Lecture 2

Glauber Model

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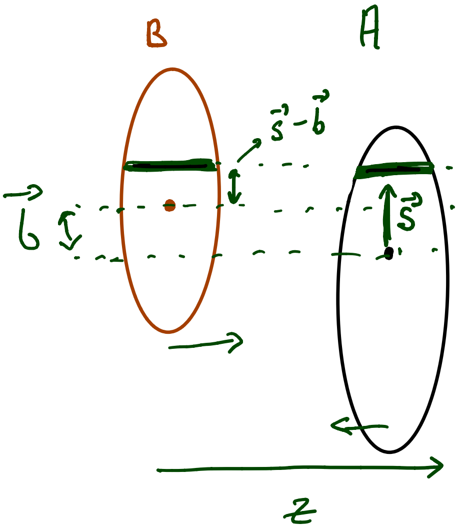
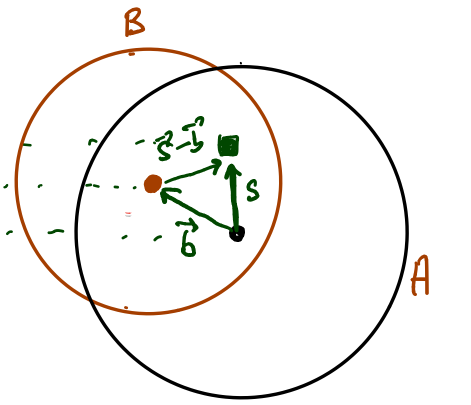
Glauber Model



side view



beam line vew

Nuclear density function.

- Woods-Saxon density

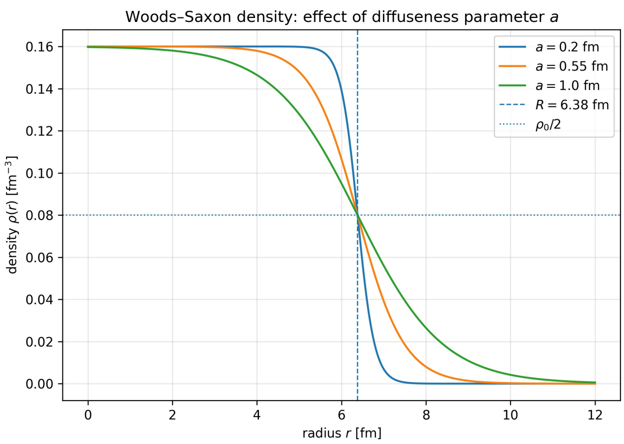
$$\rho_A(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$\rho_A(r) = \frac{dN}{dV}$; number of nucleons per unit volume

$a \sim 0.5$ fm $\to$ surface diffuseness

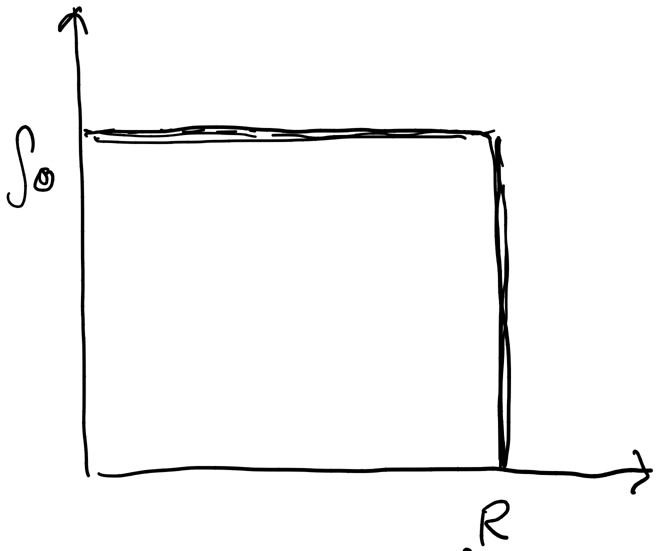
for $r=R$; $\exp\left[\frac{r-R}{a}\right] = 1$

$$\boxed{\rho_A(r=R) = \frac{\rho_0}{2}}$$



- Sharp-shere approx.

$$\rho_A(r) = \begin{cases} \rho_0, & r \leq R \\ 0, & r > R \end{cases}$$



Heaviside step function

$$\Theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & 1 < 0 \end{cases}$$

$$\rho_A(r) = \rho_0 \, \Theta(R-r)$$

ρ_0 - is defined from normalization: $A = \int d^3r\, \rho_A(r)$

$$A = \rho_0 \int d^3r \frac{1}{1+\exp\left(\frac{r-R}{a}\right)}$$

$$\boxed{d^3r = 4\pi r^2 dr}$$

$$A = \rho_0 \int d^3r \qquad r \leq R$$

$$A = 4\pi \rho_0 \int r^2 dr$$

$$A = \frac{4}{3}\pi r^3 \cdot \rho_0$$

$$\boxed{\rho_0 = \frac{3A}{4\pi r^3}}$$

Example: $R_{Au} = 6.38$ fm, $a_{Au} = 0.54$

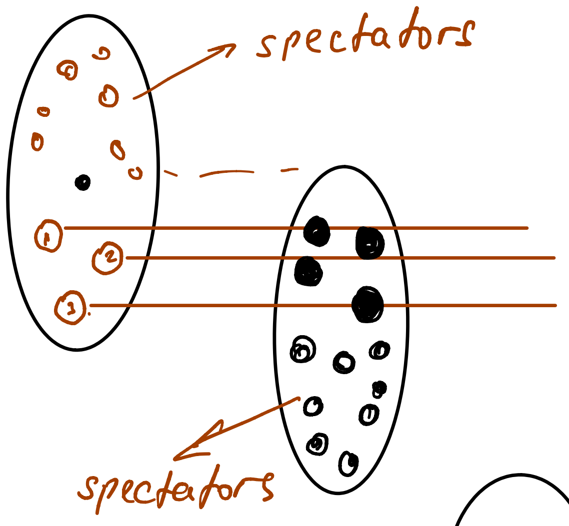
Woods-Saxon: $\rho_0^{W-S} = 0.170$

Hard-sphere: $\rho_0 = 0.181$

ρ_0^{W-S} - is smaller because of the diffuce surface

Basic dictionary

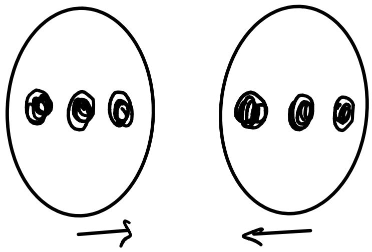
- Spectators: Nucleons which do not meet any other nucleon on their way
- Wounded nucleons N_W: Nucleons which underwent at least one inelastic collision
- Participants N_{part}: Nucleons which in addition collided with the produced particles (can be computed using e.g. generators)
- Binary collisions, N_{coll}: total number of binary collisions.



$N_W^1 = 1 + 2 = 3, \quad N_{coll}^1 = 2$

$N_W^2 = 1 + 1 = 2, \quad N_{coll}^2 = 2$

$$\underbrace{N_W^3 = 1+1 = 2}_{N_W = 6}, \quad \underbrace{N_{coll}^2 = 1}_{N_{coll} = 5}$$



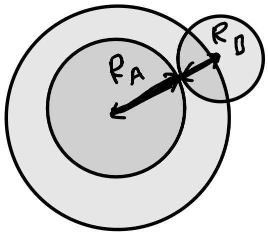
$N_W = 6, \quad N_{coll} = 9$

Black disc (pure geometry)

↓
next page

Black disk (pure geometry)

Nucleus: a hard sphere with a uniform density of nucleons



$$R_A \approx r_0 A^{1/3} \qquad r_0 \approx 1.12 \text{ fm}$$

More precise parametrisation

$$R_A \approx 1.12 A^{1/3} - 0.86 A^{-1/3}$$

$$R_{Au} \approx 6.38 \text{ fm} \; ; \quad R_{Pb} \approx 6.62 \text{ fm}$$

$$P_{inel}(b) = \begin{cases} 1, & b < 2R \\ 0, & b > 2R \end{cases} \qquad A = \int d^3r \, \rho_A(r) = \frac{4}{3}\pi R^3 \rho_0$$

$$\rho_0 = \frac{3A}{4\pi R^3}$$

$$\sigma_{inel}^{A+B} \approx \sigma_{geo} = \pi (R_A + R_B)^2$$

$$n\% = \frac{\pi b_n^2}{\pi (R_A + R_b)^2} \qquad R_A = R_B = R$$

$$\frac{n}{100} = \frac{b_n^2}{4R^2} \qquad b_n^2 = 4R^2 \cdot \frac{n}{100} \qquad b_n = 2R \cdot \sqrt{\frac{n}{100}}$$

$$0 - n\% \Rightarrow 0 \le b \le 2R\sqrt{\frac{n}{100}}$$

$n_1 - n_2\,\%$

$$2R\sqrt{\frac{n_1}{100}} \le b \le 2R\sqrt{\frac{n_2}{100}}$$

Example: $0-5\%$: $b = 2 \cdot 6.38 \cdot \sqrt{0.05} = 2.85$ fm

$R = R_{Au} = 6.38$ fm

$0-10\%$: $0 \le b \le 2 \cdot 6.38 \sqrt{0.1} = 4.03$ fm

$0-20\%$: $0 \le b \le 2 \cdot 6.38 \sqrt{0.2} = 5.71$ fm

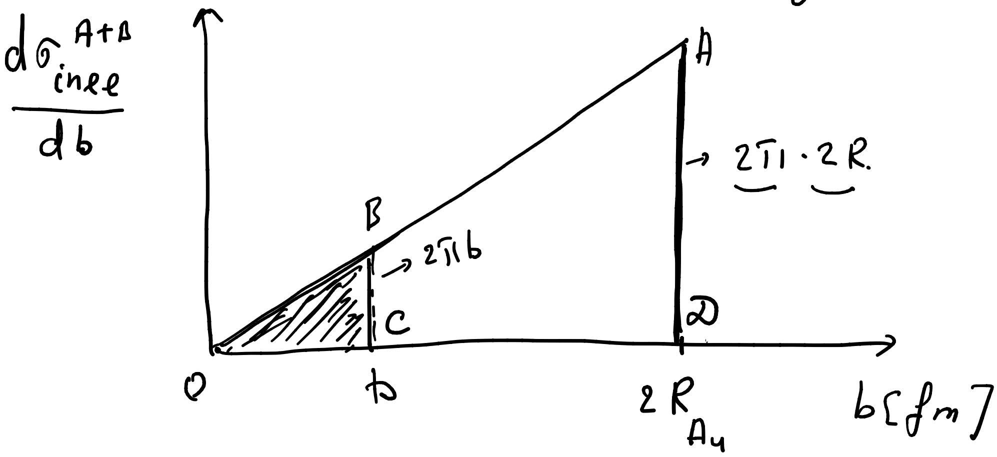
$$\sigma_{ine}^{Au+Au} \approx 4\pi R_{Au}^2 \approx 511 \text{ fm}^2 \approx 5.1 \text{ barn}$$

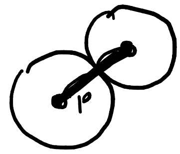
$1 \text{ fm} = 10^{-15}$ m $\qquad$ $1 \text{ barn} = 10^{-28} \text{ m}^2$

$1 \text{ fm}^2 = 10^{-30} \text{ m}^2$ $\qquad$ $1 \text{ barn} = 100 \text{ fm}^2$

$$\sigma_{inel}^{A+B} = \pi b^2 \qquad d\sigma_{inel}^{A+B} = 2\pi b\, db$$

$$\frac{d\sigma_{inel}^{A+B}}{db} = 2\pi b \qquad 0-n\% = \frac{\int_0^b \frac{d\sigma}{db}\, db}{\int_0^\infty \frac{d\sigma}{db}\, db}$$

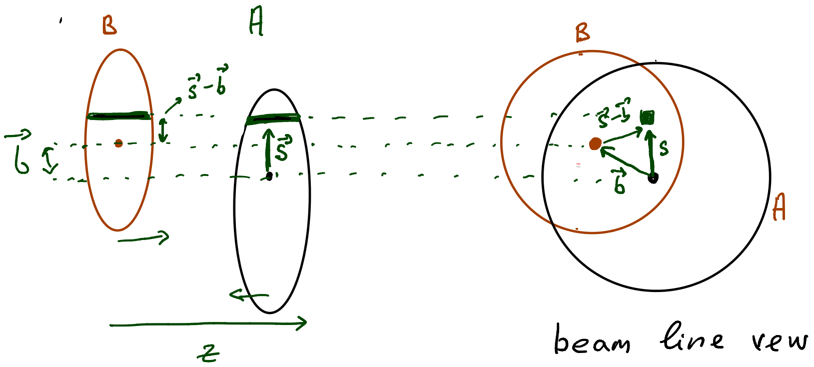




$$n\% = \frac{\text{Area}[OBC]}{\text{Area}[OAD]}$$

$$\therefore n\% = \frac{\cancel{0.5}\cdot \cancel{2}\pi b\cdot b}{\cancel{0.5}\, \cancel{2}\pi\cdot 2R\cdot 2R} = \boxed{\frac{b^2}{4R^2}}$$

Optical Glauber model



1) Start with $\rho_A(\vec{r})$
number of nucleons per unit volume.

we want to find:
number of nucleons per unit transverse area:

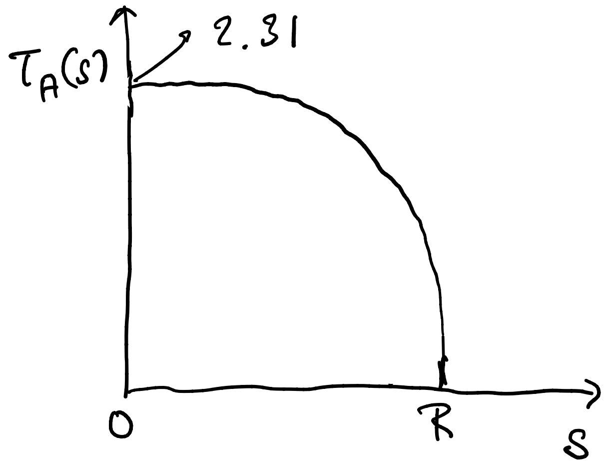
$$\rho_A(\vec{r}) = \rho_A(\vec{s}', z) \qquad |\vec{s}'| = \sqrt{x^2+y^2}$$

$$\boxed{\int d^2s\, T_A(s) = A}$$

$$\int \rho_A(\vec{s}', z)\, dz = T_A(\vec{s}') \rightarrow \text{nuclear thickness function.}$$

$T_A(\vec{s}')$ - amount of nuclear matter per unit transverse position $\vec{s}$.

$[T_A] = \frac{\text{nucleons}}{\text{area}}$



For sharp-sphere

$$\rho_A(\vec{r}) = \rho_0 \cdot \Theta(R-r)$$

$$T_A(\vec{s}) = 2\rho_0 \sqrt{R^2 - s^2}$$

$$s = R; \quad T_A(\vec{s}) = 0;$$

$$s = 0; \quad T_A(0) = 2\rho_0 R$$

$$T_{Au}(0) = 2 \cdot 0.181 \cdot 6.38 \simeq 2.31 \, fm^{-2}$$

2) Consider one nucleon from nucleus A:

→ pick one nucleon from A at transverse pos. $\vec{s}$

What is the probability that this nucleon suffers at least one inelastic collision with nucleus B?

for each target nucleon:

- The collision probability: p
- No-collision probability: 1-p

Assuming independence:

Th probability that one nucleon from A has no collision with any of th B nucleons in B is

$$P_0 = (1-p)^B$$

- What is p?

Nucleon from A at transverse position $\vec{s}$, sees nucleus B at a relative transverse position $\vec{s} - \vec{b}$.

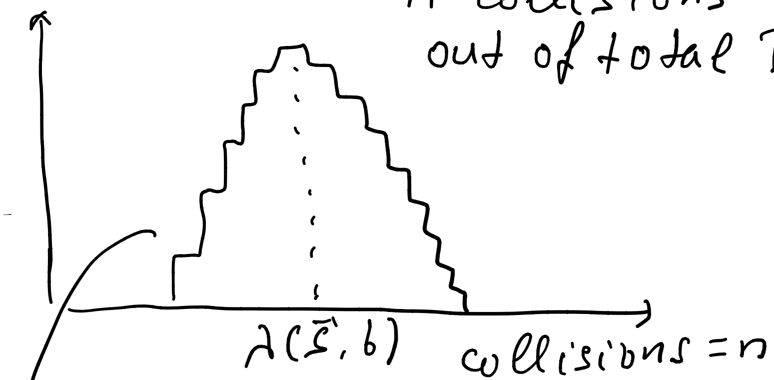
The areal density of nucleons from B there is:

$$T_B(|\vec{s} - \vec{b}|)$$

For nucleon-nucleon inelastic cross section σ_{NN}^{inel}, the mean number of collision suffered by this nucleon

$$\lambda(\vec{s}, b) = \sigma_{NN}^{inel} \cdot T_B(s-b)$$

What this means?



p - probability to have one collision

$$p = \frac{\lambda(\vec{s}, \vec{b})}{B}$$

if p is small   Poissonian: $\boxed{e^{-\lambda} \frac{\lambda^n}{n!}}$

thus:

$$p = \frac{\lambda(\vec{s}, \vec{b})}{B} = \frac{\sigma_{NN}^{inel} \cdot T_B(\vec{s}-\vec{b})}{B}$$

No-collision probability:

$$P_0^A(\vec{s}, \vec{b}) = \left(1 - \frac{\sigma_{NN}^{inel} \cdot T_B(\vec{s}-\vec{b})}{B}\right)^B$$

This is the binomial-optical-limit expression!

$$\lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^n = e^{-x};$$

for large B: $\boxed{P_0^A(\vec{s}, \vec{b}) = e^{-\sigma_{NN}^{inel} T_B(\vec{s}-\vec{b})}}$

Alternatively:

$\underbrace{e^{-\lambda} \cdot \frac{\lambda^n}{n!}}$ → this is true when $p = \frac{\lambda(\vec{s}; \vec{b})}{B}$ becomes small (Binomial → Poisson) thus for large B.

Therefore probability that nucleon has zero collision $(n=0)$ (9)

$$P_0^A(\vec{s}';\vec{b}') = e^{-\lambda}\frac{\lambda^n}{n!}\Big|_{n=0} = e^{-\lambda} = e^{-\sigma_{NN}^{inel}\cdot T_B(\vec{s}'-\vec{b}')}$$

Probability that nucleon is wounded:

$$P_w^A(\vec{s}';\vec{b}') = 1 - P_0^A(\vec{s}';\vec{b}')$$

A small transverse area d^2s contains on average $T_A(s)\,d^2s$ nucleons from A.

Eeach nucleon is wounded with prob $P_w^A(\vec{s}';\vec{b}')$

Hence, the number of wounded nucleons in this patch is

$$dN_w^A(\vec{s}',\vec{b}') = T_A(\vec{s}')\,P_w^A(\vec{s}',\vec{b}')\,d^2s$$

$$N_w^A(\vec{b}') = \int d^2s\, T_A(\vec{s}')\cdot P_w^A(\vec{s}',\vec{b}')$$

$$\boxed{\begin{aligned} N_w^A(b) &= \int d^2s\, T_A(\vec{s})\left[1 - e^{-\sigma_{NN}^{inel} T_B(\vec{s}-\vec{b})}\right] \\ N_w^B(b) &= \int d^2s\, T_B(\vec{s}-b)\left[1 - e^{-\sigma_{NN}^{inel} T_A(\vec{s})}\right] \end{aligned}}$$

$$N_w(b) = N_w^A(b) + N_w^B(b)$$

For identical nuclei: A + A

$$N_w(b) = 2 \int d^2s \, T_A(\vec{s}') \left[1 - e^{-\sigma_{NN} T_A(\vec{s}' - \vec{b}')} \right]$$

at $b=0$, for a very opaque large nucleus the exponential is nearly 0

$$N_w(0) \simeq 2 \int d^2s \, T_A(\vec{s}') = 2A$$

almost all nucleons participating!

Nuclear overlap function:

Recall: $T_A(\vec{s}') = \int dz \, \rho_A(\vec{s}', z) \rightarrow$ amount of nuclear matter per unit transrverse area at transverse position $\vec{s}$.

Units: $[T_A] = \dfrac{\text{nucleons}}{\text{area}}$

At a given transverse point s:

- $T_A(\vec{s}') \rightarrow$ how many nucleons from A are there
- $T_B(\vec{s}' - \vec{b}') \rightarrow$ how many nucleons from B are there

The local density of possible N-N encounters is proportional to:

$$T_A(\vec{s}') \, T_B(\vec{s}' - \vec{b})$$

Collision can happen anywhere in tr. plane

$$T_{AB}(b) = \int d^2s \, \underbrace{T_A(\vec{s})}\, \underbrace{T_B(\vec{s}' - \vec{b}')}$$

Nuclear overlap function.

$$[T_{AB}(b)] = \frac{1}{[\text{area}]}$$

$T_{AB}(\ddot{b}) \to$ The transverse density of possible nucleon-nucleon pairs.

Hence the total mean number of binary collisions:

$$N_{coll}(b) = \underbrace{\sigma_{NN}^{inel} \cdot T_{AB}(b)}$$

$$[T_{AB}] = \frac{1}{(\text{area})}$$

dimensionless, as required for a number of particles.

at $b=0$: maximum overlap. $T_{AB}(0) \to$ maximal.

Hard sphere:

$$T(s) = 2\rho_0 \sqrt{R^2 - s^2} \qquad T_{AA}(b) = \int d^2s \cdot 4\rho_0^2 \sqrt{R^2 - s^2}\sqrt{R^2 - |\vec{s} - \vec{b}|^2}$$

$$T_{AA}(0) = \frac{9A^2}{8\pi R^2} \simeq 341$$

Interaction probability:

$$N_{coll}(b) = \sigma_{NN}^{inel} \cdot T_{AB}(b)$$

$$\boxed{P_{inel}(b) = 1 - e^{-\sigma_{NN}^{inel} T_{AB}(b)}}$$

binomial:
$$P_{inel}(b) = 1 - \left(1 - \frac{\sigma_{NN}^{inel} T_{AB}(b)}{AB}\right)^{AB}$$

$$\sigma_{AB}^{inel} = \int d^2b \left[1 - e^{-\sigma_{NN}^{inel} \cdot T_{AB}(b)}\right] = \int d^2b \, P_{int}(b)$$

Reminder: Black disc: $\sigma_{AB}^{inel} = \pi b^2$ $\quad d\sigma = 2\pi b\, db$

$$\sigma = \int \frac{d\sigma}{db} db = \int 2\pi b\, db = \int d^2 b$$

$\boxed{P_{inel} = 1}$

Centrality:

$$C_{n-m} = \frac{\int_{b_n}^{b_m} \frac{d\sigma}{db} db}{\sigma_{AB}^{inel}} = \frac{\int_{b_n}^{b_m} 2\pi b\, db\, P_{int}(b)}{\int_0^\infty 2\pi b\, db\, P_{int}(b)}$$

Averedge number of nuceons.

$$\langle N_{part} \rangle_{c_1 - c_2} = \frac{\int_{b_1}^{b_2} 2\pi b\, db\, P_{int}(b) N_{part}(b)}{\int_{b_1}^{b_2} 2\pi b\, db\, P_{int}(b)}$$

If you are given a probability density function $P(x)$, then the mean of quantity $m(x)$ over the interval $[x_1, x_2]$

$$\langle m(x) \rangle = \frac{\int_{x_1}^{x_2} m(x)\, P(x)\, dx}{\int_{x_1}^{x_2} P(x)\, dx}$$

Why optical? Probability of having no collision

$$P_0 = e^{-\sigma_{NN}^{inel} T}$$

has the same structure as optical absorbtion

$I = I_0 \cdot e^{-\mu L}$ → Beer-Lambert Law

L - path length, μ → absorbtion probability.

- Optics: light travels through matter

- the medium absorbs/scatters light

In Glauber Model:

- nucleon travels through nucleus
- nucleus actts as absorbing medium.