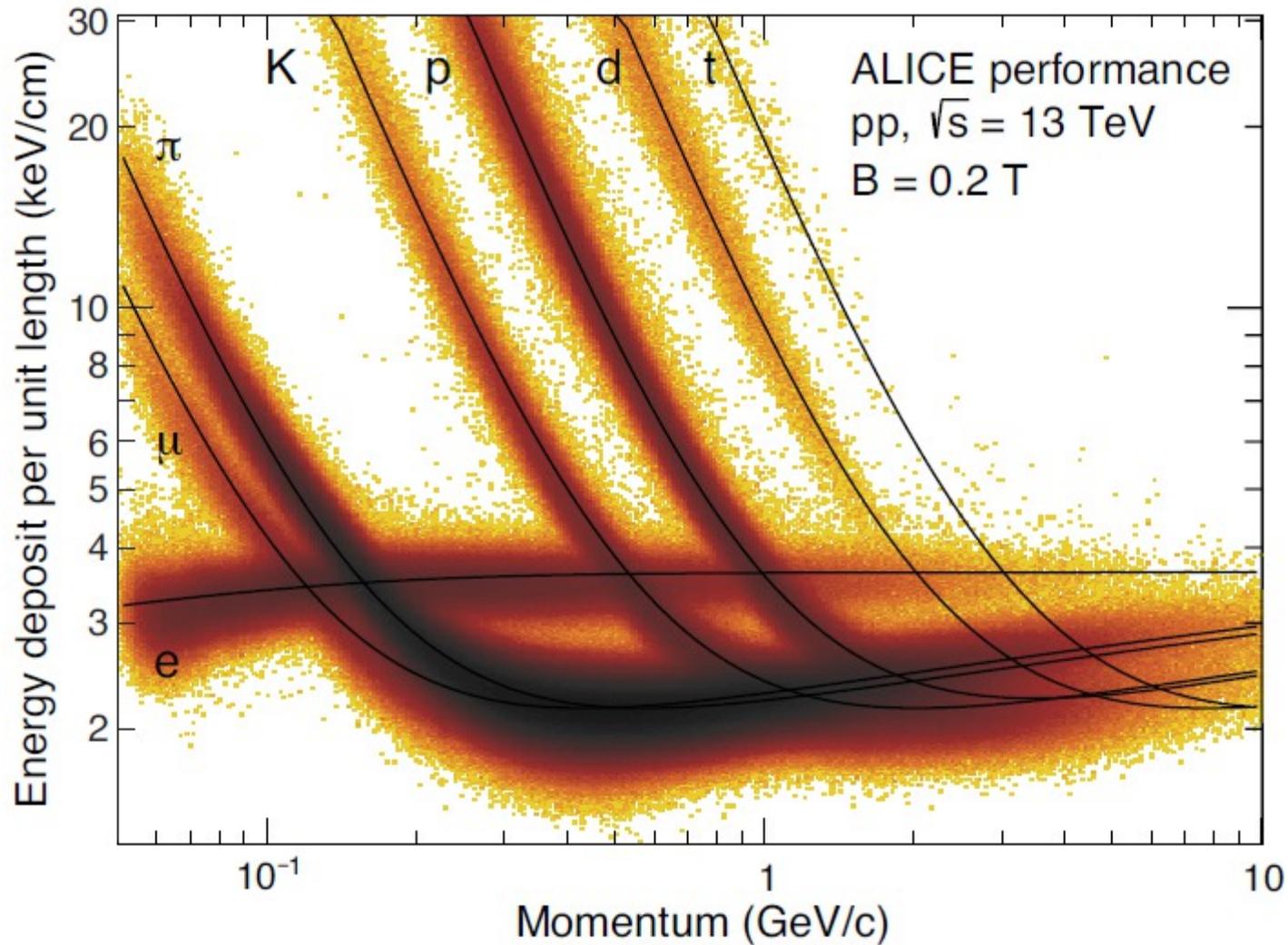


ALICE



# particle identification with the ALICE TPC

from 50 MeV to 50 GeV



now PDG standard

# **hadron production and the QCD phase boundary**

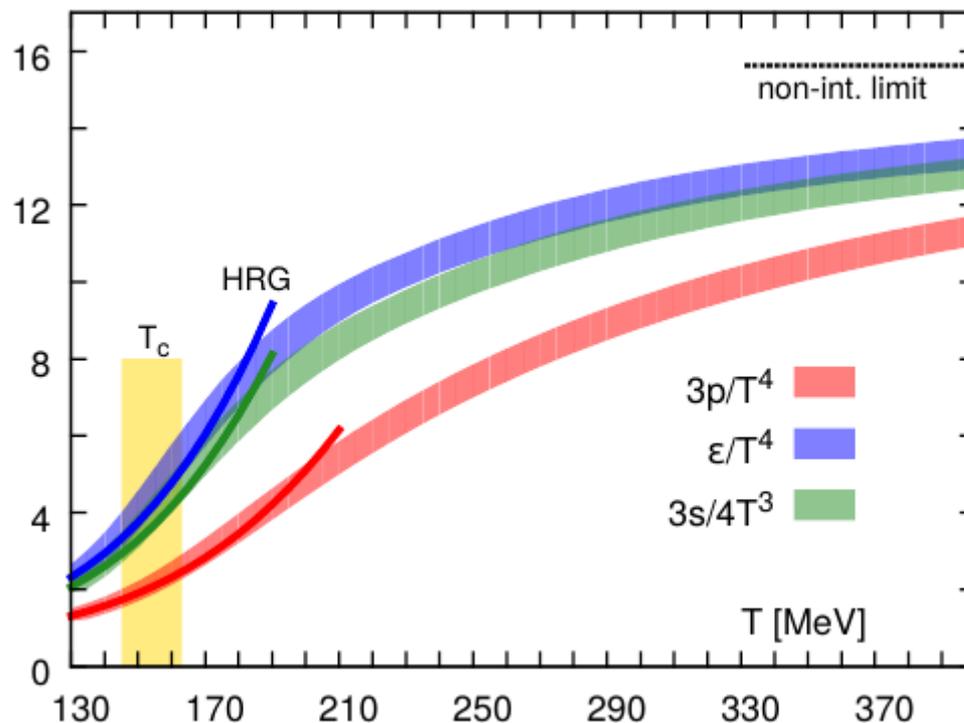
**measure the momenta and identity of all produced particles  
at all energies and look for  
signs of equilibration, phase transitions, regularities, etc**

# duality between hadrons and quarks/gluons (I)

comparison of equation of state from  
LQCD  
Phys.Rev. D90 (2014) 094503  
HOTQCD coll.

with hadron resonance gas predictions  
(colored lines)

essentially the same results also from  
Wuppertal-Budapest coll.  
Phys.Lett. B730 (2014) 99-104



pseudo-critical  
temperature  
 $T_c = 156 \pm 1.5$  MeV, very new and improved  
 arXiv:1807.05607

$$\epsilon_{\text{crit}} = 340 \text{ MeV/fm}^3$$

$$\epsilon_{\text{nucl}} = 450 \text{ MeV/fm}^3$$

# duality between hadrons and quarks/gluons (II)

in the dilute limit  $T < 165$  MeV:

$$\ln Z(T, V, \mu) \approx \sum_{i \in \text{mesons}} \ln \mathcal{Z}_{M_i}^M(T, V, \mu_Q, \mu_S) + \sum_{i \in \text{baryons}} \ln \mathcal{Z}_{M_i}^B(T, V, \mu_b, \mu_Q, \mu_S)$$

where the partition function of the hadron resonance model is expressed in mesonic and baryonic components. The chemical potential  $\mu$  reflects then the baryonic, charge, and strangeness components  $\mu = (\mu_b, \mu_Q, \mu_S)$ .

# thermal model of particle production and QCD

partition function  $Z(T,V)$  contains sum over the full hadronic mass spectrum and is fully calculable in QCD

for each particle  $i$ , the statistical operator is:

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

particle densities are then calculated according to:

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

from analysis of all available nuclear collision data we now know the energy dependence of the parameters  $T$ ,  $\mu_b$ , and  $V$  over an energy range from threshold to LHC energy and can confidently extrapolate to even higher energies

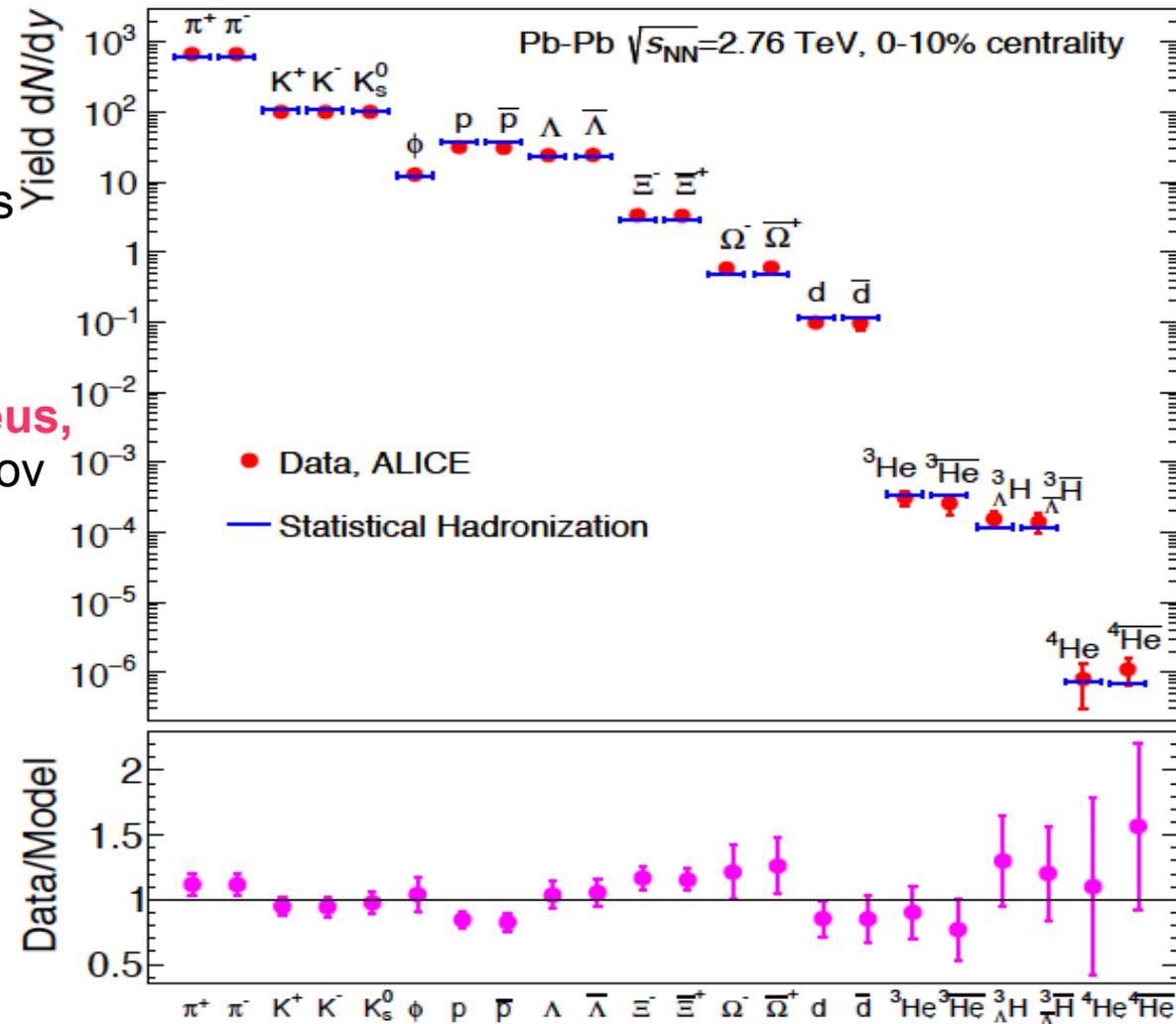
in practice, we use the full experimental hadronic mass spectrum from the PDG compilation (vacuum masses) to compute the 'primordial yield'

comparison with measured hadron yields needs evaluation of all strong decays

# Oct. 2017 update: excellent description of ALICE@LHC data

fit includes loosely bound systems such as deuteron and hypertriton  
 hypertriton is bound-state of ( $\Lambda$ ,p,n),  
 $\Lambda$  separation energy about 130 keV  
 size about 10 fm, the **ultimate halo nucleus**,  
 produced at  $T=156$  MeV. close to an Efimov  
 state

proton discrepancy about 2.8 sigma



Andronic, pbm, Redlich, Stachel, arXiv:1710.09425, Nature 561 (2018) 321

J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich, Confronting LHC data with the statistical hadronization model, J.Phys.Conf.Ser.509 (2014) 012019, arXiv:1311.4662 [nucl-th].

## the proton anomaly and the Dashen, Ma, Bernstein S-matrix approach

R. Dashen, S. K. Ma, and H. J. Bernstein, Phys. Rev. **187**, 345 (1969).

The S-matrix formalism [20–24] is a systematic framework for incorporating interactions into the description of the thermal properties of a dilute medium. In this scheme, two-body interactions are, via the scattering phase shifts, included in the leading term of the S-matrix expansion of the grand canonical potential. The resulting interacting density of states is then folded into an integral over thermodynamic distribution functions, which, in turn, yields the interaction contribution to a particular thermodynamic observable.

$$\langle R_{I,J} \rangle = d_J \int_{m_{th}}^{\infty} dM \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\pi} B_{I,J}(M) \times \frac{1}{e^{(\sqrt{p^2+M^2}-\mu)/T} + 1},$$

thermal yield of an (interacting) resonance with mass  $M$ , spin  $J$ , and isospin  $I$

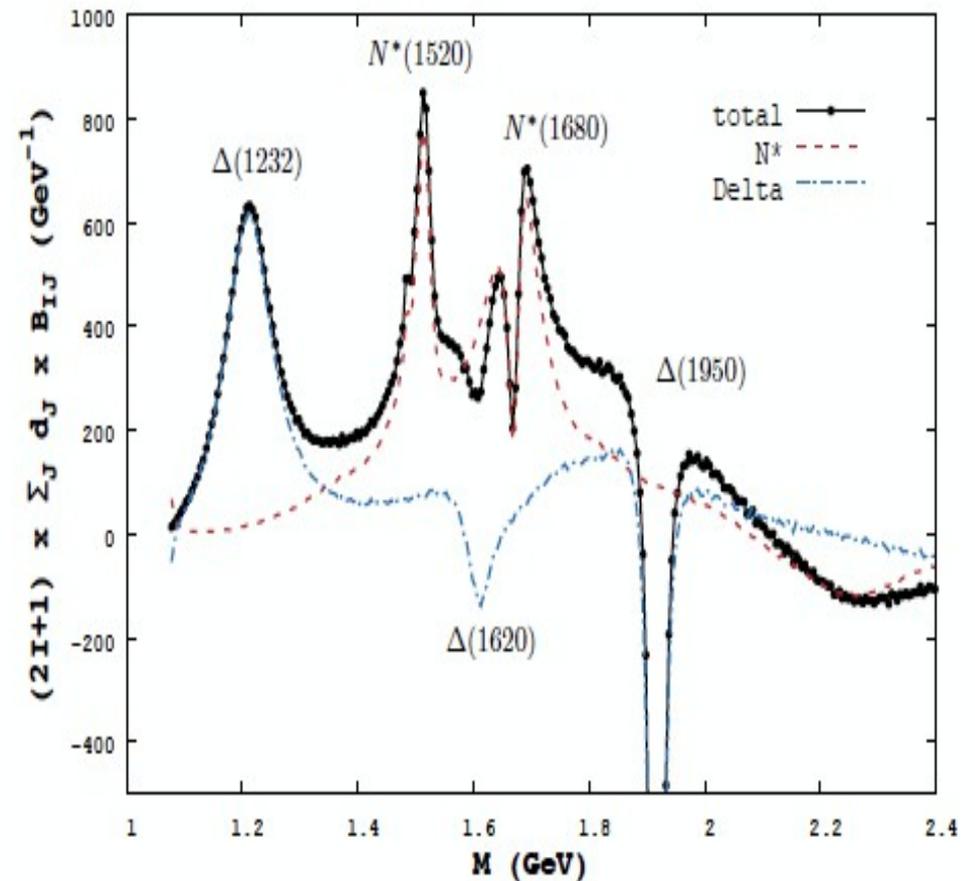
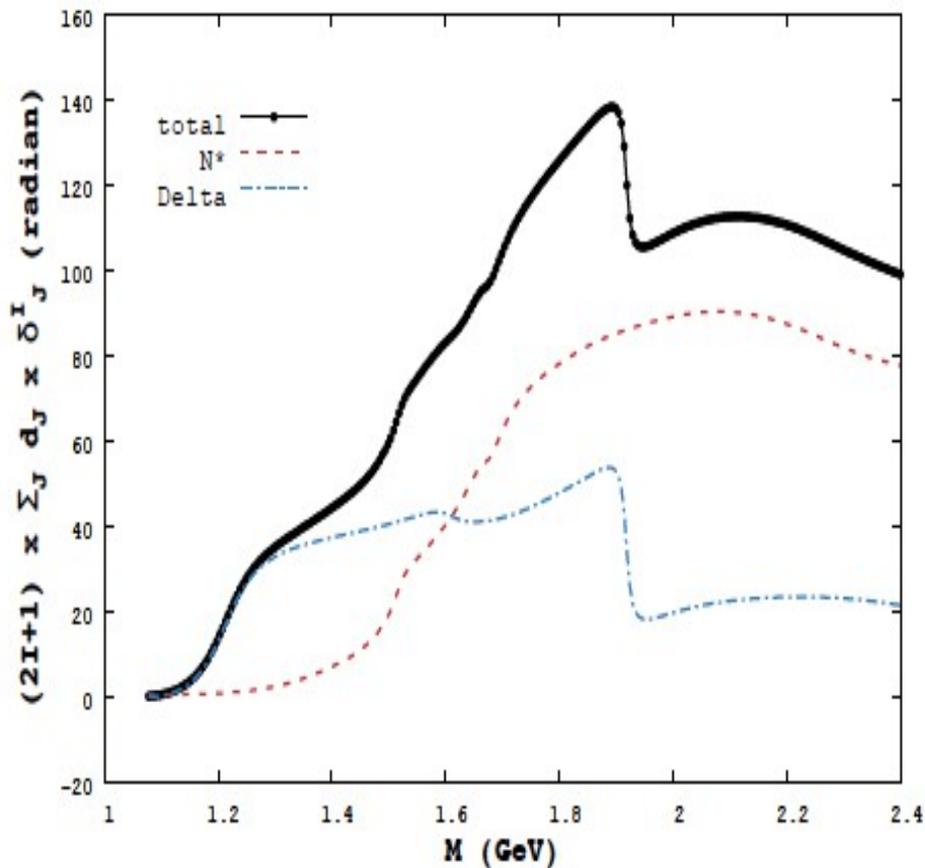
need to know derivatives of phase shifts with respect to invariant mass

$$B_{I,J}(M) = 2 \frac{d\delta_J^I}{dM}.$$

A. Andronic, pbm, B. Friman, P.M. Lo, K. Redlich, J. Stachel, arXiv:1808.03102, update Jan. 2019

# pion nucleon phase shifts and thermal weights for $N^*$ and $\Delta$ resonances

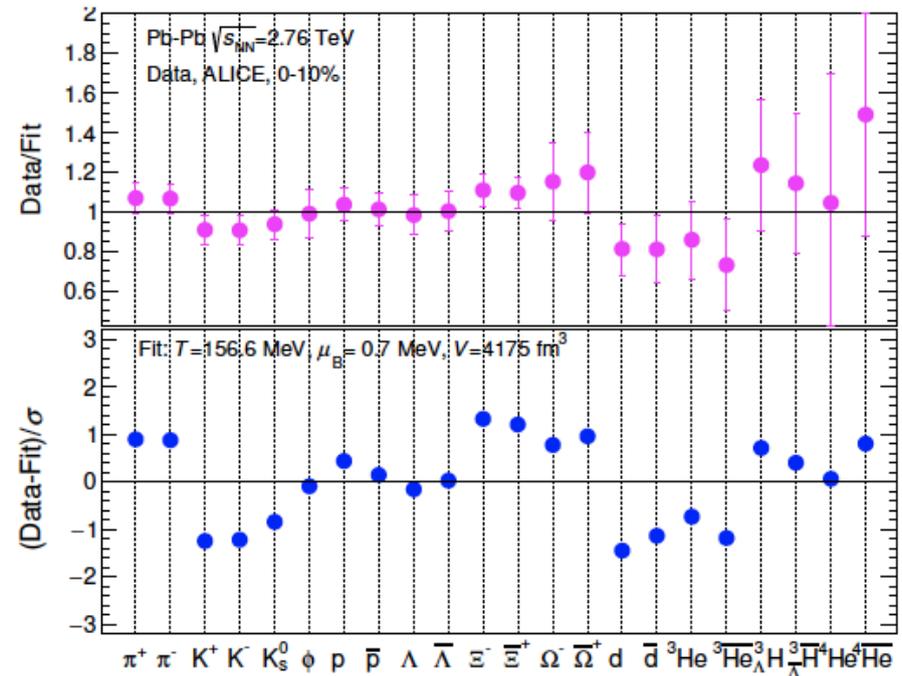
GWU/SAID phase shift analysis, 15 partial waves for each isospin channel



# Jan. 2019 update: excellent description of ALICE@LHC data

proton discrepancy of 2.8 sigma is now explained in arXiv:1808.03102  
 explicit phase shift description of baryon resonance region  
 (Andronic, pbm, Friman, Lo, Redlich, Stachel)

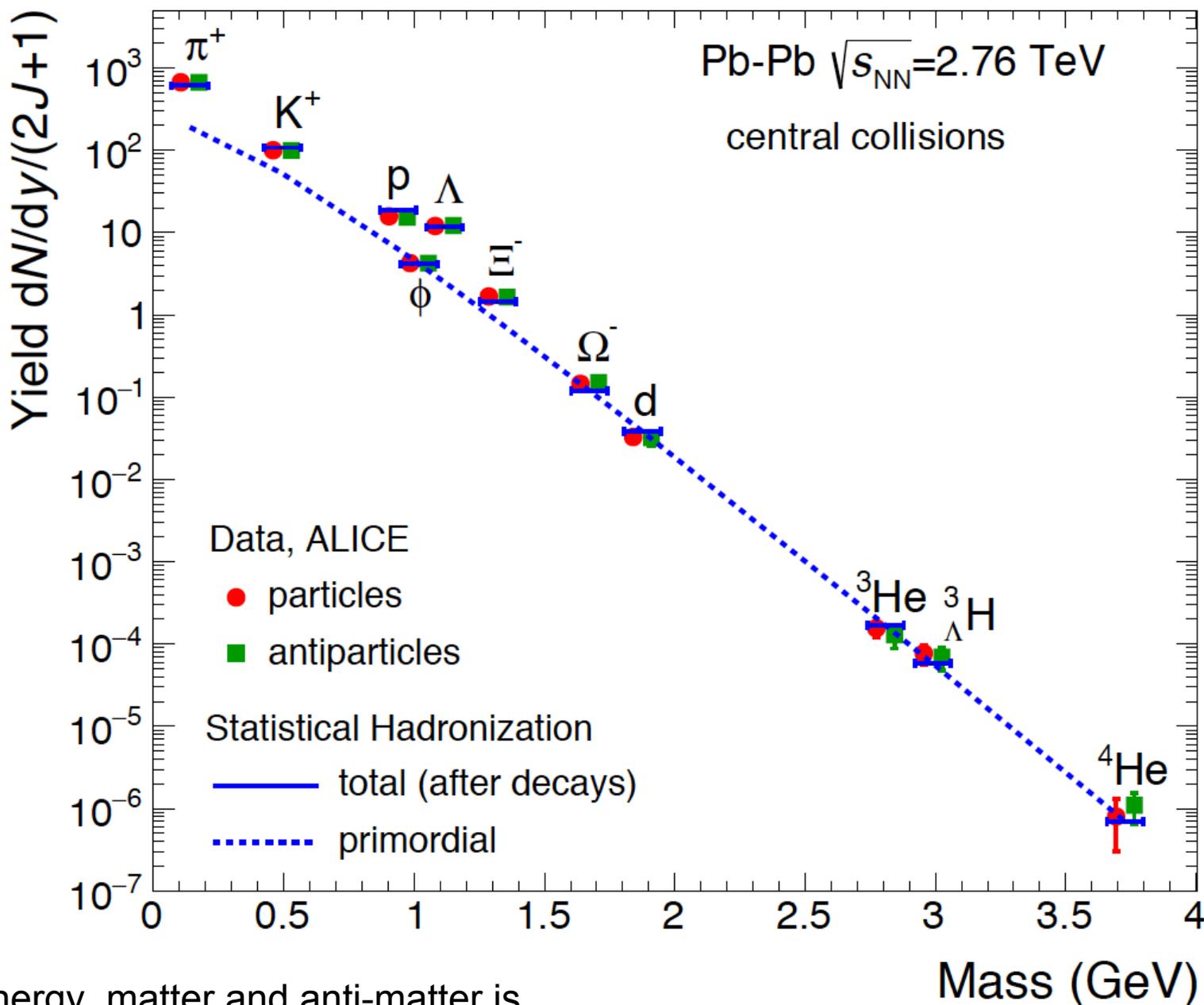
Contributions of three- and higher resonances and inelastic channels are taken into account with normalization with normalization to LQCD susceptibilities



$$\chi^2 = 19.7 \text{ per } 19 \text{ dof}$$

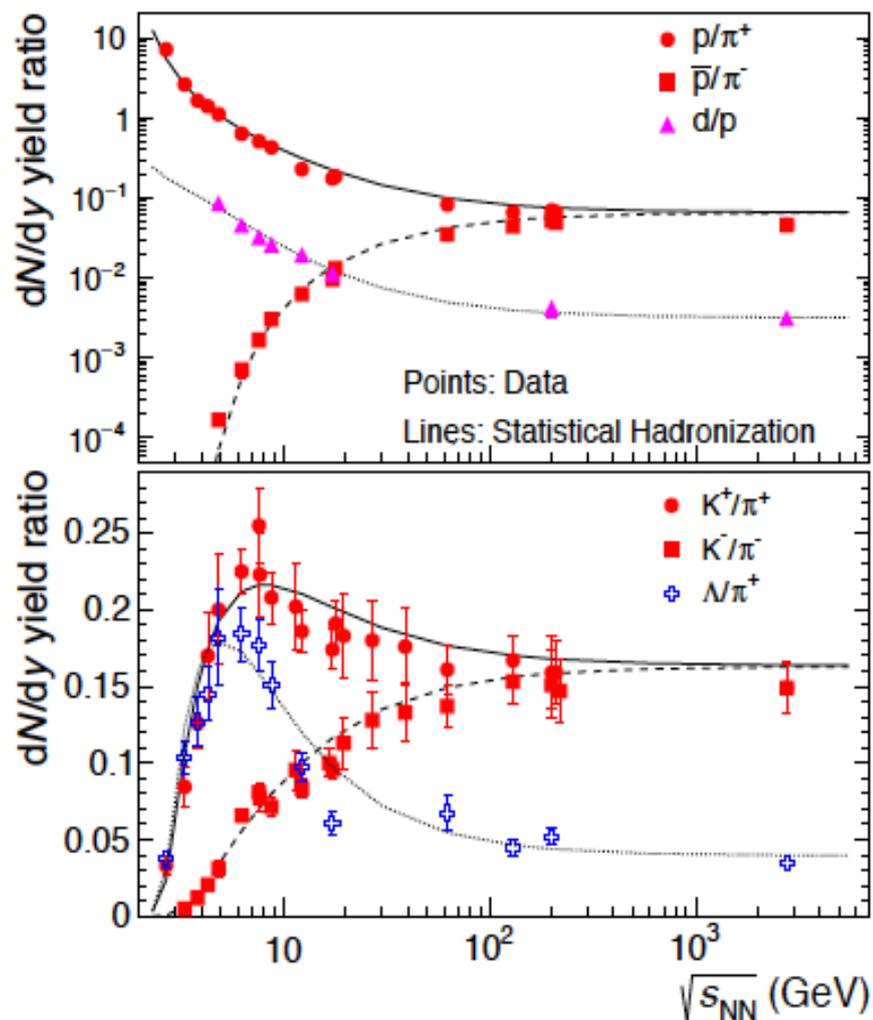
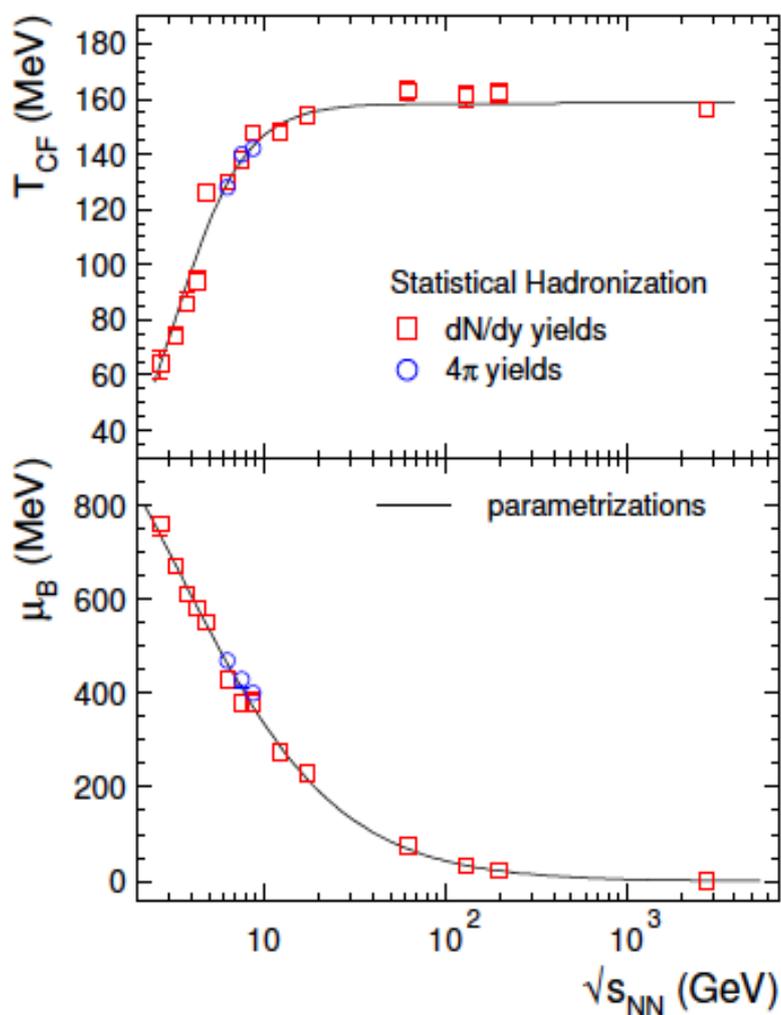
very good fit!

at LHC energy, production of (u,d,s) hadrons is governed  
by mass and quantum numbers only  
quark content does not matter



at LHC energy, matter and anti-matter is  
produced with equal yields

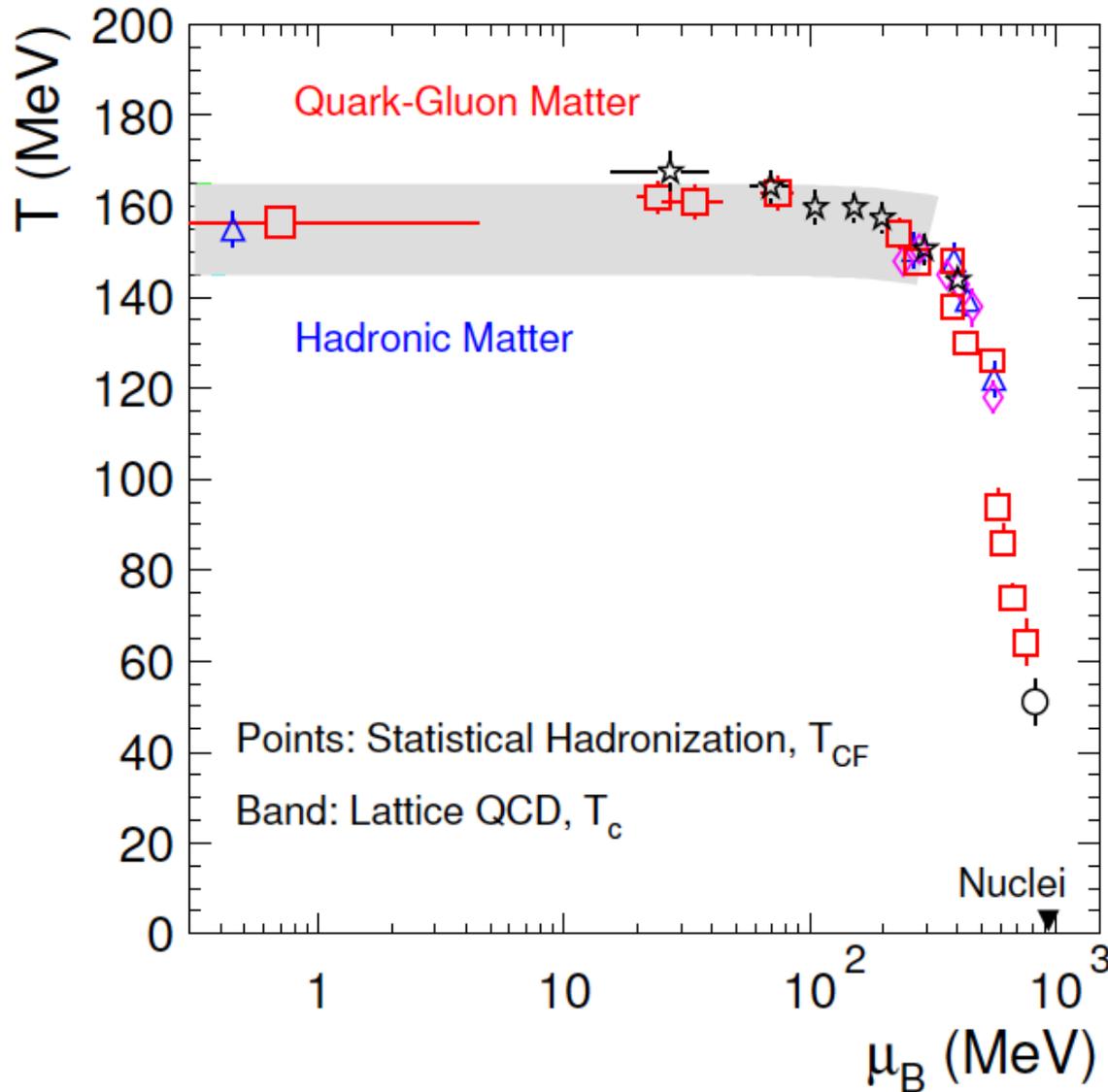
# energy dependence of hadron production described quantitatively



together with known energy dependence of charged hadron production in Pb-Pb collisions we can predict yield of all hadrons at all energies with < 10% accuracy

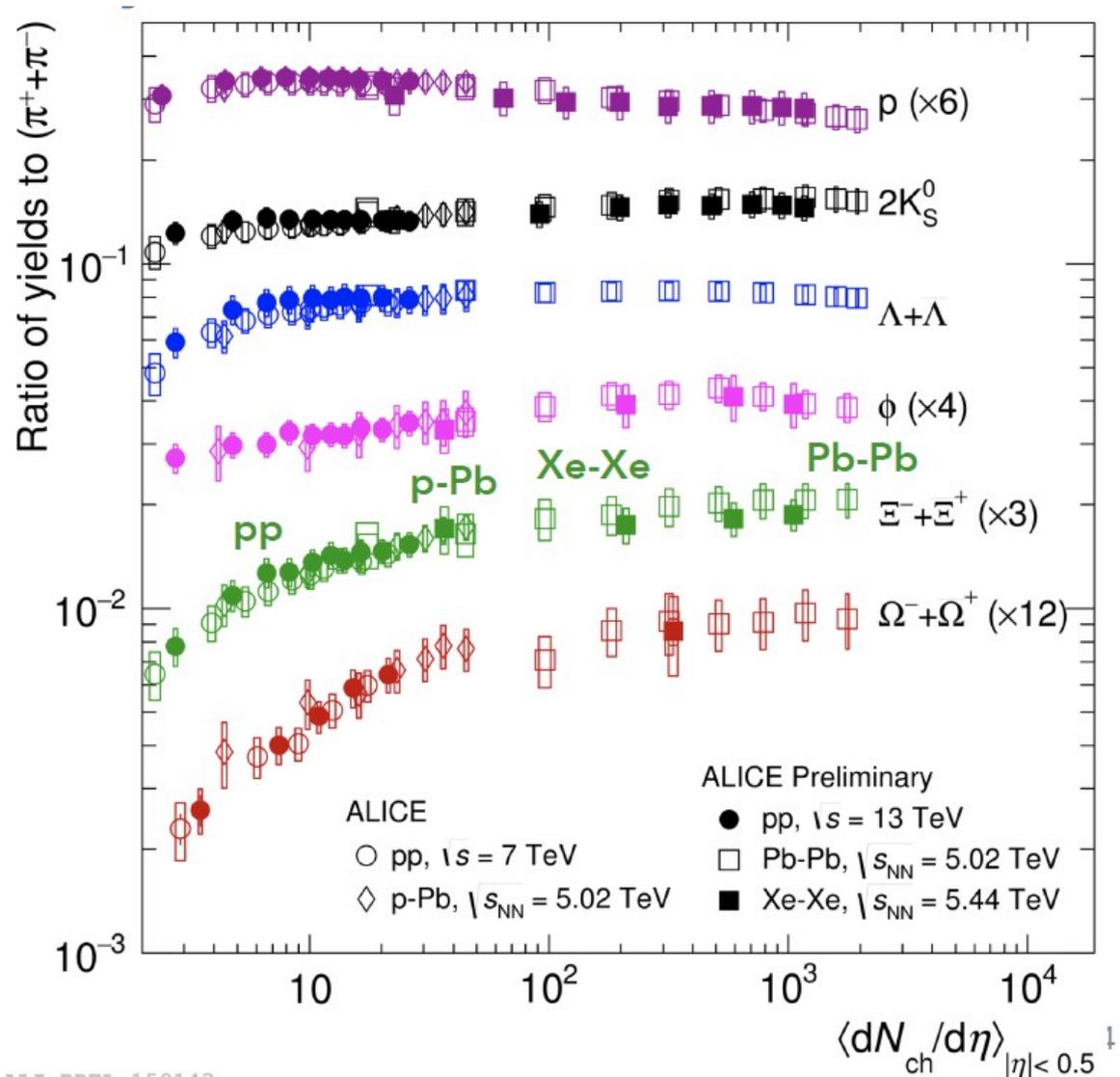
no new physics needed to describe  $K^+/\pi^+$  ratio including the 'horn'

# the QGP phase diagram, LQCD, and hadron production data



quantitative agreement of chemical freeze-out parameters with LQCD predictions for baryochemical potential  $< 300$  MeV

# from pp to Pb-Pb collisions: smooth evolution with system size



universal hadronization can be described with few parameters in addition to  $T$  and  $\mu_B$   
 transition from canonical to grand-canonical thermodynamics

# The Hypertriton

mass = 2990 MeV, binding energy = 2.3 MeV

Lambda sep. energy = 0.13 MeV

molecular structure: (p+n) + Lambda

2-body threshold: (p+p+n) + pi- =  ${}^3\text{He}$  + pi-

rms radius =  $(4 \text{ B.E. } M_{\text{red}})^{-1/2} = 10.3 \text{ fm} =$

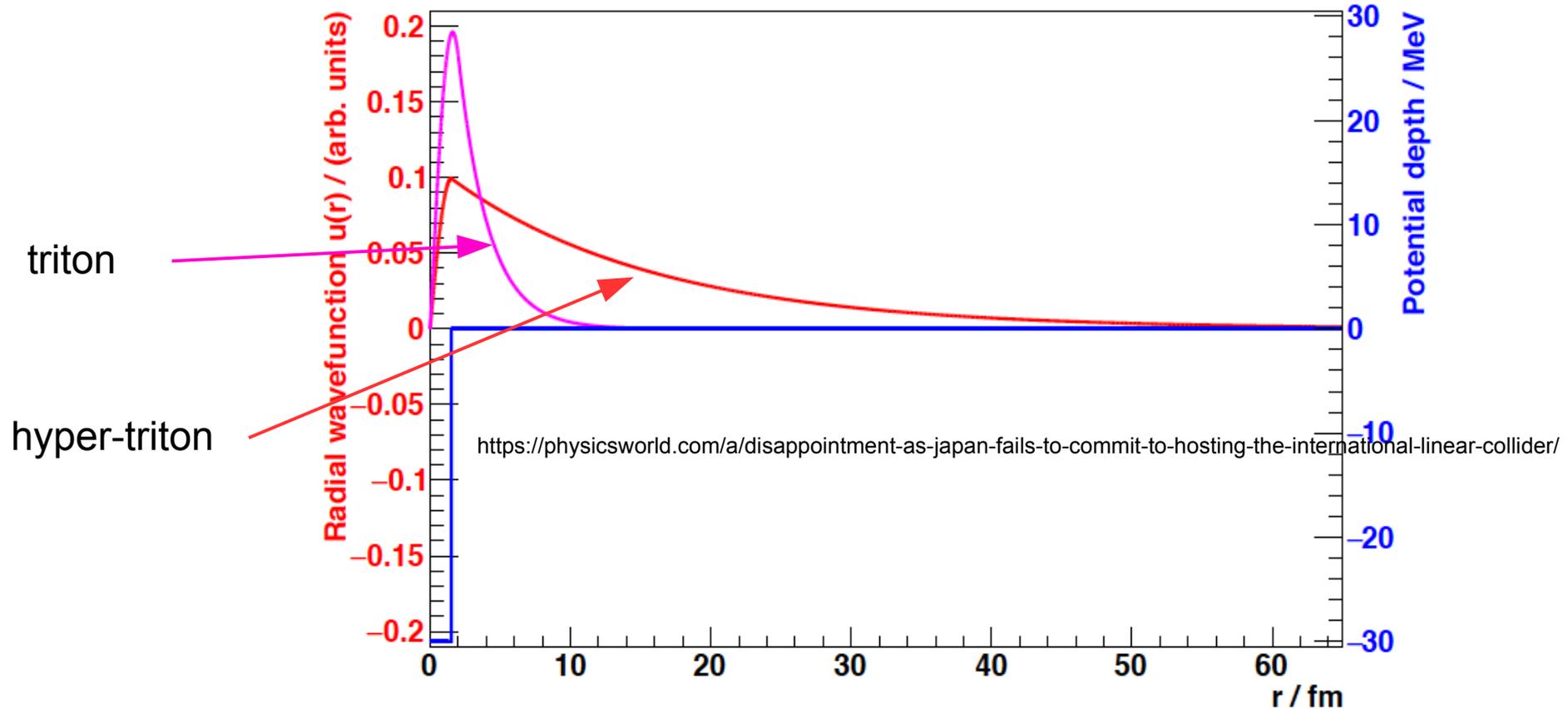
rms separation between d and Lambda

in that sense: hypertriton = (p n Lambda) =  
(d Lambda) is the ultimate halo state

yet production yield is fixed at 156 MeV temperature  
(about 1000 x Lambda separation energy.)

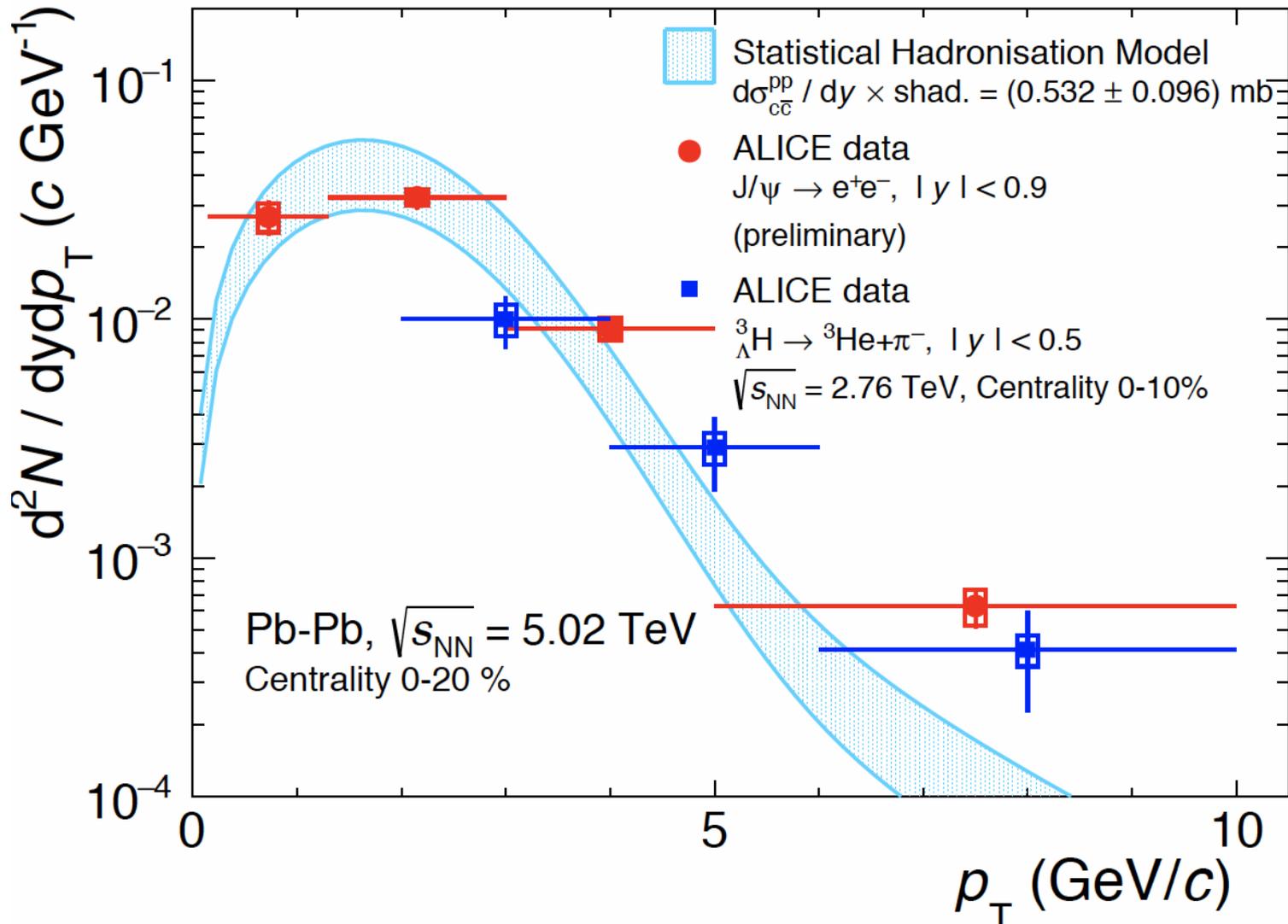
# wave function of the hyper-triton – schematic picture

figure by Benjamin Doenigus, August 2017



Wavefunction (red) of the hypertriton assuming a s-wave interaction for the bound state of a  $\Lambda$  and a deuteron. The root mean square value of the radius of this function is  $\sqrt{\langle r^2 \rangle} = 10.6$  fm. In blue the corresponding square well potential is shown. In addition, the magenta curve shows a "triton" like object using a similar calculation as the hypertriton, namely a deuteron and an added nucleon, resulting in a much narrower object as the hypertriton.

# J/psi and hyper-triton described with the same flow parameters in the statistical hadronization model



binding energies:  
 J/psi 600 MeV  
 hypertriton 2.2 MeV  
 Lambda S.E. 0.2 MeV

from review: hypernuclei and other loosely bound objects produced in nuclear collisions at the LHC, pbm and Benjamin Doenigus, to appear in Nucl. Phys. A, arXiv:1809.04681

**doorway state hypothesis:  
all nuclei and hyper-nuclei, penta-quark and X,Y,Z states  
are formed as virtual, compact multi-quark states at the  
phase boundary. Then slow time evolution into hadronic  
representation. Excitation energy about 20 MeV, time  
evolution about 10 fm/c**

Andronic, pbm, Redlich, Stachel, arXiv :1710.09425

**how can this be tested?**

precision measurement of spectra and flow pattern for light  
nuclei and hyper-nuclei, penta-quark and X,Y,Z states from pp  
via pPb to Pb-Pb

**a major new opportunity for ALICE Run3/4  
and beyond LS4 for X,Y,Z and penta-quark states**

**also new opportunities for  
GSI/FAIR and JINR/NICA  
experiments**

## summary

- statistical hadronization model is an effective tool to understand the phenomenology of hadron production in relativistic nuclear collisions from SIS to LHC energy with predictive power for future facilities
- deeply rooted in duality 'hadrons – quarks' near QCD phase boundary
- present precision is mostly limited by incomplete knowledge of hadron mass spectrum and related branching ratios for decays
- measurements from ALICE at the 5% accuracy level show deviations for protons, now quantitatively understood by using experimental pion-nucleon phase shifts
- yields of light nuclei and hyper-nuclei successfully predicted  
→ maybe produced as quark bags?
- coalescence approach not microscopic enough for loosely bound states

### key results:

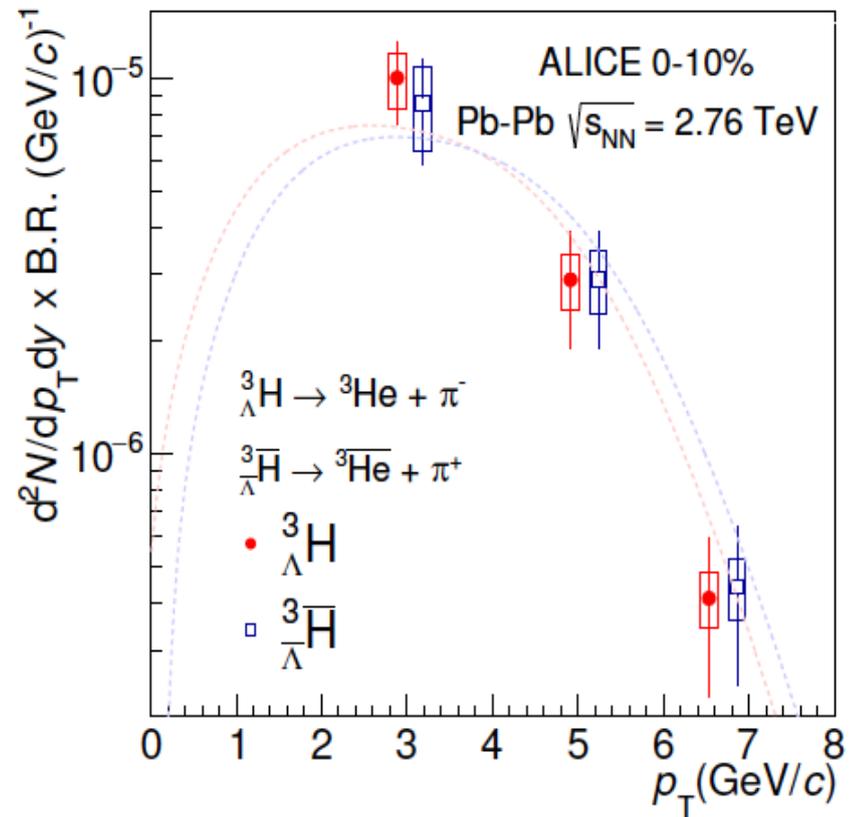
experimental location of QCD phase boundary for  $\mu_b < 300$  MeV:

$$T_c = 156 \pm 3 \text{ MeV}$$

new insight into hadronization

additional slides

# even hyper-triton flows with same common fluid velocity

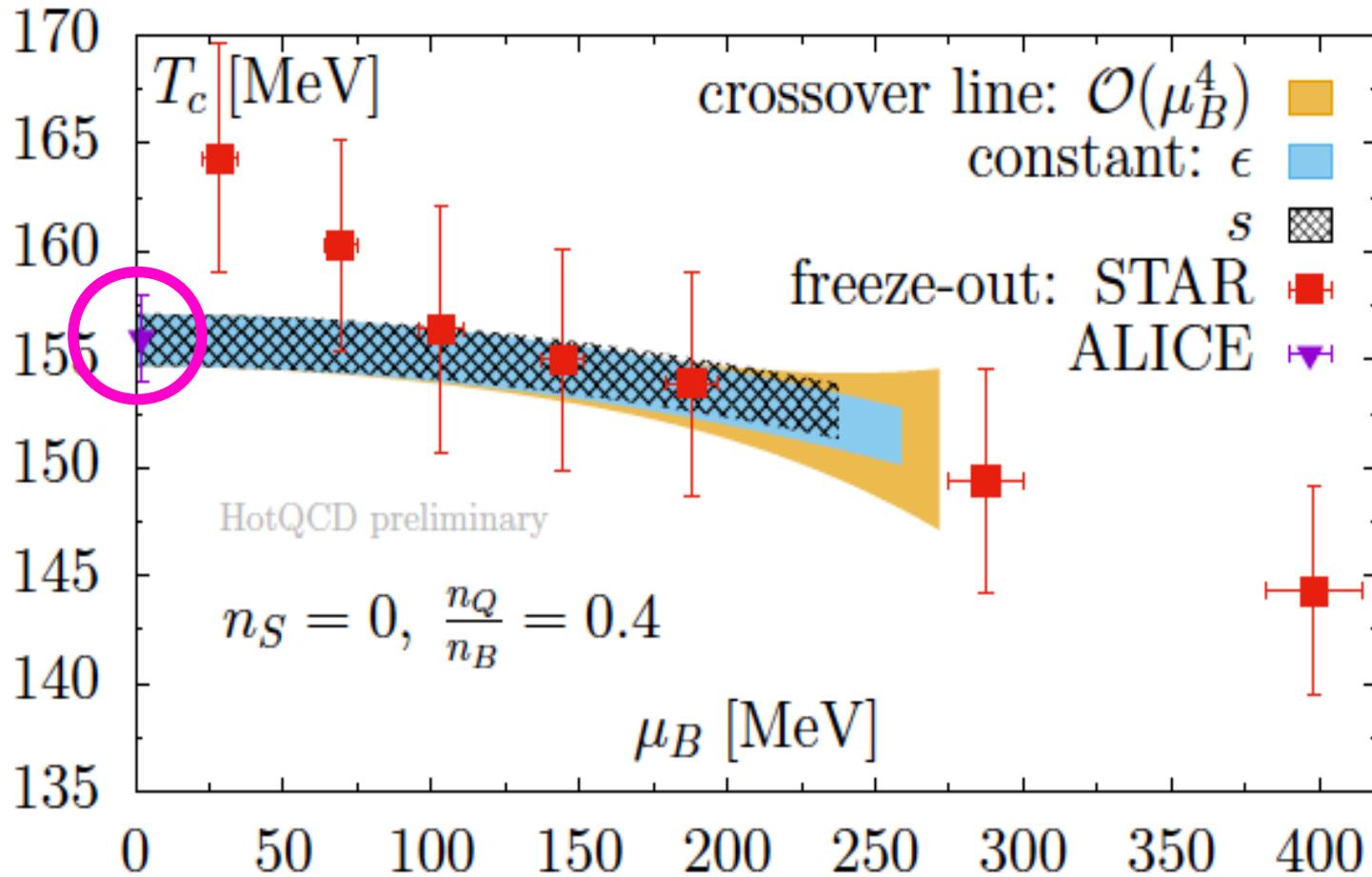


# chemical freeze-out and the chiral crossover line

ALICE point:  $156 \pm 1.5 \pm 3$  (sys) MeV, measured with TPC and Si vertex detector

STAR points: measured with TPC only, feeding from weak decays

lattice:  $156 \pm 1.5$  MeV



lattice: BNL-Bielefeld coll. 1807.05607

# a note on the chemical freeze-out temperature

$$T_{\text{chem}} = 156.5 \pm 1.5 \text{ MeV from fit to all particles}$$

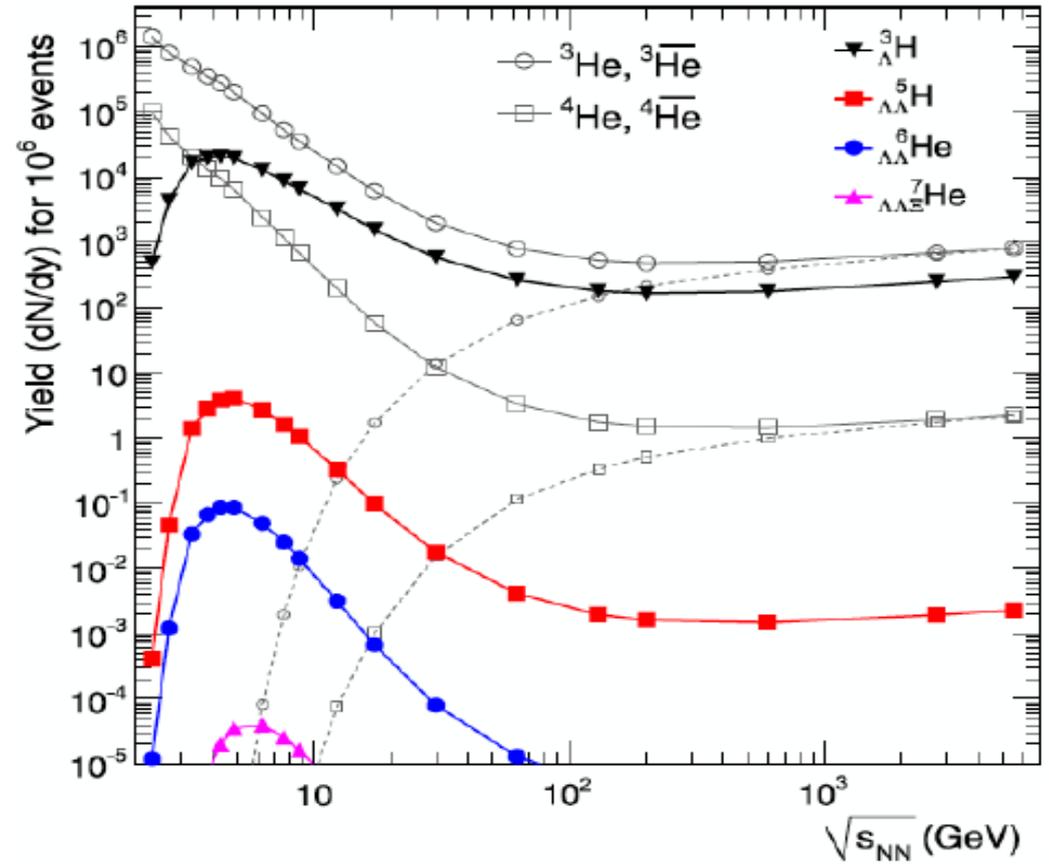
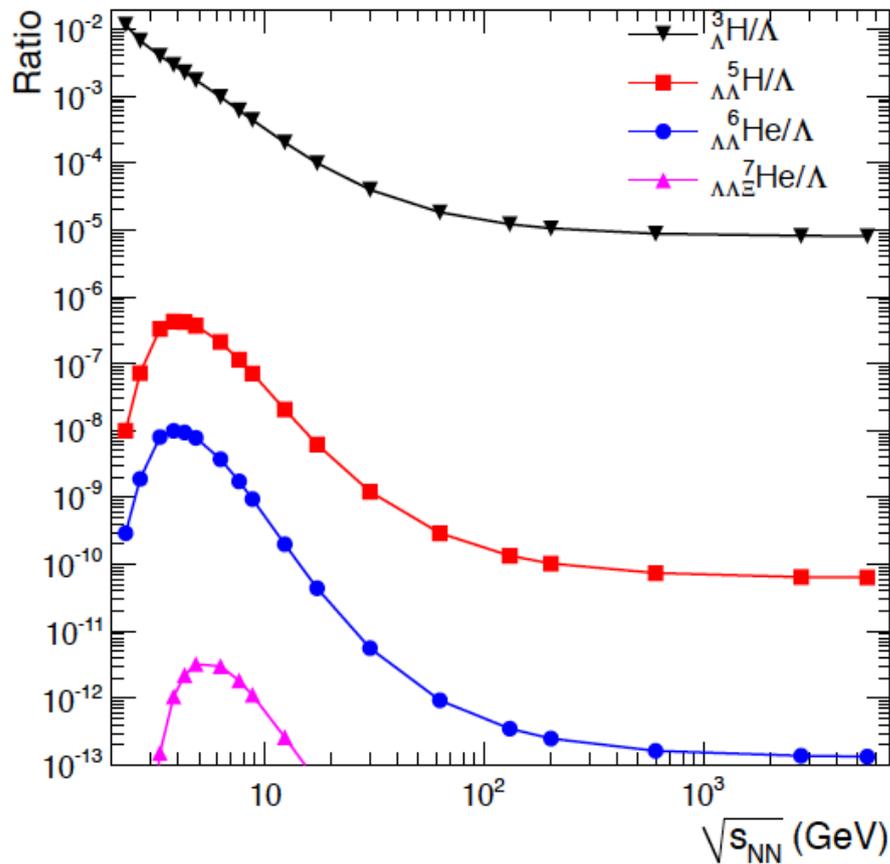
there is an additional uncertainty because of the poorly known hadronic mass spectrum for masses  $> 2 \text{ GeV}$

for d,  $^3\text{He}$ , hypertriton and alpha, there is very little feeding from heavier states and none from high mass states in the hadronic mass spectrum, for these particles the temperature  $T_{\text{nuc}}$  can be determined 'on the back of an envelope' :

$$T_{\text{nuc}} = 159 \pm 5 \text{ MeV, independent of hadronic mass spectrum}$$

# now loosely bound objects

exciting opportunities for the upcoming accelerator facilities  
NICA, FAIR/CBM, J-Parc



Andronic, pbm, Stachel, Stoecker  
Phys.Lett. B697 (2011) 203-207

# implementation

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to  $m=2$  GeV;  
our code: 555 species (including light nuclei, charm and bottom hadrons)

for resonances, the width is considered in calculations

$$\text{Minimize: } \chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$$

$N_i$  hadron yield,  $\sigma_i$  experimental uncertainty (stat.+syst.)

$$\Rightarrow (T, \mu_B, V)$$

canonical treatment whenever needed (small abundances)

[Braun-Munzinger and Stachel, PLB 490 (2000) 196]

[Andronic, Braun-Munzinger and Stachel, NPA 789 (2007) 334]

- ▶ Charm quarks are produced in initial hard scatterings ( $m_{c\bar{c}} \gg T_c$ ) and production can be described by pQCD ( $m_{c\bar{c}} \gg \Lambda_{\text{QCD}}$ )
- ▶ Charm quarks survive and *thermalise* in the QGP
- ▶ Full screening before  $T_{\text{CF}}$
- ▶ Charmonium is formed at phase boundary (together with other hadrons)
- ▶ Thermal model input ( $T_{\text{CF}}, \mu_b \rightarrow n_X^{\text{th}}$ )

$$N_{c\bar{c}}^{\text{dir}} = \underbrace{\frac{1}{2} g_c V \left( \sum_i n_{D_i}^{\text{th}} + n_{\Lambda_i}^{\text{th}} + \dots \right)}_{\text{Open charm}} + \underbrace{g_c^2 V \left( \sum_i n_{\psi_i}^{\text{th}} + n_{\chi_i}^{\text{th}} + \dots \right)}_{\text{Charmonia}}$$

- ▶ Canonical correction is applied to  $n_{\text{oc}}^{\text{th}}$
- ▶ Outcome  $N_{J/\psi}, N_D, \dots$

going beyond the non-interacting HRG – next 3 slides from K. Redlich, QM18

## HRG in the S-MATRIX APPROACH

Pressure of an interacting,  $a+b \Leftrightarrow a+b$ , hadron gas in an equilibrium

$$P(T) \approx P_a^{id} + P_b^{id} + P_{ab}^{int}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{int} = \sum_{I,j} \int_{m_{th}}^{\infty} dM B_j^I(M) P^{id}(T, M)$$

$$B_j^I(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^I(M)$$

R. Dashen, S. K. Ma and H. J. Bernstein,  
Phys. Rev. 187, 345 (1969)

R. Venugopalan, and M. Prakash,  
Nucl. Phys. A 546 (1992) 718.

W. Weinhold, and B. Friman,  
Phys. Lett. B 433, 236 (1998).

Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

*Effective weight function*

*Scattering phase shift*

- Interactions driven by narrow resonance of mass  $M_R$

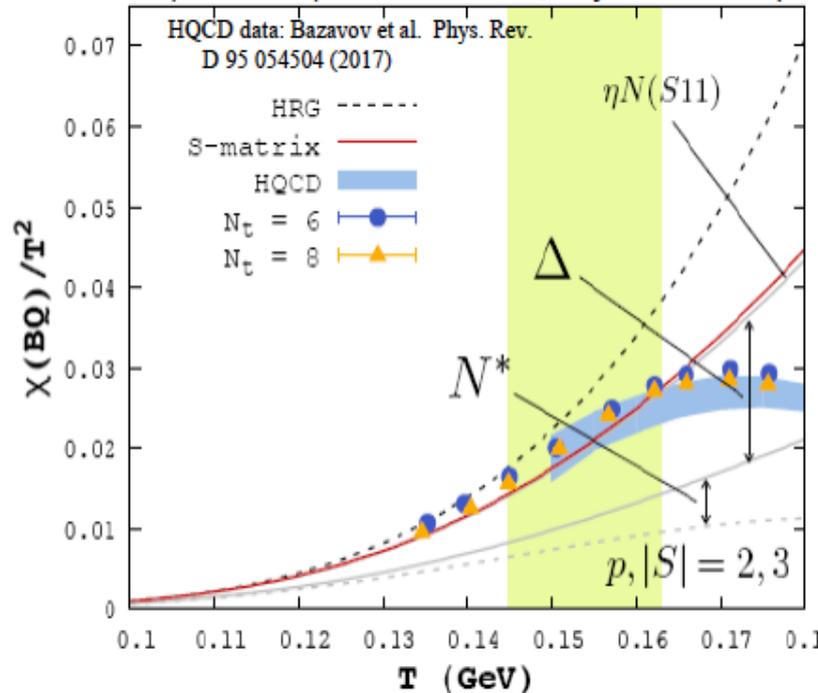
$$B(M) = \delta(M^2 - M_R^2) \Rightarrow P^{int} = P^{id}(T, M_R) \Rightarrow HRG$$

- For non-resonance interactions or for broad resonances the HRG is too crude approximation and  $P^{int}(T)$  should be linked to the phase shifts

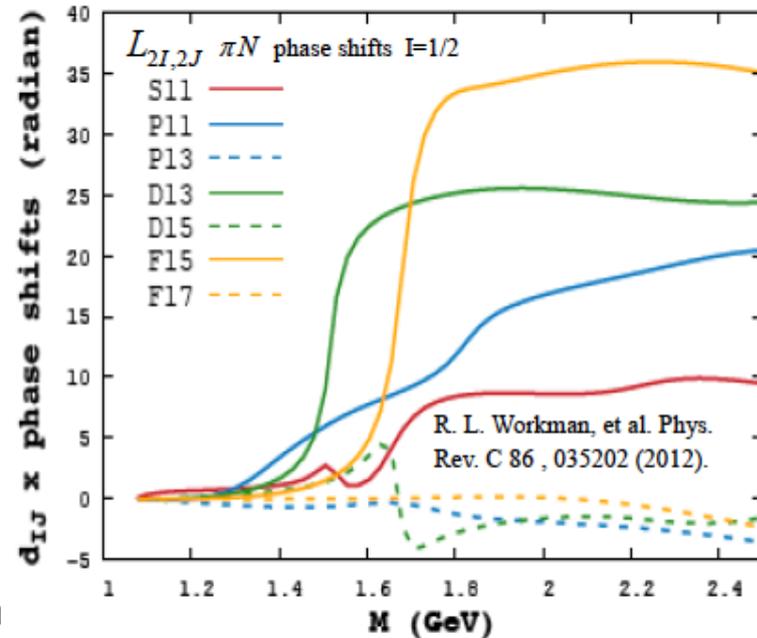
considering all pion-nucleon phase shifts with isospin 1/2 and 3/2

## Probing non-strange baryon sector in $\pi N$ - system

Pok Man Lo, B. Friman, C. Sasaki & K.R., Phys.Lett. B778 (2018)



$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|) / 2$$

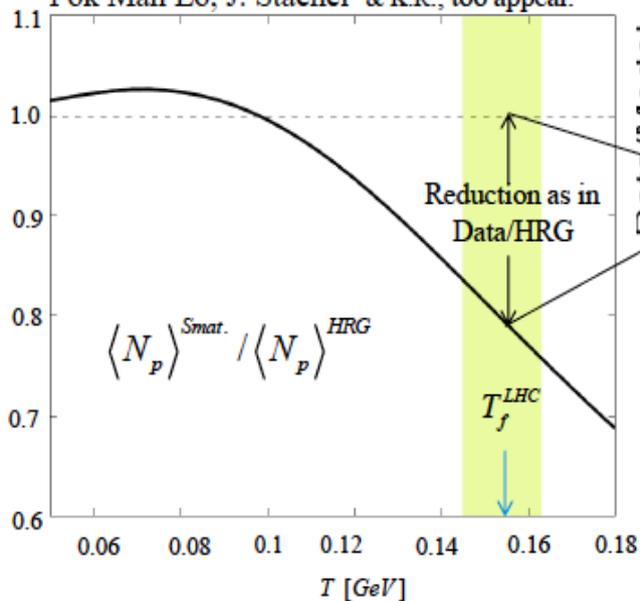


$$\Delta\chi_{BQ} \approx \sum_{I_z, j, B} d_j BQ \int dM \int d^3p \frac{1}{T} \frac{d\delta_j^I}{dM} \times e^{-\beta\sqrt{p^2+M^2}} (1 + e^{-\beta\sqrt{p^2+M^2}})^{-2}$$

- Considering contributions of all  $\pi N$   $\delta_j^{I=(1/2), (3/2)}$  ( $N^*$ ,  $\Delta^*$  resonances) to  $\chi_{BQ}$  within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover  $0.15 < T < 0.16$  GeV

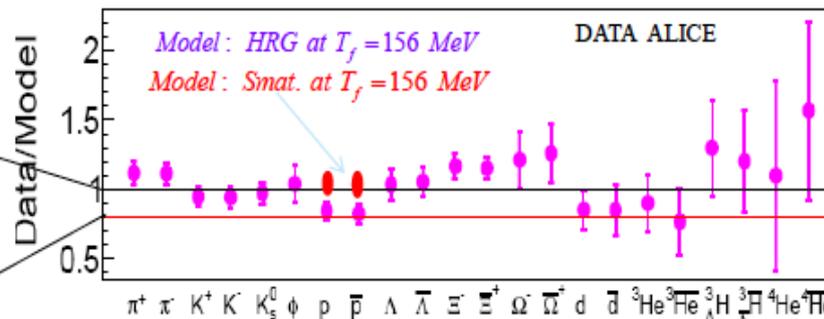
# Phenomenological consequences: proton production yields

A. Andronic, P. Braun-Munzinger, B. Friman, Pok Man Lo, J. Stachel & K.R., too appear.



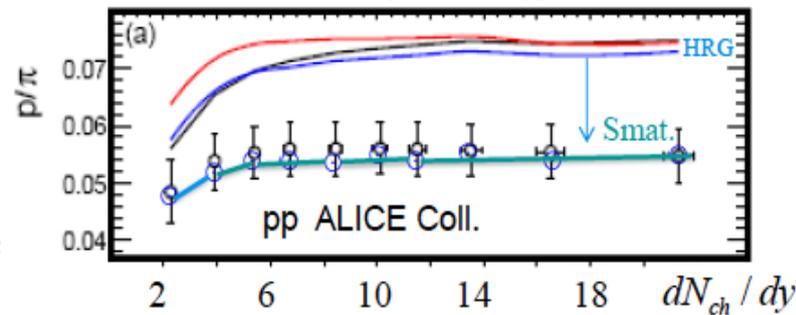
- Yields of protons in the S-matrix is suppressed relative to HRG. For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018) P. Huovinen, poster QM2018

HRG: A. Andronic, P. Braun-Munzinger, J. Stachel & K.R.



- Yields of protons in AA collisions at LHC is consistent with S-matrix result within  $1\sigma$

HRG: N. Sharma, J. Cleymans, B. Hippolite, arXiv: 1803.05409



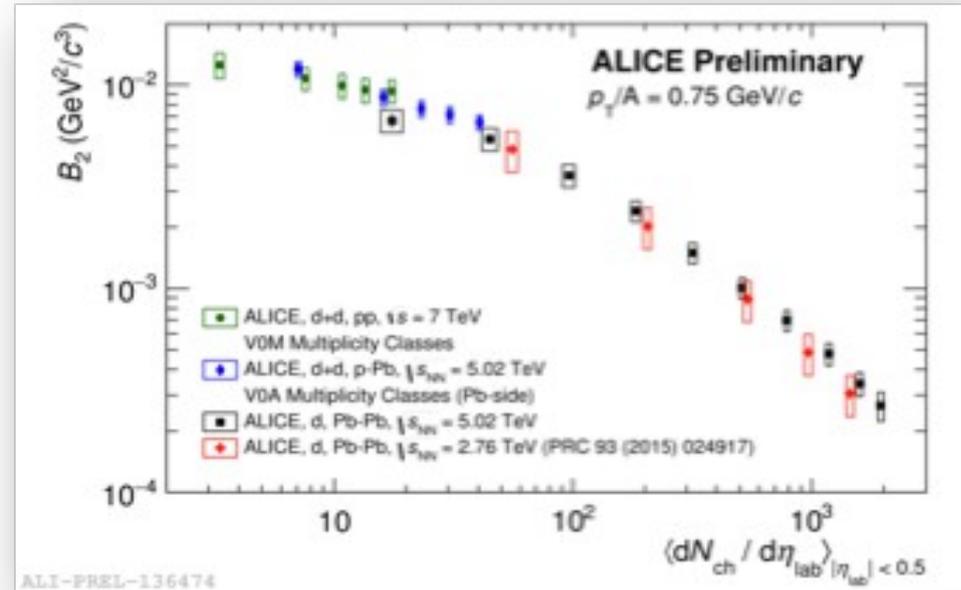
- S-matrix results well consistent with pp data

points a way to explain 'proton puzzle', new description to appear soon

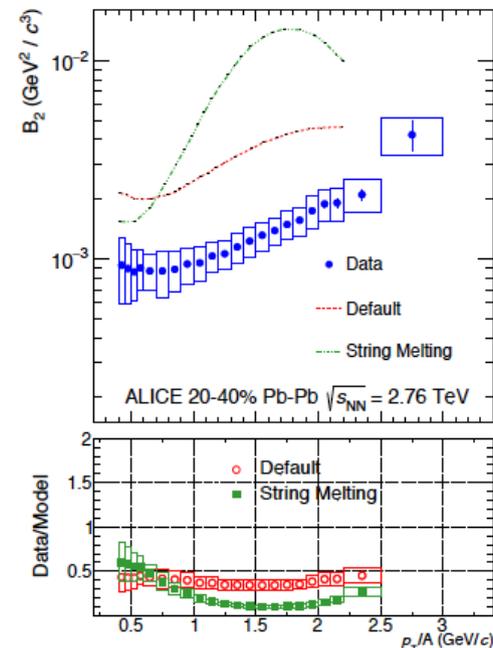
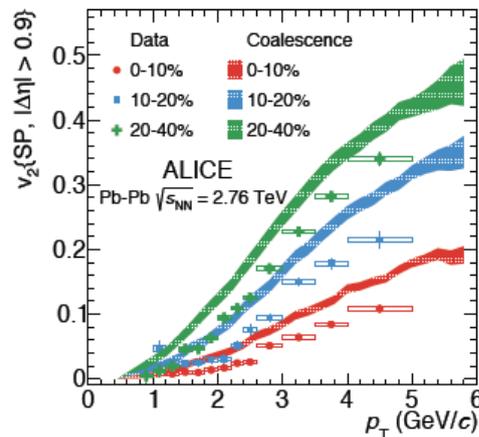
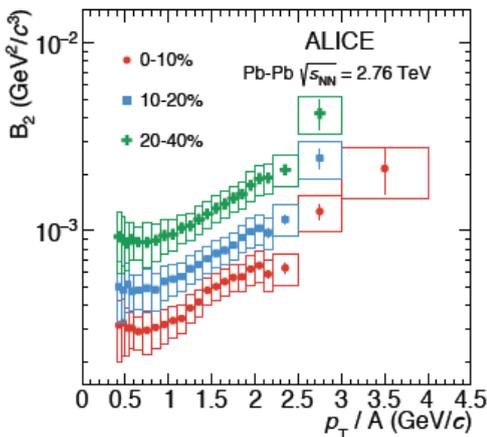
# is coalescence approach an alternative?

$$E_i \frac{d^3 N_i}{dp_i^3} = B_A \left( E_p \frac{d^3 N_p}{dp_p^3} \right)^A \quad B_A = \left( \frac{4\pi}{3} p_0^3 \right)^{A-1} \frac{M}{m^A}$$

centrality and  $p_T$  dependence of coalescence parameter not understood and not well reproduced by models such as AMPT



ALICE: arXiv:1707.07304



# coalescence approach, general considerations for loosely bound states

- production yields of loosely bound states is entirely determined by mass, quantum numbers and fireball temperature.
- hyper-triton and  $^3\text{He}$  have very different wave functions but essentially equal production yields.
- energy conservation needs to be taken into account when forming objects with baryon number  $A$  from  $A$  baryons
- coalescence of off-shell nucleons does not help as density must be  $\ll$  nuclear matter density, see below
- delicate balance between formation and destruction; maximum momentum transfer onto hyper-triton before it breaks up:  $\Delta Q_{\text{max}} < 20 \text{ MeV}/c$ , typical pion momentum  $p_{\text{pi}} = 250 \text{ MeV}/c$ , typical hadronic momentum transfer  $> 100 \text{ MeV}/c$ .
- hyper-triton interaction cross section with pions or nucleons at thermal freeze-out is of order  $\sigma > 70 \text{ fm}^2$ . For the majority of hyper-tritons to survive, the mfp  $\lambda$  has to exceed  $15 \text{ fm} \rightarrow$  density of fireball at formation of hyper-triton  $n < 1/(\lambda \sigma) = 0.001/\text{fm}^3$ . Inconsistent with formation at kinetic freeze-out, where  $n \approx 0.05/\text{fm}^3$ .

# is large size of light nuclei and hypernuclei an issue for statistical hadronization model?

note: in thermal approach, the only scale is temperature  $T$   
at LHC energy and below,  $T < 160$  MeV

at such a scale, momentum transfer  $q=T$ , form factors of hadrons are sampled  
at  $q^2 = T^2$

this implies that sizes of hadrons  $< 2$  fm cannot be resolved

since  $G(q) \sim 1 - q^2 R^2 / 6$

and since all (rms) radii for nuclei with  $A = 2, 3,$  and  $4$  are smaller than  $2$  fm,  
the correction due to the finite size of nuclei will not exceed  $35\%$

the actual change from this on thermal model results should be much less as  
only the relative change between normal hadrons and light nuclei matters, the  
overall change only leads to a volume correction, so the correction for nuclei is  
estimated to be less than  $25\%$

but hyper-triton has much larger radius  $> 5$  fm?

measured yield of hyper-triton and  $^3\text{He}$  is well compatible with thermal  
prediction, even though wave function is very different – any wave function  
correction must be small

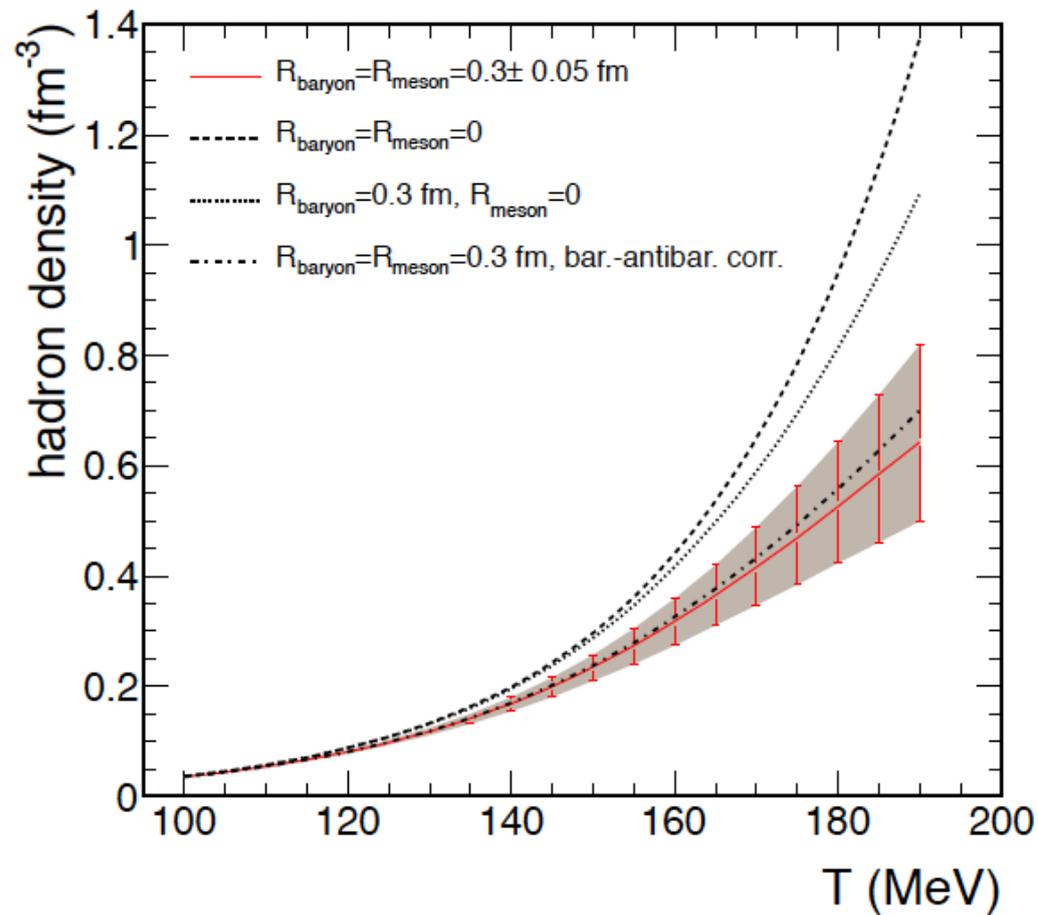
the agreement of the baryon number  $3$  states is also big problem for  
coalescence model

see also the detailed analysis by Francesca Bellini and Alexander Kalweit,  
arXiv:1807.05894,  
Benjamin Doenigus and Nicole Loehner, GSI-EMMI meeting, Feb. 2018

**How can 'thermal production near the phase boundary' i.e. at  $T \sim 155$  MeV be reconciled with binding energies  $< 5$  MeV and large break-up cross sections?**

**a possible way out**

# Hadron resonance gas and interactions



for  $T < 165 \text{ MeV}$ , the details of the interactions don't matter and the 'low density approximation' is a good assumption

# Quark Model Spectroscopy

Why does the quark model work so well?

Why do M and B body plans dominate?

Why don't multibaryons make one big bag?

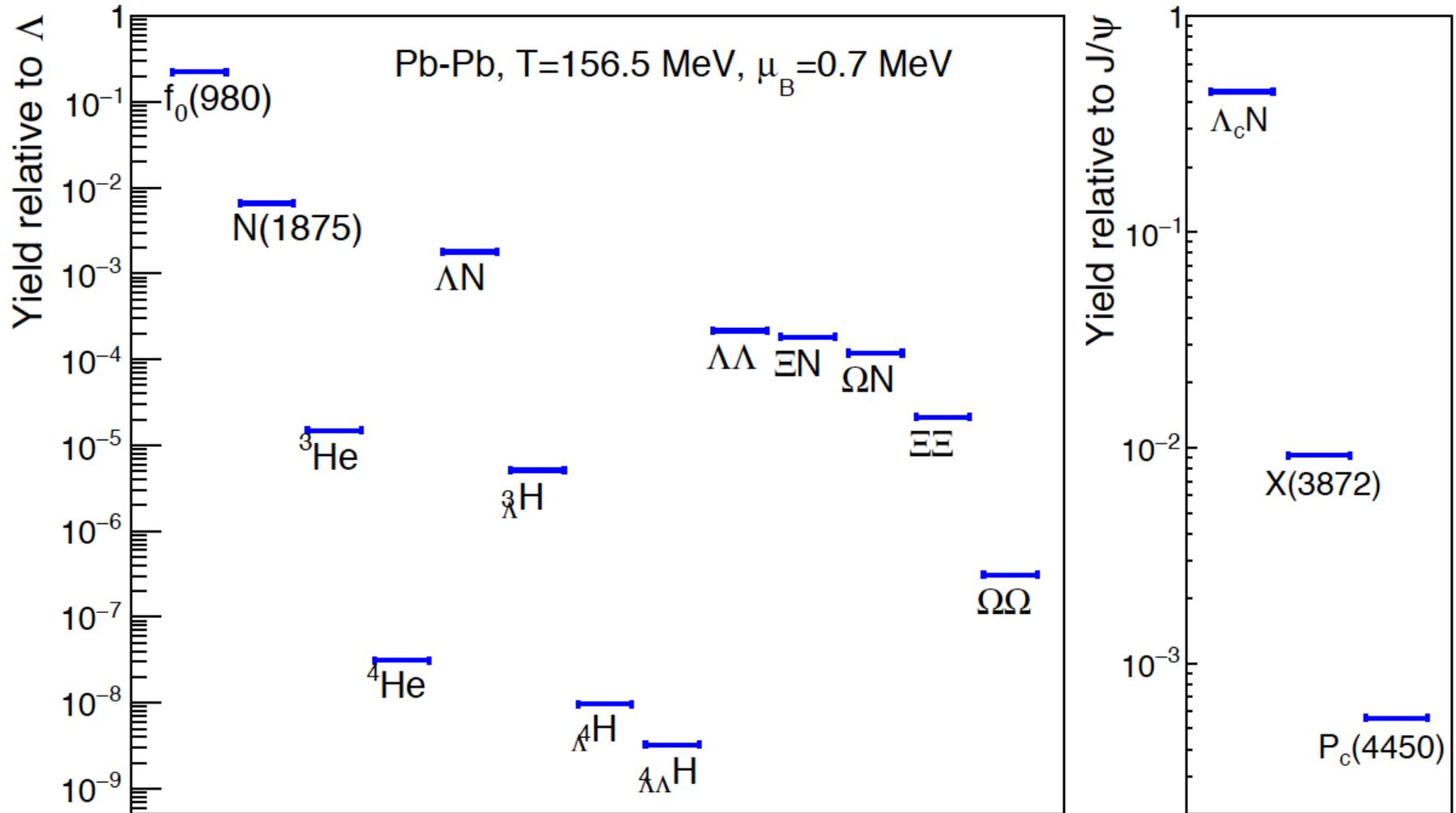
Frank Wilczek, QM2014 introductory talk

see also the recent review:

Marek Karliner, Jonathan L. Rosner, Tomasz Skwarnicki, arXiv:1711.10626

# thermal production yields of exotic states in central Pb-Pb collisions at 5 TeV/u

Andronic, pbm, Koehler, Redlich, Stachel  
preprint in preparation



# example: X(3872)

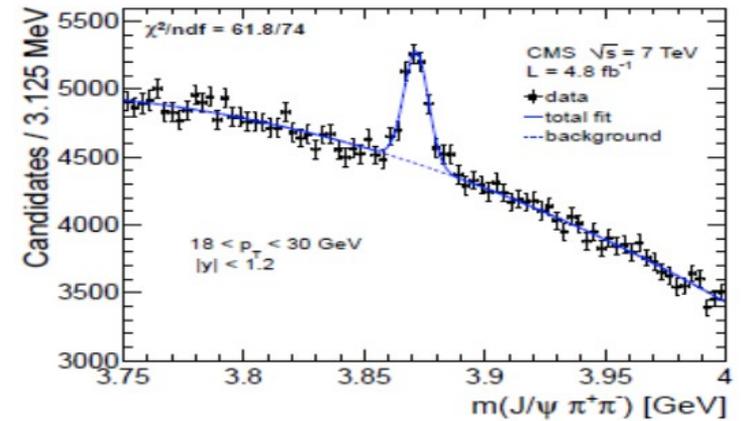
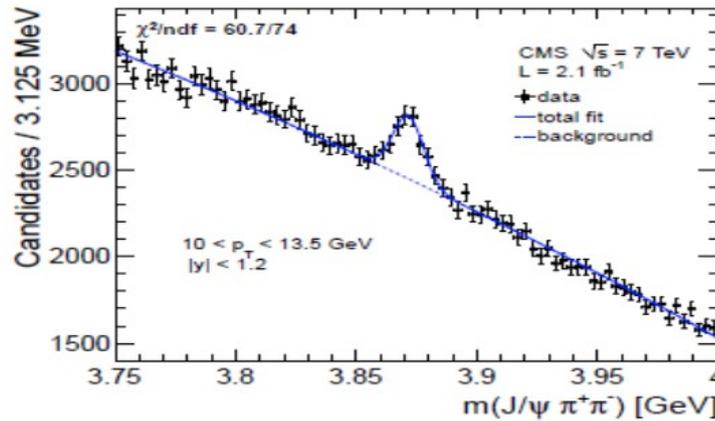
X(3872)

- 2003 -



$$M = 3872.0 \pm 0.6 \pm 0.5 \text{ MeV}$$

- 2013 -



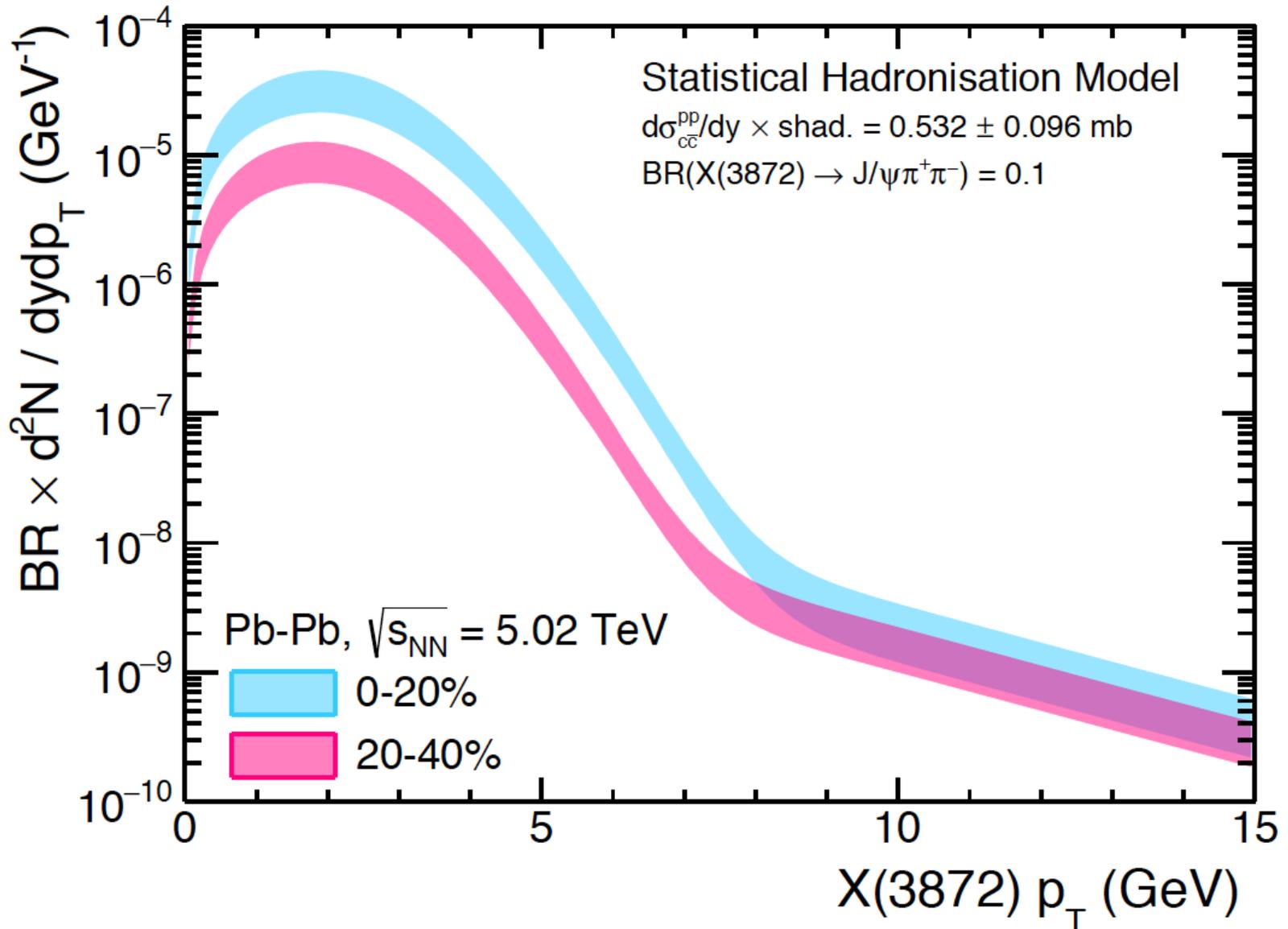
**X(3872)**

$$I^G(J^{PC}) = 0^+(1^{++})$$

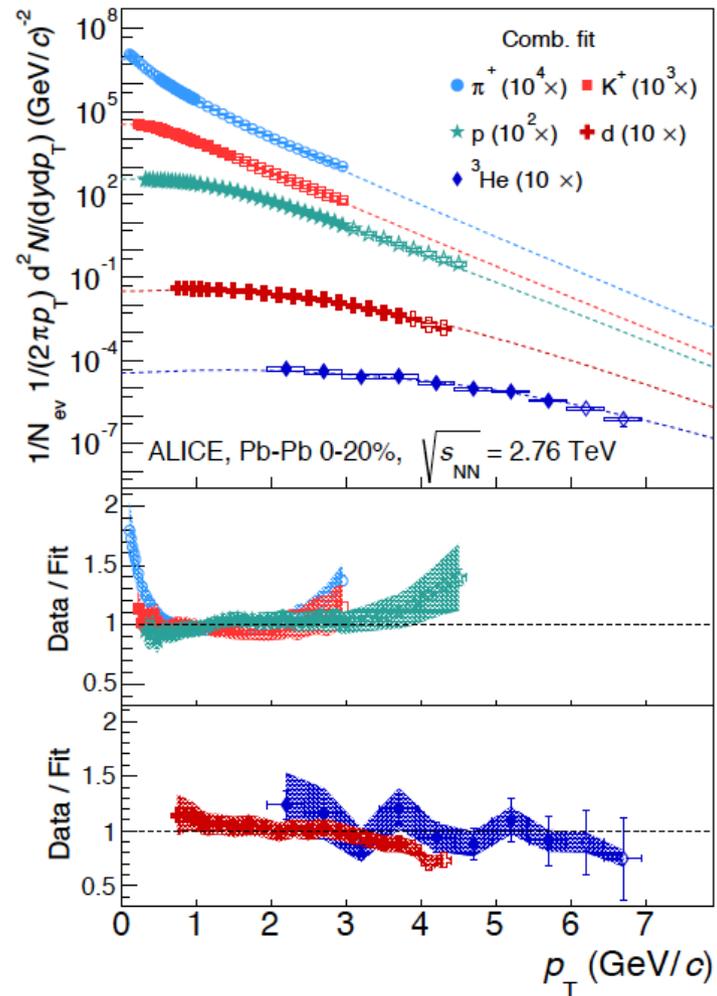
Mass  $m = 3871.69 \pm 0.17 \text{ MeV}$   
 $m_{X(3872)} - m_{J/\psi} = 775 \pm 4 \text{ MeV}$   
 $m_{X(3872)} - m_{\psi(2S)}$   
 Full width  $\Gamma < 1.2 \text{ MeV}$ , CL = 90%



# transverse momentum spectrum for X(3872) in the statistical hadronization model Pb-Pb collisions at 5 TeV/u



# light nuclei flow with same fluid velocity as pions, kaons, and protons



# outlook

ALICE is currently upgraded:

GEM based read-out chambers for the TPC  
new inner tracker with ultra-thin Si layers  
continuous read of (all) subdetectors

**increase of data rates by factor 100**

focus on rare objects, exotic quarkonia, low mass lepton pairs and low  $p_t$  photons to address a number of fundamental questions and issues such as:

- are there colorless bound states in a deconfined medium?
- are complex, light nuclei and exotic charmonia (X,Y,Z) produced as compact multi-quark bags?
- can fluctuation measurements shed light on critical behavior near the phase boundary?

deciphering QCD in the strongly coupled regime

# duality between hadrons and quarks/gluons (I)

Z: full QCD partition function

all thermodynamic quantities derive from QCD partition functions

for the pressure we get:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu)$$

comparison of trace anomaly from LQCD  
Phys.Rev. D90 (2014) 094503  
HOTQCD coll.

with hadron resonance gas prediction  
(solid line)

LQCD: full dynamical quarks with realistic  
pion mass

