Particle-plus-Rotor Model

In the **strong coupling limit** or **deformation aligned limit**, the odd particle adiabatically follows the rotations of the even mass core. It is realized if the coupling to the deformation is much stronger than the perturbation of the single particle motion by the Coriolis interaction. In a semiclassical picture, the angular momentum **j** of the valence particle precesses around the 3-axis, which is shown in the coupling scheme of fig. 1. Assuming that the rotor has the 3-axis as axis of symmetry, it follows immediately that K, the 3-component of the total angular momentum **I**, has to be equal to Ω, the 3-component of **j**. In this case **K is a good quantum number**.

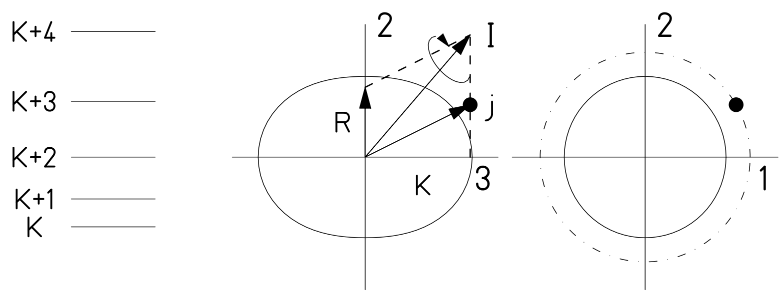

Figure 1: Energy spectrum (left) and shape (right) of an odd-mass axial symmetric rotator. The odd particle follows adiabatically the rotation of the even mass core (strong coupling limit).

The energy spectrum is given by

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathcal{J}}[I(I+1) - K^2 + \delta_{K,1/2}\ a(-)^{I+1/2}(I+1/2)] \qquad (1)$$

where a is the so-called *decoupling factor*. The lowest possible spin is I=K. The spectrum has a spacing △I=1 and its moment of inertia is that of the rotor. For a positive decoupling factor the levels with odd values of $I+\frac{1}{2}$ (I=$\frac{1}{2}$, $\frac{5}{2}$, $\frac{9}{2}$,...) are shifted downwards.

For the reduced transition matrix elements (Eq. **??**) between states with the same K value we obtain

$$< I-2, K||M(E2)||I, K >= \sqrt{\frac{15}{32\pi}}\sqrt{\frac{(I+K-1)(I+K)(I-K-1)(I-K)}{(I-1)(2I-1)I}}\ Q_{20}\ e \qquad (2)$$

$$< I-1, K||M(E2)||I, K >= -\sqrt{\frac{5}{16\pi}}\sqrt{\frac{3(I+K)(I-K)}{(I-1)I(I+1)}}\ K\ Q_{20}\ e \qquad (3)$$

$$< I, K||M(E2)||I, K >= -\sqrt{\frac{5}{16\pi}}\sqrt{\frac{2I+1}{(2I-1)I(I+1)(2I+3)}}(I^2-3K^2+I)\ Q_{20}\ e \qquad (4)$$

$$< I+1,K||M(E2)||I,K >= \sqrt{\frac{5}{16\pi}}\sqrt{\frac{3(I+K+1)(I-K+1)}{I(I+1)(I+2)}}\ K\ Q_{20}\ e \qquad (5)$$

$$< I+2,K||M(E2)||I,K >= \sqrt{\frac{15}{32\pi}}\sqrt{\frac{(I+K+1)(I+K+2)(I-K+1)(I-K+2)}{(I+1)(I+2)(2I+3)}}\ Q_{20}\ e \qquad (6)$$

According to the separation of the system into valence particles and a core, we have two contributions for the intrinsic quadrupole moment. Since the core contribution is much larger than the particle contribution, the latter can be neglected and Q_{20} is given by Eq. **??**.

Intraband M1 transitions are induced if the band has a g_K factor different from g_R.

$$\begin{aligned} < I-1,K||M(M1)||I,K > &= -\sqrt{\frac{3}{4\pi}}\sqrt{\frac{(I+K)(I-K)}{I}}\ K(g_K-g_R) \\ &\quad [1+\delta_{K,1/2}(-)^{I+1/2}b_0]\ \mu_N \end{aligned} \qquad (7)$$

$$\begin{aligned} < I,K||M(M1)||I,K > &= \sqrt{\frac{3}{4\pi}}\mu_N\sqrt{2I+1}\{(g_K-g_R)[\frac{K^2}{\sqrt{I(I+1)}} \\ &\quad +\frac{2I+1}{4\sqrt{I(I+1)}}b_0(-1)^{I+1/2}\delta_{K,1/2}] \\ &\quad +g_R\sqrt{I(I+1)}\} \end{aligned} \qquad (8)$$

$$\begin{aligned} < I+1,K||M(M1)||I,K > &= \sqrt{\frac{3}{4\pi}}\sqrt{\frac{(I+K+1)(I-K+1)}{I+1}}\ K(g_K-g_R) \\ &\quad [1+\delta_{K,1/2}(-)^{I+1/2}b_0]\ \mu_N \end{aligned} \qquad (9)$$

The quantity b_0 depends on the magnetic decoupling parameter (Eq. 13). For a band with $K > 1/2$, the E2/M1 mixing ratio (Eq. **??**) can be written

$$\delta_{if}(E2,M1) = 0.835\sqrt{\frac{5}{4}}\frac{1}{\sqrt{(I-1)(I+1)}}\frac{Q_{20}\ e}{(g_K-g_R)} \qquad (10)$$

For the static M1 and E2 moments we obtain

$$\mu(I) = \{g_R I + (g_K-g_R)\frac{K^2}{I+1}[1+\delta_{K,1/2}(2I+1)(-)^{I+1/2}b_0]\}\mu_N \qquad (11)$$

$$Q(I) = \frac{3K^2-I(I+1)}{(I+1)(2I+3)}Q_{20} \qquad (12)$$

The second term in the bracket of Eq. 11 (b_0) depends on the decoupling factor. For $\Omega = 1/2$, the magnetic decoupling parameter is given by

$$(g_K - g_R)b_0 = -(g_\ell - g_R)a - \frac{1}{2}(-1)^\ell (g_s + g_K - 2g_\ell) \tag{13}$$

For the one-quasiparticle state, the g_K factor can be estimated in the extreme single-particle model from the expression

$$g_K = g_\ell \pm \frac{1}{2\ell + 1}(g_s - g_\ell) \qquad for\ j = \ell \pm \frac{1}{2} \tag{14}$$

where g_s and g_l are the spin and orbital g-factor for the last odd nucleon. The constants for a free single-proton are $g_\ell = 1$, $g_s = 5.586$ and for a free single-neutron are $g_\ell = 0$, $g_s = -3.826$, respectively. If we assume that the free nucleon moments persist in the nucleus, the above formula gives the so-called Schmidt moments ([Sch37]).

Once we know the magnetic moment of a single-particle state, the question arises what values to expect for the magnetic moments of two- or more-particle states. We have the generalized Landé formula

$$g(j_1 x j_2; J) = \frac{1}{2}(g_1 + g_2) + \frac{j_1(j_1+1) - j_2(j_2+1)}{2J(J+1)}(g_1 - g_2) \tag{15}$$

A special case is the two-particle state $(jxj; J)$ configuration, where we have the identity $g(jxj; J) = g(j)$.

In the high-spin limit with $I \gg K$ the transition probabilities and static moments can be replaced by their classical analogs

$$B(M1; I \to I \pm 1) \simeq \frac{3}{4\pi}\frac{K^2}{2}(g_K - g_R)^2[1 + b_0(-1)^{I+1/2}\delta_{K,1/2}]^2\ \mu_N^2 \tag{16}$$

$$\mu(I) \simeq g_R I\ \mu_N \tag{17}$$

$$B(E2; I \to I \pm 2) \simeq \frac{5}{16\pi}\frac{3}{8}Q_{20}^2\ e^2 \tag{18}$$

$$B(E2; I \to I \pm 1) \simeq \frac{5}{16\pi}\frac{3}{8}(\frac{2K}{I})^2 Q_{20}^2\ e^2 \tag{19}$$

$$Q(I) \simeq -\frac{1}{2}Q_{20} \tag{20}$$

Thus in the high-spin limit the $B(M1; I \to I \pm 1)$, $B(E2; I \to I \pm 2)$ and Q(I) are independent of I while the magnetic moment increases with increasing spin. Furthermore the E2 transitions between states with I and $I \pm 2$ are favoured compared to the $I \to I \pm 1$ transitions.

Bibliography

[Sch37] T. Schmidt: Z.Phys., **106** (1937), 358