Review of critical fluctuations

from few GeV to few TeV

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$$\hat{\chi}_2^B = \frac{\langle \Delta n_B^2 \rangle - \langle \Delta n_B \rangle^2}{VT^3}$$

- **Critical phenomena**
  - Strongly Interacting Matter
  - Experimental Plan
  - Results/interpretations
- **Summary**

Critical phenomena

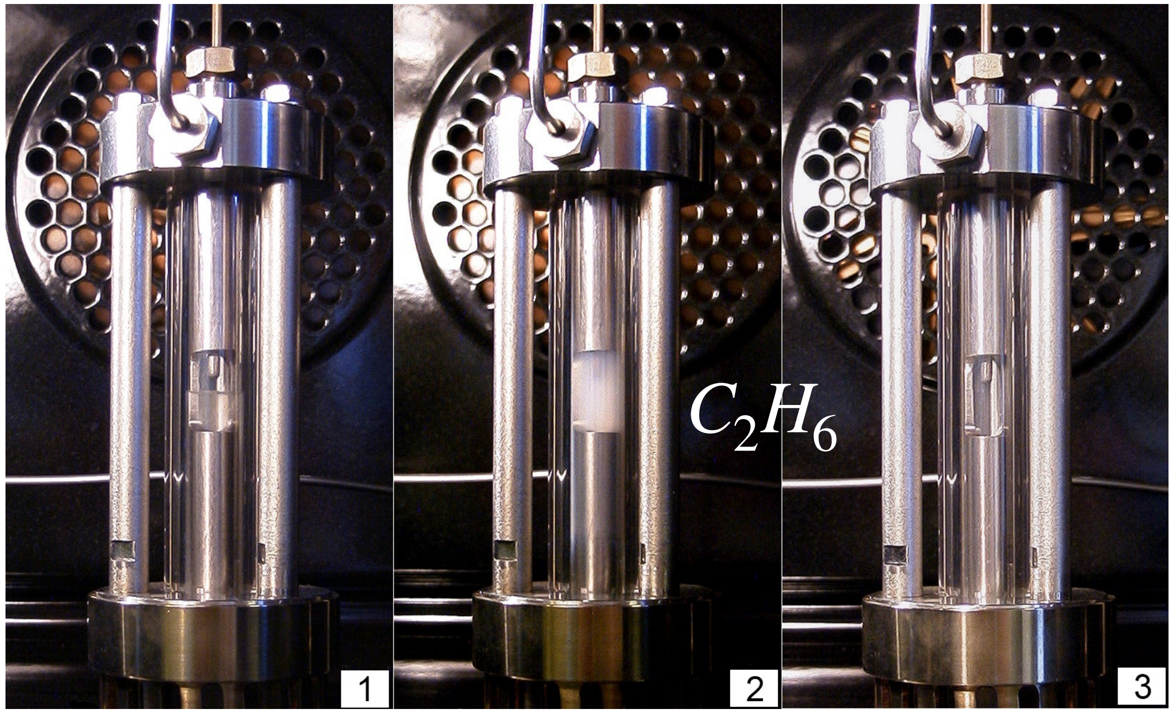
discovered ~ 200 years ago

Cagniard de la Tour (1777-1859)

Ann. Chim. Phys., 21 (1822) 127-132

using steam digester
invented by Denis Papin in 1679

$T < T_c$ $T = T_c$ $T > T_c$

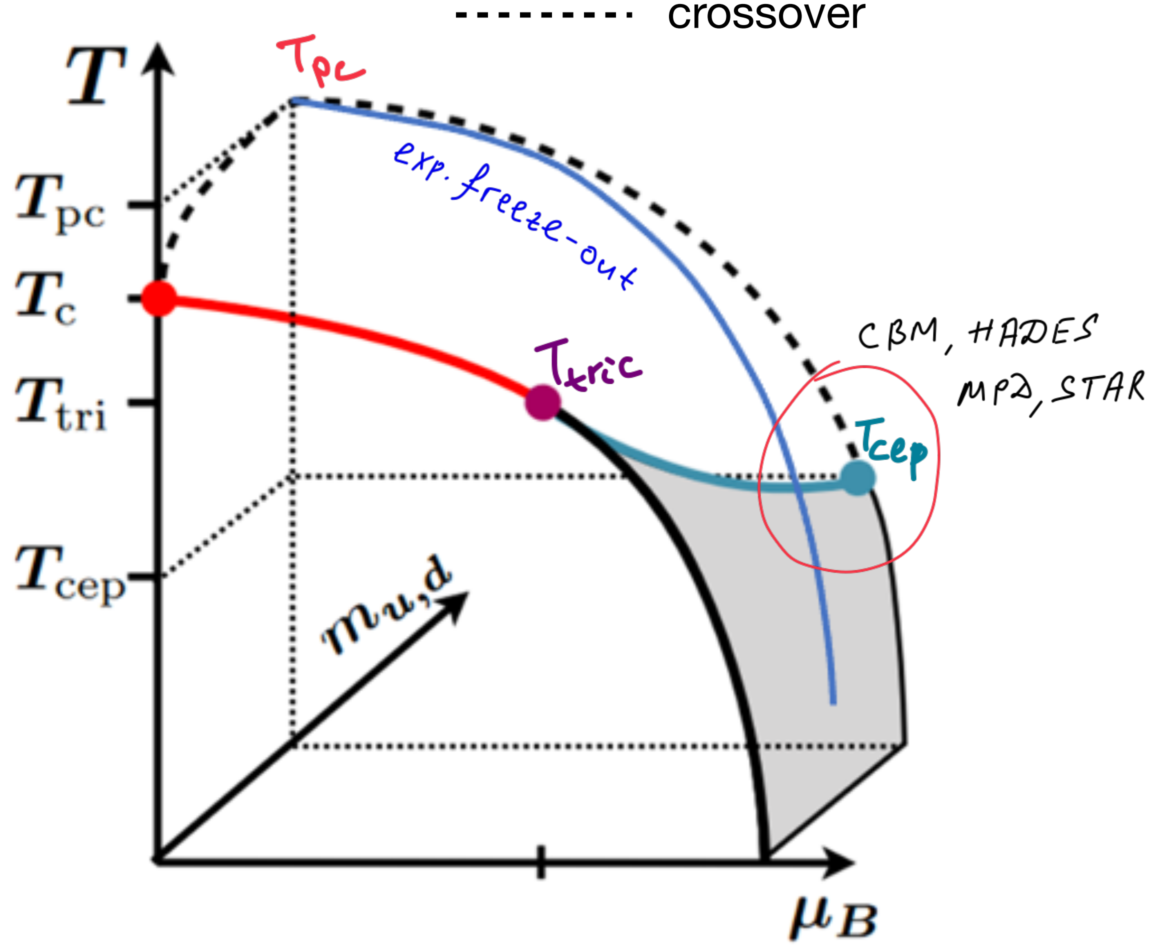


$$\frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle^2} = \frac{T\chi_T}{V} \qquad \chi_k = -\frac{1}{V\left(\frac{\partial P}{\partial V}\right)_T}$$

A. Einstein, Annalen der Physik, Volume 338, Issue 16, 1910: $h \sim \frac{1}{\lambda^4}\chi_T$

probing criticality with fluctuations

Strongly interacting matter



E-by-E fluctuations

- Locating phase boundaries
- Search for critical phenomena
- …

E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{\chi_T}{V} \qquad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

direct link to EoS

$T_{cep} < T_{tric} < T_c = 132^{+3}_{-2}$ MeV $< T_{pc} = 156 \pm 1.5$ MeV

H. T. Ding et al [HotQCD], arXiv:1903.04801
A. Bazavov et al [HotQCD], arXiv:1812.08235

$$\langle N^2\rangle - \langle N\rangle^2 = \kappa_2(N) = T^2\frac{\partial^2 lnZ}{\partial\mu^2}$$

probing the response of the system to external perturbations (V, μ)

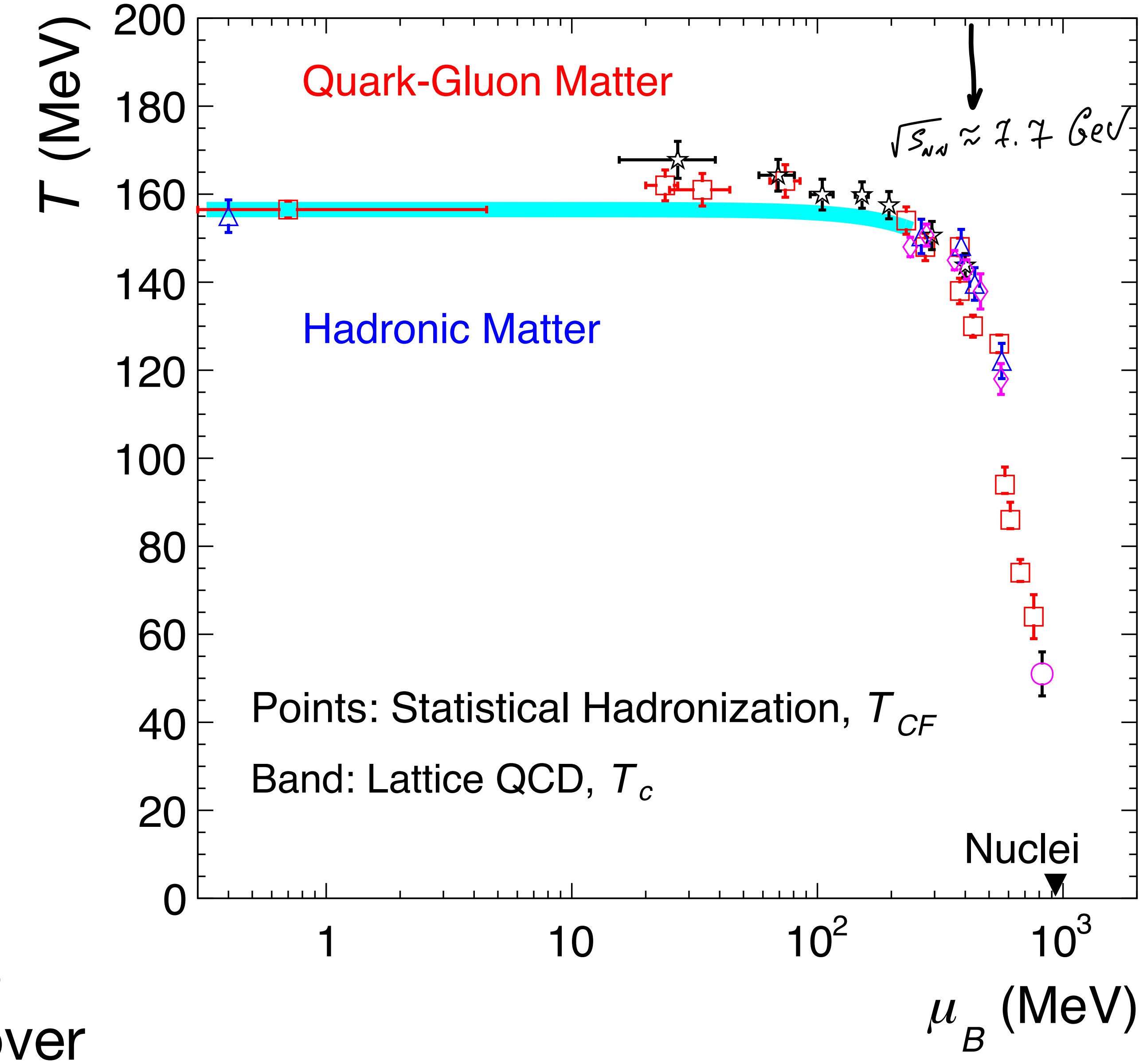
Freeze-out at the phase boundary

$T_{fo}^{ALICE} = 156.5 \pm 1.5 MeV \pm 3 MeV(sys)$

$T_{C}^{LQCD} = 156.5 \pm 1.5 MeV$

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)

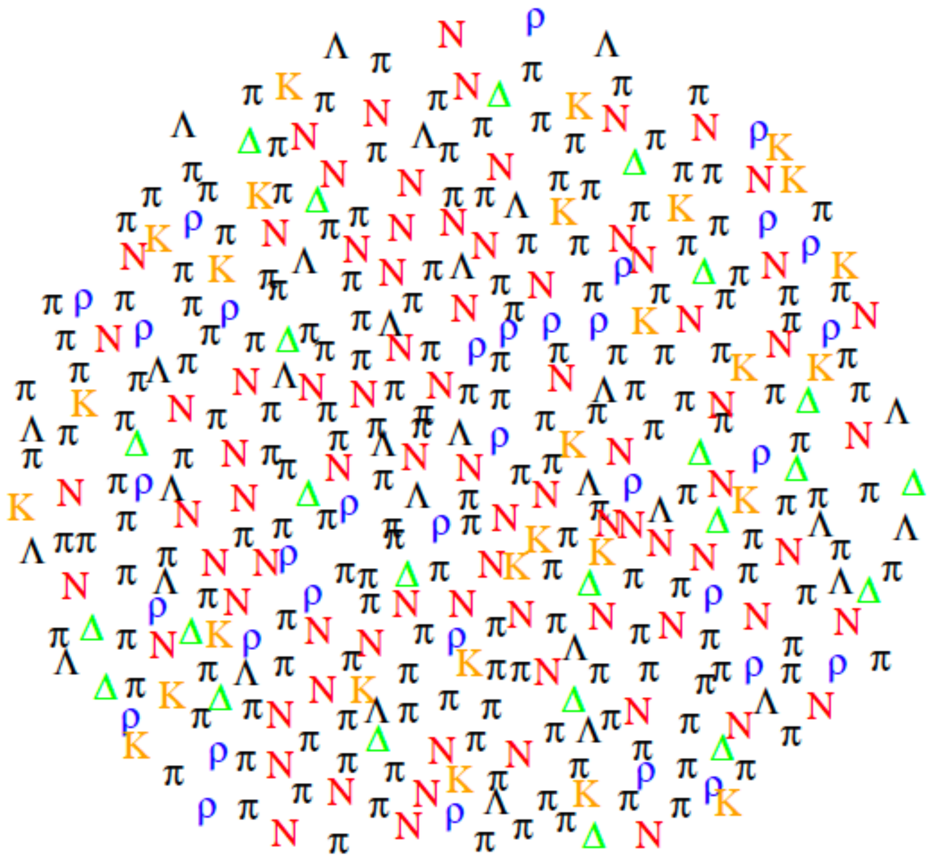
A. Bazavov et al., Phys.Rev. D85 (2012) 054503



- measuring fluctuations of net-baryons along the QCD phase boundary

- the order of the phase transitions
- experimental evidence for crossover
- existence of the critical endpoint
- ...

Hadron Resonance Gas model (HRG)



J.Cleymans and H. Satz, Z. fuer Physik C57, 135, 1993

partition function for particle type i $(\hbar = c = 1)$:

- $lnZ_i = \frac{Vg_i}{(2\pi)^3}\int_0^\infty d^3p ln(1 \pm e^{-(E_i-\mu_i)/T})^{\pm 1}$
  - +1 fermions
  - -1 bosons
  - g_i - spin degeneracy
- $\langle n_i \rangle = \frac{T}{V}\frac{\partial lnZ_i}{\partial \mu_i} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{exp[(E_i-\mu_i)/T] \pm 1}$

chemical potential for every conserved quantity

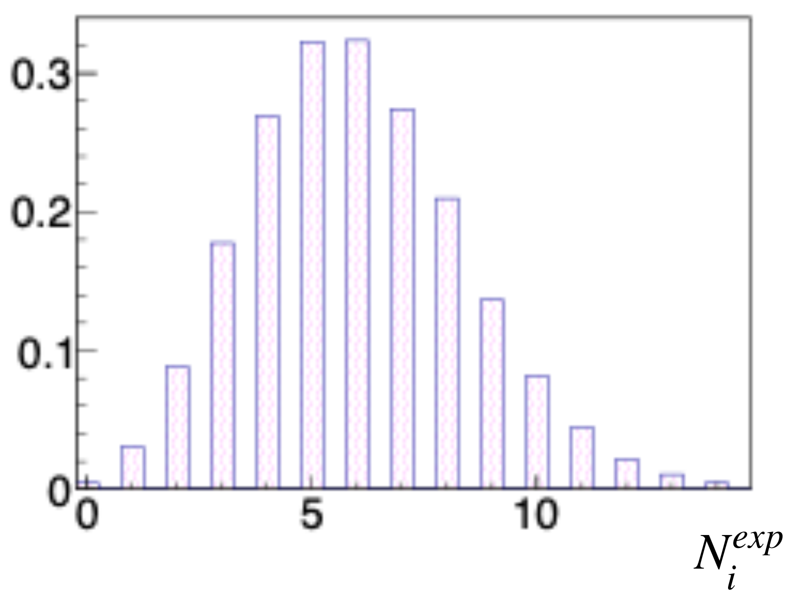
- $\mu_i = B_i\mu_B + S_i\mu_s + I_{3i}\mu_{I_3}$

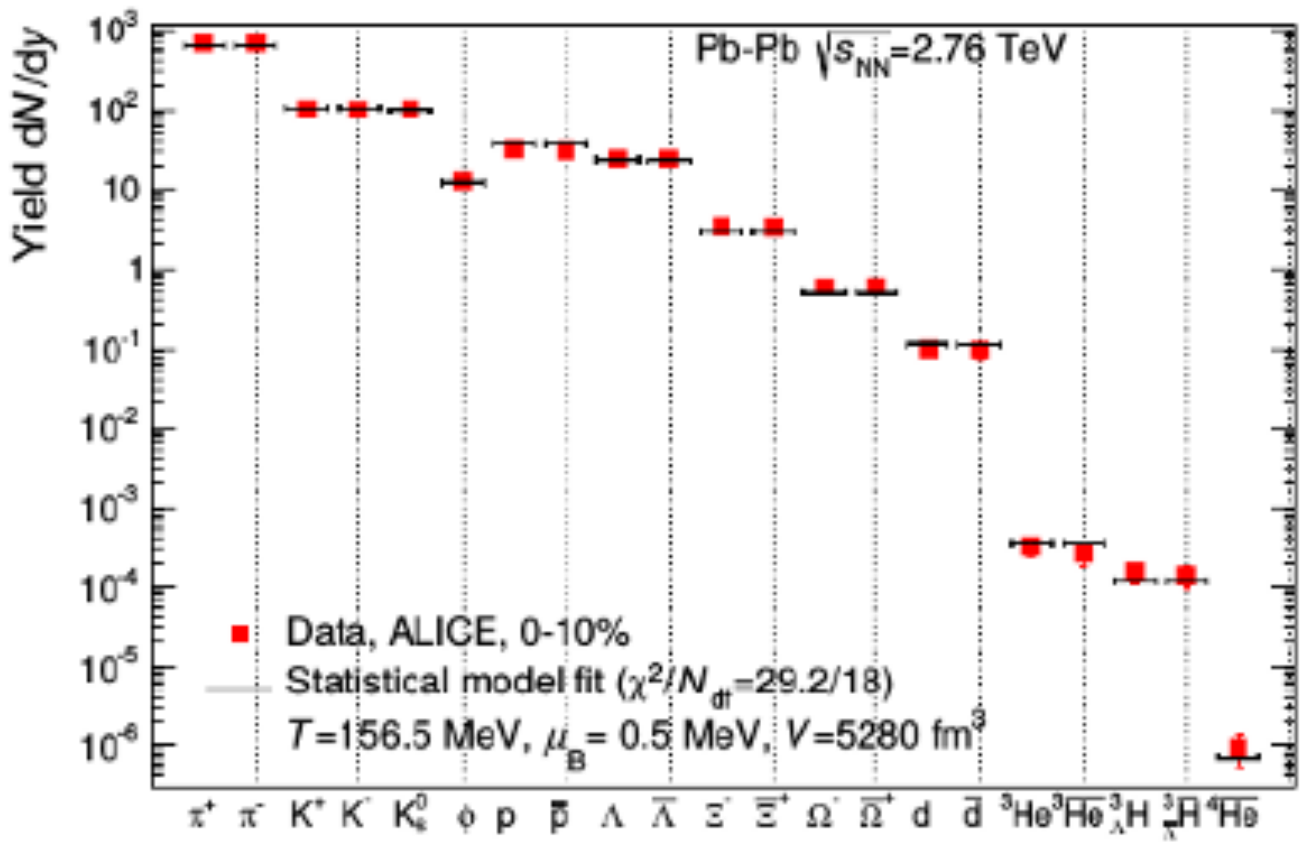
$$V\sum_i n_i B_i = Z + N \qquad \sum_i n_i(\mu_s)S_i = 0 \qquad V\sum_i n_i(I_{3i})I_{3i} = \frac{Z-N}{2}$$

only two (three) independent variables are left: T, μ_B, (V)

non-interacting HRG ⟹ **approximation for interacting hadron gas**

Hadron Resonance Gas model (HRG)





$$\langle N_i \rangle^{therm}(T, \mu_B) = \langle N_i \rangle + \sum_j Br(j -> i) \langle N_j \rangle$$

Minimize:

$$\chi^2 = \sum_i \frac{\langle N \rangle_i^{exp} - \langle N \rangle_i^{therm}(T, \mu_B)}{\sigma_i^2}$$

$\langle N \rangle_i$ - first moments of hadron i

σ_i - experimental uncertainty (sys.+stat.)

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)

- $T_{fo}^{ALICE} = 156.5 \pm 1.5 MeV \pm 3 MeV(sys)$
- $T_C^{LQCD} = 156.5 \pm 1.5 MeV$

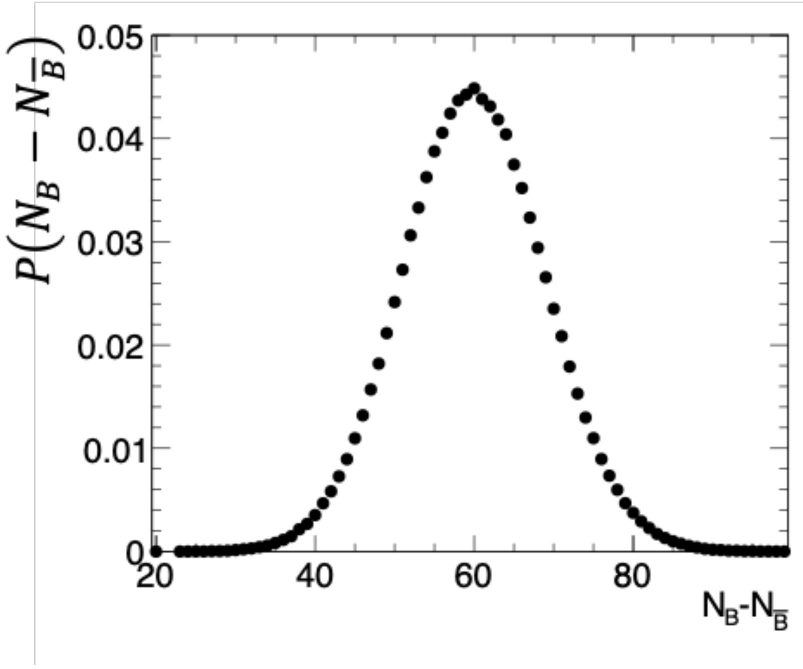
what about higher moments: $\kappa_2(N) = \langle N^2 \rangle - \langle N \rangle^2 = T^2 \frac{\partial^2 lnZ_{GCE}}{\partial \mu^2}$

Cumulants and minimal baseline

$X = N_B - N_{\bar{B}}$

r^{th} **order central moment**

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$



first 4 cumulants

$$\kappa_1 = \langle X \rangle, \ \kappa_2 = \mu_2, \ \kappa_3 = \mu_3, \ \kappa_4 = \mu_4 - 3\mu_2^2$$

uncorrelated Poisson limit ⇒ **Skellam distribution for** X

$$\kappa_1 = \lambda_1 - \lambda_2, \quad \kappa_2 = \lambda_1 + \lambda_2, \quad \kappa_3 = \lambda_1 - \lambda_2, \ \ldots, \ \kappa_n = \lambda_1 + (-1)^n \lambda_2$$

high energy limit ($\lambda_1 \neq 0, \lambda_2 \neq 0$)

Skellam

$$\frac{\kappa_{2n}}{\kappa_{2k}} = \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} = 1$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \neq 1$$

low energy limit ($\lambda_1 \neq 0, \lambda_2 = 0$)

Poisson

$$\frac{\kappa_{2n}}{\kappa_{2k}} = \frac{\lambda_1}{\lambda_1} = 1$$

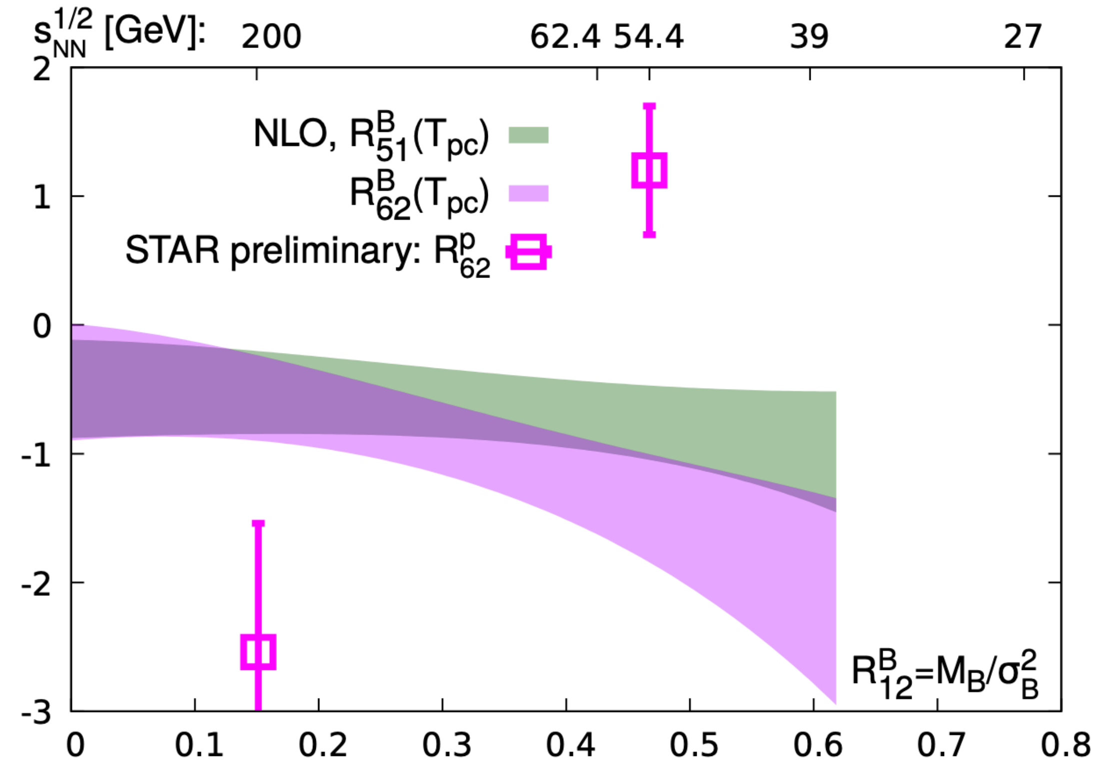
$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\lambda_1}{\lambda_1} = 1$$

Predictions for crossover region

for a thermal system in a fixed volume V within the Grand Canonical Ensemble (GCE)

probing the response of the system to variation of μ_B

$$\hat{\chi}_n^B = \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} \equiv \frac{\kappa_n(B)}{VT^3}$$



A. Bazavov et al [HotQCD], PRD 101 (2020) 074502

assumptions

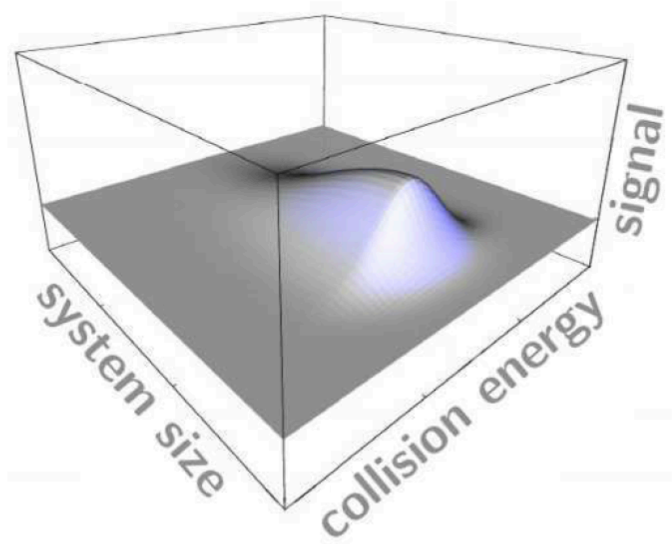
- volume is fixed
- conservations are imposed on the averages

artefacts of volume fluctuations and conservation laws in experiments

$$\frac{\kappa_4^{exp}(n_B - n_{\bar{B}})}{\kappa_2^{exp}(n_B - n_{\bar{B}})} \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

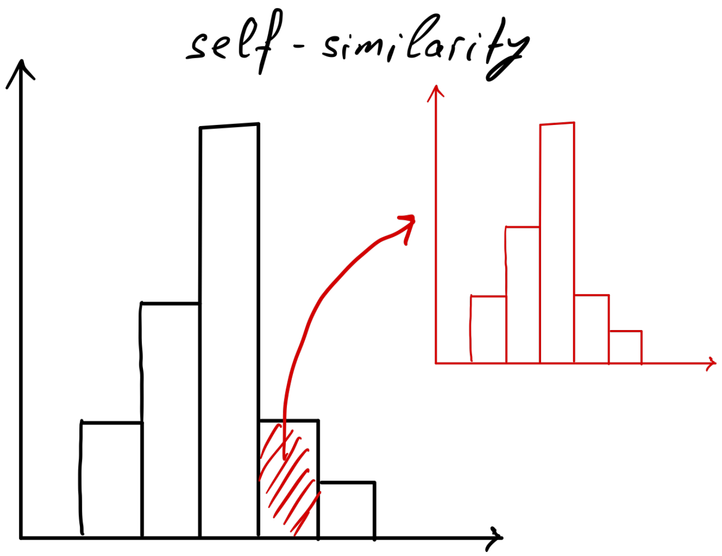
$$\frac{\kappa_6^{exp}(n_B - n_{\bar{B}})}{\kappa_2^{exp}(n_B - n_{\bar{B}})} \neq \frac{\hat{\chi}_6^B}{\hat{\chi}_2^B} \equiv R_{62}^B$$

P. Braun-Munzinger, AR., J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911
M. I. Gorenstein, M. Gazdzicki, PRC 84 (2011) 014904



search for non-monotonic behaviour in (2D) excitation functions of fluctuation signals

Expected signals near the critical point



- scale invariance, onset of self-similarity
- increase in the range of density-density correlation functions
- divergence of correlation length ξ

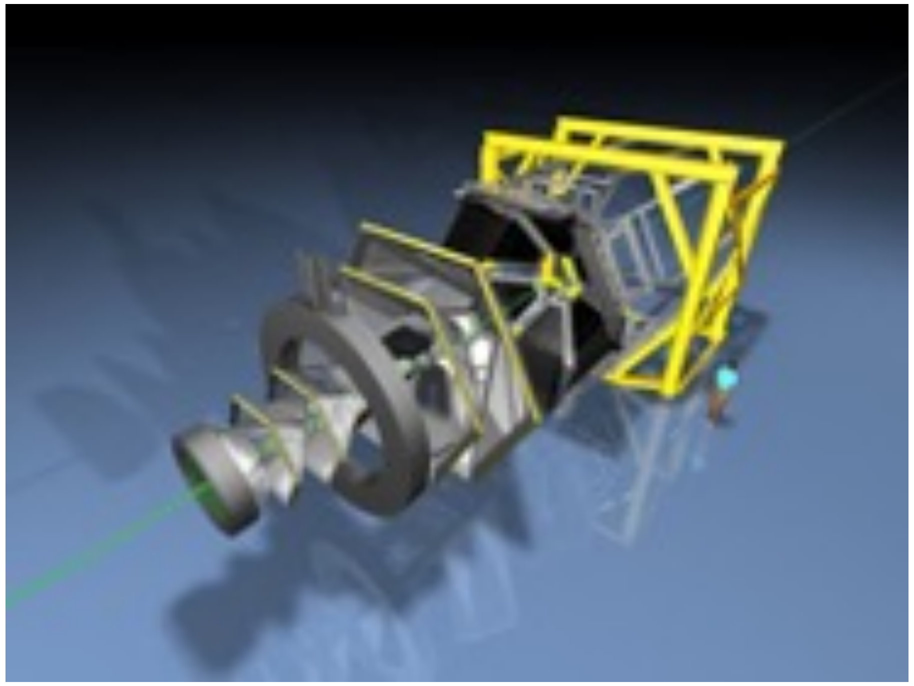
higher order cumulants are more sensitive to ξ: $\kappa_2 \sim \xi^2, \kappa_4 \sim \xi^7$

M. A. Stephanov, PRL 102 (2009), 032301

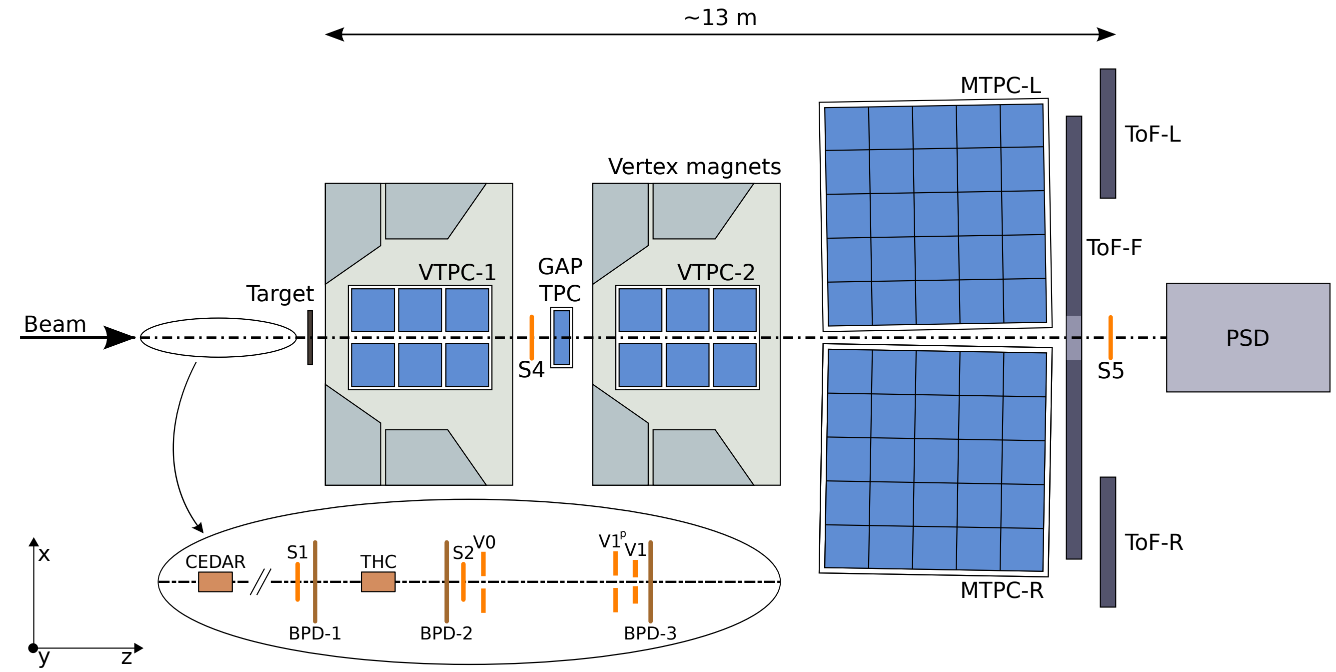
Experimental results and their interpretations

Experiments

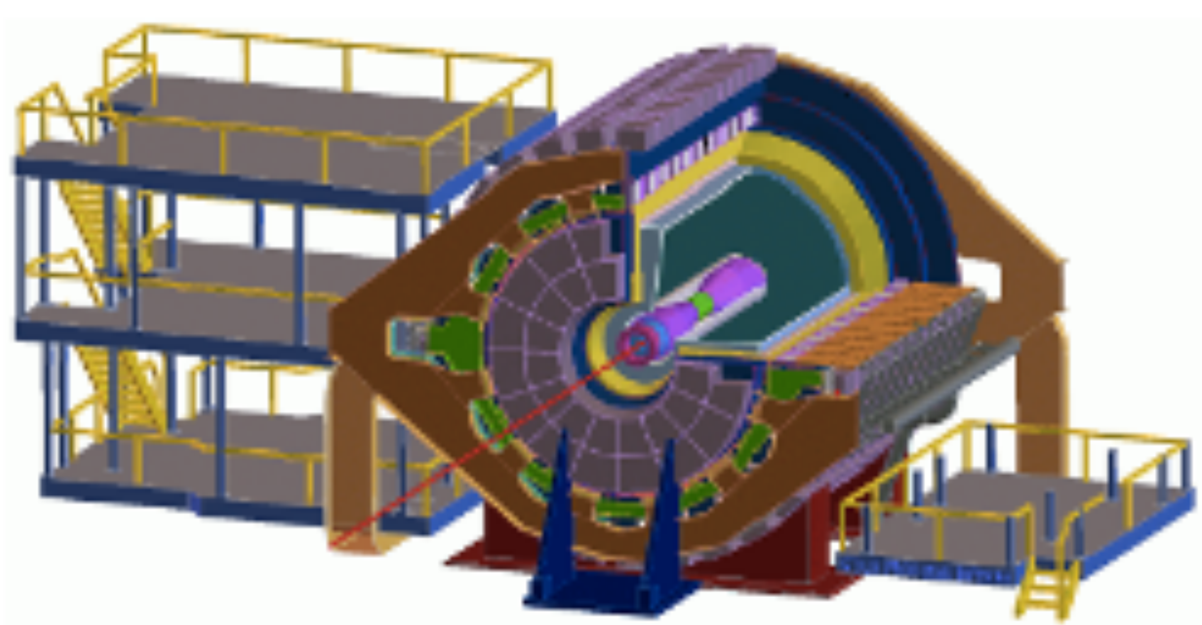
HADES@GSI SIS18 (few GeV)



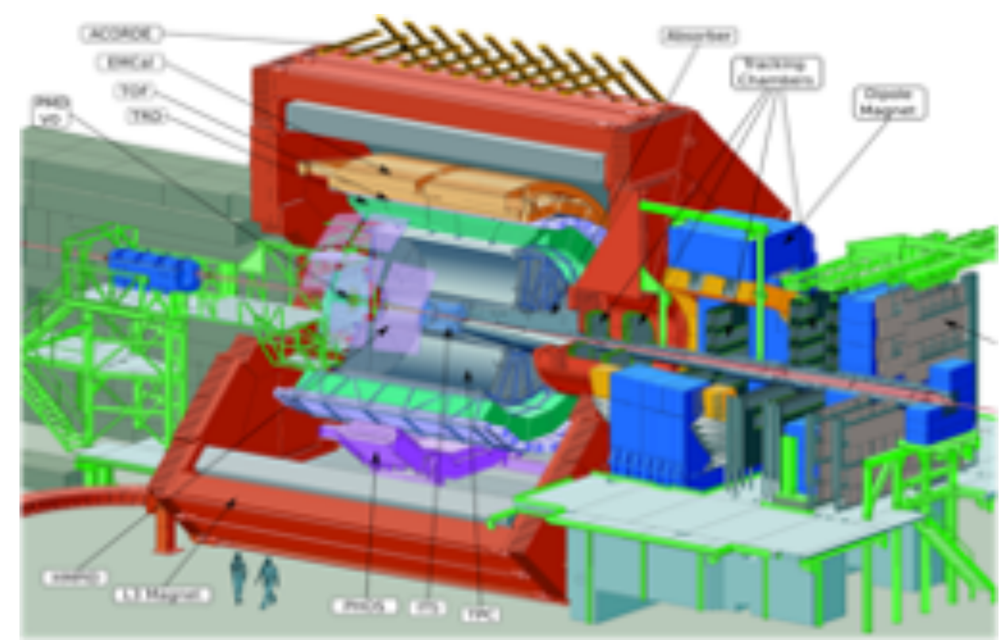
NA61/SHINE@CERN SPS (5-17 GeV)



STAR@BNL RHIC (3 - 200 GeV)



ALICE@ CERN LHC (few TeV)



Experimental challenges

Event-by-event identification issues

- Cut based approach (hard cut)
- Identity Method
- PSET Identity Method

M. Gazdzicki, K. Grebieszkow, M. Mackowiak and S. Mrowczynski, PRC 83 (2011), 054907
M. Mackowiak-Pawlowska and P. Przybyla, EPJ. C78 (2018) no.5, 391
AR. and M. I. Gorenstein, PRC 86 (2012), 044906
M. Arslandok and AR., NIM A946 (2019), 162622
M. Gazdzicki, M. I. Gorenstein, M. Mackowiak and AR., NPA 1001 (2020), 121915
…

Non-dynamical contributions

- E-by-e fluctuations of wounded nucleons
- Depends on centrality selection methods

P. Braun-Munzinger, AR., J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911
M. I. Gorenstein and M. Gazdzicki, Phys. Rev. C 84 (2011) 014904
X. Luo, J. Xu, B. Mohanty, and N. Xu, J.Phys. G40 (2013) 105104
…

Contributions from pileup events

Y. Zhang, Y. Huang, T. Nonaka and X. Luo, arXiv:2108.10134
T. Nonaka, M. Kitazawa and S. Esumi, NIM. A 984 (2020), 164632

these topics are not covered
in this presentation,
see the following talks for the details

Toshihiro NONAKA (STAR)

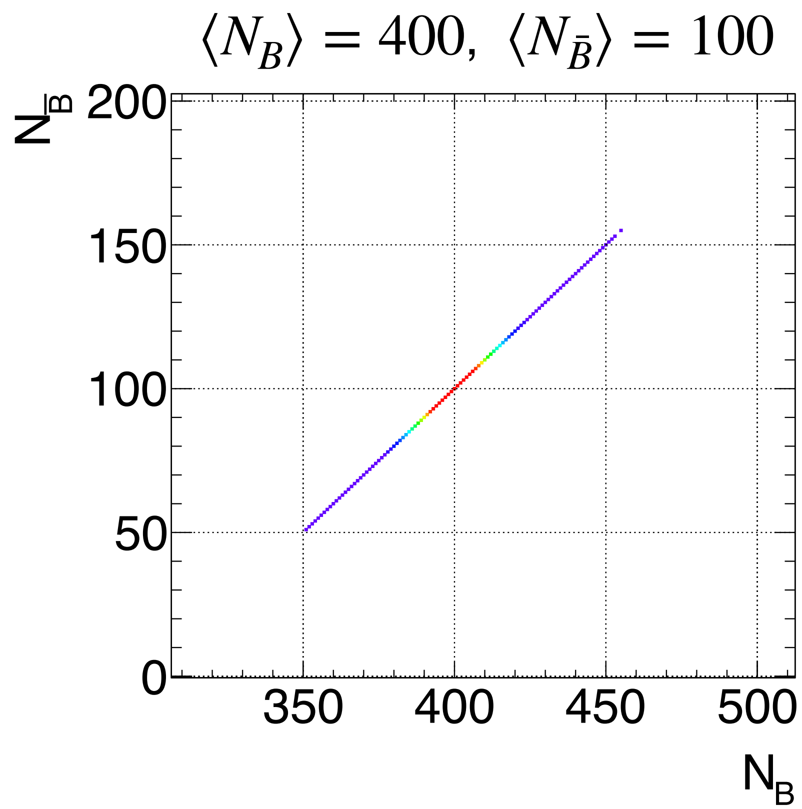
Romain Holzmann (HADES)

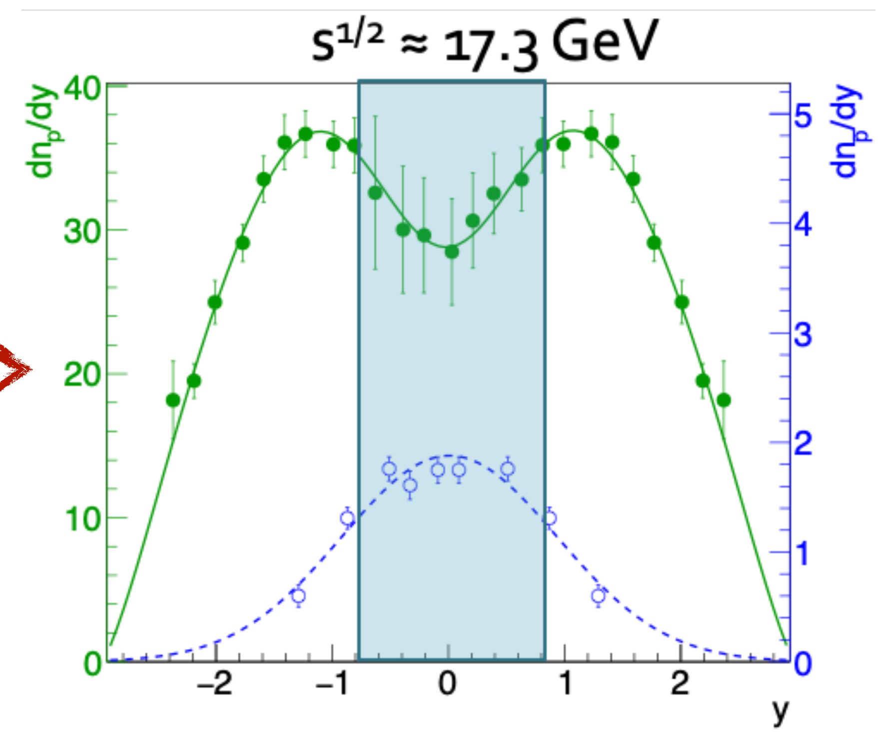
ShinIchi Esumi (STAR)

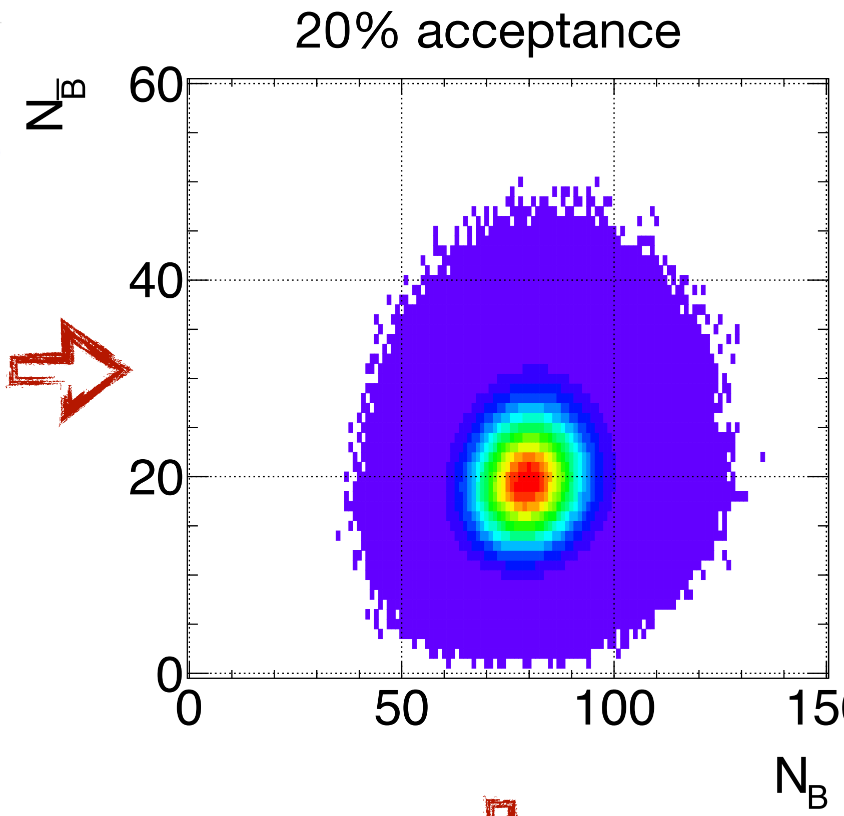
Maja Pawłowską (NA61/SHINE)

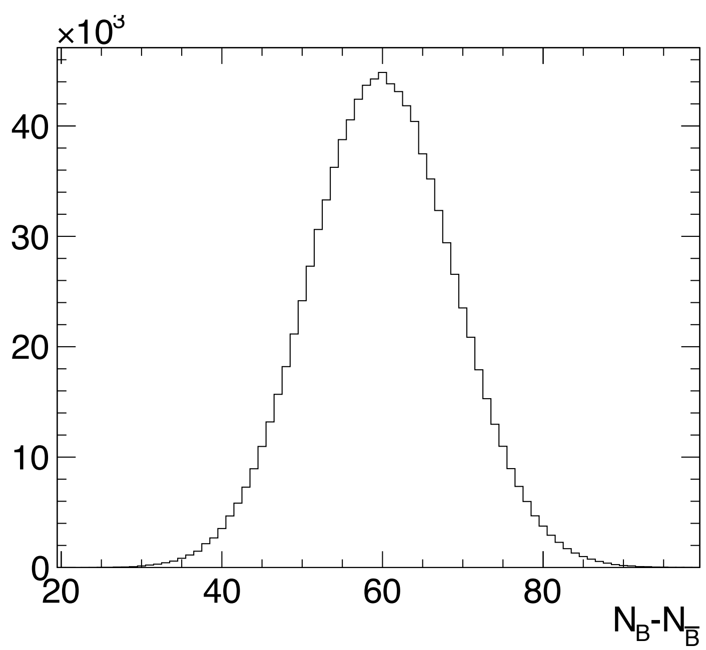
Mesut Arslandok (ALICE)

Acceptance selection is crucial









to achieve requirements of GCE

- cuts on p_T, y or η are imposed
  - $\Delta y_{acc} > \Delta y_{thr}$ - conservations dominate
  - $\Delta y_{acc} < \Delta y_{thr}$ - dynamical fluctuations are masked by Poisson statistics

constant cut in rapidity introduces

- different acceptances for N_B and $N_{\bar{B}}$
- energy dependent acceptances
- **both effects are implemented in our model**

$$\alpha_B = \frac{\langle N_B \rangle_{acc}}{\langle N_B \rangle_{4\pi}} \qquad \alpha_{\bar{B}} = \frac{\langle N_{\bar{B}} \rangle_{acc}}{\langle N_{\bar{B}} \rangle_{4\pi}}$$

AR. NPA 1005 2021,121858

The non-critical baseline

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2z\sqrt{\lambda_B \lambda_{\bar{B}}})$$

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

B net baryon number, conserved in each event

I_B modified Bessel function of the first kind

z_B, $z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

λ_B, $\lambda_{\bar{B}}$ auxiliary parameters for calculating mean number of baryons, anti baryons

$$z = \sqrt{z_B z_{\bar{B}}} = \sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}$$

$\langle N_B\rangle_{GCE}$, $\langle N_{\bar{B}}\rangle_{GCE}$ - are in GCE, experiments measure canonical multiplicities, $\langle N_B\rangle$, $\langle N_{\bar{B}}\rangle$

$$\langle N_B\rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B, \lambda_{\bar{B}} = 1} = z\frac{I_{B-1}(2z)}{I_B(2z)} \qquad \langle N_{\bar{B}}\rangle = \lambda_{\bar{B}} \frac{\partial \ln Z_B}{\partial \lambda_{\bar{B}}}\bigg|_{\lambda_B, \lambda_{\bar{B}} = 1} = z\frac{I_{B+1}(2z)}{I_B(2z)}$$

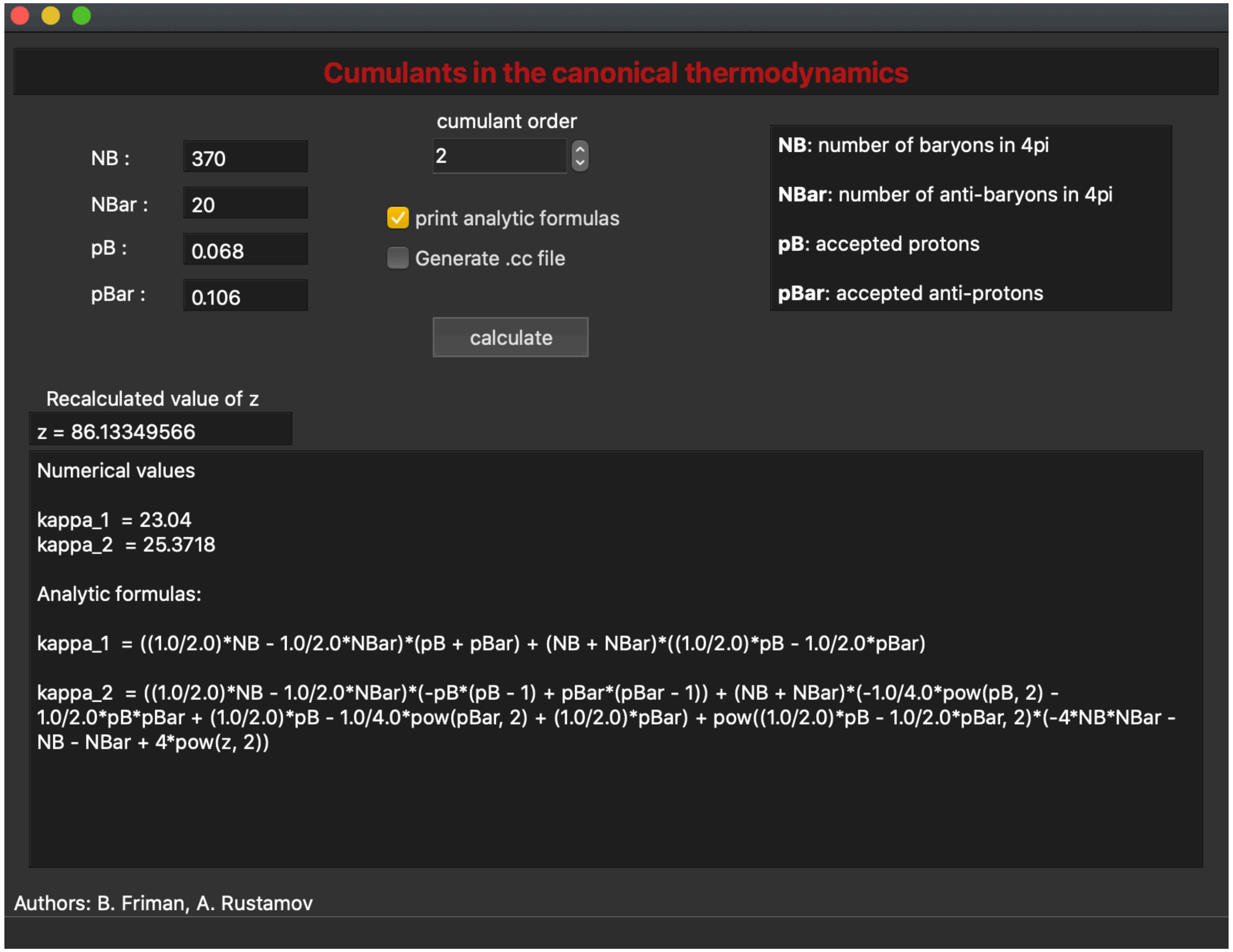
z is recalculated by solving Eq. for $\langle N_B\rangle$ or $\langle N_{\bar{B}}\rangle$

see also: V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901,
B. P. Braun-Munzinger, AR., J. Stachel, NPA982, (2019), 307-310

Graphical User Interface

$$\kappa_1 = \langle N_B \rangle \alpha_B - \langle N_{\bar{B}} \rangle \alpha_{\bar{B}}$$

$$\kappa_2 = \langle N_B \rangle \alpha_B (1-\alpha_B) + \langle N_{\bar{B}} \rangle \alpha_{\bar{B}} (1-\alpha_{\bar{B}}) + \left(z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle\right) \left(\alpha_B - \alpha_{\bar{B}}\right)^2$$



a Python package for calculating both analytic formulas and numerical values for net-baryon cumulants of any order in the finite acceptance is available for download

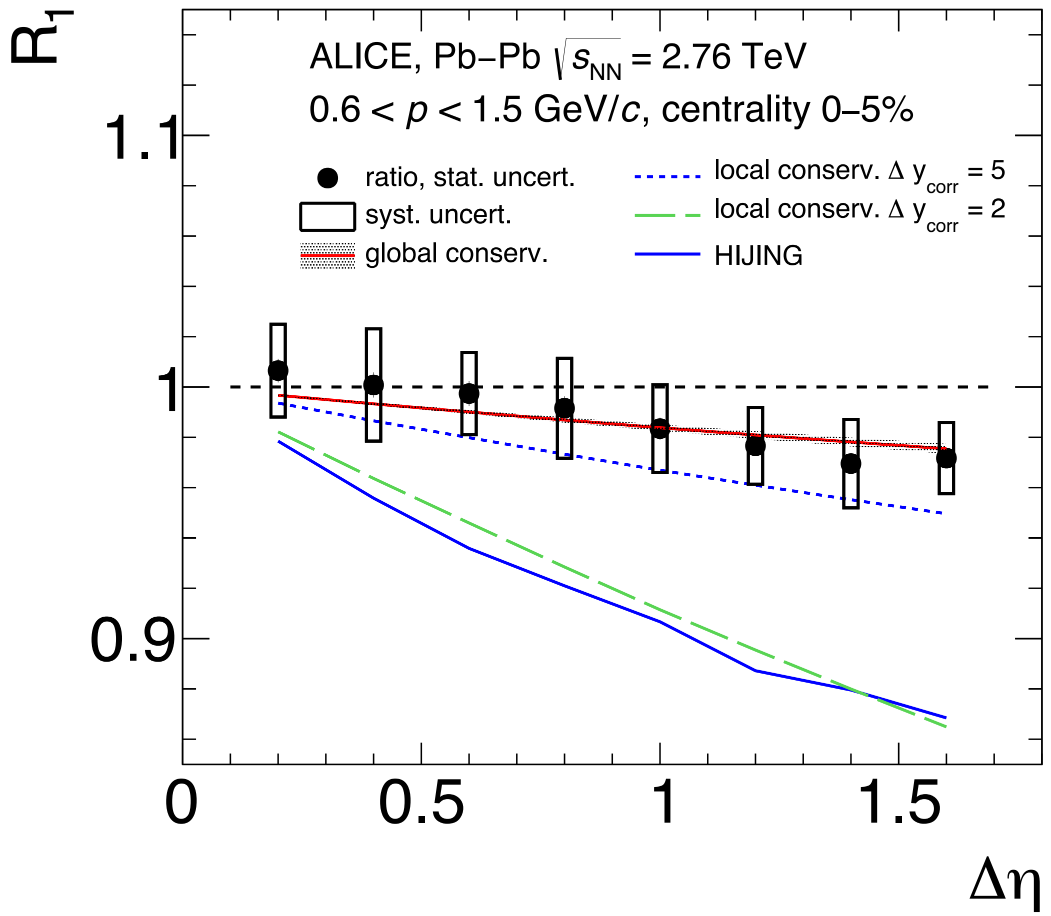
git clone https://github.com/e-by-e/Cumulants-CE.git

Results from ALICE

at LHC energies the R_1 does not depend on volume fluctuations

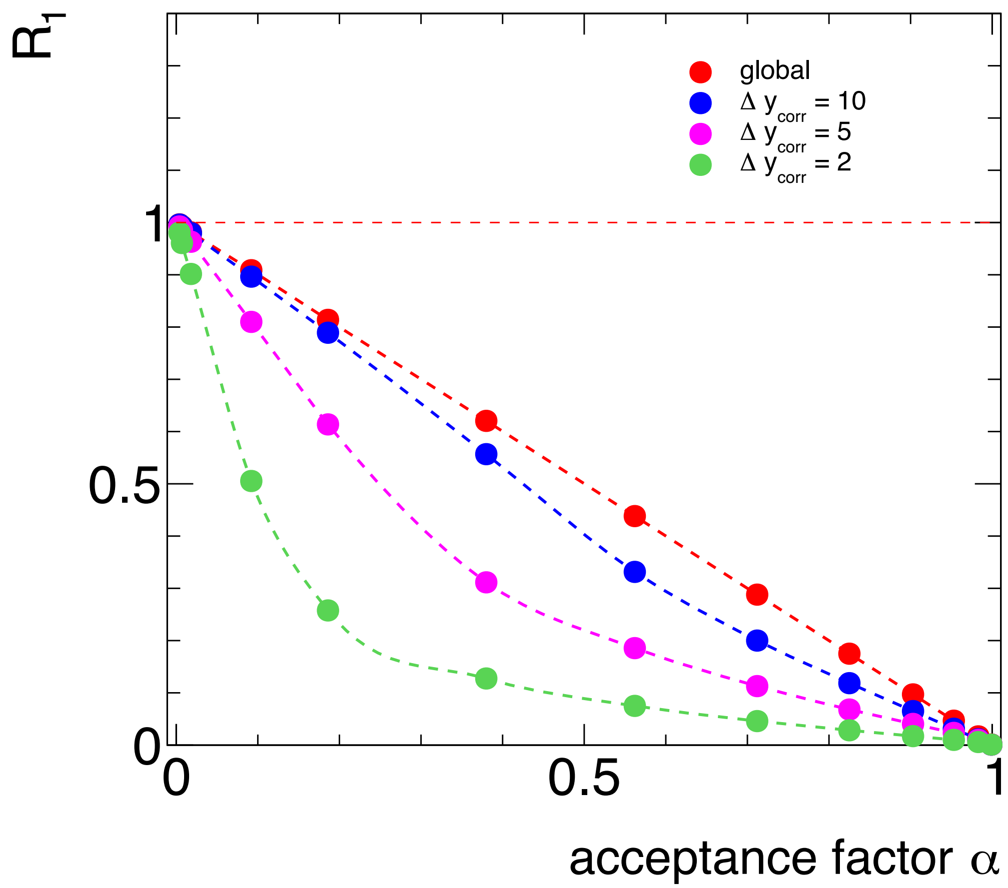
AR. NPA 967 (2017) 453-456

$R_1 = \kappa_2(p - \bar{p})/\langle p + \bar{p} \rangle$



ALICE: Phys. Lett. B 807 (2020) 135564

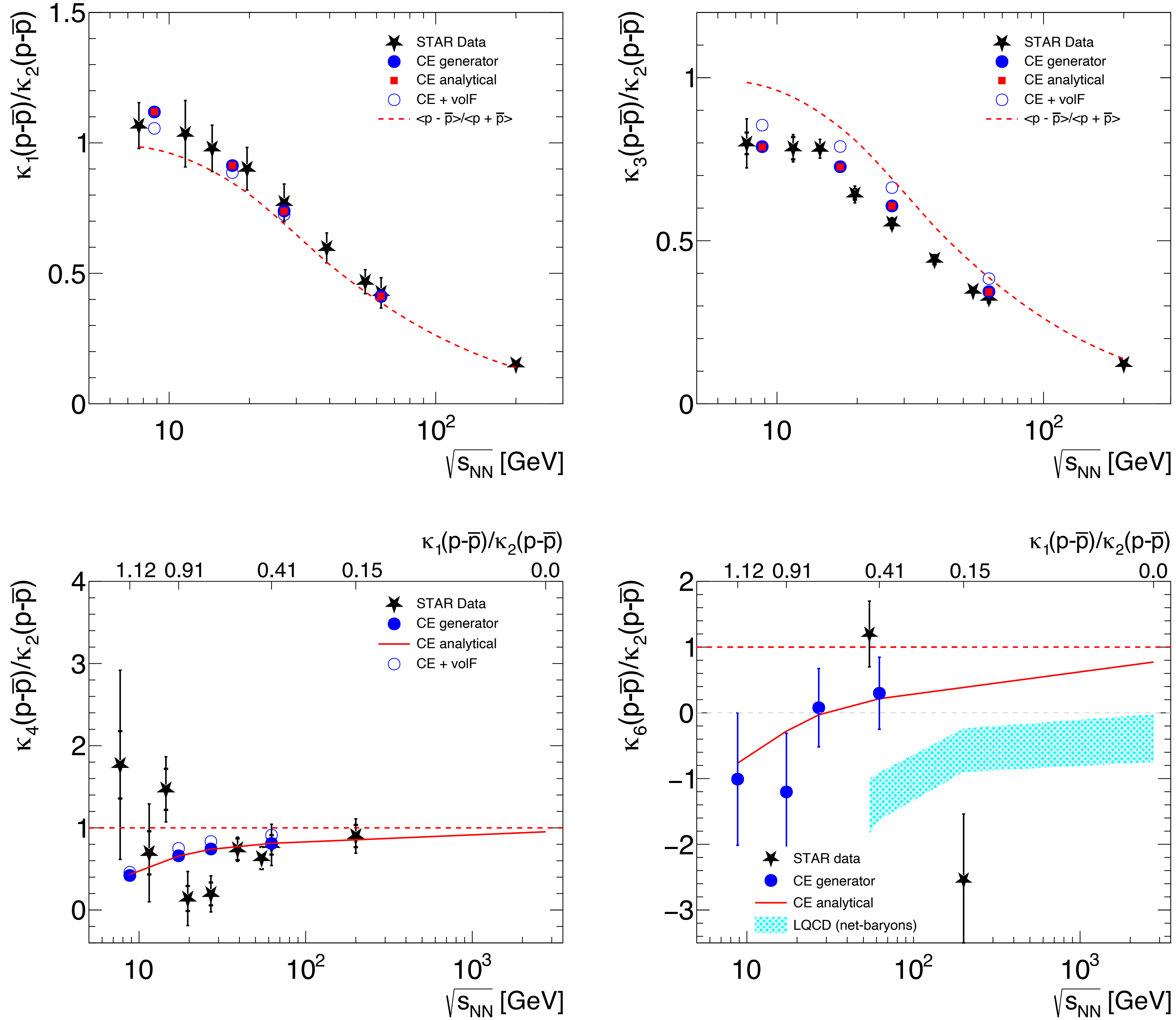
$R_1 = \kappa_2(p - \bar{p})/\langle p + \bar{p} \rangle$



P. Braun-Munzinger, AR., J. Stachel, arXiv:1907.03032

- the data are best described by global baryon number conservation: $\sim 1 - \alpha$
  - **Indication of long range correlations**
- HIJING corresponds to $\Delta y_{corr} = 2$, not consistent with data
  - **baryon production in string models is not consistent with data**

Results from STAR



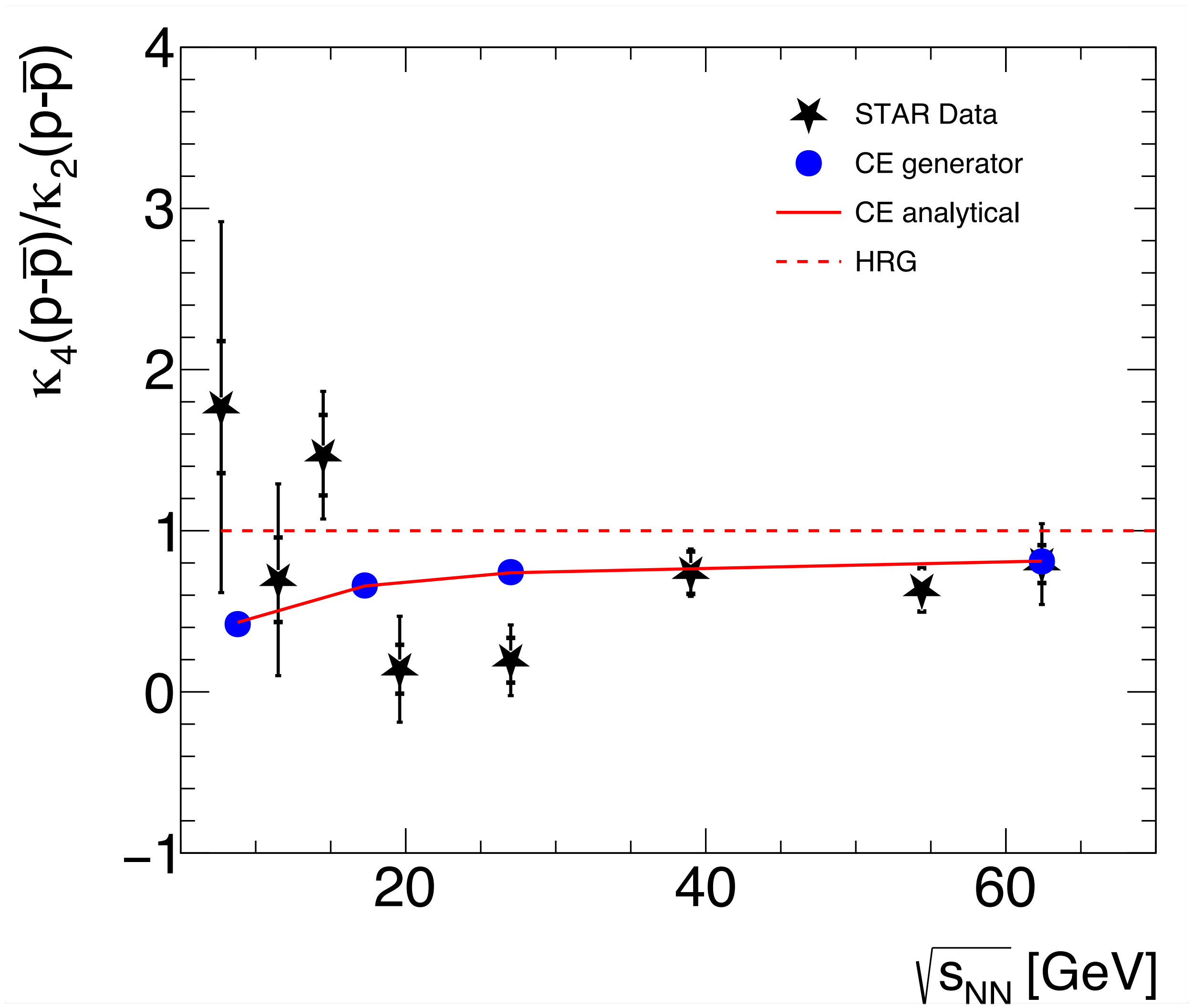
- remarkable agreement between calculations and STAR data
- for higher energies the ratios approach the GCE baseline **(artefact of fixed acceptance in rapidity)**
- significant reduction of κ_6/κ_2 going from positive values at LHC to negative values at lower energies
- LQCD results for κ_6/κ_2 are negative for all energies

STAR: Phys.Rev.Lett. 126 (2021) 9, 092301
predictions: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141
LQCD: A. Bazavov et al., Phys.Rev.D 101 (2020) 7, 074502

See also: V. Vovchenko, V. Koch and C. Shen, arXiv:2107.00163

Hypothesis testing for κ_4/κ_2

$\sqrt{s_{NN}} = 8.8 - 62.4$ GeV



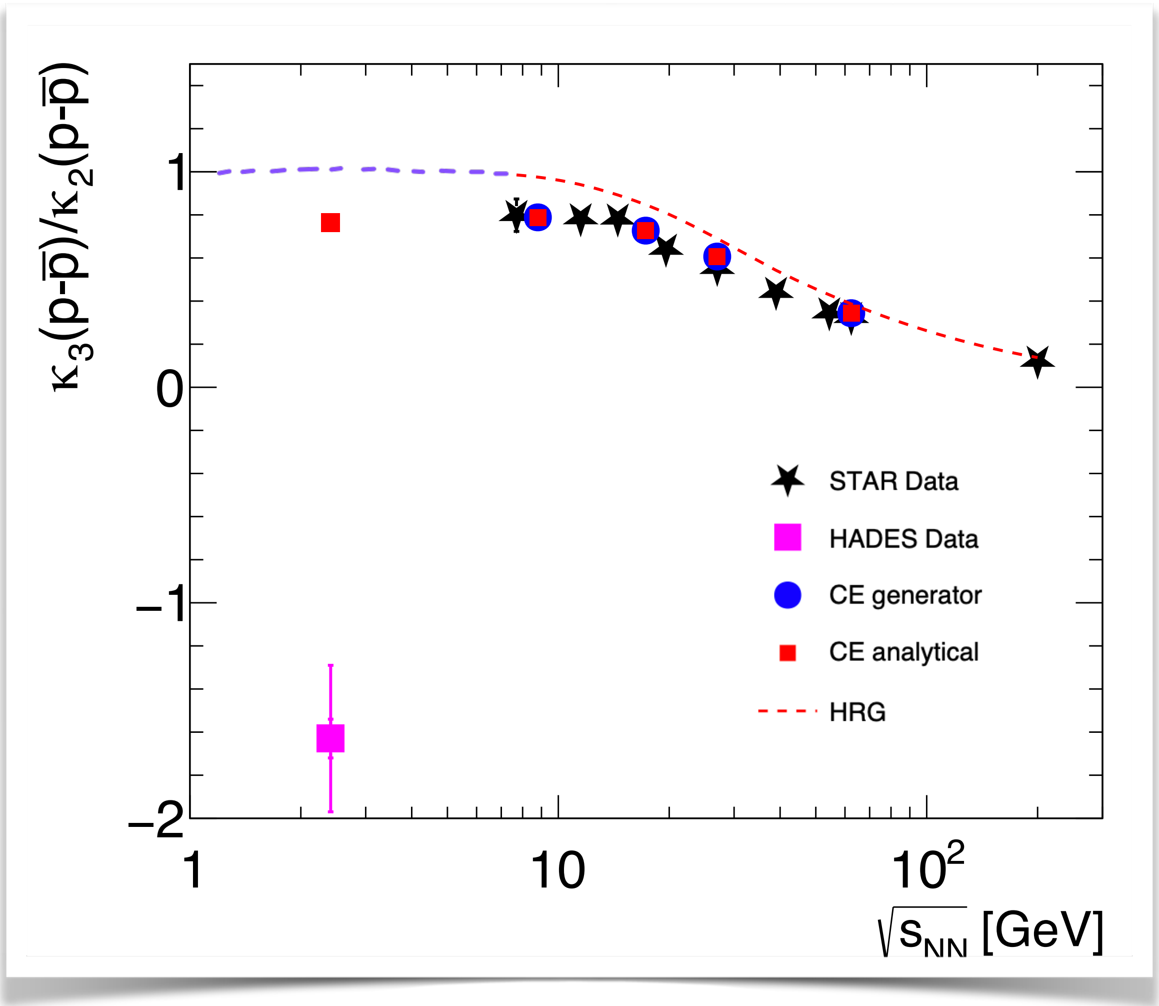
Kolmogorov-Smirnov test

- null-hypothesis: the data and CE baseline are consistent
  - rejected when p-value < 0.1
- obtained p-value: > 0.3

the observed deviations between the STAR data and the canonical baseline are not statistically significant

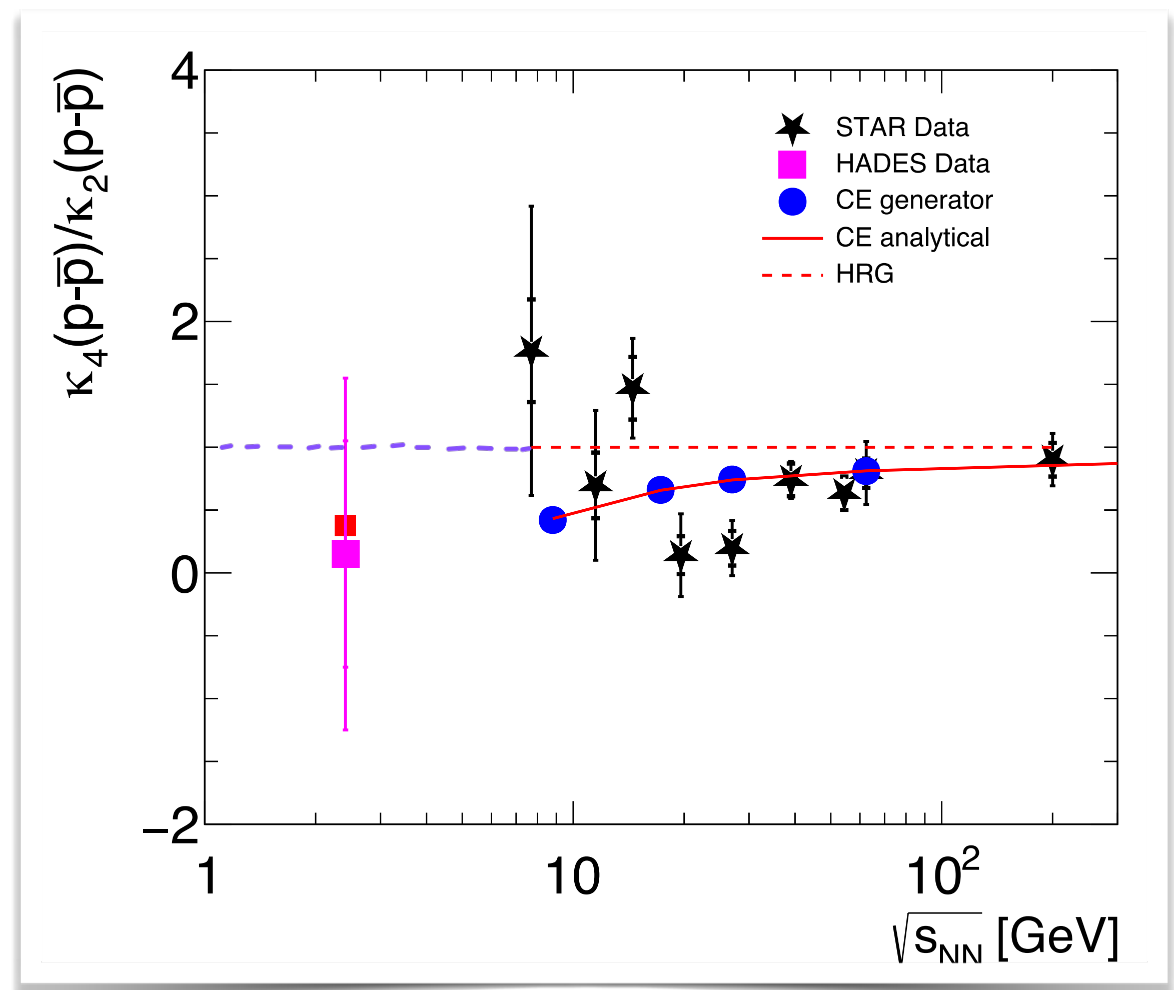
P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

Results from HADES



HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

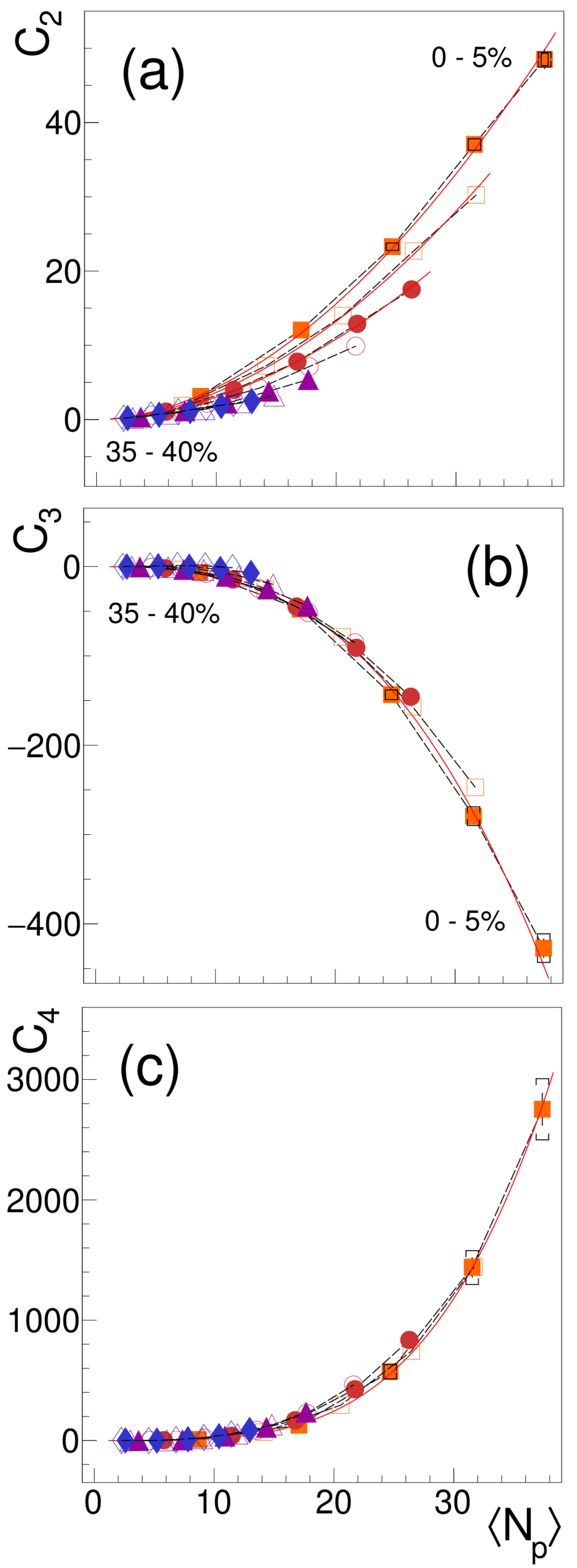
- **HADES**
  - Au-Au@$\sqrt{s_{NN}}$ = 2.4 GeV, 0-10%
  - $0.4 < p_T < 1.6$ GeV/c, $|y_{cm}| < 0.4$
- **STAR**
  - Au-Au@$\sqrt{s_{NN}} = 7.7 - 200$ GeV, 0-5%
  - $0.4 < p_T < 2$ GeV/c, $|y_{cm}| < 0.5$



- κ_3/κ_2
  - significantly lower at HADES
  - becomes negative
- κ_4/κ_2
  - HADES and STAR data are consistent
  - consistent with the CE predictions

CE predictions: PBM, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

Results from HADES



Au-Au@$\sqrt{s_{NN}}$ = 2.4 GeV

$\kappa_2 = \kappa_1 + C_2$ $\quad\quad \kappa_3 = \kappa_1 + 3C_2 + C_3$ $\quad\quad \kappa_4 = \kappa_1 + 7C_2 + 6C_3 + C_4$

C_n - integrated n-particle correlation function

e.g. for m-particle clusters: $C_{1,2,...,m} \neq 0$, $C_{n>m} = 0$

- $\langle N_p \rangle$ - mean number of protons in selected $y_0 \pm \Delta y$
- Δy = 0.1, 0.2, 0.3, 0.4, 0,5

large values for integrated correlation functions

- do data imply multi-cluster formation?
- what is the mechanism behind?

cf. talk by: Romain Holzmann

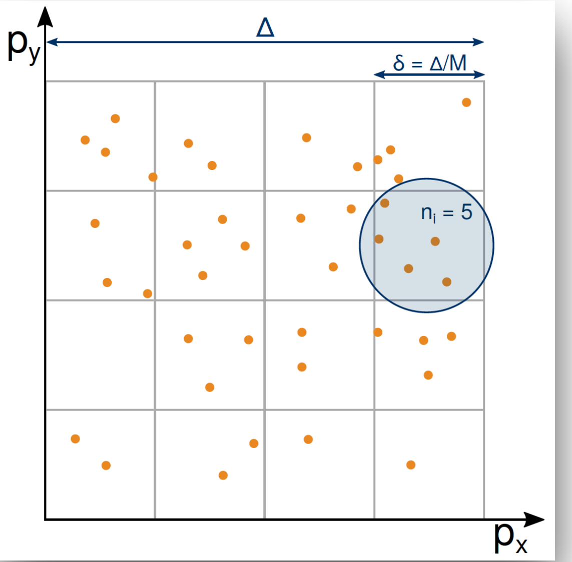
HADES: Phys.Rev.C 102 (2020) 2, 024914

Results from NA61/SHINE

$$F_2(M) = \frac{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_1 - 1) \rangle}{\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \rangle^2}$$

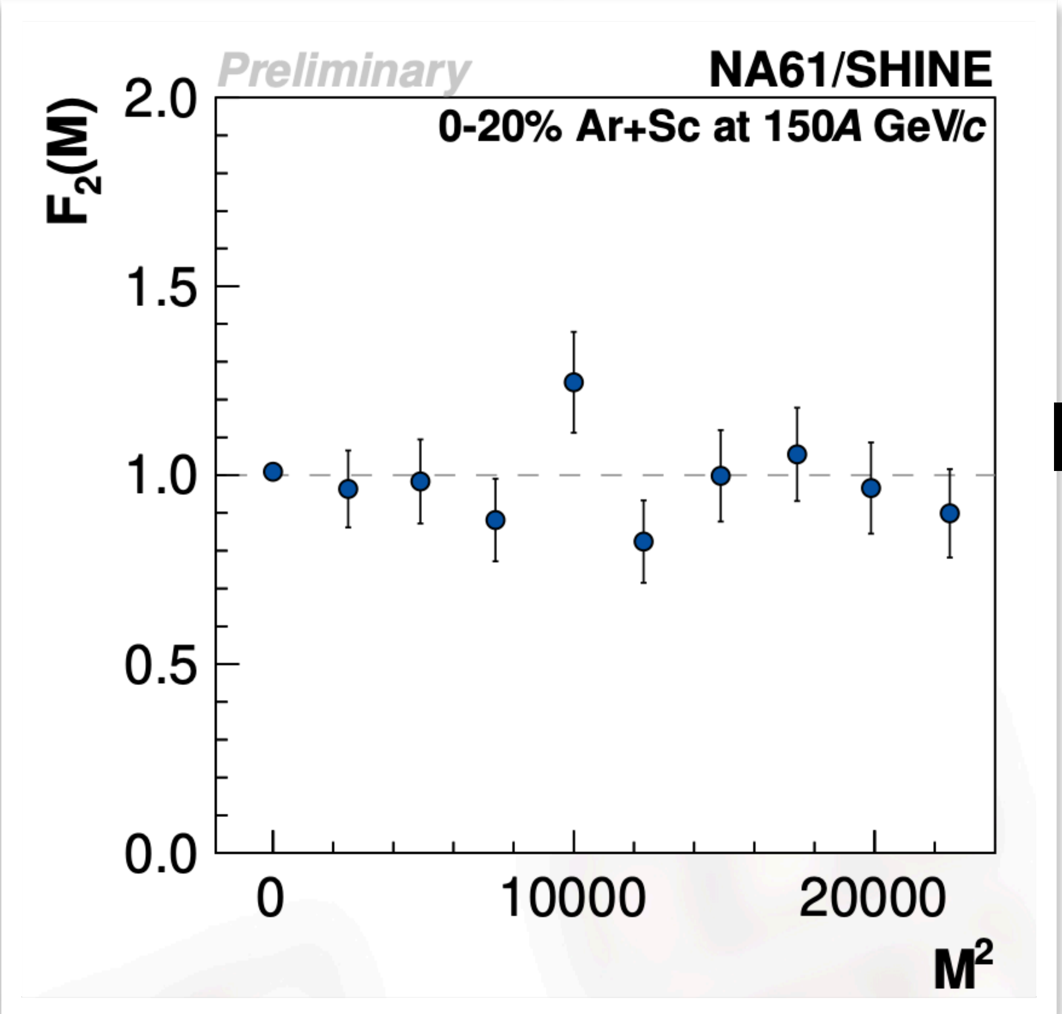
δ - width of the i^{th} bin

n_i - number of particles in i^{th} bin

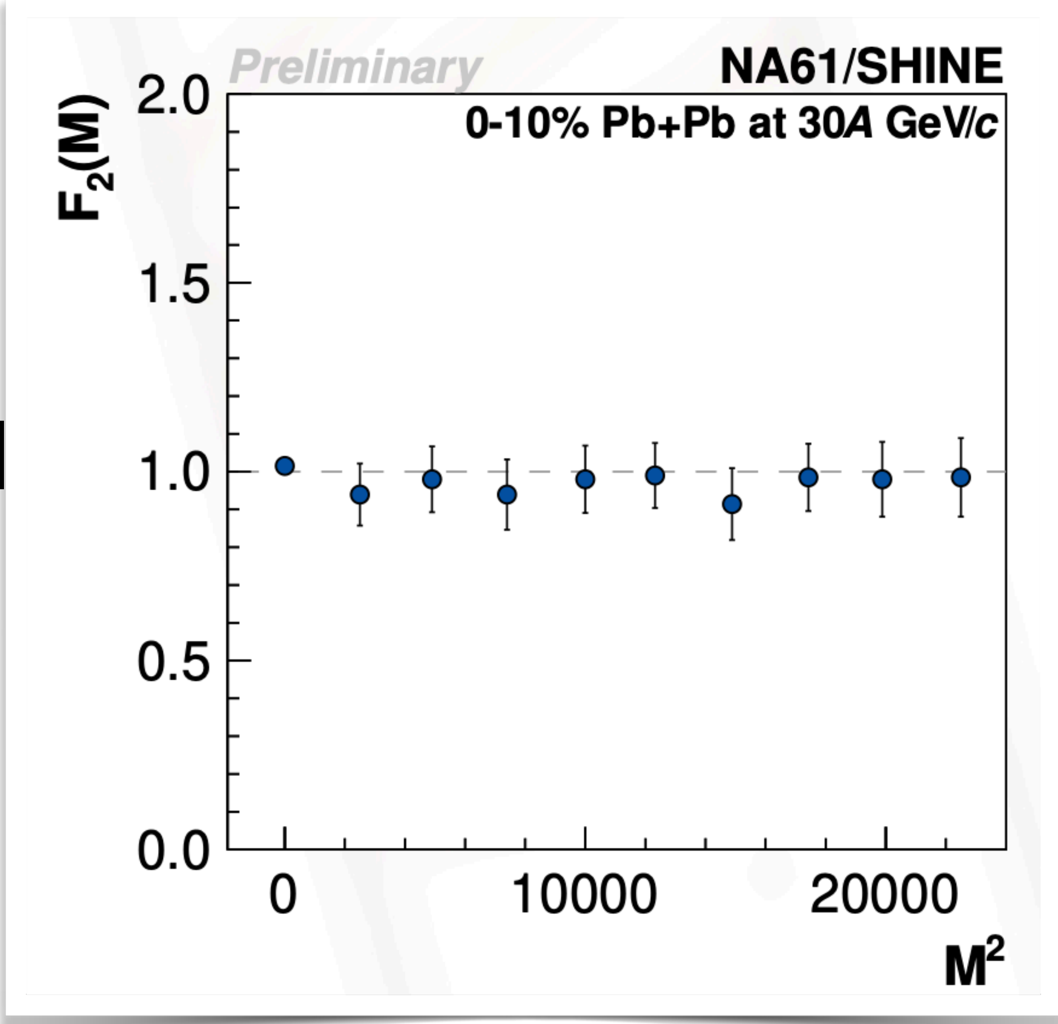


near the QCD critical point (assuming 3D Ising universality class)

$$F_2(M) \sim (M^2)^{\phi_2}, \quad \phi_2 = 5/6$$



cf. talk by: Tobiasz Czopowicz



no indication of power-low behaviour

Summary

- E-by-e fluctuation signals are promising tools to explore the phase structure of strongly interacting matter
- To confront experiment with theory a number of non-dynamical contributions are to be accounted for:
    - Fluctuations of wounded nucleons
    - Contributions from pileup events
    - Conservation of baryon number

- Overall, the experimental results from **STAR** and **ALICE** follow the non-critical baseline predictions
- Contributions due to local baryon number conservation at LHC energies are negligible, if present at all
    - The **ALICE** data strongly indicate long range correlations, implying sensitivity to early stages of collisions

- The data from **HADES** is not consistent with the canonical baseline and may indicate strong multi-particle correlations

- The **NA61/SHINE** data does not show indications for critical behaviour


THANKS FOR YOUR
ATTENTION